RF Pulse Design

RF Pulses / Adiabatic Pulses

M229 Advanced Topics in MRI

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Class Business

- Homework 1 is due on 4/23 (Friday)

Outline

- Review of RF pulses
- Adiabatic passage principle
- Adiabatic inversion

Review of RF Pulses

Notation and Conventions

$$\vec{B} = B_0\hat{k} + B_1(t)[\cos\omega t\hat{i} - \sin\omega t\hat{j}]$$

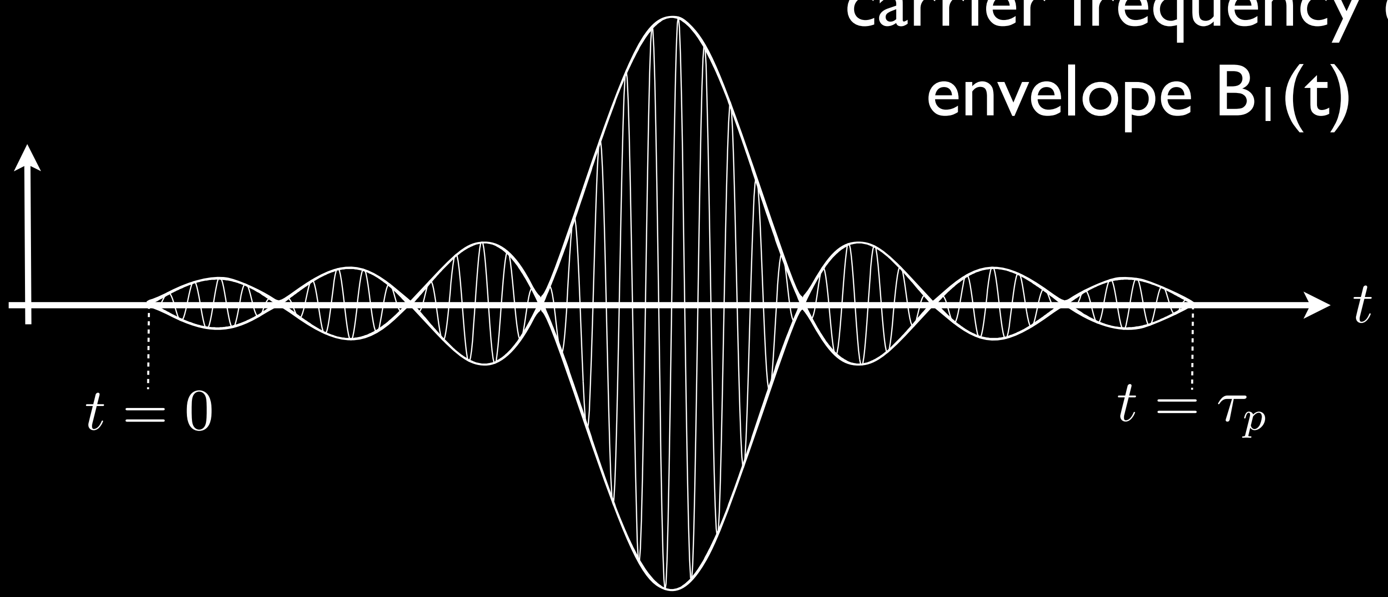
- ω = carrier frequency
- ω_0 = resonant frequency
- $B_1(t)$ = complex valued envelop function

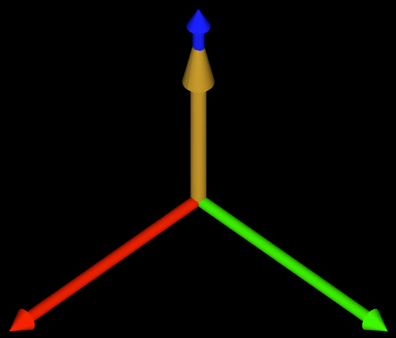
RF Pulse - Excitation

$$\vec{B} = B_0\hat{k} + B_1(t)[\cos\omega t\hat{i} - \sin\omega t\hat{j}]$$

$$B_1(t) \cdot [\cos(\omega t)\hat{i} - \sin(\omega t)\hat{j}]$$

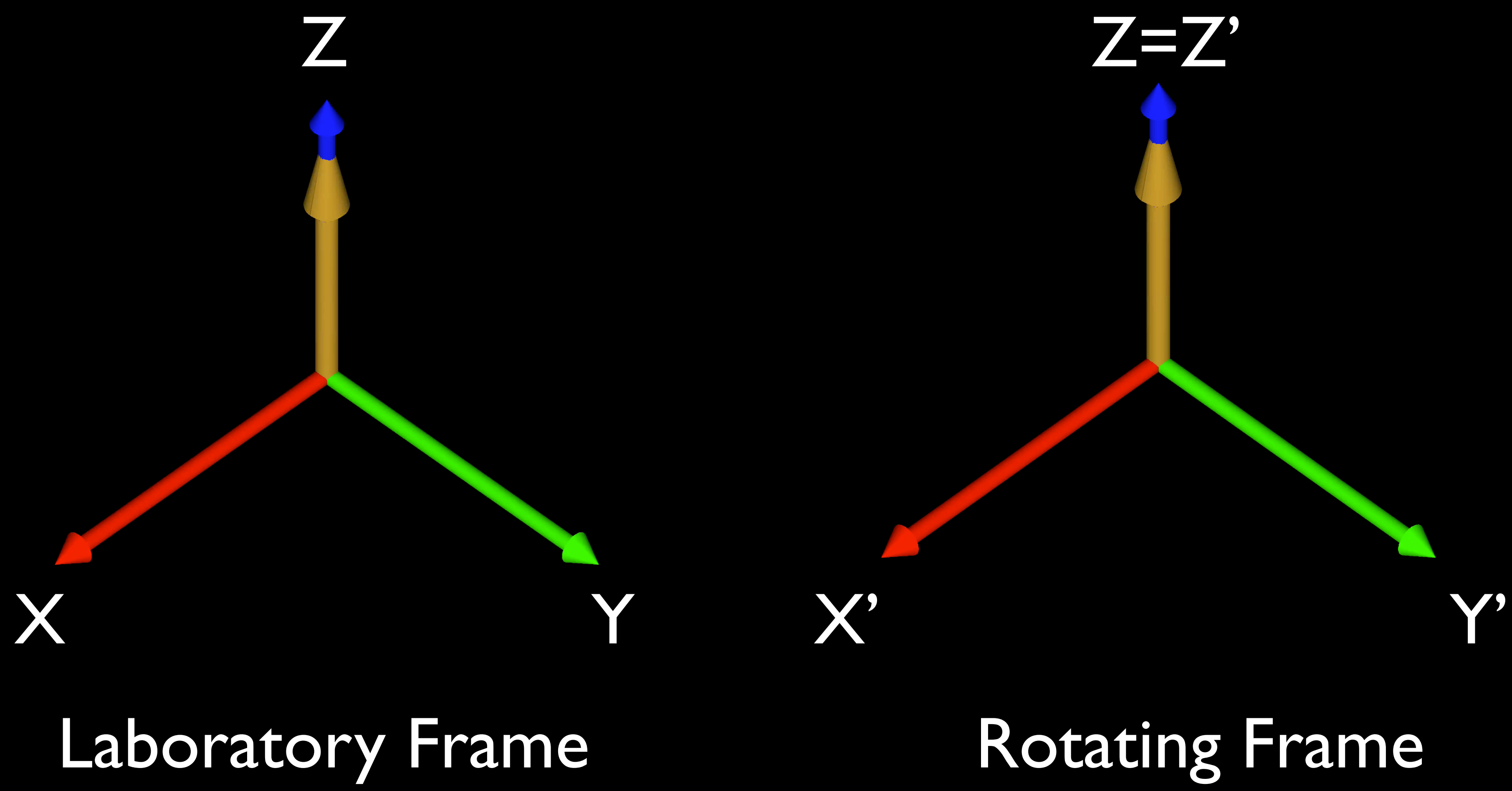
carrier frequency ω
envelope $B_1(t)$





Lab vs. Rotating Frame

- The rotating frame simplifies the mathematics and permits more intuitive understanding.



Rotating Frame

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\vec{M}_{lab}(t) = R_Z(w_0 t) \cdot \vec{M}_{rot}(t)$$

$$\vec{B}_{lab}(t) = R_Z(w_0 t) \cdot \vec{B}_{rot}(t)$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Rightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{w}_{rot}}{\gamma}$$ fictitious field

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix}$$

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{w}_{rot}}{\gamma}$$

$$\vec{B}_{lab} = \begin{pmatrix} B_1(t)\cos\omega_0 t \\ B_1(t)\sin\omega_0 t \\ B_0 \end{pmatrix} \qquad \vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ B_1(t) \\ B_0 \end{pmatrix}$$

Assume real-valued $B_1(t)$

$$\vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix}$$

To the board ...

Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix} \rightarrow \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$$

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

To the board ...

B1 Variations

- In MRI, B1 field is not always uniform across the imaging volume
- B1 inhomogeneity can cause:
  - Image shading
  - Incomplete saturation (e.g. in fat suppression)
  - Incomplete inversion (e.g. CSF suppression, myocardium suppression in cardiac scar imaging)
  - Inaccurate/imprecise quantification in T1 mapping

B1 Variations

- It is highly desirable if we can excite tissue homogeneously and produce a uniform flip angle throughout

➔ **Adiabatic Pulses!**

"Adiabatic pulses are a special class of RF pulses that can excite, refocus or invert magnetization vectors uniformly, even in the presence of a spatially nonuniform B1 field."

Adiabatic Passage Principle

Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied B1 field

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Can be used for excitation, inversion and refocusing

Adiabatic vs. Non-Adiabatic Pulses

Adiabatic Pulses:

$$\theta \neq \int_0^T B_1(\tau)d\tau$$

- Amplitude and frequency/phase modulation
- Long duration (8-12 ms)
- Higher B1 amplitude (>12 µT)
- Generally NOT multi-purpose (inversion pulse cannot be used for refocusing, etc.)

Non-Adiabatic Pulses:

$$\theta = \int_0^T B_1(\tau)d\tau$$

- Amplitude modulation, constant carrier frequency (constant phase)
- Short duration (0.3 ms to 1 ms)
- Lower B1 amplitude
- Generally multi-purpose

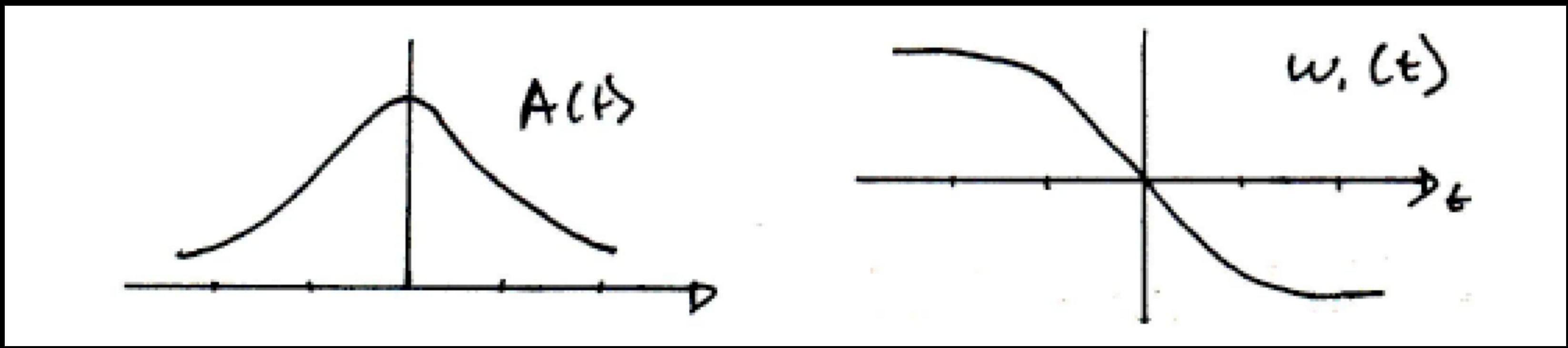
Adiabatic Pulses

- Frequency modulated pulses:

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t')dt'}$$

envelop

frequency sweep



- Or phase modulation:

$$B_1(t) = A(t) \exp^{-i\phi(t)}$$

Bloch Equation (at on-resonance)

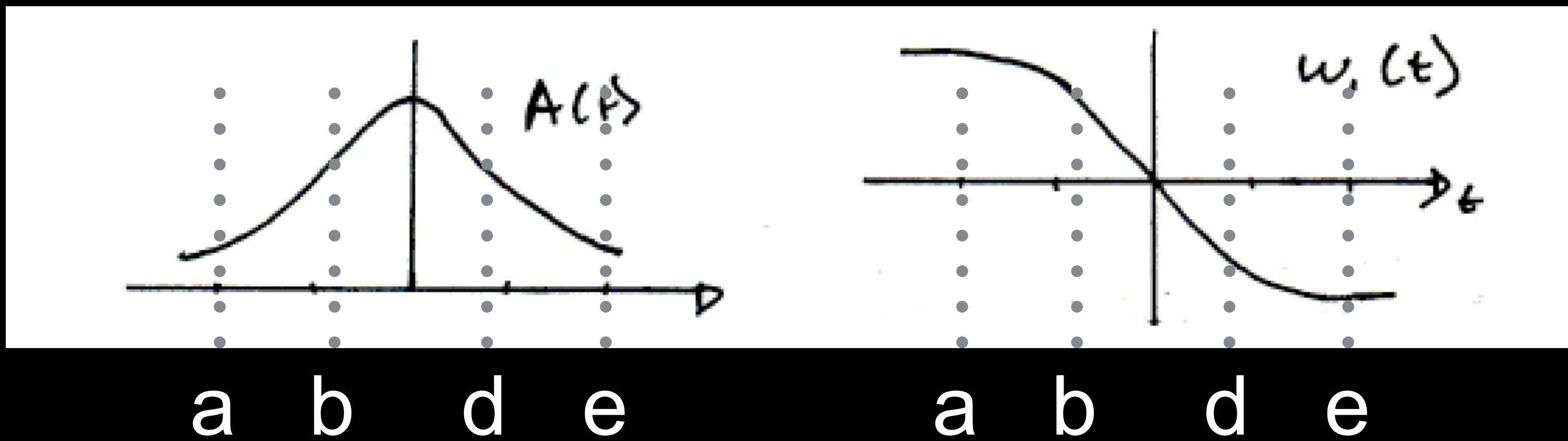
$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + \frac{\omega_1(t)}{\gamma} \end{pmatrix}$

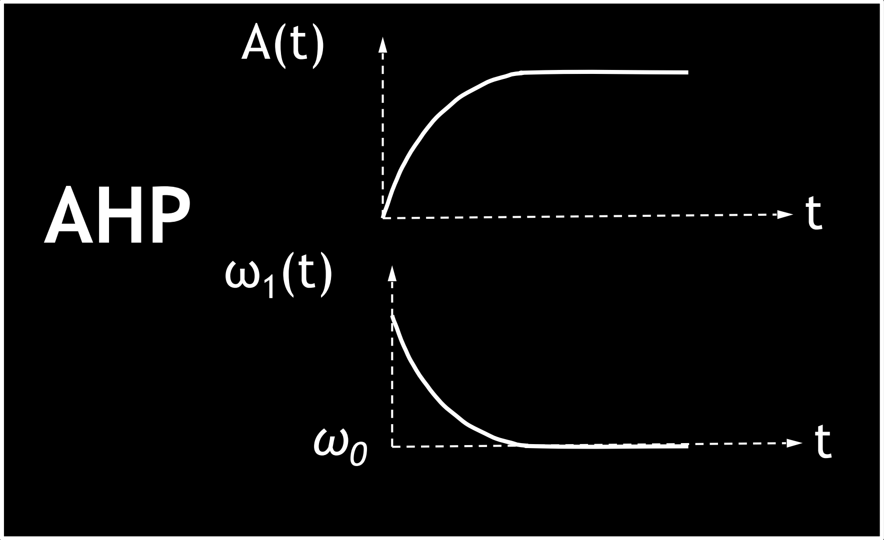
$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega_1(t) & 0 \\ -\omega_1(t) & 0 & \gamma A(t) \\ 0 & -\gamma A(t) & 0 \end{pmatrix} \vec{M}$$

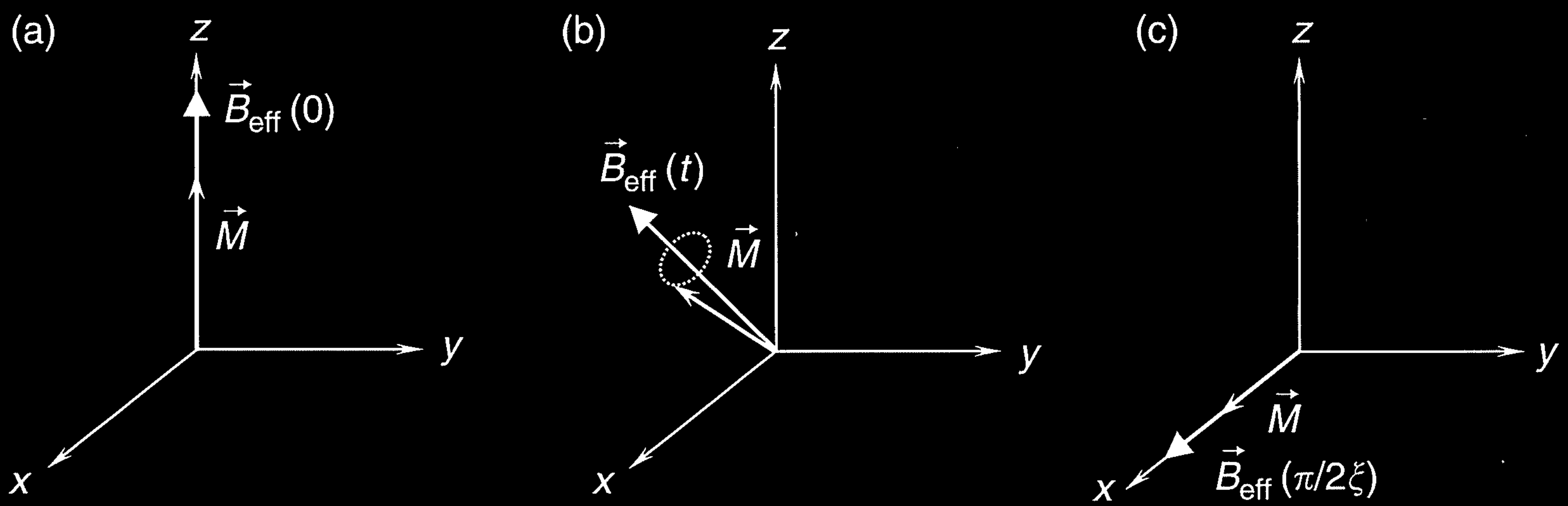
Magnetization Plot



To the board ...

Adiabatic Excitation



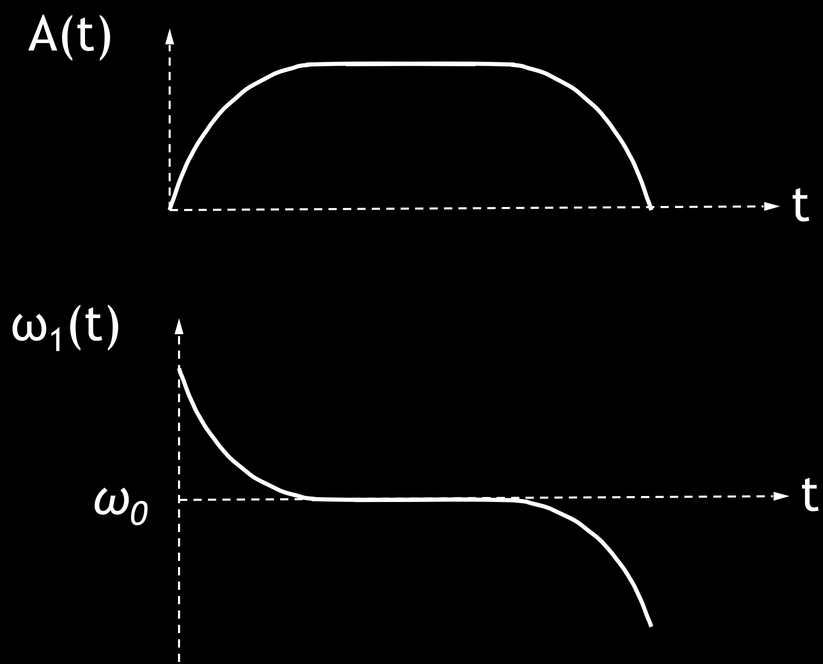


- At the end of the pulse, all the magnetization is in the transverse plane → so we have adiabatic excitation!
- This is also called an **adiabatic half passage (AHP)**

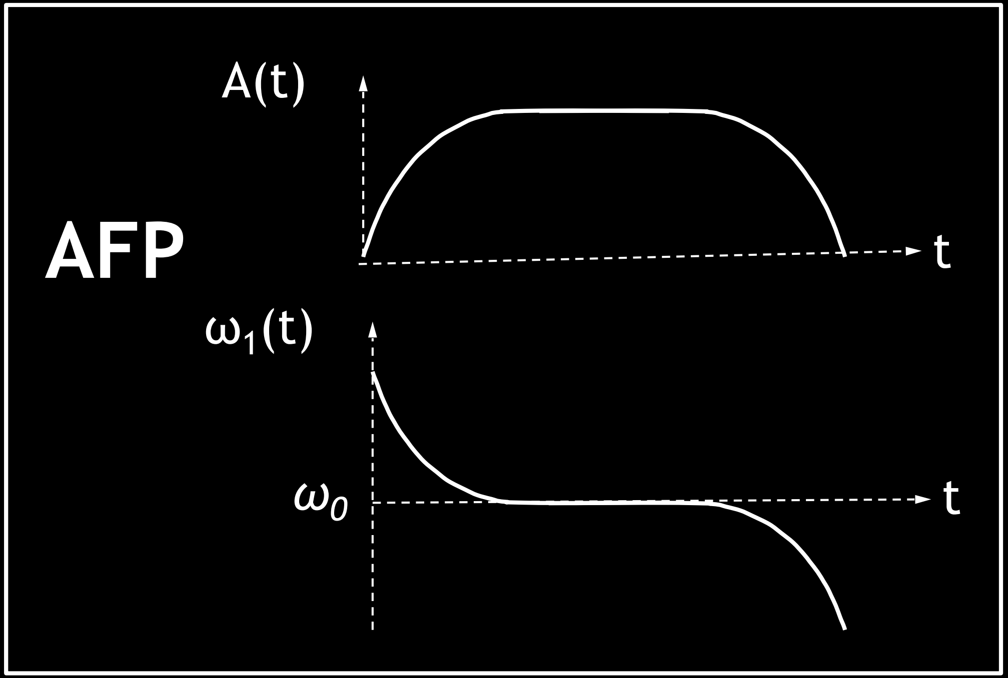
Adiabatic Inversion

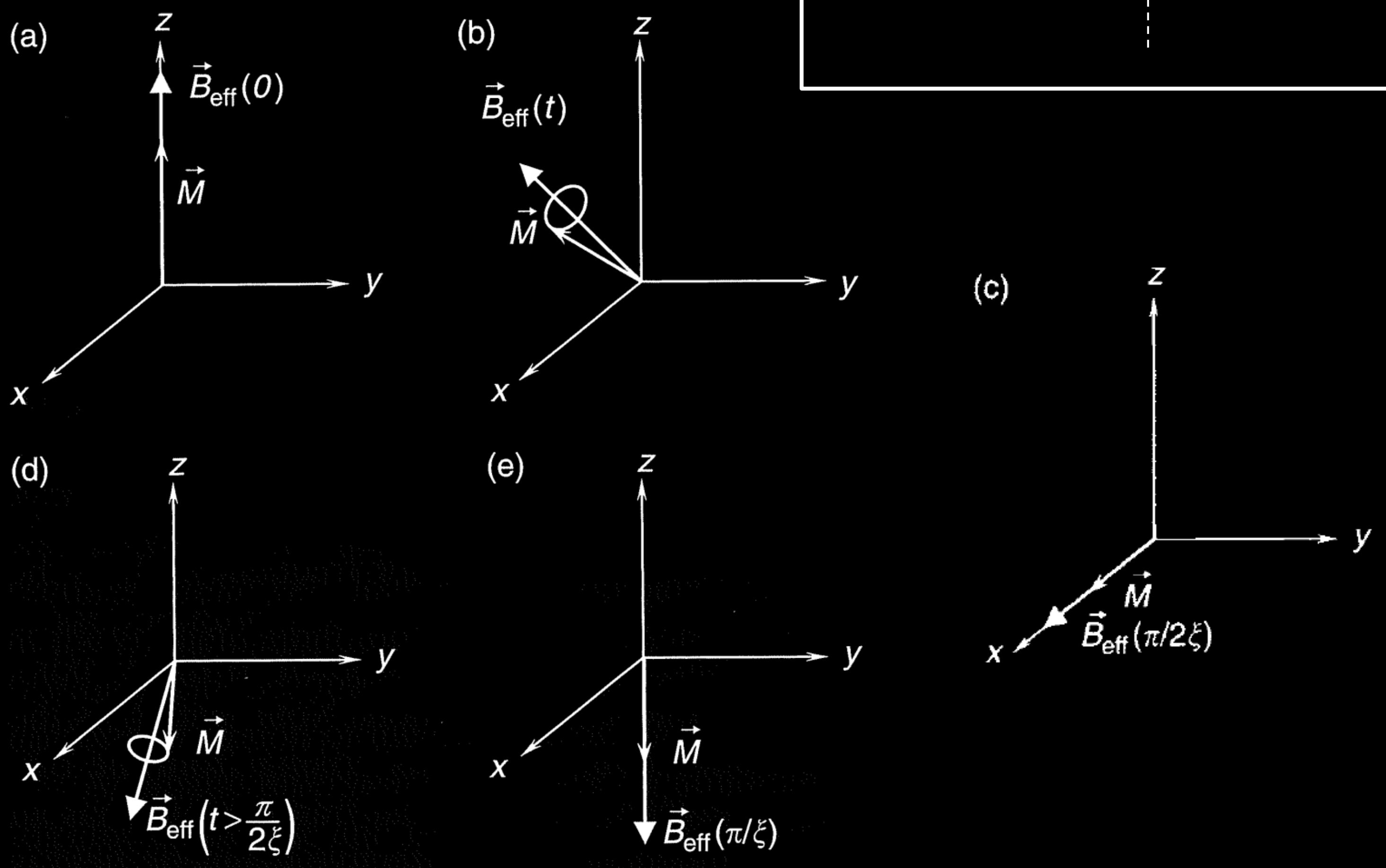
- An adiabatic inversion requires an adiabatic full passage (AFP) pulse:

AFP



Adiabatic Inversion





Adiabatic Inversion

Design of Adiabatic Inversion

- General equation for an adiabatic pulse:

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t')dt'}$$

- Many different types of adiabatic pulses can be designed by choosing different amplitude and frequency modulation functions

- The most famous one is…

The Hyperbolic Secant Inversion Pulse!

Hyperbolic Secant Pulse Equations

$$B_1(t) = A(t)\exp^{-i\int \omega_1(t')dt'}$$

where

$$A(t) = A_0 sech(\beta t)$$

$$\omega_1(t) = -\mu\beta tanh(\beta t)$$

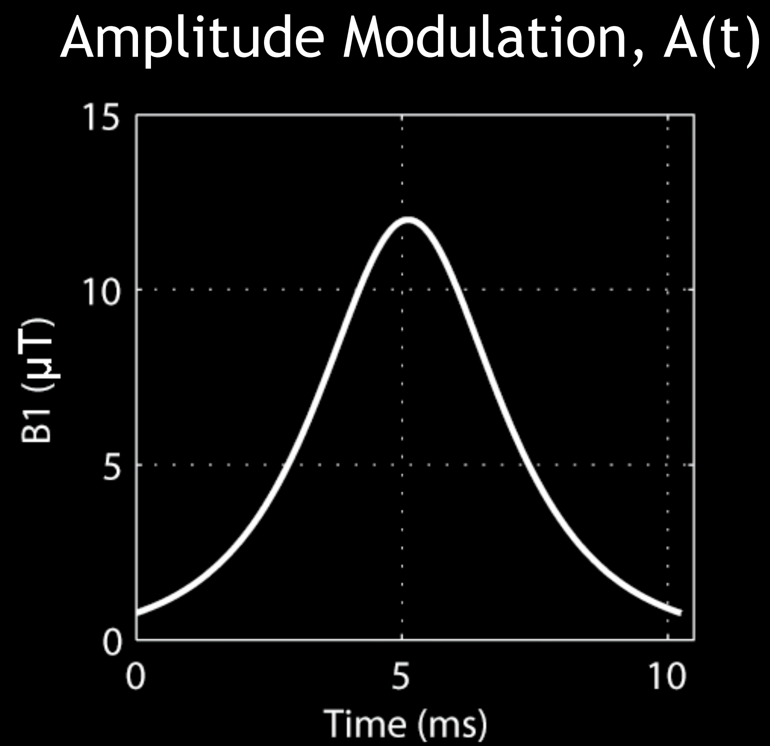
A_0: peak amplitude (μT)

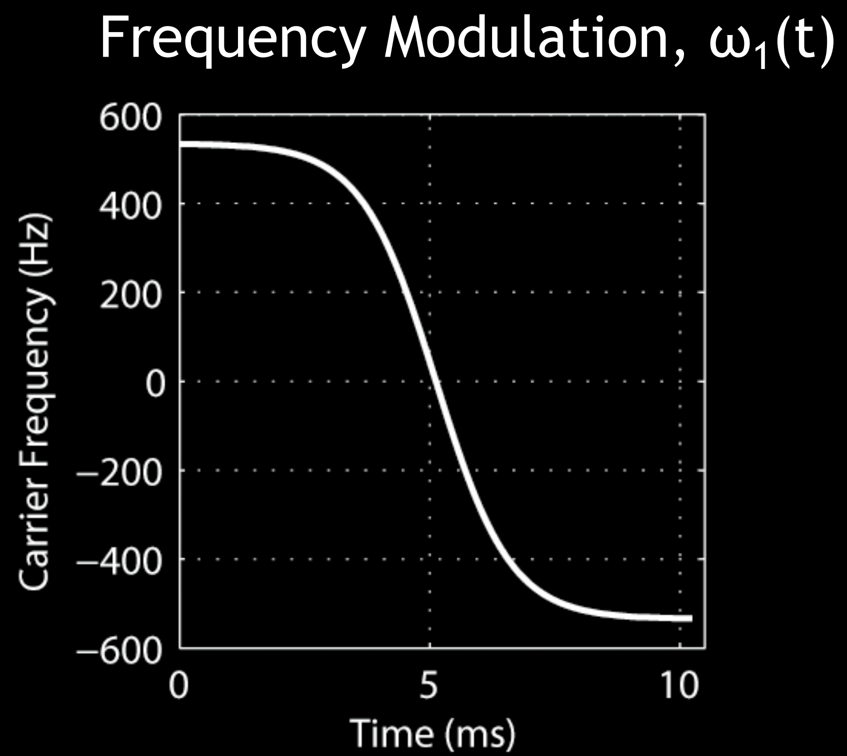
β: frequency modulation parameter (rad/s)

μ: phase modulation parameter (dimensionless)

Handbook of MRI Pulse Sequences, Ch 6.2, pp 194-195

Hyperbolic Secant Pulse Example





Pulse Parameters:

A_0 = 12 μT

μ = 5

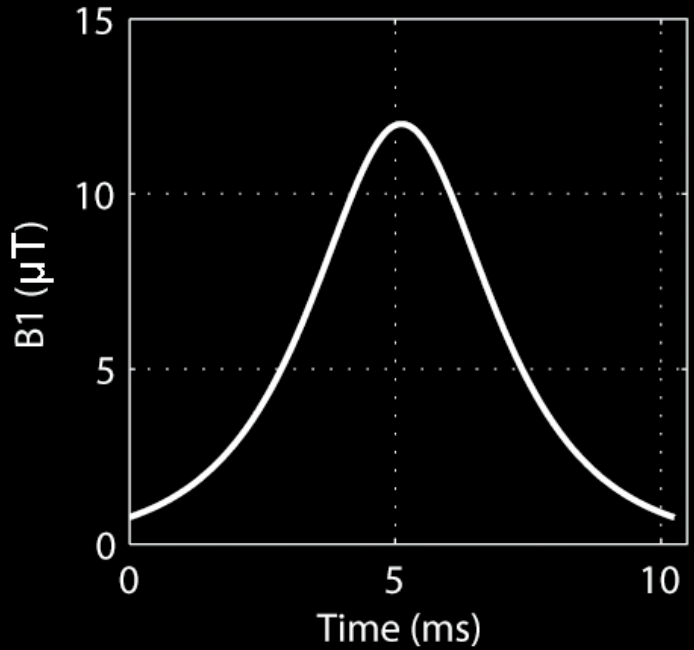
β = 672 rad/s

Duration = 10.24 ms

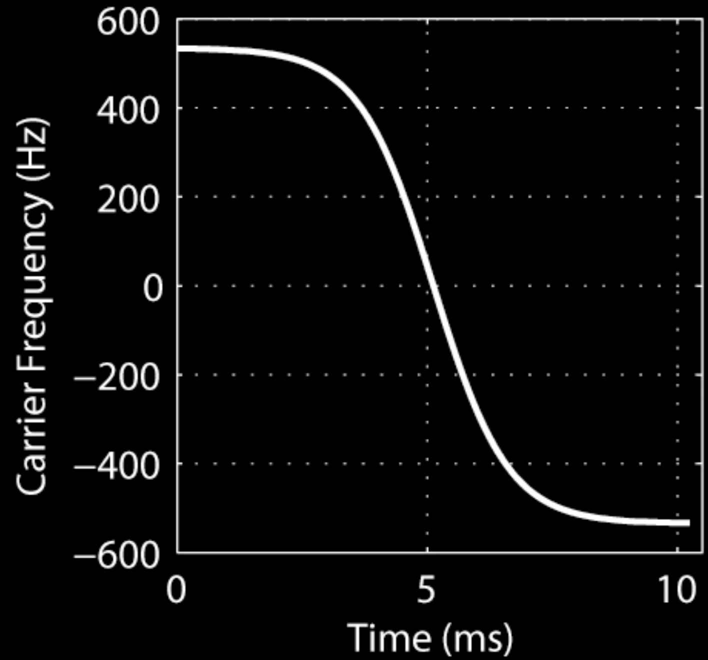
Comparing Hyperbolic Secant with an AFP Example

Hyperbolic Secant Pulse

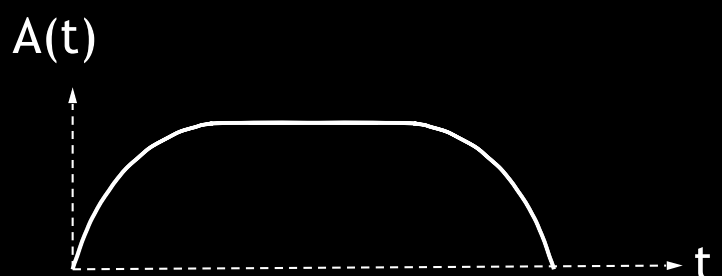
Amplitude Modulation, A(t)

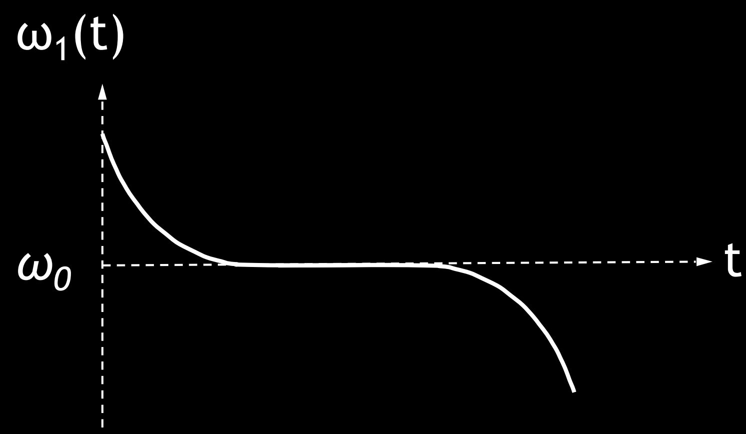


Frequency Modulation, $\omega_1(t)$



General Adiabatic Full Passage pulse





Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	A(t)	$\omega_1(t)$
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2}+\frac{1}{\sqrt{\beta}}\tan^{-1}(\sqrt{\beta}\tau)$
HS	$\mathrm{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$
Gauss[c]	$\exp\left(-\frac{\beta^2\tau^2}{2}\right)$	$\frac{\mathrm{erf}(\beta\tau)}{\mathrm{erf}(\beta)}$
Hanning	$\frac{1+\cos(\pi\tau)}{2}$	$\tau+\frac{4}{3\pi}\sin(\pi\tau)\left[1+\frac{1}{4}\cos(\pi\tau)\right]$
HSn[c] ($n=8$)	$\mathrm{sech}(\beta\tau^n)$	$\int \mathrm{sech}^2(\beta\tau^n)\,d\tau$
Sin40[d] ($n=40$)	$1-\left\|\sin^n\left(\frac{\pi\tau}{2}\right)\right\|$	$\tau-\int \sin^n\left(\frac{\pi\tau}{2}\right)\left(1+\cos^2\left(\frac{\pi\tau}{2}\right)\right)d\tau$

Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423

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Sin40[d] ($n=40$)	$1-\left\lvert \sin^n\left(\frac{\pi\tau}{2}\right)\right\rvert$	$\tau-\int \sin^n\left(\frac{\pi\tau}{2}\right)\left(1+\cos^2\left(\frac{\pi\tau}{2}\right)\right)d\tau$

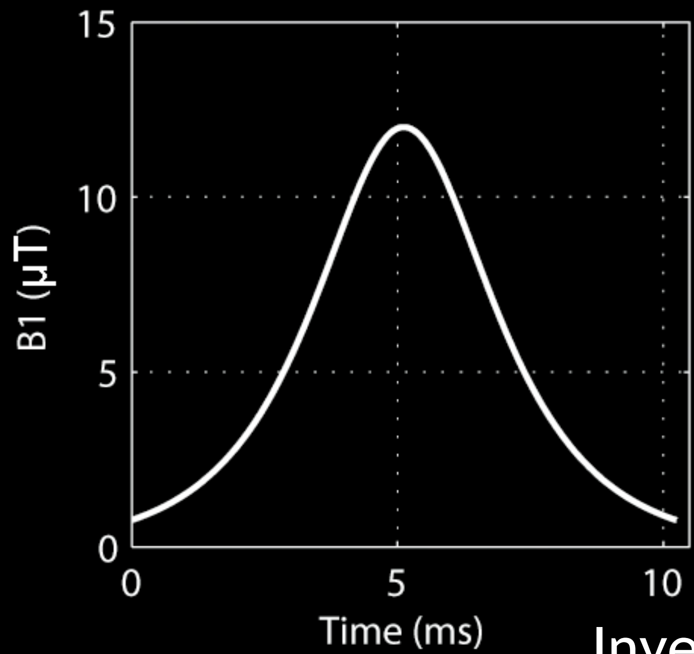
The shape of the inversion profile depends on the choice A(t) and $\omega_1(t)$!

Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, vol. 10, p423

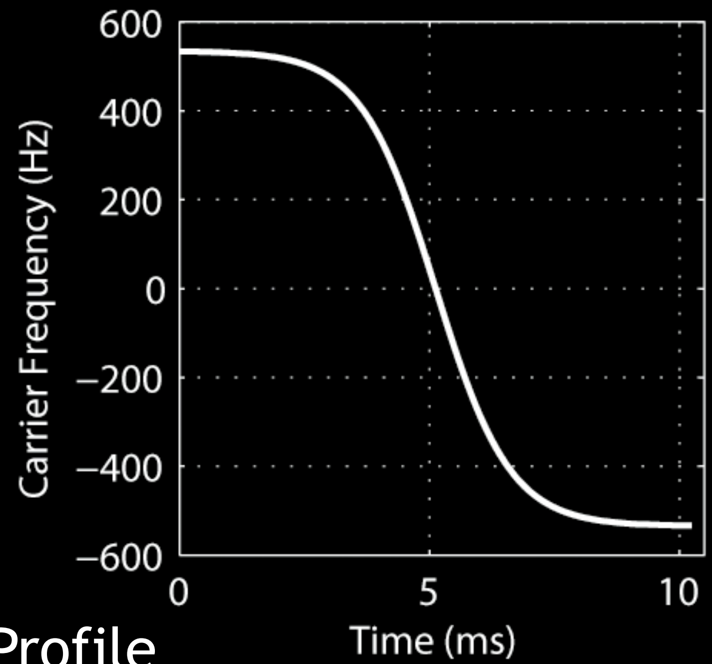
What Will Inversion Profile Look Like?

Hyperbolic Secant Pulse

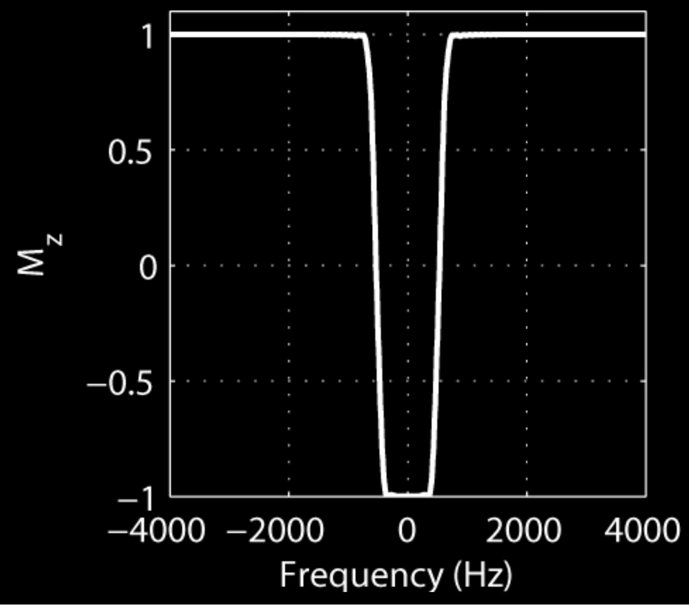
Amplitude Modulation, A(t)



Frequency Modulation, $\omega_1(t)$



Inversion Profile



Inversion Profiles

- The inversion profile typically calculated using Bloch simulation of the RF pulse (will be covered later) shows us the inversion efficiency and RF bandwidth
- The inversion efficiency depends strongly on the B1 amplitude (as well as pulse duration, T1, T2 and pulse shape)
- For the hyperbolic secant pulse,

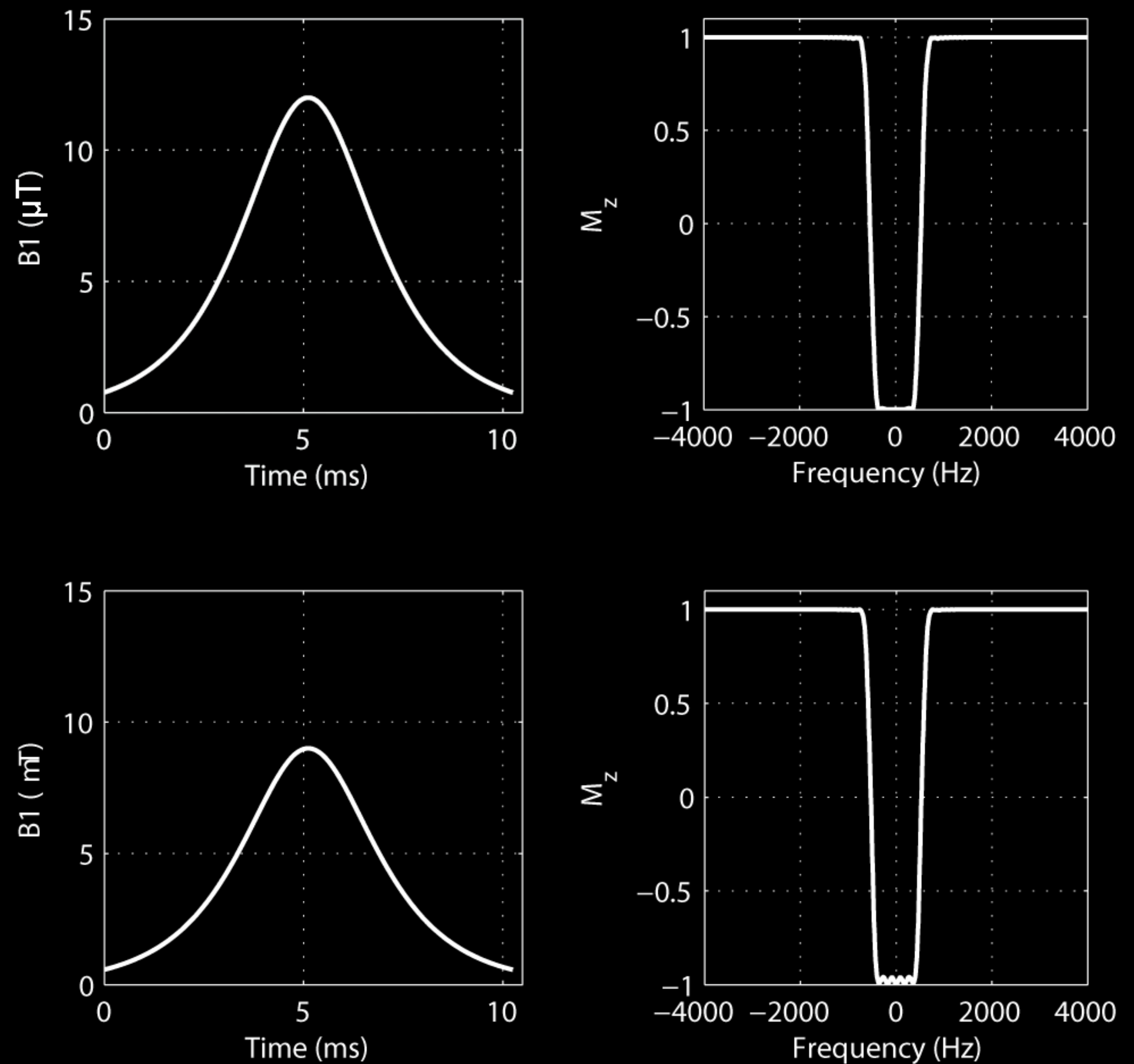
$$\text{RF spectral bandwidth} = \mu\beta$$

$$B_{1max} \gg (\beta\sqrt{\mu})/\gamma \quad (B_1 \text{ threshold for adiabaticity})$$

Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
$B1_{max}$ = 12 μT

75% Attenuated Pulse
$B1_{max}$ = 9 μT



Hyperbolic Secant: Adiabatic Property

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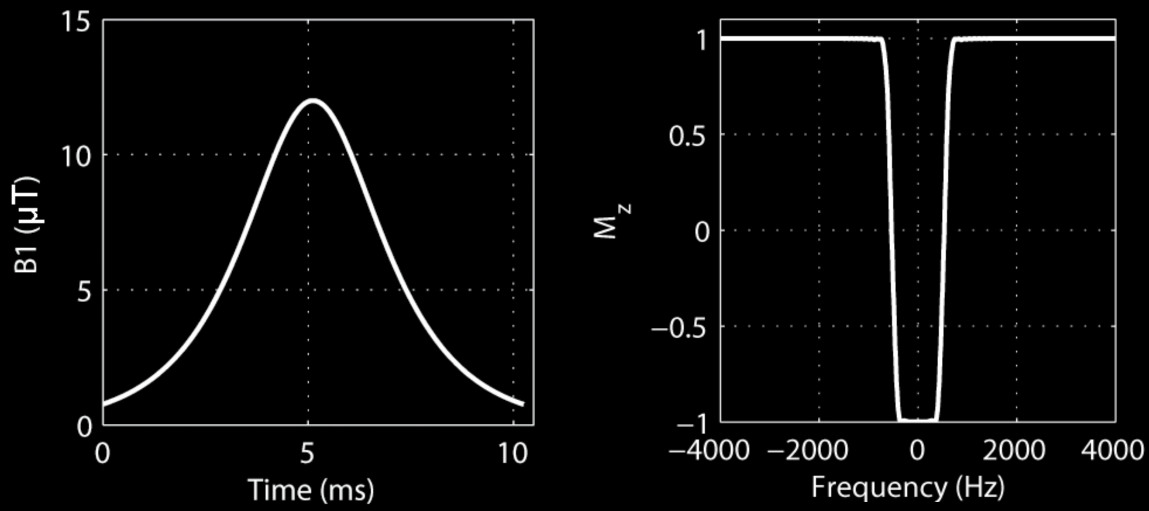
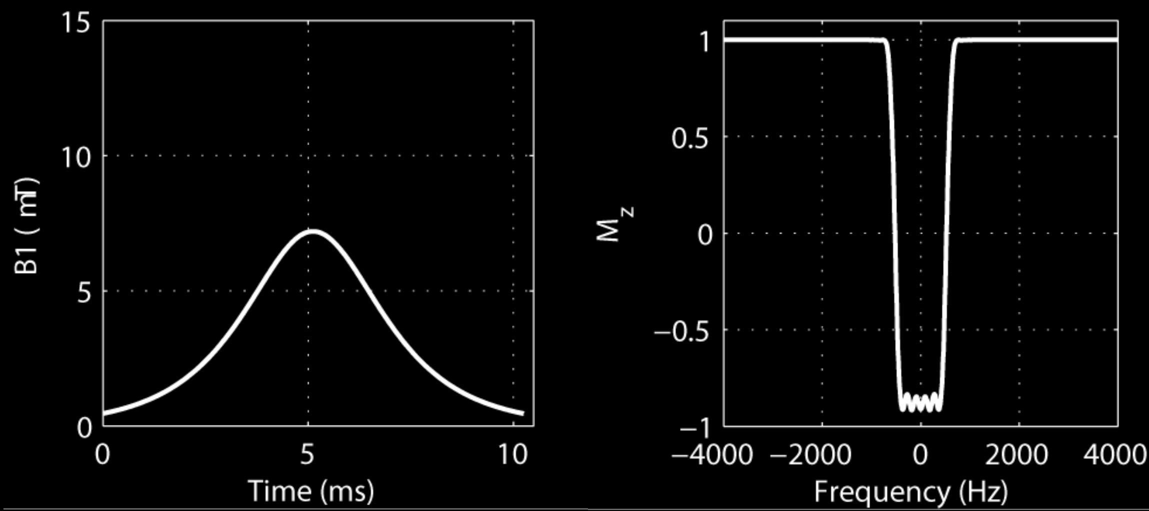
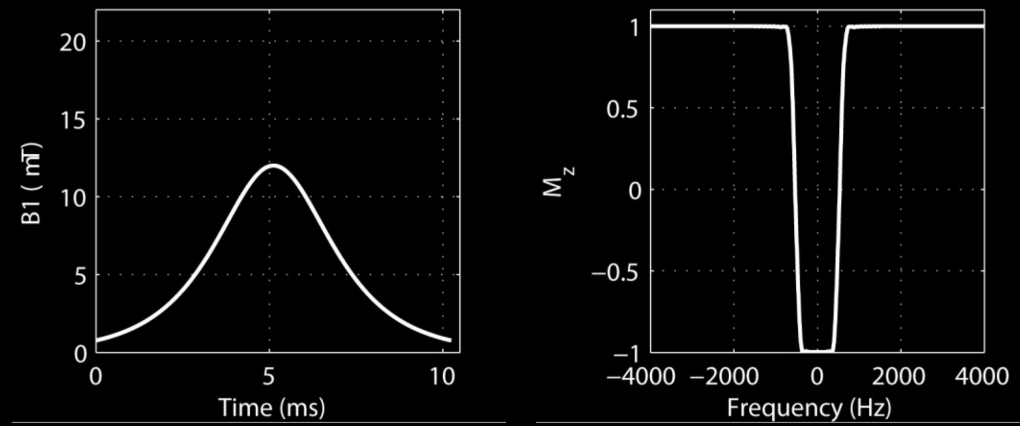


60% Attenuated Pulse
$B1_{max}$ = 7.2 μT

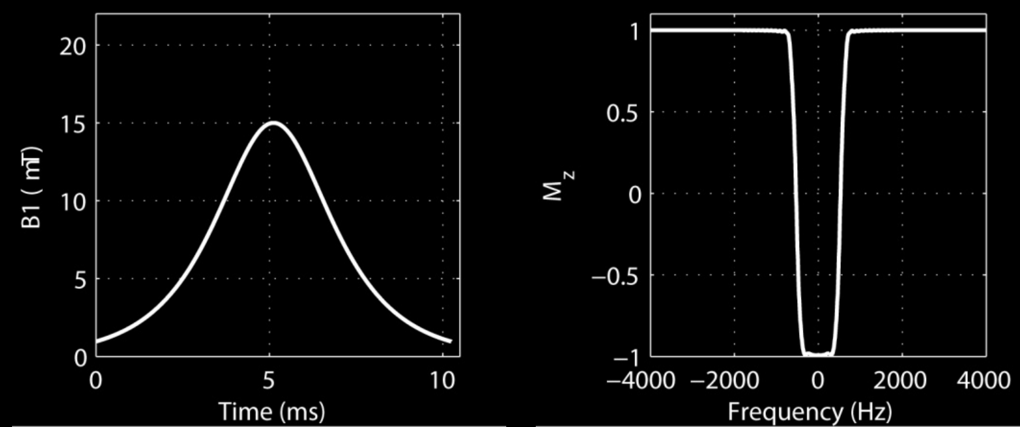


B1 Threshold ≈ 6 μT

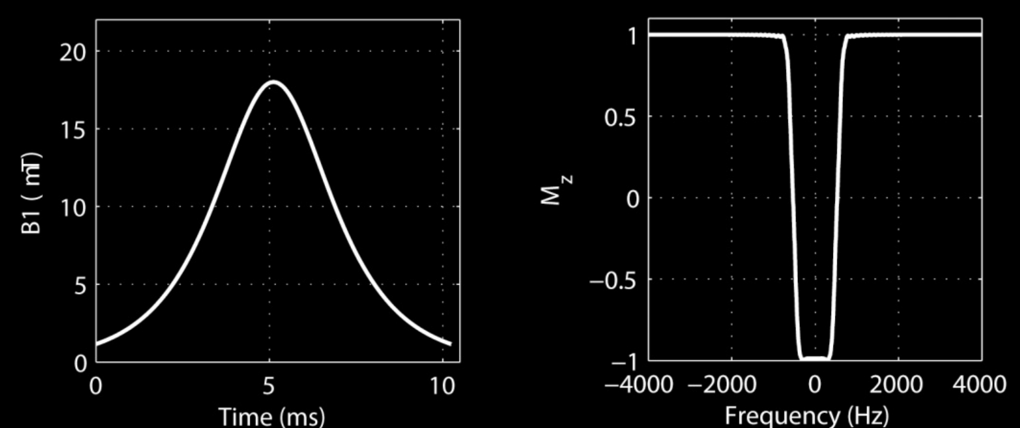
Original Pulse (100%)
B1 = 12 μT



125% Amplified Pulse
B1 = 15 μT



150% Amplified Pulse
B1 = 18 μT



Comments

- Many envelope/modulation functions work
- If a range of adiabaticity is required, optimization can help reduce pulse length
- Hyperbolic Sech needs to be truncated, which can affect the overall performance

Thank You!

- Further reading
  - Read "Adiabatic Refocusing Pulses" p.200-212
  - Tannus et al., “Adiabatic Pulses”, NMR in Biomedicine, Vol. 10, 423-434 (1997)

- Acknowledgments
  - John Pauly’s EE469b (RF Pulse Design for MRI)

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