Image Reconstruction

Parallel Imaging II

M229 Advanced Topics in MRI

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Class Business

- Final project abstract / presentation
  - Abstract due on 6/4 by 5pm
  - Recorded presentation file due on 6/7 by 5pm
  - Final Project Presentation and Q&A Session on 6/8 (10-12pm)

- Office hours
  - Instructors: Fri 10-12 noon
  - email beforehand would be helpful

Today's Topics

- Parallel Imaging
  - SMASH review
  - Auto-SMASH
  - GRAPPA

Parallel Imaging
(SMASH)

SMASH

- Simultaneous Acquisition of Spatial Harmonics (SMASH) uses linear combinations of acquired k-space data from multiple coils to generate multiple data sets with offsets in k-space

SMASH Review

- Signal Equation:

$$\hat{m}_j(k_x, k_y) = \int_y \int_x C_j(x,y) m(x,y) \exp^{-i2\pi(k_x \cdot x + k_y \cdot y)} dx dy$$

m(x,y) = image

$C_j(x,y)$ = j^{th} coil sensitivity

$$\hat{m}_j(k_y) = \int_y C_j(y) m(y) \exp^{-i2\pi(k_y \cdot y)} dy$$

SMASH Review

- The linear combination of coil sensitivities looks like sinusoids:

$$e^{-i2\pi(m\Delta k_y)y} = \sum_{j=0}^{L-1} a_{j,m} C_j(y)$$

- Once we have $a_{j,m}$,

$$\hat{m}(k_y + m\Delta k_y) = \int_y m(y) e^{-i2\pi k_y y} e^{-2\pi(m\Delta k_y)y} dy$$

$$\hat{m}(k_y + m\Delta k_y) = \int_y m(y) e^{-i2\pi k_y y} \sum_{j=0}^{L-1} a_{j,m} C_j(y) dy$$

SMASH Review

$$\hat{m}(k_y + m\Delta k_y) = \int_y m(y) e^{-i2\pi k_y y} \sum_{j=0}^{L-1} a_{j,m} C_j(y) dy$$

$$\hat{m}(k_y + m\Delta k_y) = \sum_{j=0}^{L-1} a_{j,m} \int_y C_j(y) m(y) e^{-i2\pi k_y y} dy$$

$$\hat{m}(k_y + m\Delta k_y) = \sum_{j=0}^{L-1} a_{j,m} m_j(k_y)$$

To the board ...

Key Points of SMASH

- k-space lines are synthesized by combining signals from multiple coils such that it creates a partial replacement for a phase encoding gradient
- Decreases acquisition time by 1/N
  - N is the number of generated spatial Harmonics

$$\sum_j a_{j,m} C_j(y) = e^{-i2\pi\Delta k_y y}$$

Sodickson et al. MRM 1997

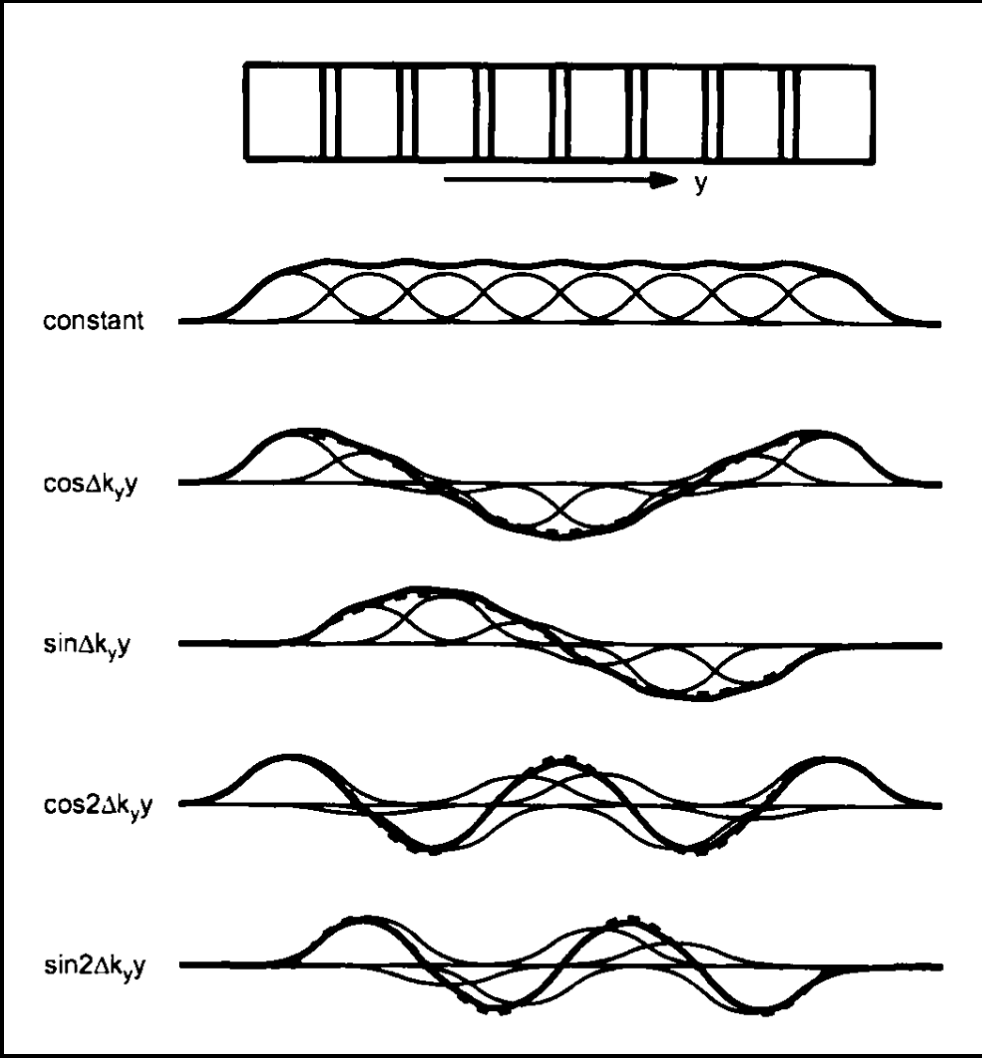
Spatial Harmonic Generation Using Coil Arrays

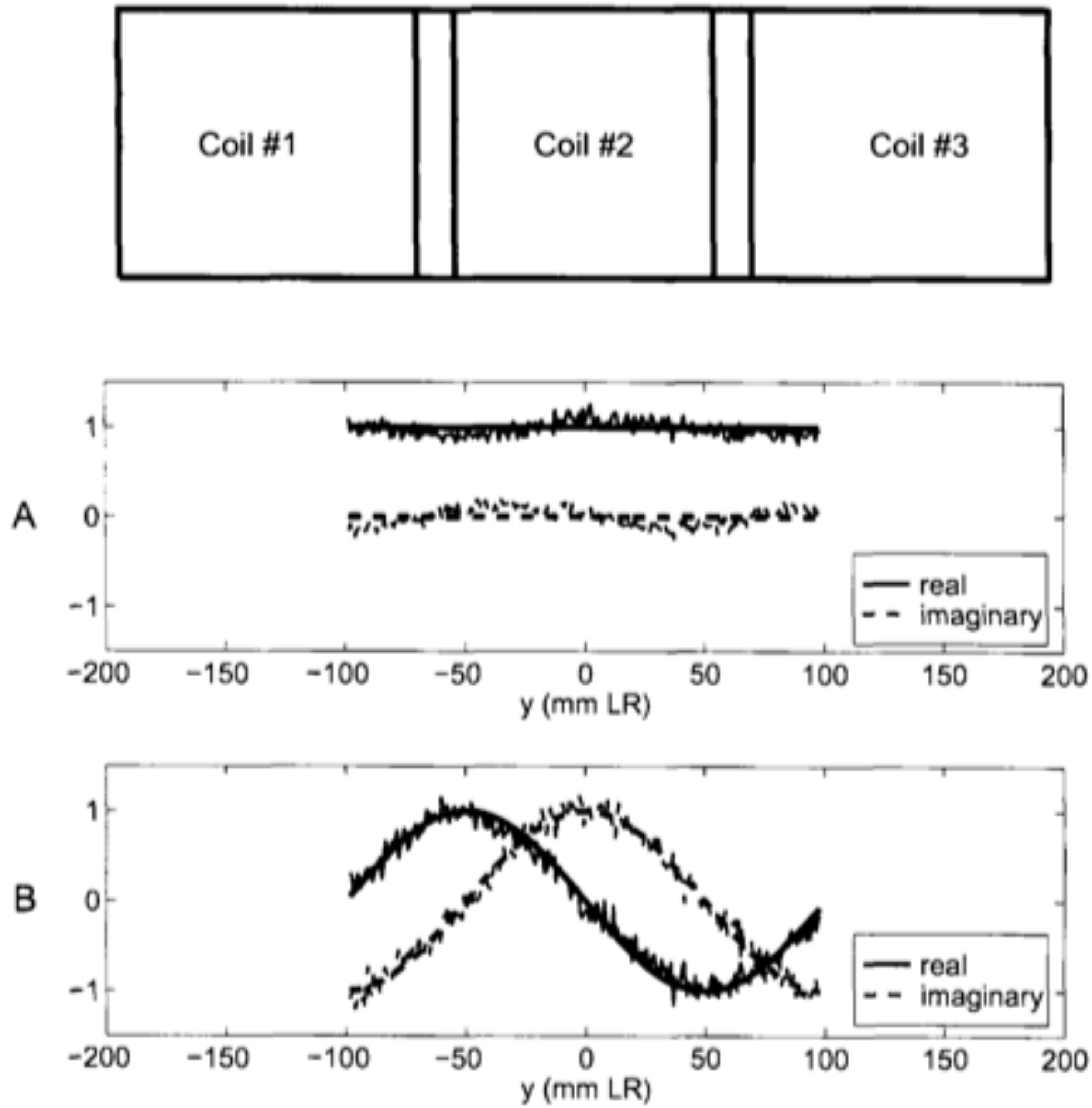
$$C_m^{comp}(y) = \sum_j a_{j,m} C_j(y) = e^{-i2\pi m \Delta k_y y}$$

- Linear surface coil array sensitivities C_j are combined with linear weights, $a_{j,m}$, to produce composite sinusoidal sensitivity
- Composite sensitivities are arranged to be spatial harmonics
- m is an integer, chosen to be a desired harmonic

Theory: Spatial Harmonics

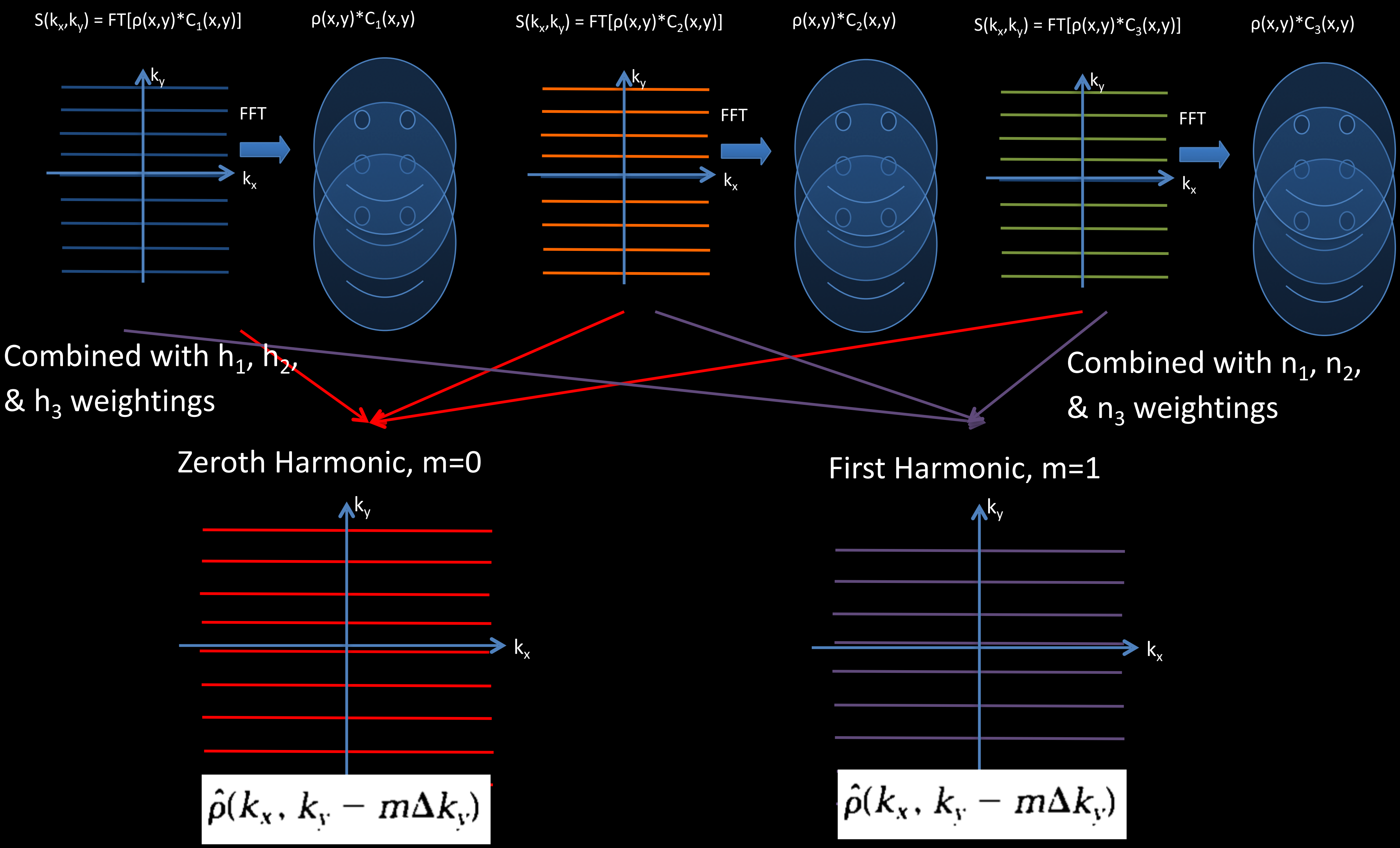
- 8 coil array
- Gaussian coil sensitivity distribution used
- m = 0, 1, -1, 2, -2
- Each spatial harmonic generated is shifted by $-m\Delta k_y$





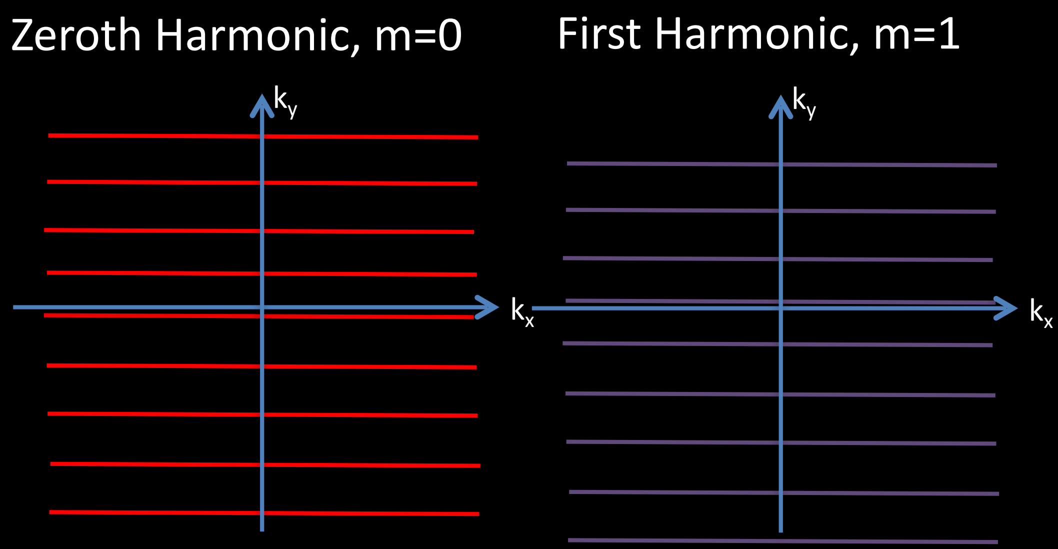
Coil #1
Coil #2
Coil #3
A
real
imaginary
y (mm LR)
B
real
imaginary
y (mm LR)

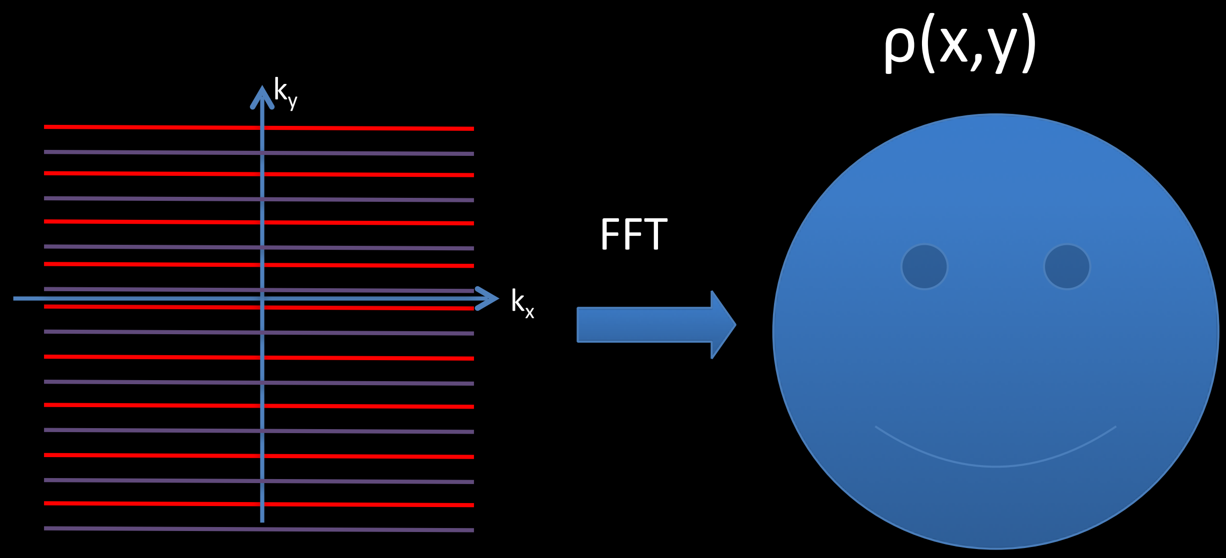
SMASH Reconstruction



SMASH Reconstruction

$$\hat{\rho}(k_x, k_y - m\Delta k_y)$$





Auto-SMASH

- Estimate $a_{j,m}$ directly

calibration

$$\hat{m}(k_y + m\Delta k_y) = \sum_{j=0}^{L-1} a_{j,m} m_j(k_y)$$

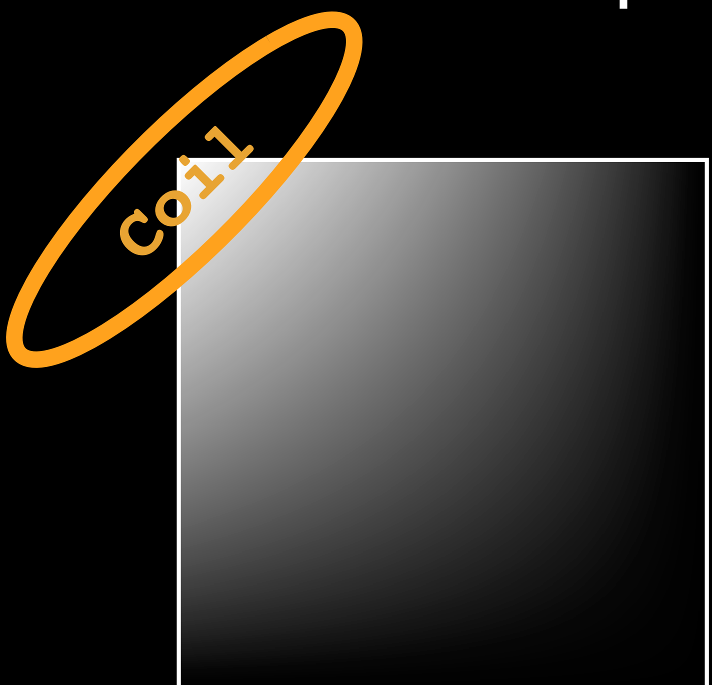
synthesis

- Solve for $a_{j,m}$ from calibration data & synthesize the missing data with $a_{j,m}$

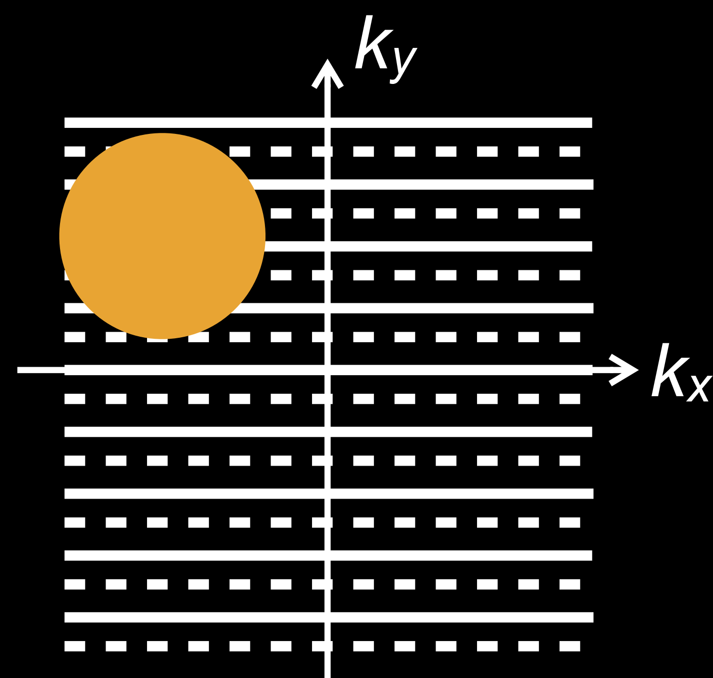
Parallel Imaging
(GRAPPA)

GRAPPA

- Coil sensitivities are
  - Smooth in image space
  - Local in k-space



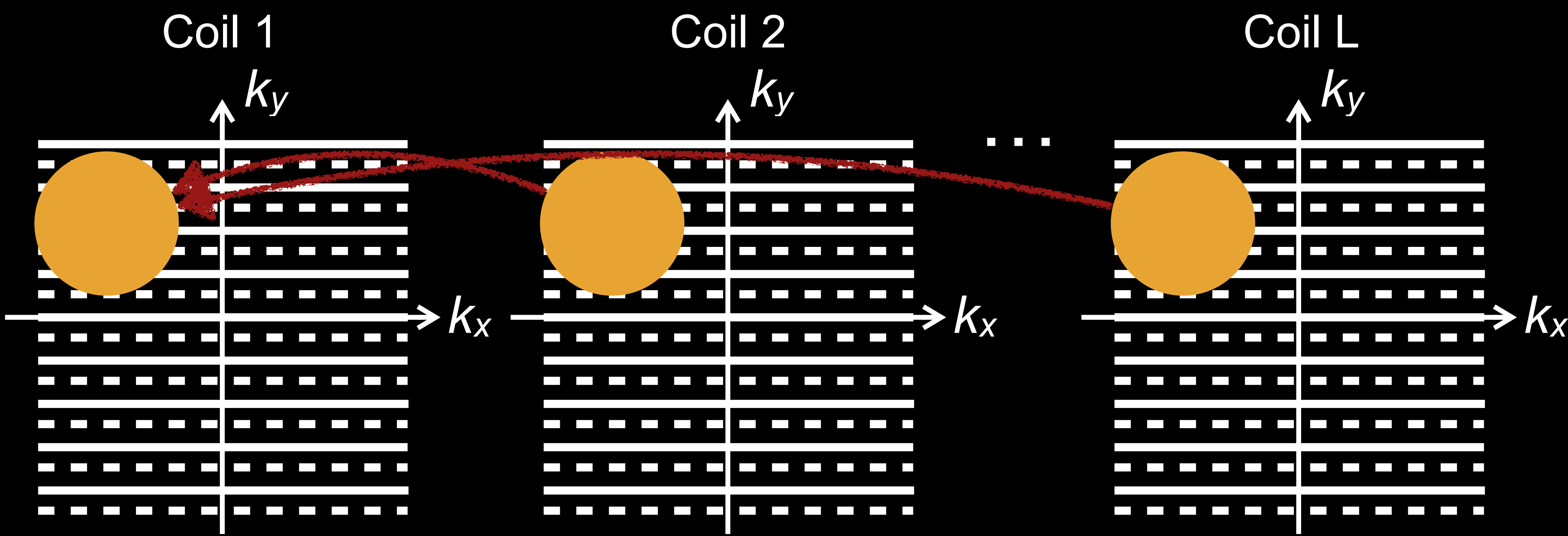
$$m(\vec{x})C_j(\vec{x})$$



$$M(\vec{k}) * C_j(\vec{k})$$

GRAPPA

- Missing information is implicitly contained by adjacent data



$M(\vec{k}) * C_1(\vec{k})$ $M(\vec{k}) * C_2(\vec{k})$ $M(\vec{k}) * C_L(\vec{k})$

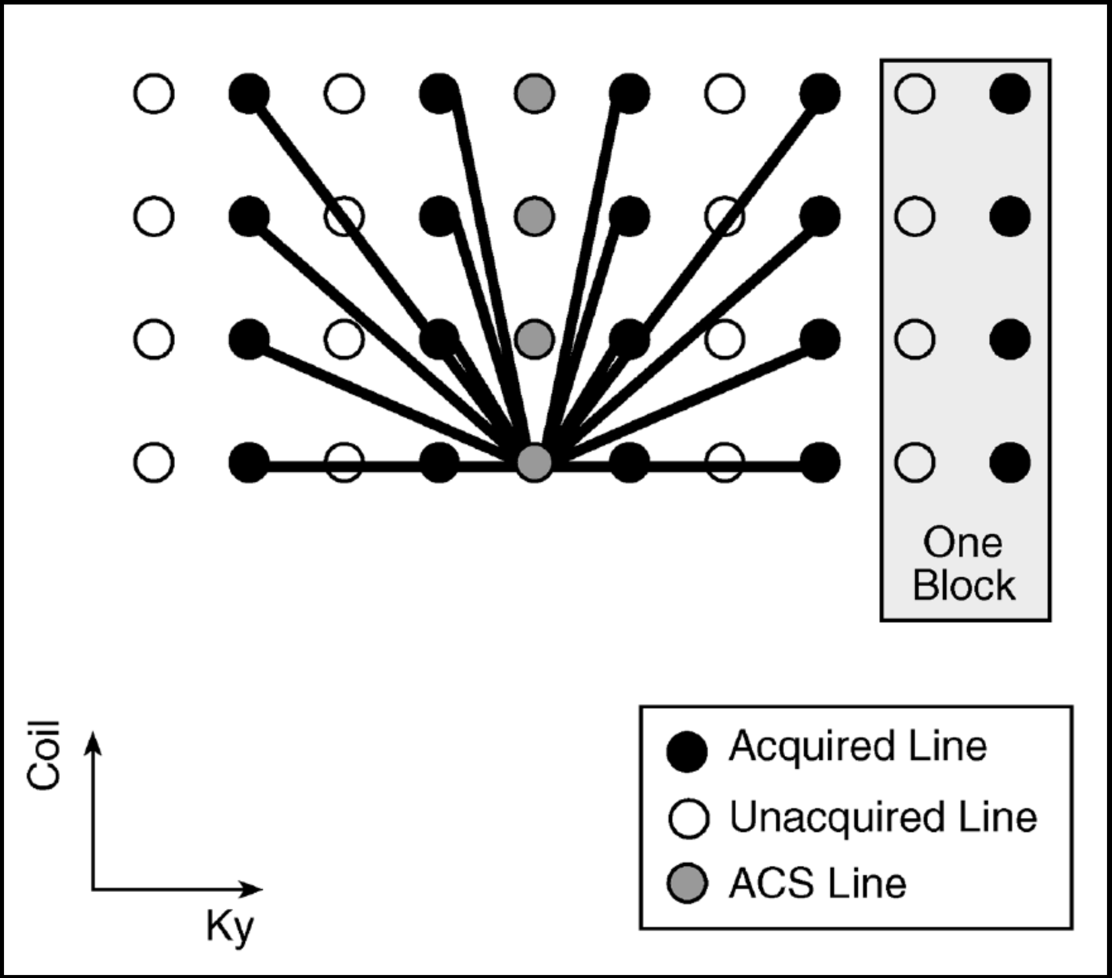
GRAPPA Reconstruction

- How do we find missing data from these samples?

$$\underbrace{\hat{m}_k(k_x, k_y)}_{\text{missing data for each coil}} = \sum_{i,j,k} \underbrace{a_{i,j,k}}_{\text{weights}} \cdot \underbrace{m_k(k_x + i\Delta k_x, k_y + j\Delta k_y)}_{\text{neighborhood data for each coil}}$$

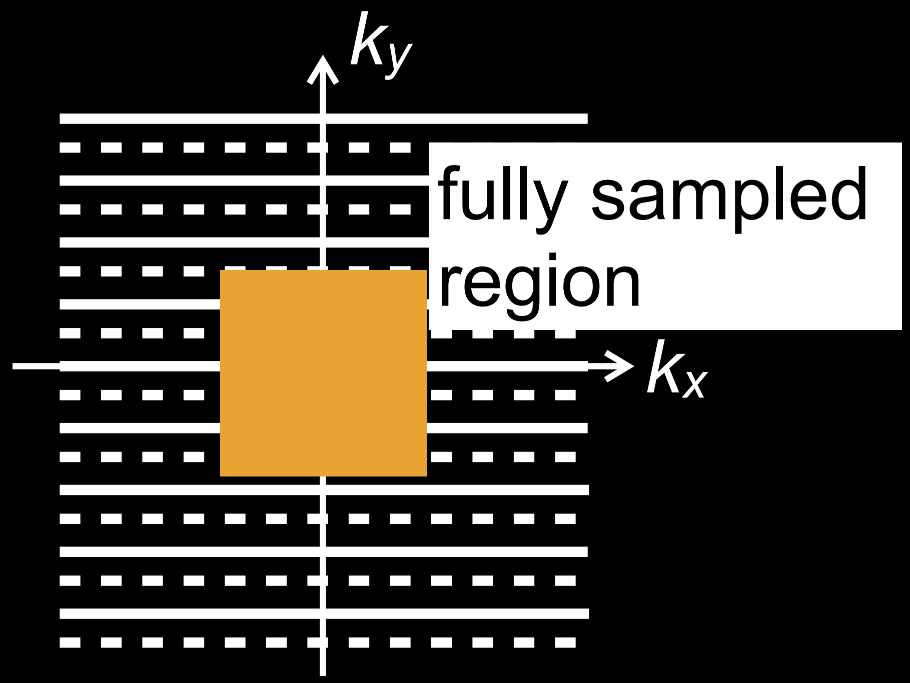
Auto-Calibration

$$\hat{m}_k(k_x, k_y) = \sum_{i,j,k} a_{i,j,k} \cdot m_k(k_x + i\Delta k_x, k_y + j\Delta k_y)$$



Auto-Calibration

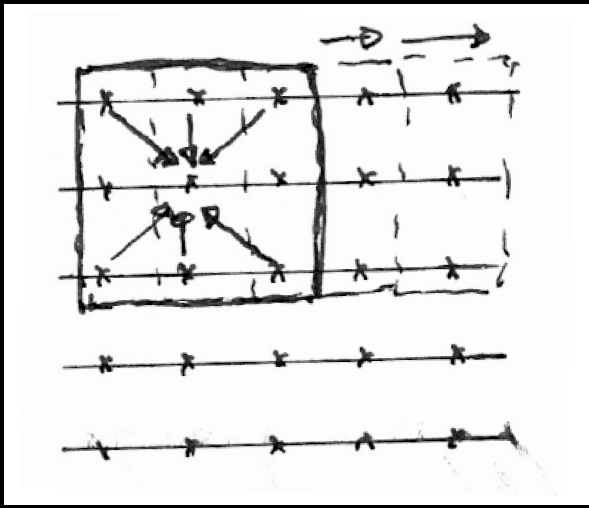
- Assume there is a fully sampled region
- We have samples of what the GRAPPA synthesis equations should produce



- Invert this to solve for GRAPPA weights

Auto-Calibration

- Calibration area has to be larger than the GRAPPA kernel
- Each shift of kernel gives another equation



- Here, 3x3 kernel, 5x5 calibration area gives 9 equations

Auto-Calibration

$$\hat{m}_k(k_x, k_y) = \sum_{i,j,k} a_{i,j,k} \cdot m_k(k_x + i\Delta k_x, k_y + j\Delta k_y)$$

- Write as a matrix equation

GRAPPA Coefficients

$$M_{k,c} = M_A \cdot a_k$$

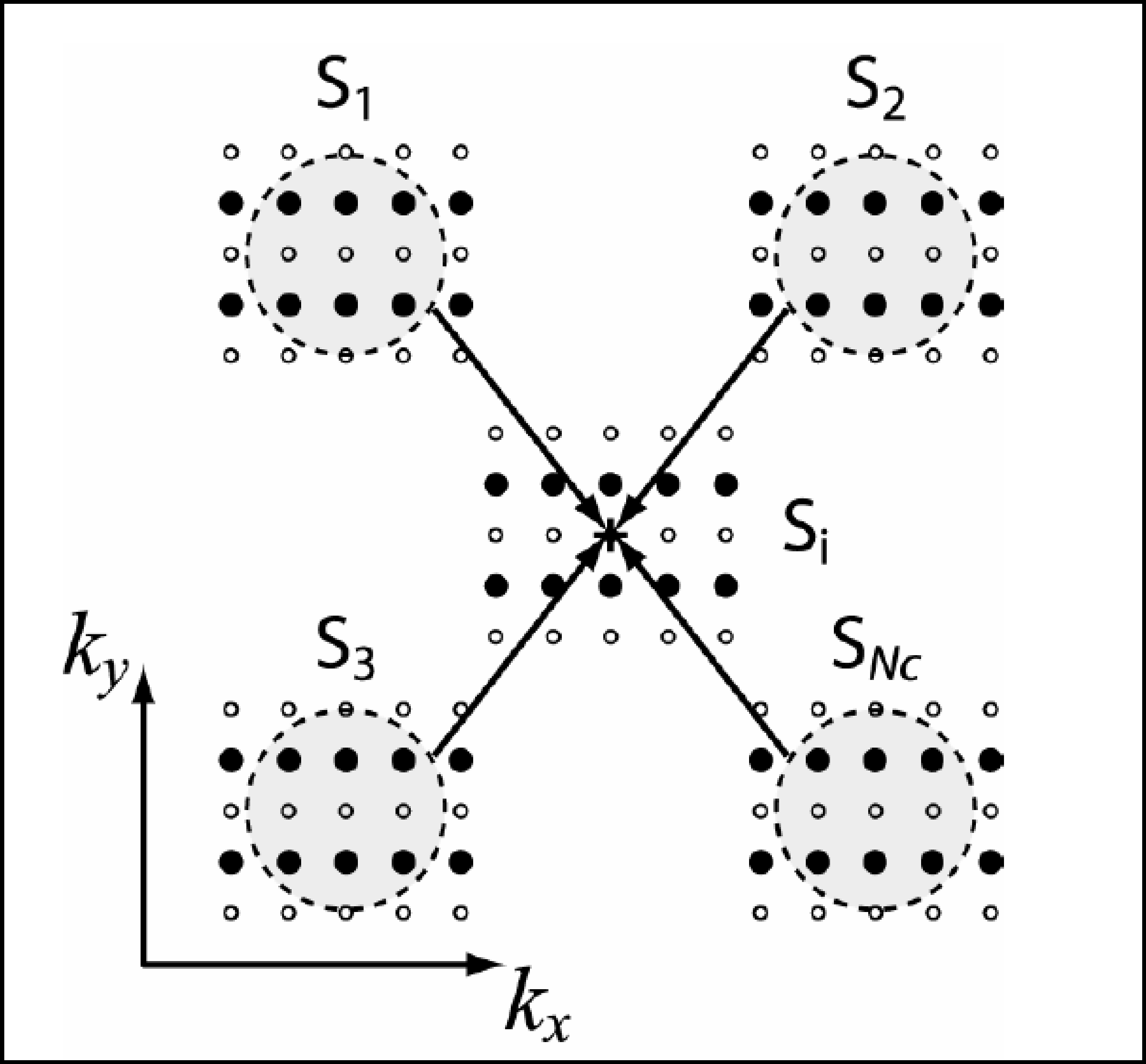
Calibration Data

Neighborhood Data

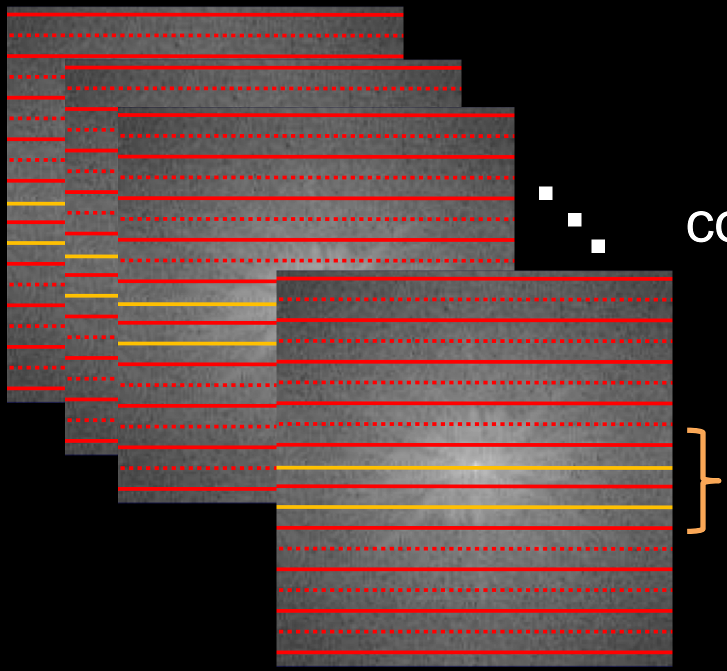
- GRAPPA weights are:

$$a_k = (M_A^* M_A + \lambda I)^{-1} M_A^* M_{k,c}$$

GRAPPA - Synthesis



Auto-Calibration Parallel Imaging



ACS (Auto-Calibration Signal) lines

$$\sum_{l=1}^{L} S_l^{ACS}(k_y - m\Delta k_y) = \sum_{l=1}^{L} n(l, m) S_l(k_y)$$

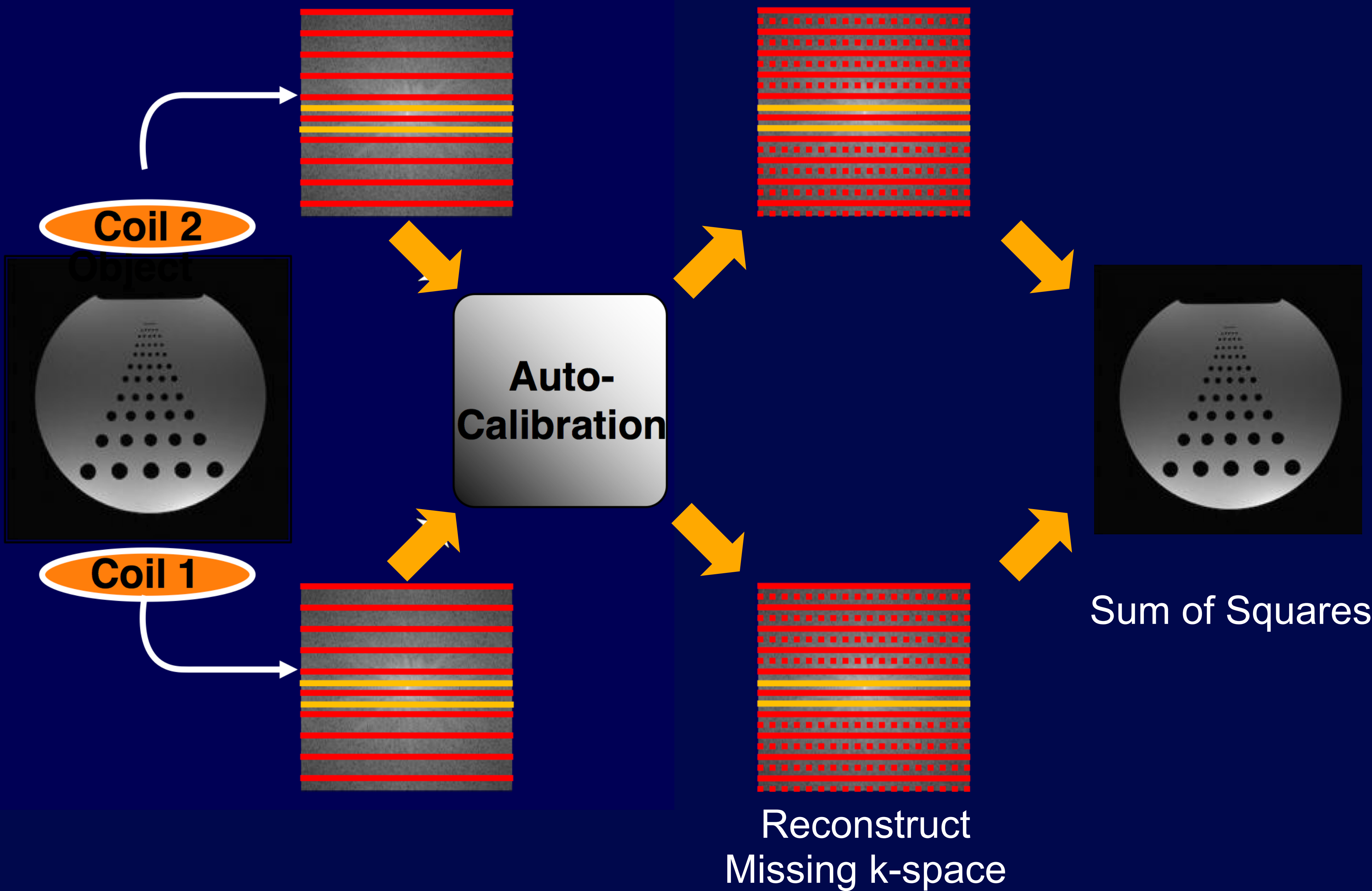
GRAPPA formula to reconstruct signal in one channel

$$S_j(k_y - m\Delta k_y) = \sum_{l=1}^{L} \sum_{b=0}^{N_b-1} n(j, b, l, m) S_l(k_y - bA\Delta k_y)$$

A: Acceleration factor
n(j,b,l,m): GRAPPA weights

Griswold et al. MRM, 47(6):1202-1210 (2002)

GRAPPA Reconstruction

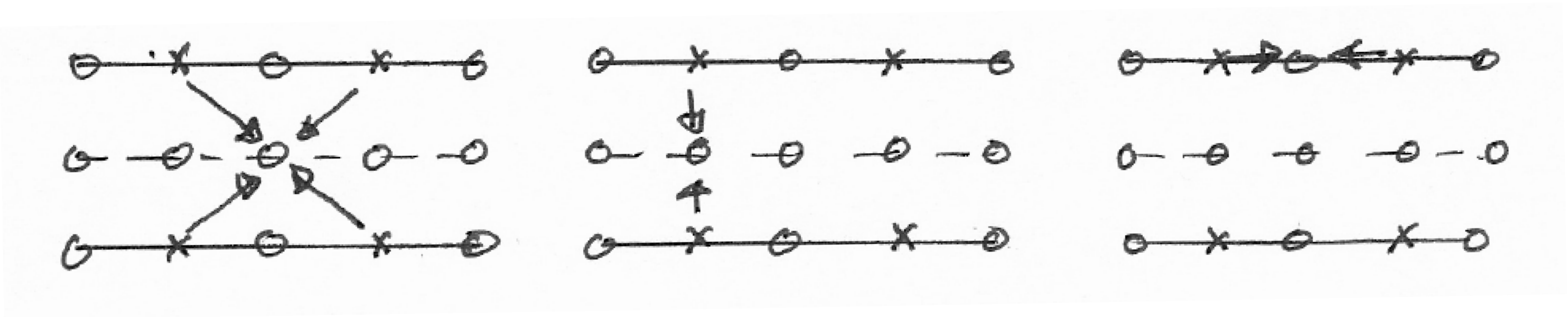


GRAPPA

- Compute GRAPPA weights from calibration region
- Compute missing k-space data using the GRAPPA weights
- Reconstruct individual coil images
- Combine coil images

Considerations of GRAPPA

- Calibration region size
- GRAPPA kernel size
- Sample geometry dependence



GRAPPA

- Compute GRAPPA weights from calibration region
- Compute missing k-space data using the GRAPPA weights
- Reconstruct individual coil images
- Combine coil images

Summary

- Parallel imaging utilizes coil sensitivities to increase the speed of MRI
- Cases for parallel imaging
  - Higher patient throughput,
  - Real-time imaging/Interventional imaging
  - Motion suppression
- Cases against parallel imaging
  - SNR starving applications

Summary

- Many approaches:
  - Image domain - SENSE
  - k-space domain - SMASH, GRAPPA
  - Hybrid - ARC

- We will focus on two:
  - SENSE: optimal if you know coil sensitivities
  - GRAPPA: autocalibrating / robust

Further Reading

- Multi-coil Reconstruction
  - http://onlinelibrary.wiley.com/doi/10.1002/mrm.1910160203/abstract
- SENSE
  - http://www.ncbi.nlm.nih.gov/pubmed/10542355
- SMASH
  - http://www.ncbi.nlm.nih.gov/pubmed/9324327
- Parallel Imaging Overview
  - http://www.ncbi.nlm.nih.gov/pubmed/17374908

Thanks!

- Next time
  - Compressed Sensing & Artificial Intelligence

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https://mrrl.ucla.edu/sunglab/