M219, Winter 2018

Homework Assignment #1 (15 Points)

Due via E-mail on Tuesday, January 23rd by 9pm

To submit the assignment, e-mail `DEnnis@mednet.ucla.edu` a PDF entitled `M219_HW01_[First Initial]_[Last Name].pdf` (*e.g.* `M219_HW01_D_Ennis.pdf`). Please only submit neat and clear solutions. Late assignments will be discounted by $e^{-t/\tau}$, where $\tau = 72$ hours.

For all problems – Clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

If your assignments are hard to read, poorly commented, or sloppy, then points may be deducted. As appropriate, each solution should be obtained using Matlab. Please comment and submit your code as individual files that run for each problem.

Problem #1 (5 points, plus 1 *Extra Credit* point) – Design the Main (B_0) Field. For this problem you will design the main (B_0) magnet that meets the following specifications: 1) 1.5T field strength (at isocenter); 2) 70cm bore; 3) Length $< 2m$; 4) Field variation $< 100,000ppm$ for $50cm$ along the z-axis.

A. Modify the `PAM_Lec01_Bz_Uniformity.m` function to design the length and current needed to meet these design specifications. This Matlab function use the following expression:

$$B_z(z) = \frac{\mu_0 N I}{2L} \left(\cos\alpha_2 - \cos\alpha_1\right) \tag{1}$$

Note, that according to this expression there is an axial (z), but no radial (x or y) dependence on the magnetic field strength and the field remains z-oriented. Make a plot of $B_z(z)$ for the length and current you have designed. [2 points]

B. What is the magnetic field variation (maximum, minimum, mean) for $50cm$? Calculate and report the field homogeneity $[(B_{0,max} - B_{0,min})/B_{0,mean}]$ in PPM for $50cm$. What is the vRMS error for $50cm$ relative to the target field strength of 1.5T? [2 points]

C. How would you improve the design of your magnet to improve the field homogeneity to $< 1,000ppm$? [1 point]

D. *Extra Credit*: Use the principle of superposition and Eqn. 1 to improve the field homogeneity to $< 1,000ppm$. [2 points]

Problem #2 (5 points, plus 1 *Extra Credit* point) – B_0 vs. B_1 fields. Assume a hard RF pulse with a flip angle of $\alpha = \pi/2$, phase of $\pi/4$, and $B_{1,max} = 20$ gauss for ^{31}P at $B_0 = 0.15T$.

A. What is the duration (τ_{RF}) of the RF pulse? [1/2 point]

B. Find ω_0, the frequency of *precession* in MHz for the B_0 field. [1/2 point]

C. Find ω_1, the frequency of *nutation* in MHz for the B_1 field. [1/2 point]

D. How many cycles of precession does the bulk magnetization go through during the RF pulse? How does this compare to the number of cycles of nutation? [1/2 point]

E. Use `PAM_B1_op.m` to generate the M_x, M_y, and M_z components for this RF pulse from 0 to τ_{RF} in the *rotating* frame using MATLAB. This can be done with a *for-loop*. Use 1,000 points for your simulation. Plot the results; label the axes. [1 point]

F. Now incorporate the use of `PAM_B0_op.m` to generate the M_x, M_y, and M_z components in the *laboratory* frame using MATLAB. Plot the results; label the axes. *Hint*: The RF phase is constant in the rotating frame, but not the laboratory frame. [2 points]

G. *Extra Credit*: Explain how B_1 field can be effective at perturbing the spin system when B_0 is so much larger in magnitude. [1 point]

Problem #3 (5 points) – T_1 and T_2 relaxation

A. In lecture we learned that T_1 and T_2 relaxation are tissue dependent characteristics. Using the equations for relaxation during free precession in the rotating frame, find a general expressions for T_1 contrast after an *inversion* pulse. [1/2 point]

B. Derive an analytic expression for the time that maximizes the image contrast (signal difference) between white matter ($790ms$) and gray matter ($925ms$). Assume that the proton densities are the same. [1 point]

C. Plot the T_1 relaxation results for white matter ($790ms$) and gray matter ($925ms$). Prove that your solution in (A) produces the same result as simply taking the difference between the two curves. Label the axes. [1 point]

D. Using the equations for relaxation during free precession in the rotating frame, find a general expressions for T_2 contrast after an *saturation* pulse. [1/2 point]

E. Repeat the process and derive an analytic expression for T_2 contrast after a *saturation* pulse. Assume that the proton densities are the same. [1 point]

F. Plot the T_2 relaxation results for white matter ($92ms$) and gray matter ($100ms$). Prove that your solution in (C) produces the same result as simply taking the difference between the two curves. Label the axes. [1 point]

Problem #4 (1 *Extra Credit* point) Create your own three-part question using the concepts from the first four lectures. Provide an answer. Your question may be chosen to appear on the final exam (and you'll already know the answer!).