M219 2017
HW2 solutions

1

A.

$$M_z^0 = M_0 \qquad\qquad M_{xy}^0 = 0$$

$$M_z^1 = 0 \qquad\qquad M_{xy}^1 = M_0$$

$$M_z^2 = M_0\left(1 - e^{-TE_1/2T_1}\right) \qquad\qquad M_{xy}^2 = M_0\, e^{-\frac{TE_1}{2T_2}}\, e^{-i\phi_{off_1}}$$

$$M_z^3 = -M_0\left(1 - e^{-TE_1/2T_1}\right) \qquad\qquad M_{xy}^3 = M_0\, e^{-\frac{TE_1}{2T_2}}\, e^{i\phi_{off_1}}$$

$$M_z^4 = M_0\left(1 - 2e^{-TE_1/2T_1} + e^{-TE_1/T_1}\right) \qquad\qquad M_{xy}^4 = M_0\, e^{-\frac{TE_1}{T_2}}$$

$$M_z^5 = M_0\left(1 - 2e^{-TE_2/2T_1} + e^{-(TE_1+TE_2)/T_1} \qquad\qquad M_{xy}^5 = M_0\, e^{-(TE_1+TE_2)/2T_2}\, e^{-i\phi_{off_2}}$$

$$M_z^6 = -M_0\left(1 - 2e^{-TE_2/2T_1} + e^{-(TE_1+TE_2)/T_1}\right) \qquad\qquad M_{xy}^6 = M_0\, e^{-(TE_1+TE_2)/2T_2}\, e^{i\phi_{off_2}}$$

$$M_z^7 = M_0\left(1 - e^{-TE_2/T_1} - 2e^{-(TE_2-TE_1)/2T_1} + 2e^{-(TE_2-2TE_1)/2T_1}\right)$$

$$M_{xy}^7 = M_0\, e^{-TE_2/T_2}$$

$$M_z^8 = M_0\left(1 - e^{-TR/T_1} + 2e^{-2(TR-TE_1)/2T_1} - 2e^{-(2TR-TE_1-TE_2)/2T_1}\right.$$

$$M_{xy}^8 = M_0\, e^{-TR/T_2}\, e^{-i\phi_{off_3}}$$

B.

$$M_{xy}(t = TE_1) = M_z^8\, e^{-TE_1/T_2} \qquad \text{for } TR \gg T_1 \text{ and } TE_1 \ll T_2$$

$$\approx M_0$$

Proton density weighted.

C.

$$A_E(t = TE_2) = M_z^8\, e^{-TE_2/T_2} \qquad \text{for } TR \gg T_1 \text{ and } TE_2 \approx T_2$$

T_2 weighted image

D.

Change of Bandwidth. Second gradient reduces bandwidth. SNR increases.

E.

Acquisition of proton density and T_2 weighted images in a reduced scan time.

(2)

A.

$$\vec{B} = (B_0 + \vec{G} \cdot \vec{r})\,\hat{k}$$

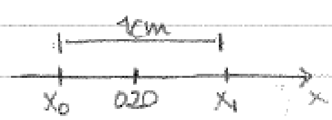
$$\vec{G} \cdot \vec{r} = (G_x, 0, 0) \cdot (x, y, z) = G_x\, x$$

$$\vec{B} = (B_0 + G_x x)\,\hat{k}$$

$$\omega = \gamma B = \gamma (B_0 + G_x x) \Rightarrow x = \left[\frac{\omega}{\gamma} - B_0\right] \frac{1}{G_x}$$

$$\boxed{x = \frac{\omega - \gamma B_0}{\gamma G_x}}$$

B.



$x_0 = 0.195$ m

$x_1 = 0.205$ m

$\gamma = 42.58\ \frac{MHz}{T}$

$B_0 = 3$ T

$G_x = 40\ \frac{mT}{m} = 0.04\ \frac{T}{m}$

$\omega_0 = \gamma(0.04 \cdot 0.195 + 3)$

$\omega_0 = 128.072$ MHz

$\omega_1 = \gamma(0.04 \cdot 0.205 + 3)$

$\omega_1 = 128.089$ MHz

C.

$$B_{Gz}(x) = 0.04x - 0.25x^3$$

$$\rightarrow B_{Gz}(x_0 = 0.195) = 0.0059\ T$$

$$\rightarrow B_{Gz}(x_1 = 0.205) = 0.0060\ T$$

$$w' = \gamma(B_{Gz} + B_0)$$

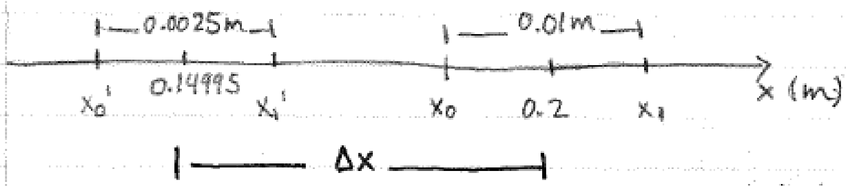
$$\boxed{\begin{aligned} w_0' &= 127.99\ \text{MHz} \\ w_1' &= 128.00\ \text{MHz} \end{aligned}}$$

D.

$$x' = \frac{w' - \gamma B_0}{\gamma G_x}$$

$$x_0' = 0.1487\ \text{m}$$

$$x_1' = 0.1512\ \text{m}$$



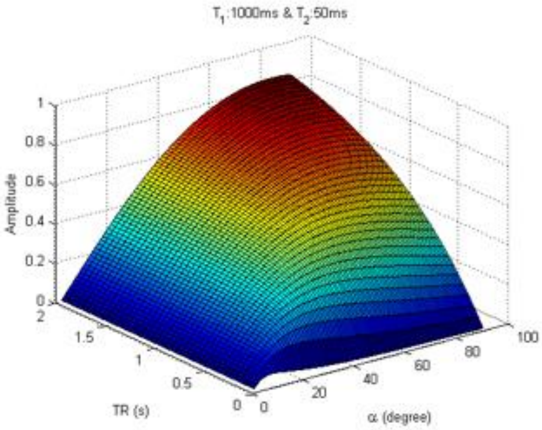
$$\boxed{\Delta x = 5\ \text{cm}}$$

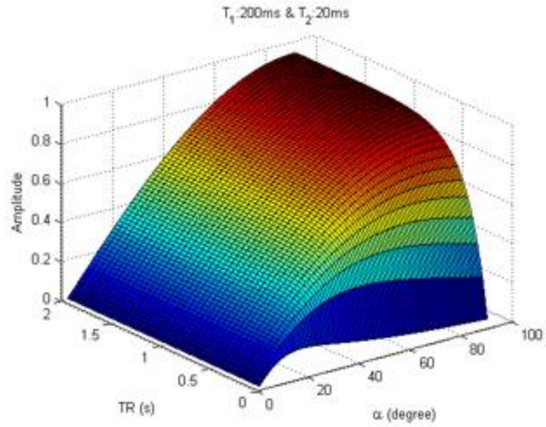
E.

Same gradient values at multiple locations. Those frequencies can't be solved for a single location.

3.

A.





B.

$$A_{echo} \propto \frac{1-e^{-TR/T_1}}{1-\cos\alpha\, e^{-TR/T_1}} \cdot \sin\alpha\, e^{-\frac{TE}{T_2^*}}$$

$$\frac{dA_{echo}}{d\alpha} = \frac{d}{d\alpha}\left(\frac{\sin\alpha}{1-\cos\alpha\, e^{-TR/T_1}}\right) = \frac{\left(1-\cos\alpha\, e^{-TR/T_1}\right)\cos\alpha - e^{-TR/T_1}\sin\alpha\sin\alpha}{\left(1-\cos\alpha\, e^{-TR/T_1}\right)^2} = 0$$

$$\cos\alpha - \cos^2\alpha\, e^{-TR/T_1} - \sin^2\alpha\, e^{-TR/T_1} = 0$$

$$\cos\alpha - e^{-TR/T_1} = 0$$

$$\cos\alpha = e^{-TR/T_1}$$

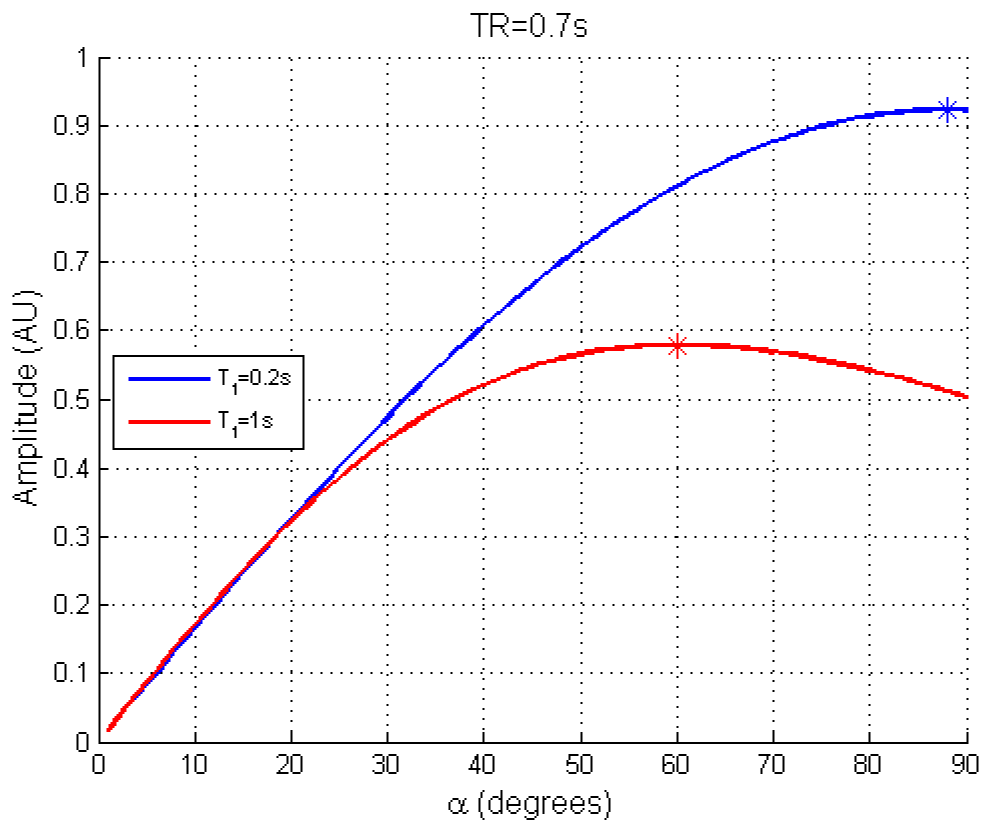
$$\boxed{\alpha = \arccos\left(e^{-TR/T_1}\right)}$$

C.

Given $TR = .7s$

$T_{1a} = 0.2s$ $\quad T_{2a} = 0.02s$ $\quad \rightarrow \alpha_1 = 88.27^\circ$

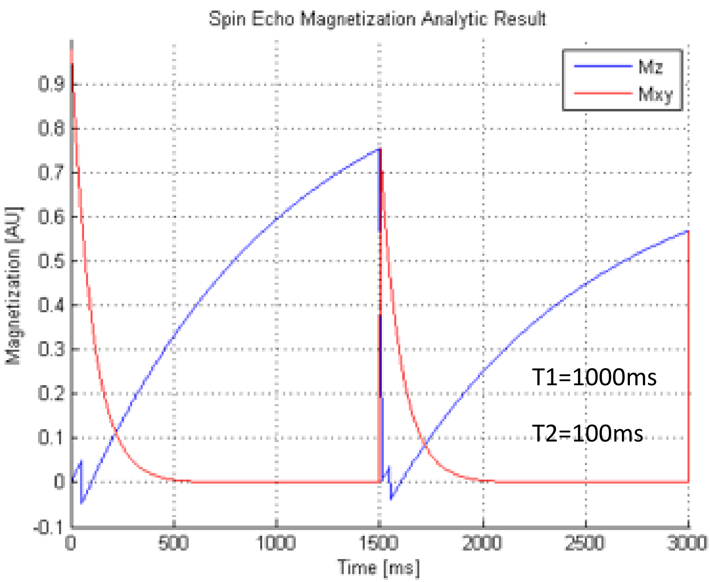
$T_{1b} = 1\ s$ $\quad T_{2b} = 0.5s$ $\quad \rightarrow \alpha_2 = 60.22^\circ$

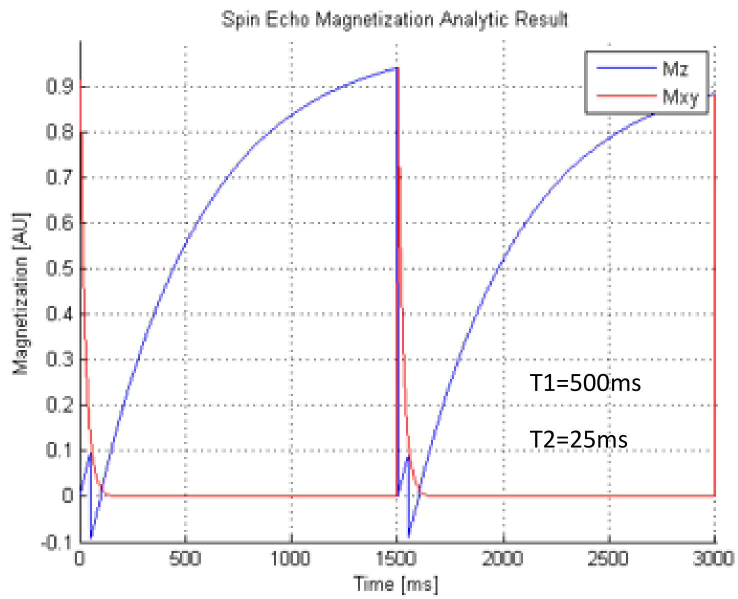


Ernst angle agrees with results.

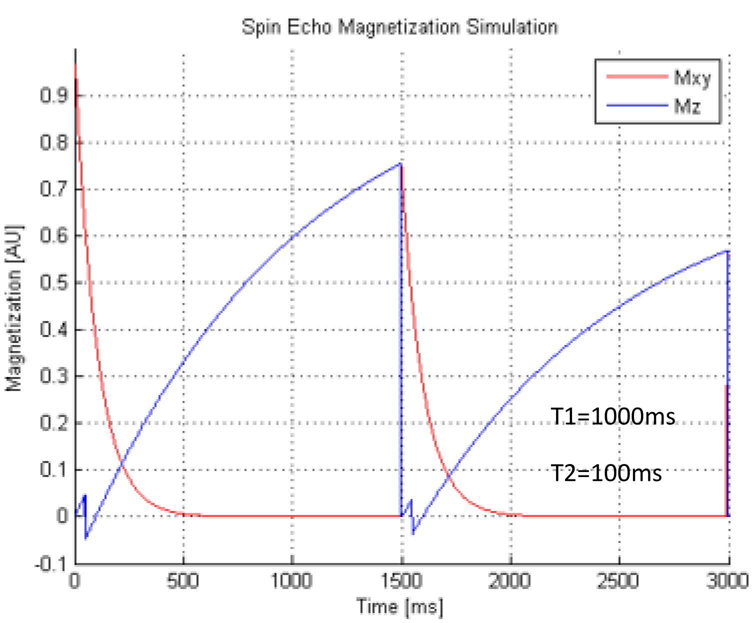
4.

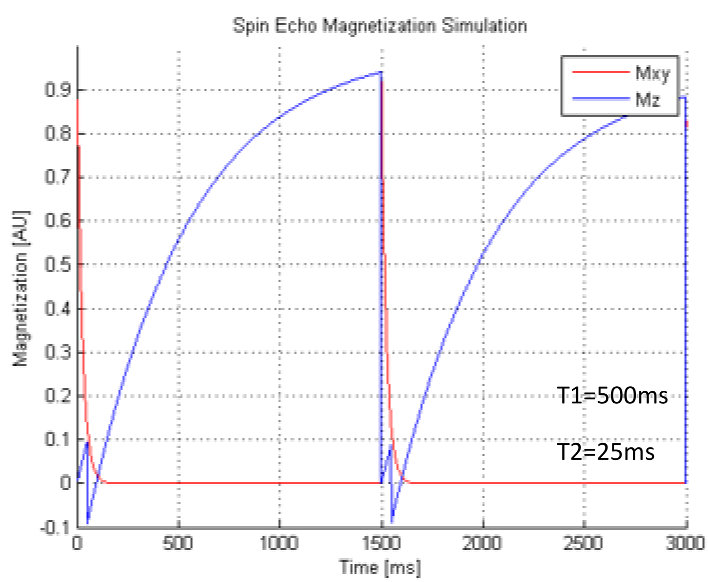
A.

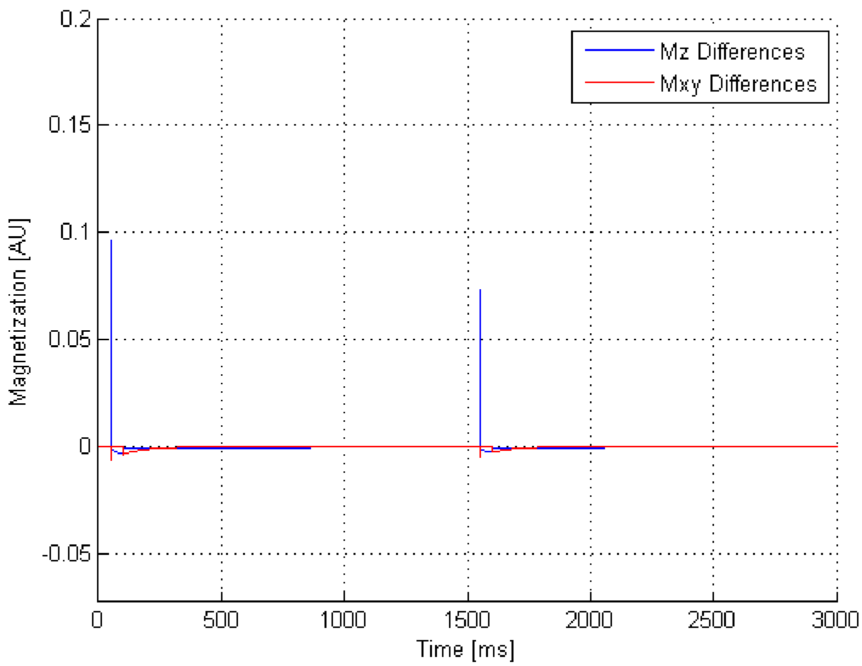

Spin Echo Magnetization Analytic Result
Mz
Mxy
T1=1000ms
T2=100ms
Magnetization [AU]
Time [ms]


Spin Echo Magnetization Analytic Result
Mz
Mxy
T1=500ms
T2=25ms
Magnetization [AU]
Time [ms]

B.


Spin Echo Magnetization Simulation
Mxy
Mz
T1=1000ms
T2=100ms
Magnetization [AU]
Time [ms]


Spin Echo Magnetization Simulation
Mxy
Mz
T1=500ms
T2=25ms
Magnetization [AU]
Time [ms]



Main difference is that the rf pulse has some duration. Magnetization rotates through different planes before settling at the right values