MRI Systems II – B1

M219 - Principles and Applications of MRI
Kyung Sung, Ph.D.
1/12/2022

Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219
- Course schedule
  - https://mrrl.ucla.edu/pages/m219_2022
- Assignments
  - Homework #1 due on 1/26 by 5pm

Course Overview

- Office Hours
  - TA (Ran Yan) - Tuesday 4-5pm
    https://uclahs.zoom.us/j/96870184581?pwd=VkczL0lyRkxsQ3FHcnIxQ1M2U3hPdz09

    Password: 900645
  - Instructor (Kyung Sung) - Friday 2-3pm
    https://uclahs.zoom.us/j/94058312815?pwd=TkI3ajhkamdGTnhqOVNnbk5RMnJGQT09

    Password: 888767

Rotations & Euler’s Formula

Vectors

- A ***vector*** ($\vec{v}$) describes a physical quantity (e.g. bulk magnetization or velocity) at a point in space and time and has a magnitude (positive real number), a direction, and physical units.

- To define a vector, we need a ***basis***:

$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- A 3D ***vector*** has components:

$$\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

2D Vectors - Euler’s Formula

- Euler’s formula provides a compact representation of a 2D vector using a complex exponential:

$$e^{i\phi} = \cos\phi + i\sin\phi$$

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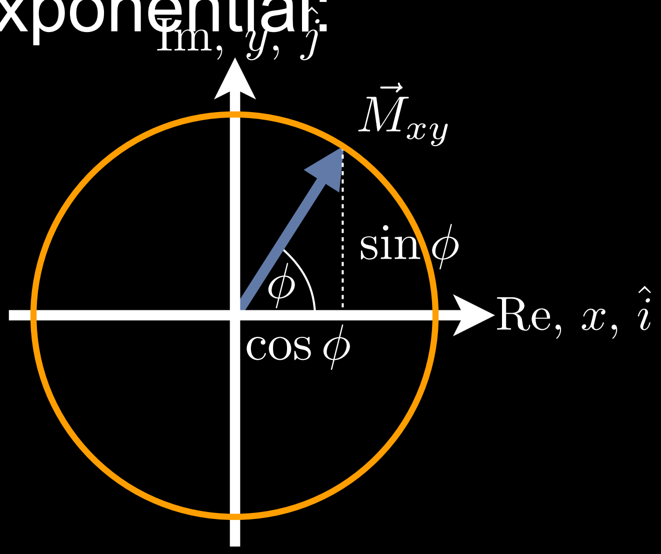


$$\vec{M}_{xy} = M_x\hat{i} + M_y\hat{j}$$

Vector components

$$= M_x + iM_y$$

Complex components

$$= |\vec{M}_{xy}|\cos\phi\hat{i} + |\vec{M}_{xy}|\sin\phi\hat{j}$$

Trigonometric components

$$= |\vec{M}_{xy}|\cos\phi + i|\vec{M}_{xy}|\sin\phi$$

Complex trigonometric components

$$= |\vec{M}_{xy}|e^{i\phi}$$

Euler’s notation

$\hat{j}$

$$= |\vec{M}_{xy}|\cos\phi + i|\vec{M}_{xy}|\sin\phi$$

$$= |\vec{M}_{xy}|e^{i\phi}$$

Euler’s formula is mathematically convenient.
There is nothing explicitly *imaginary* about M_{xy}.

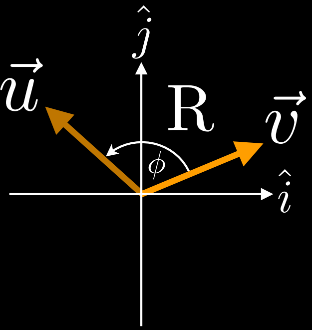
Rotations

- ***Rotations*** (R) are vector valued orthogonal transformations that preserve the magnitude of vectors and the angles between them.
- The simplest rotation matrix is the ***identity*** matrix:

$$R = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ therefore } \vec{v} = I\vec{v}$$

- More simply, R transforms (rotates) one vector to another:

$$\vec{u} = R\vec{v}$$



Rotations

- (

$\hat{i}$ ends up here

$\hat{j}$ ends up here

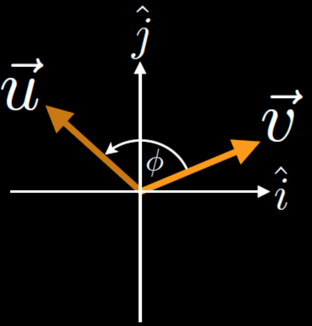
$\hat{k}$ does not change

Magnitude of rotation

$$\mathrm{R}_z^{\phi} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Axis (phase) of rotation

$$\vec{u} = \mathrm{R}\vec{v}$$



Note: Positive values of *Φ* produce right-handed (CCW) rotations.

Rotation Matrices

RIGHT-HANDED

$$\mathrm{R}_z^{\phi} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LEFT-HANDED

$$R_Z(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Y(\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

$$R_X(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Last Time...

$\vec\tau = \vec\mu \times \vec B$

$\vec S = \vec r \times \vec p$

$\vec\tau = \frac{d\vec S}{dt}$

$\vec\mu = \gamma \vec S$

$$\frac{d\vec\mu}{dt} = \vec\mu \times \gamma \vec B$$

Equation of Motion for a Magnetic Dipole

$$\vec M = \sum_{n=1}^{N_{total}} \vec\mu_n$$

$$\frac{d\vec M}{dt} = \vec M \times \gamma \vec B$$

Equation of Motion for the bulk magnetization.

$$\vec B_0 = B_0 \vec k$$

$$\frac{d\vec M}{dt} = \vec M \times \gamma \left(\vec B_0\right) = \begin{vmatrix} \hat i & \hat j & \hat k \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$
$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$
$$M_z(t) = M_z^0$$

To The Board...

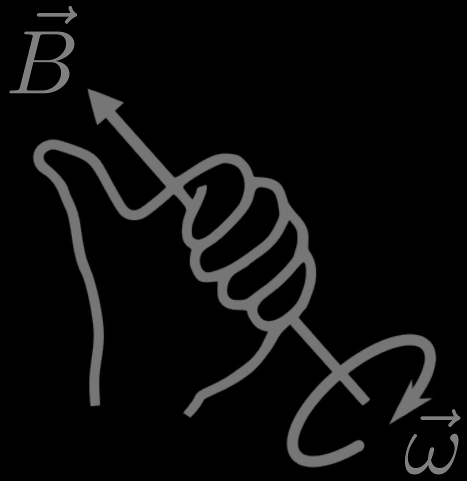
Free Precession In The Laboratory Frame Without Relaxation

Free Precession In The Laboratory Frame Without Relaxation

$$\begin{aligned}\frac{d\vec{M}}{dt} &= \vec{M} \times \gamma \left(\vec{B_0} \right) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}\end{aligned}$$

Free Precession w/o Relaxation

$$\mathbf{R}_z(\omega_0 t) = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



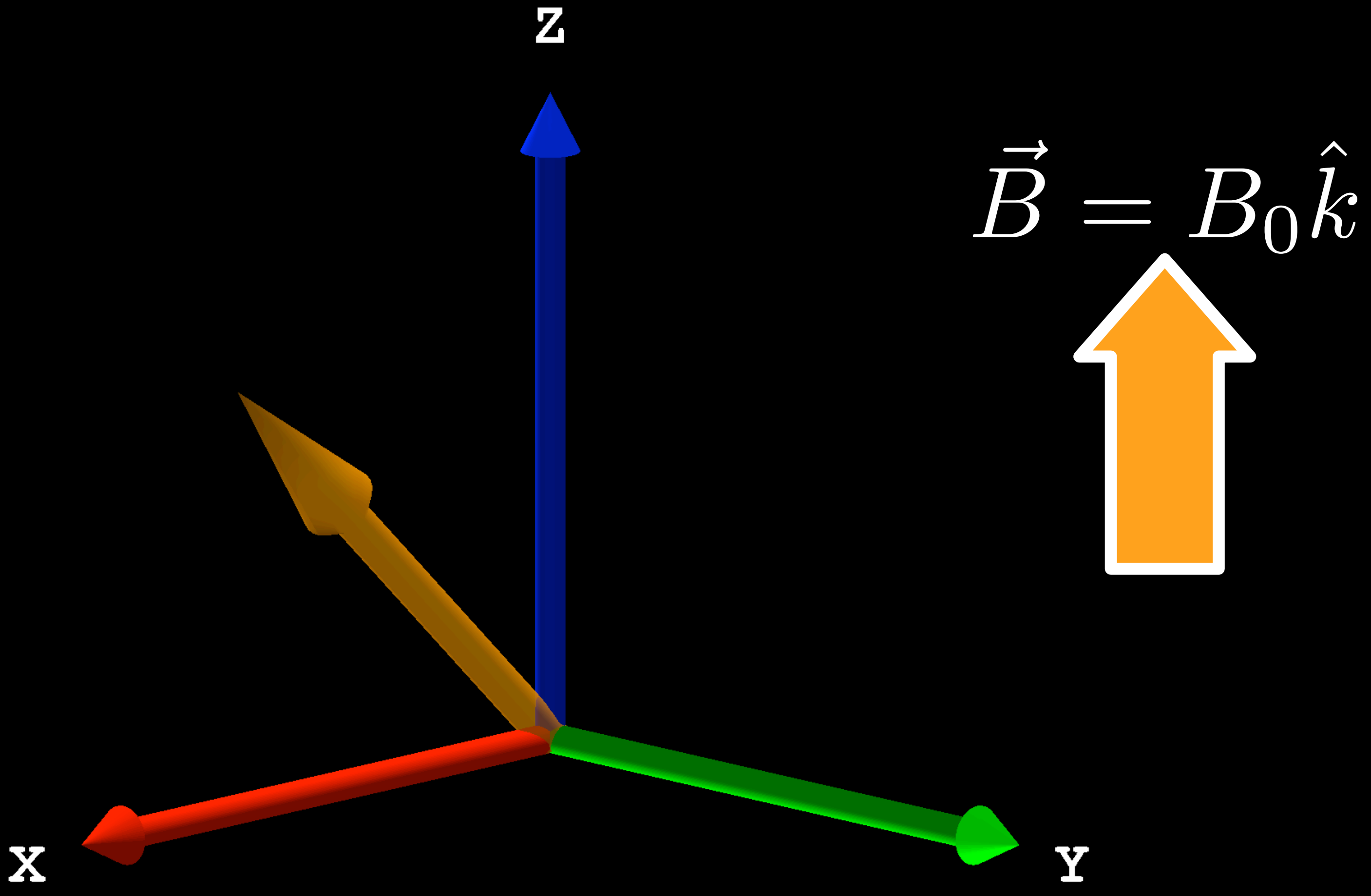
Precession is left-handed (clockwise).

$$\vec{M}(t) = \mathbf{R}_z(\omega_0 t)\vec{M}^0$$

$$\Updownarrow$$

$$\omega_0 = -\gamma B_0$$

$$\vec{\omega} = -\gamma \vec{B} = -\gamma B_0 \hat{k}$$



$$\vec{B} = B_0 \hat{k}$$

Precession only apparent when: $\vec{M} \neq \|\vec{M}\| \hat{k}$

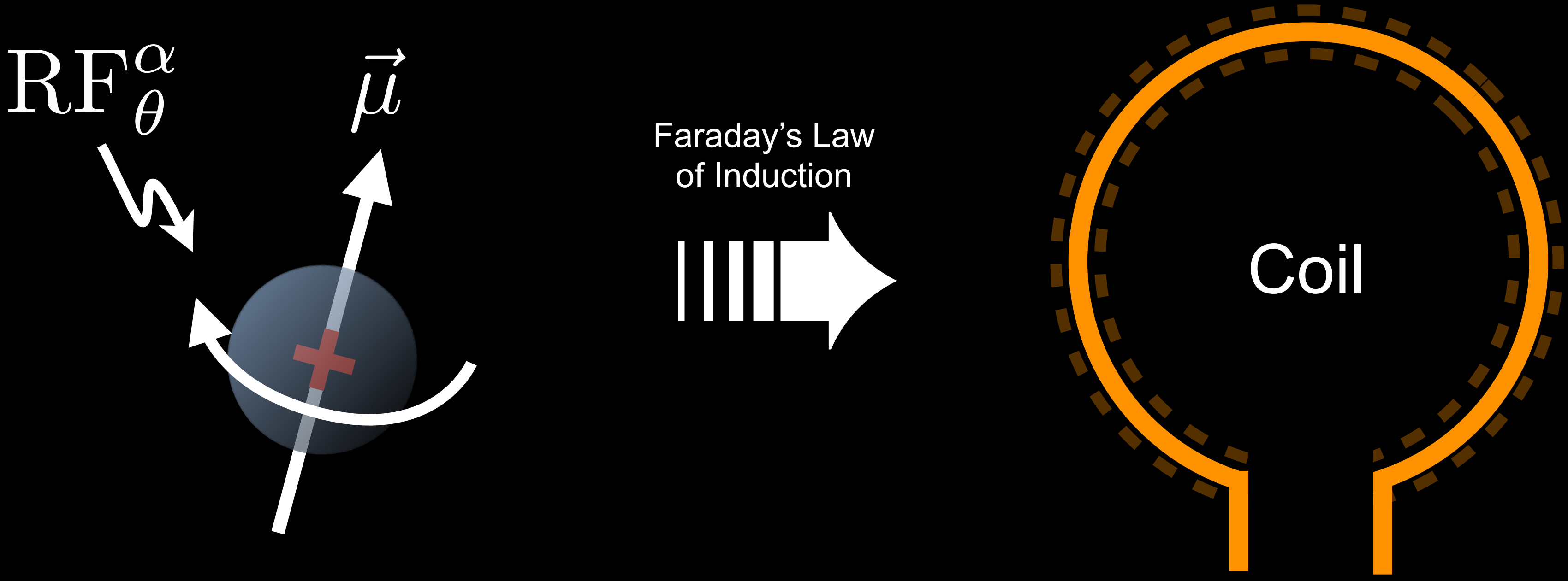
To The Board...

$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$

$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$

$$M_z(t) = M_z^0$$

Signal Reception



Faraday's Law of Induction

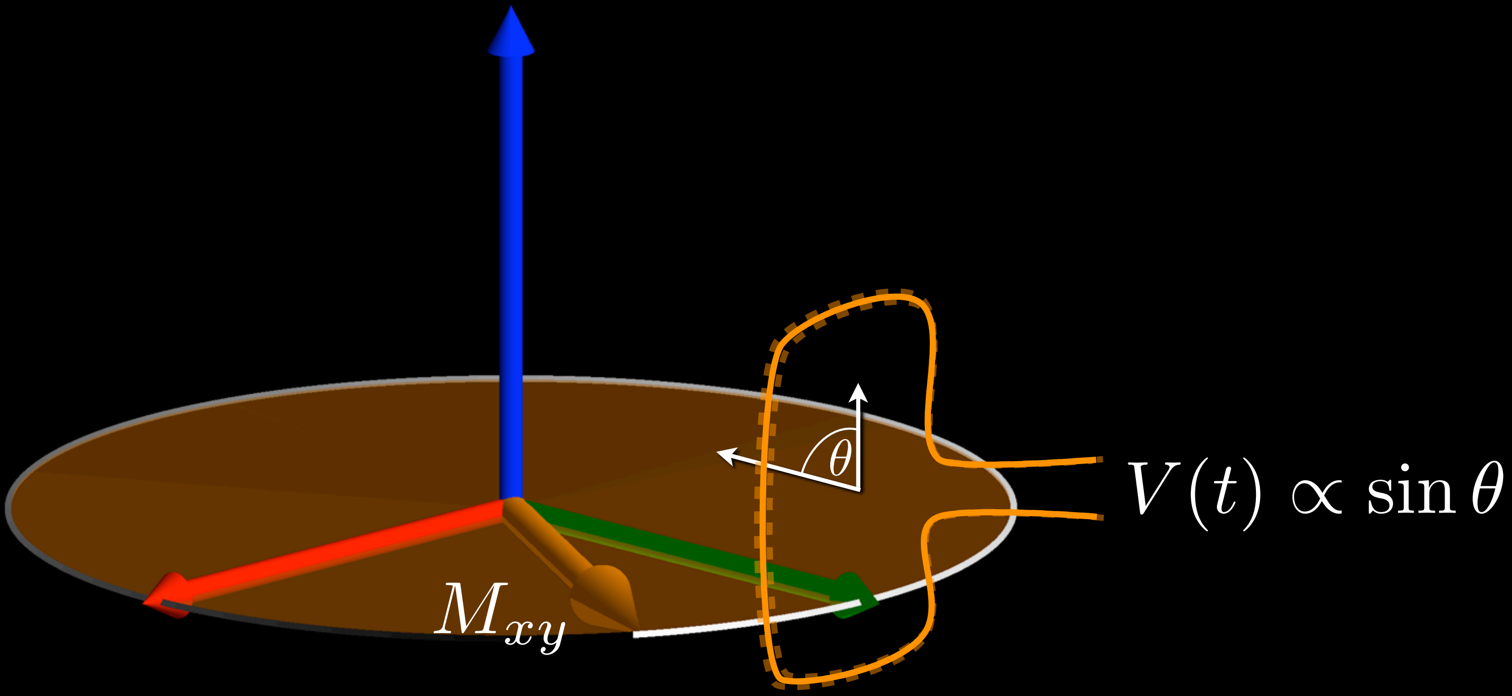
Coil

$$M_{xy}\left(\vec{r},t\right)$$

$$V(t)$$

NMR Signal Detection

- Coil only detects M_{xy}
- Coil does *not* detect M_z
- Coil must be properly oriented



How does RF alter $\vec{M}$?

$$\vec{B}_1\left(t\right)$$

Generating B_1-Fields

MRI Hardware

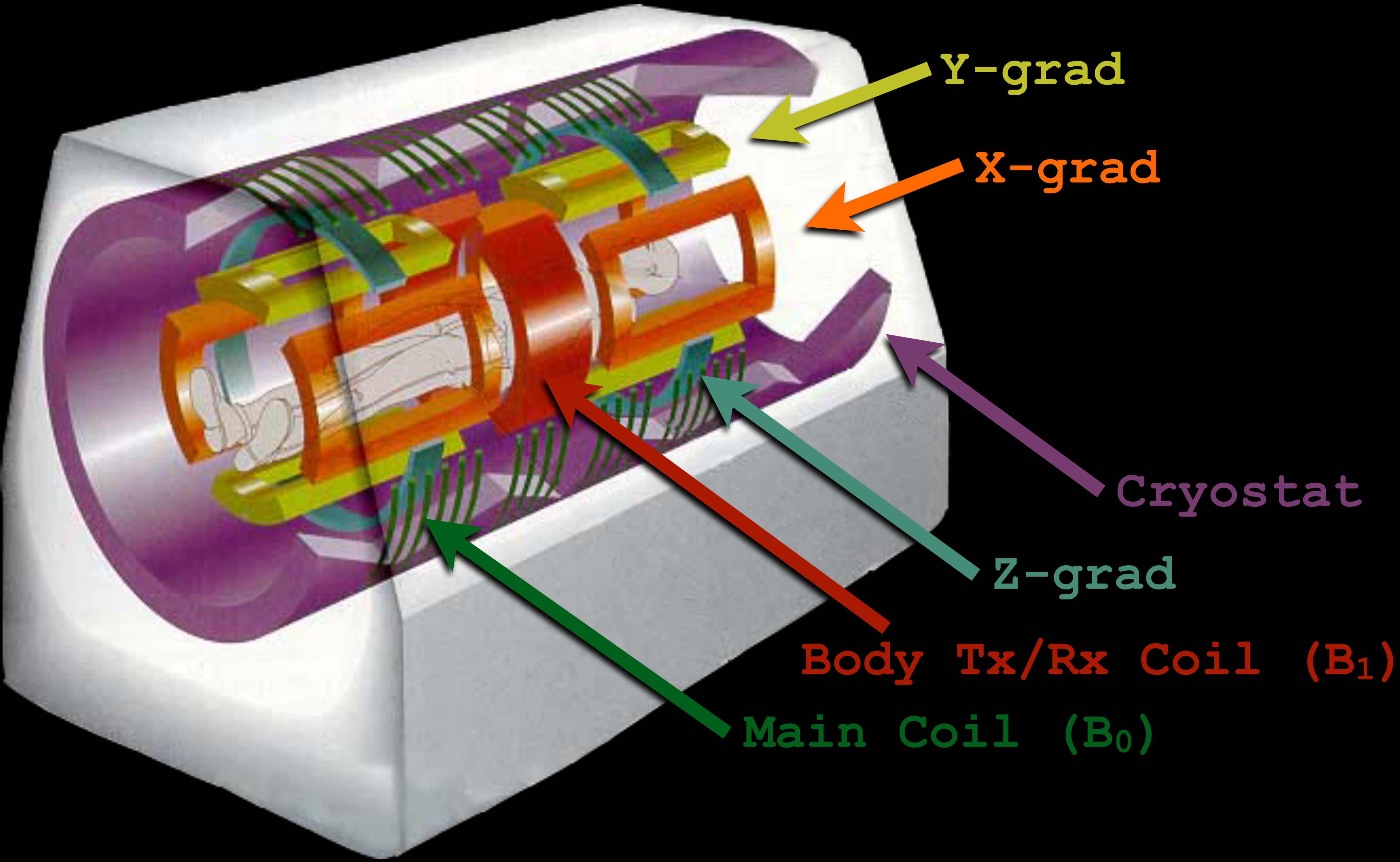
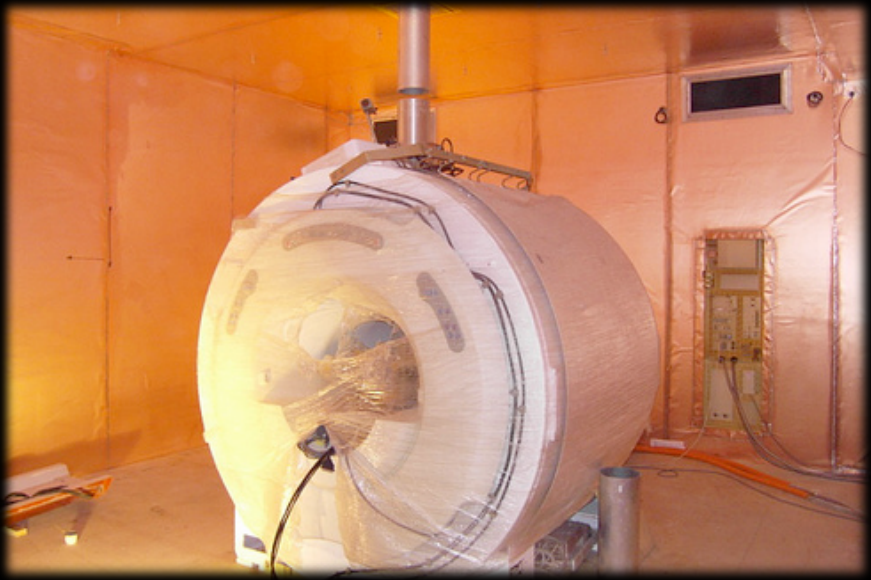


Image Adapted From: http://www.ee.duke.edu/~jshorey

RF Shielding

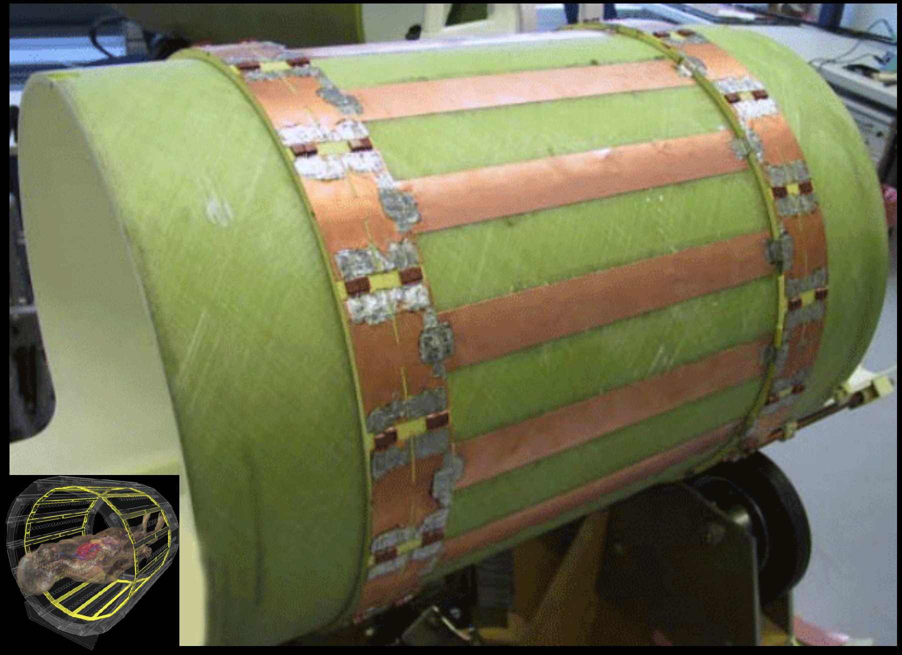
- RF fields are close to FM radio
  - ^{1}H @ 1.5T ⇒ 63.85 MHz
  - ^{1}H @ 3.0T ⇒ 127.71 MHz
  - KROQ ⇒ 106.7 MHz
- Need to shield local sources from interfering
- Copper room shielding required

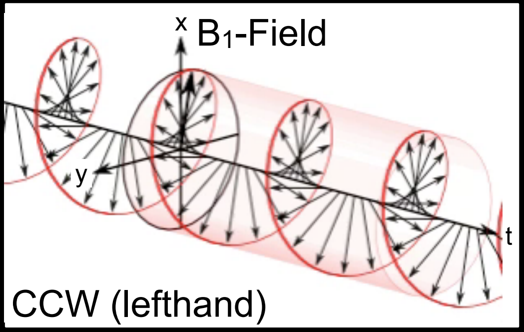


RF Birdcage Coil

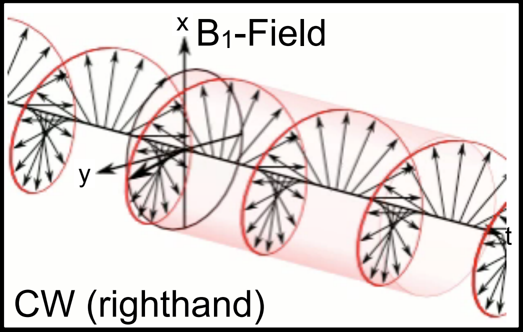
- **Most common design**
- **Highly efficient**
  - Nearly all of the fields produced contribute to imaging
- **Very uniform field**
  - Especially radially
  - Decays axially
  - Uniform sphere if L≈D
- **Generates a "quadrature" field**
  - Circular polarization

Body Tx/Rx Coil (B_1)





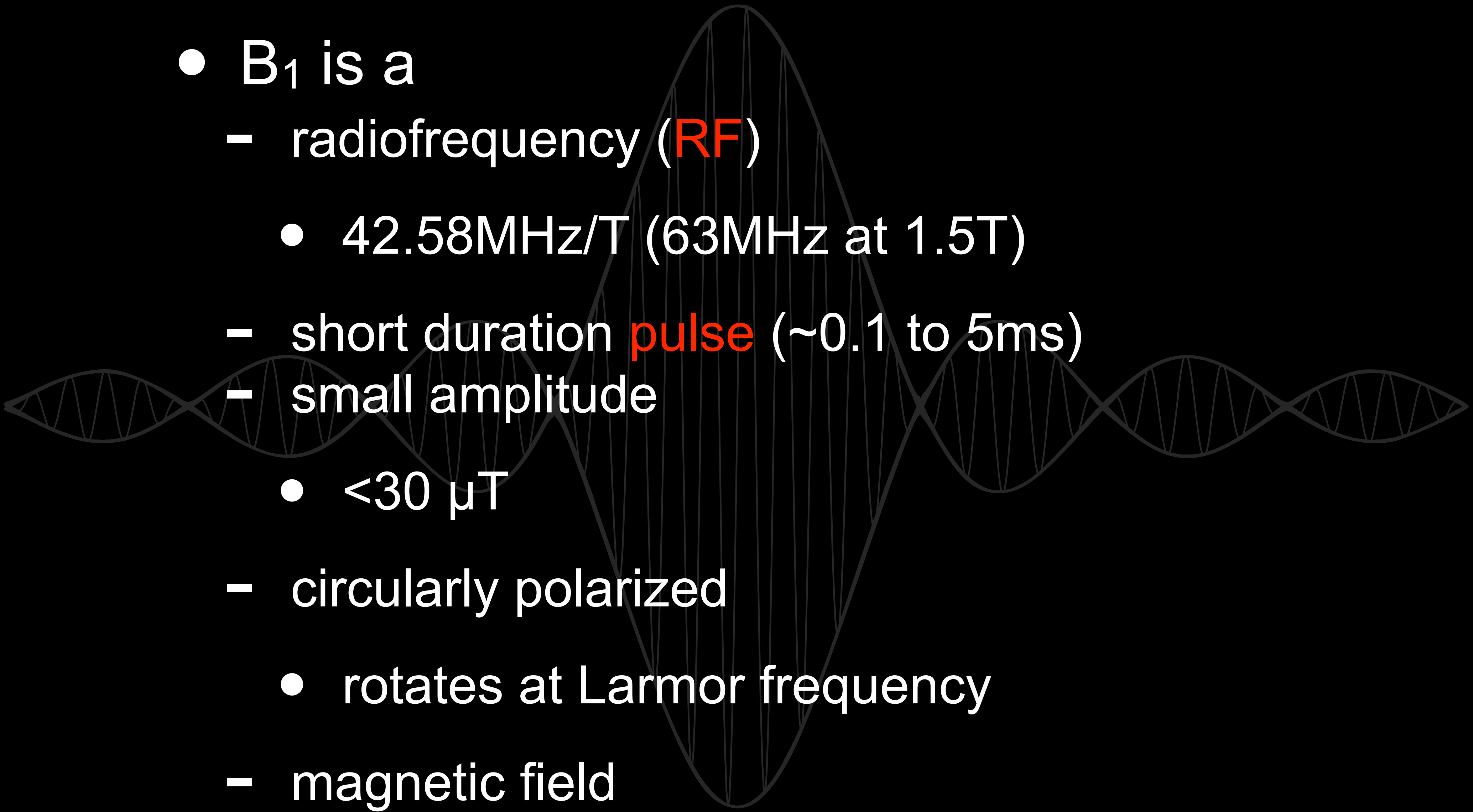
CCW (lefthand)



CW (righthand)

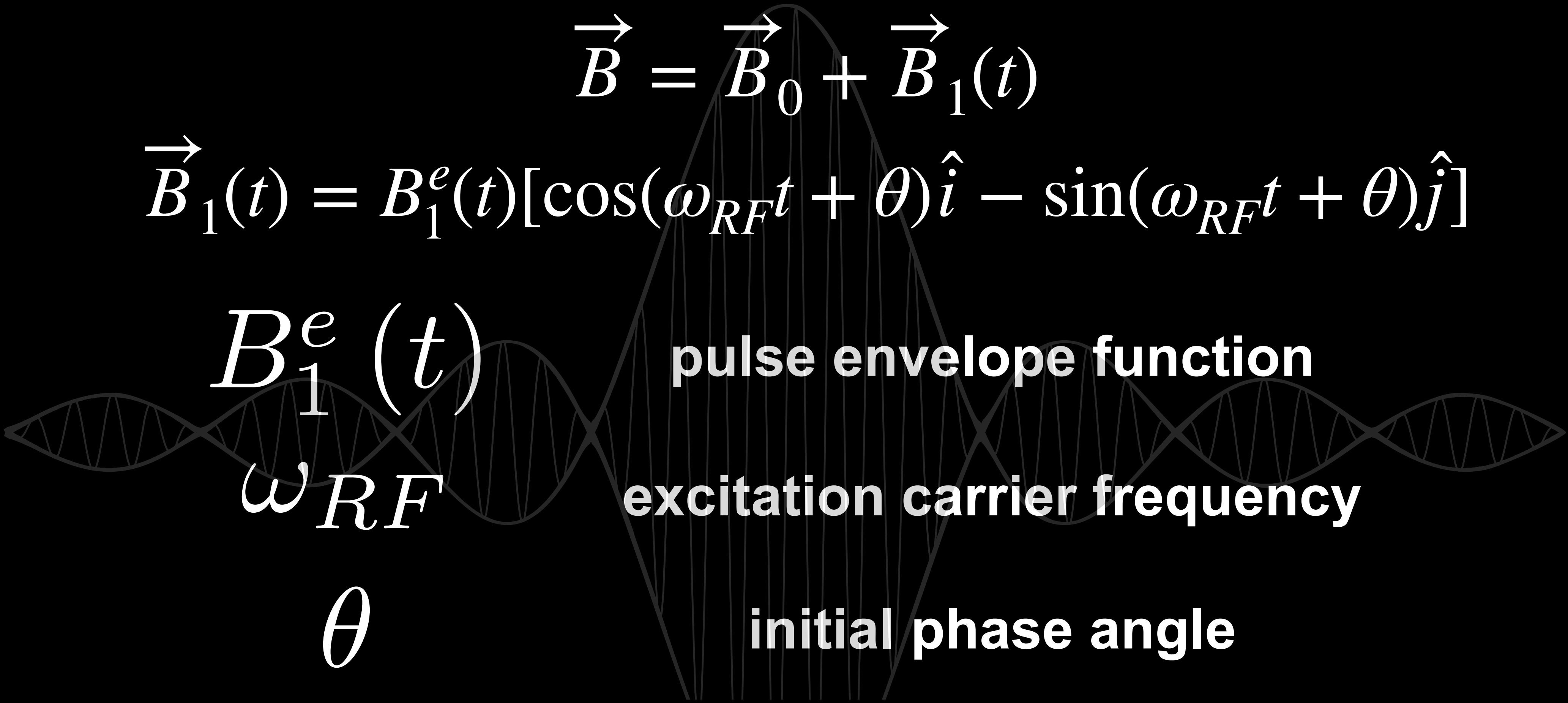
http://mri-q.com/birdcage-coil.html

B_1 Field - RF Pulse



- B_1 is a
  - radiofrequency (RF)
    - 42.58MHz/T (63MHz at 1.5T)
  - short duration pulse (~0.1 to 5ms)
  - small amplitude
    - <30 µT
  - circularly polarized
    - rotates at Larmor frequency
  - magnetic field
  - perpendicular to B_0

Basic RF Pulse



$$\overrightarrow{B} = \overrightarrow{B}_0 + \overrightarrow{B}_1(t)$$

$$\overrightarrow{B}_1(t) = B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

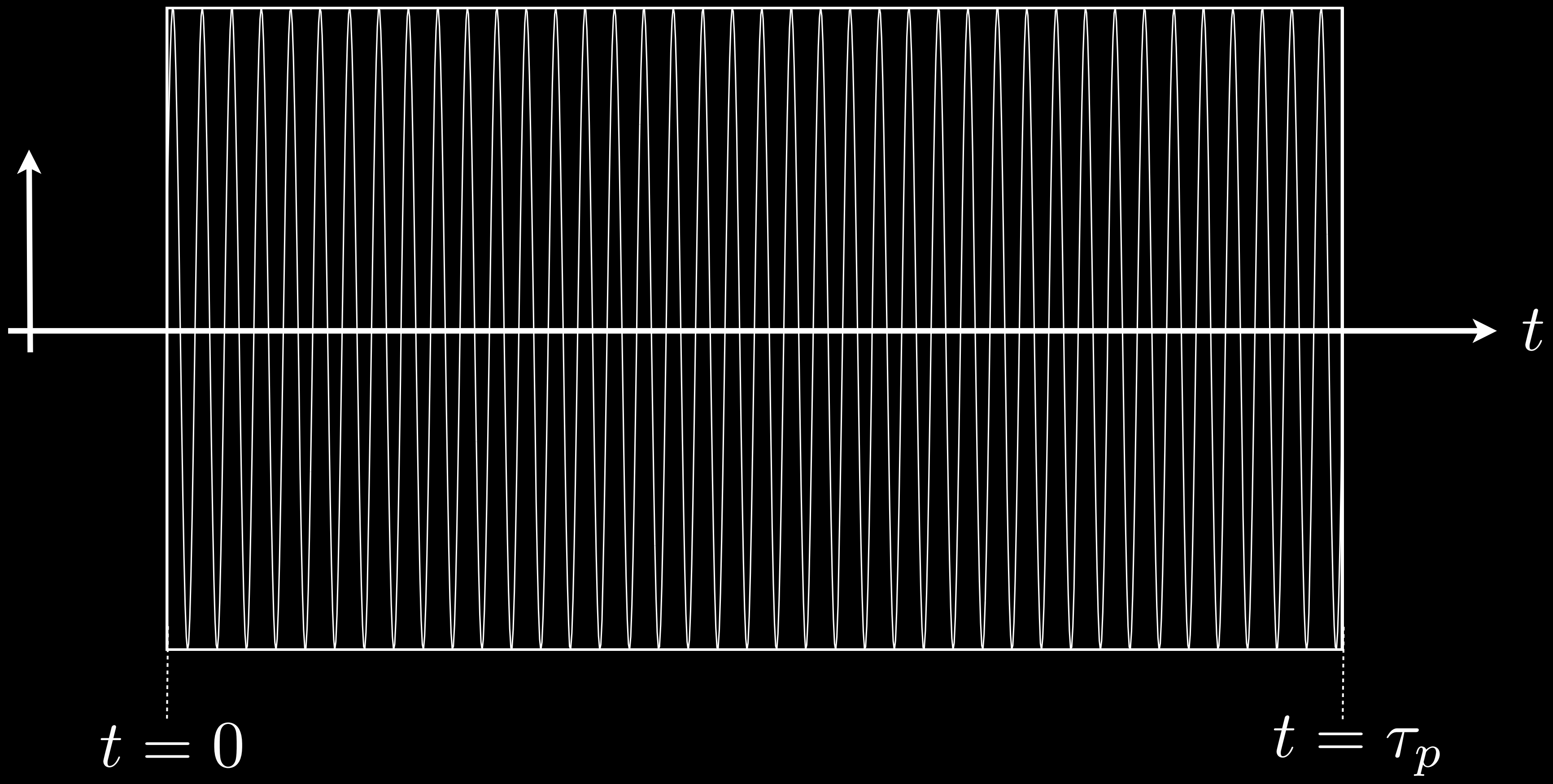
$B_1^e(t)$	**pulse envelope function**
ω_{RF}	**excitation carrier frequency**
θ	**initial phase angle**

B_1 is perpendicular to B_0.

$$\overrightarrow{B}_0 = B_0\hat{k}$$

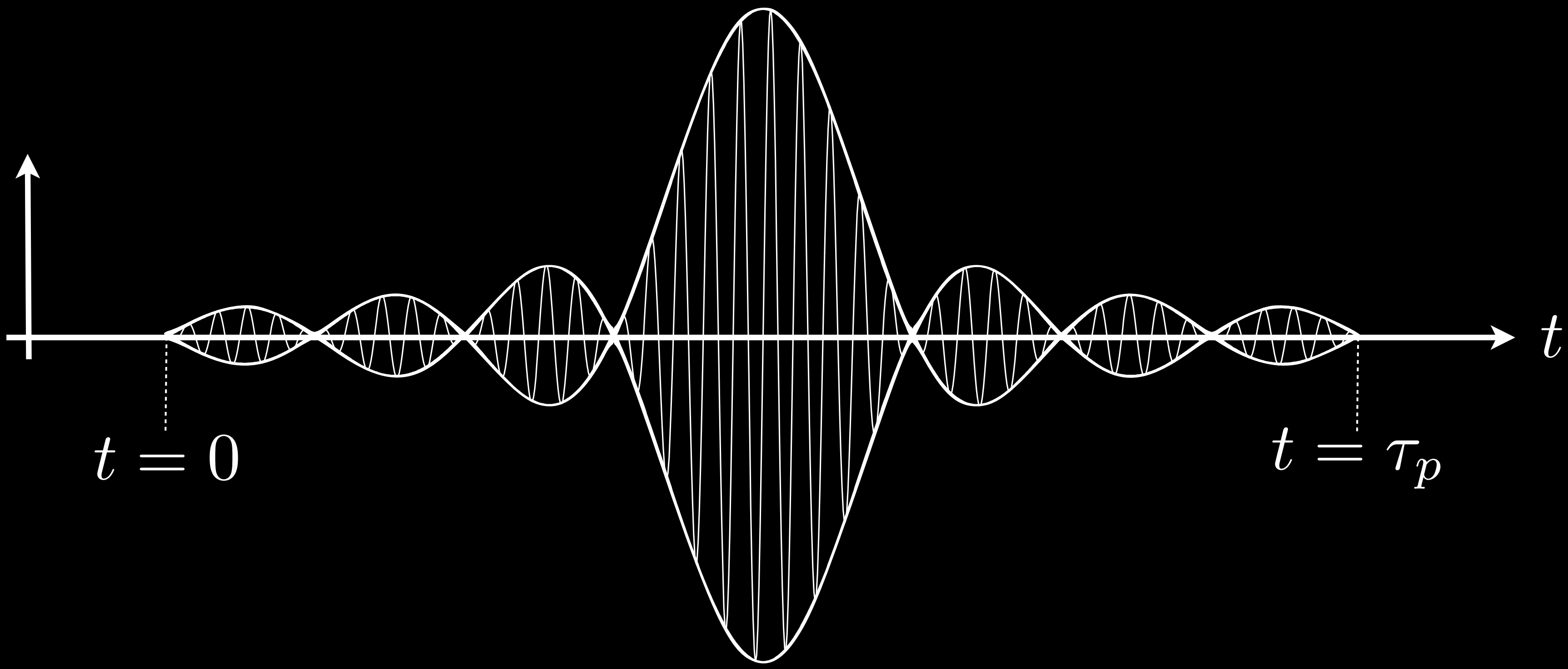
Rect Envelope Function

$$B_1^e(t) = B_1 \sqcap \left(\frac{t - \tau_p/2}{\tau_p} \right) = \begin{cases} B_1, & 0 \leq t \leq \tau_p \\ 0, & otherwise \end{cases}$$



Sinc Envelope Function

$$B_1^e(t) = \begin{cases} B_1 \text{sinc}\left[\pi f_\omega \left(t - \tau_p/2\right)\right], & 0 \le t \le \tau_p \\ 0, & otherwise \end{cases}$$



Resonance

“Establishment of a phase coherence among these ‘randomly’ precessing spins in a magnetized spin system is referred to as resonance.”

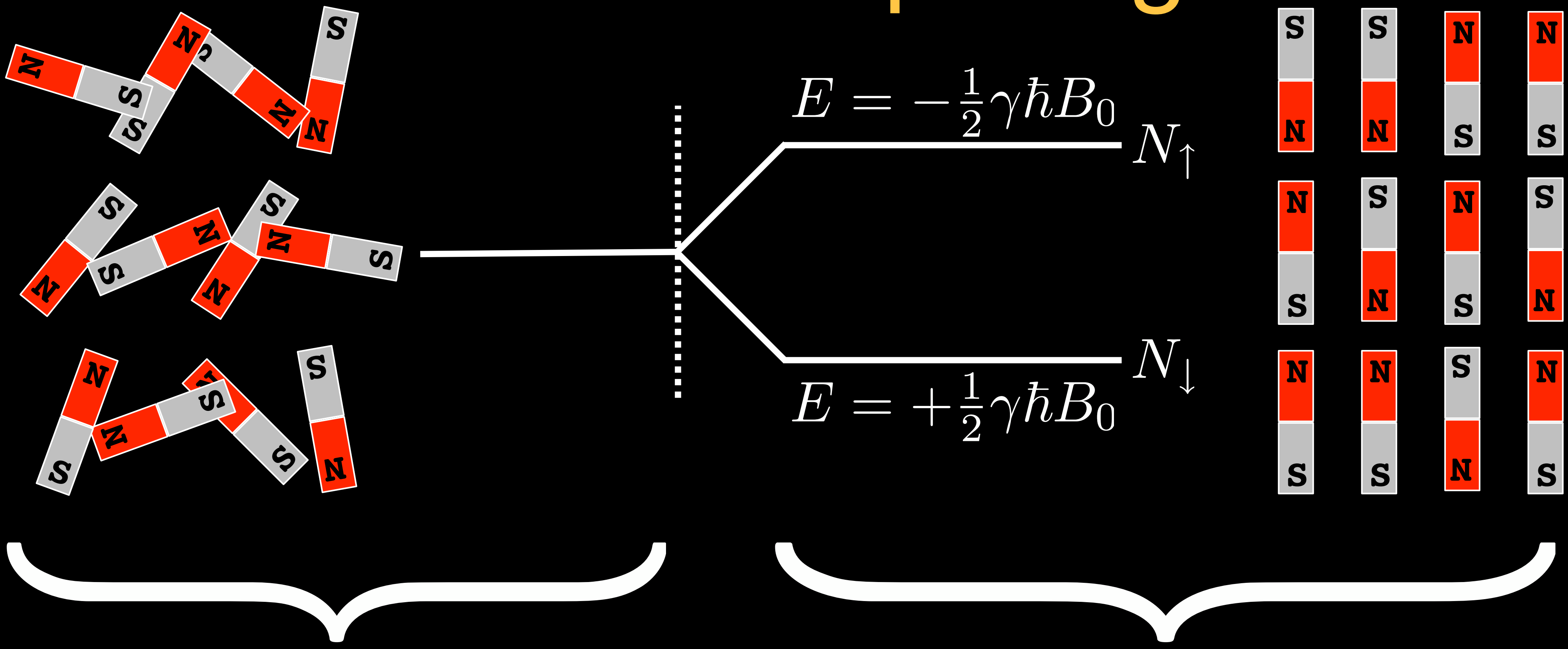
– Liang & Lauterbur p.69

Resonance

- $\overrightarrow{B}_1(t)$ provides external energy
  - RF magnetic field.

- Quantum Physics
  - Electromagnetic radiation of frequency ω_{RF} carries energy that induces a coherent transition of spins from $N_\uparrow$ to $N_\downarrow$.
- Classical Physics
  - $\overrightarrow{B}_1(t)$ rotates in the same manner as the precessing spins.
  - Coherently “pushes” on bulk magnetization.

Zeeman Splitting



$$E = -\frac{1}{2}\gamma\hbar B_0 \quad N_\uparrow$$

$$E = +\frac{1}{2}\gamma\hbar B_0 \quad N_\downarrow$$

B_0 is off

B_0 is on

$N_\uparrow$ = Spin-Up State, Low Energy

$N_\downarrow$ = Spin-Down State, High Energy

$$\frac{N_\uparrow - N_\downarrow}{N_{total}} \approx 4.5 \times 10^{-6}$$

Resonance Condition

$$\Delta E = E_{\downarrow} - E_{\uparrow} = \hbar\gamma B_0$$

Zeeman Splitting

$$E_{RF} = \hbar\omega_{RF}$$

Planck's Law

$$\hbar\gamma B_0 = \hbar\omega_{RF}$$

$$\omega_{RF} = \gamma B_0 = \omega_0$$

Resonance Condition

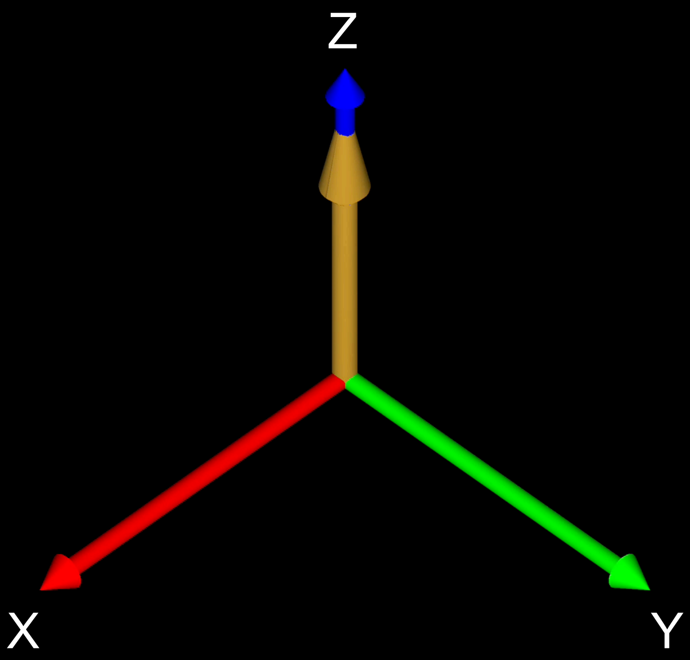
Resonance requires that the frequency of the RF energy (ω_{RF}) match the frequency of precession (ω_0).

Rotating Frame

Lab vs. Rotating Frame

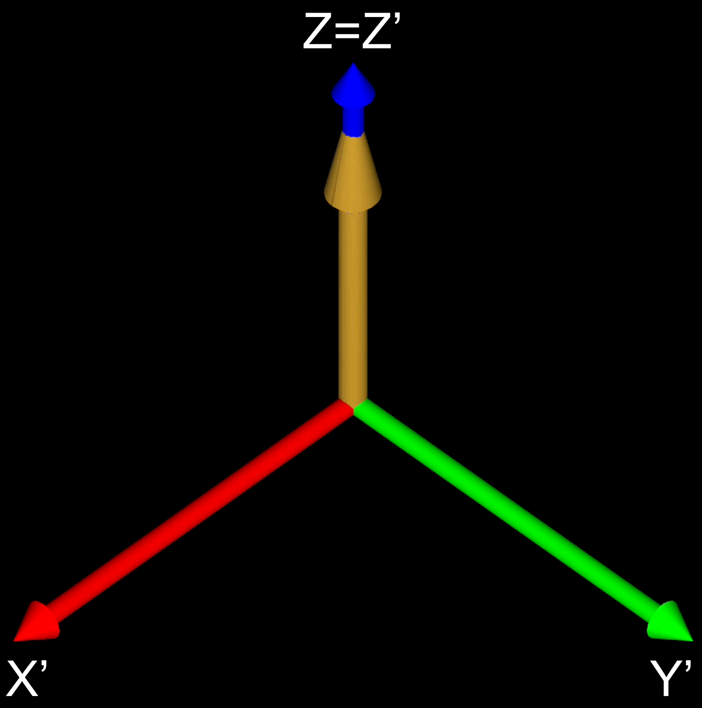
- **The rotating frame simplifies the mathematics and permits more intuitive understanding.**

90° RF (Laboratory Frame)



Spins Precess

90° RF (Rotating Frame)



Observer Precesses

Note: Both coordinate frames share the same z-axis.

Combined B$_0$ & B$_1$ Effects

$$\begin{aligned}\frac{d\vec{M}}{dt} &= \vec{M} \times \gamma\vec{B} \\ &= \vec{M} \times \gamma\left(\vec{B}_0 + \vec{B}_1\right)\end{aligned}$$

Relationship Between Lab and Rotating Frames

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\overrightarrow{M}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{M}_{rot}(t)$$

Bulk magnetization components in the rotating frame.

$$\overrightarrow{B}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{B}_{rot}(t)$$

Applied B-field components in the rotating frame.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Longrightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats). [Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats). [Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\overrightarrow{B}(t) = B_0\hat{k} + B_1^e(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

$$\overrightarrow{B}_{lab}(t) = \begin{pmatrix} B_1^e(t)\cos(\omega_{RF}t + \theta) \\ -B_1^e(t)\sin(\omega_{RF} + \theta) \\ B_0 \end{pmatrix} \qquad \overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t)\cos\theta \\ -B_1^e(t)\sin\theta \\ B_0 \end{pmatrix}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

Assume no RF phase ($\theta = 0$)

$$\overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

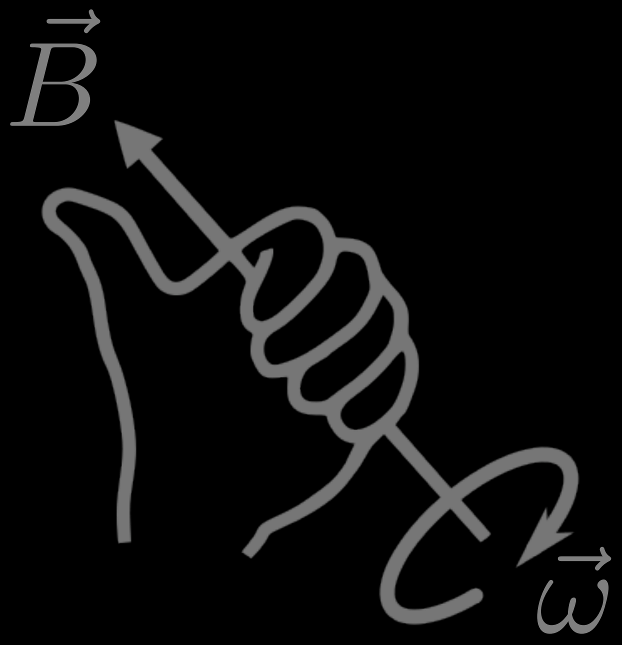
$$\overrightarrow{B}_{eff}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ \cancel{B_0 - \frac{\omega_{RF}}{\gamma}} \end{pmatrix}$$

To The Board...

Mathematics of Hard RF Pulses

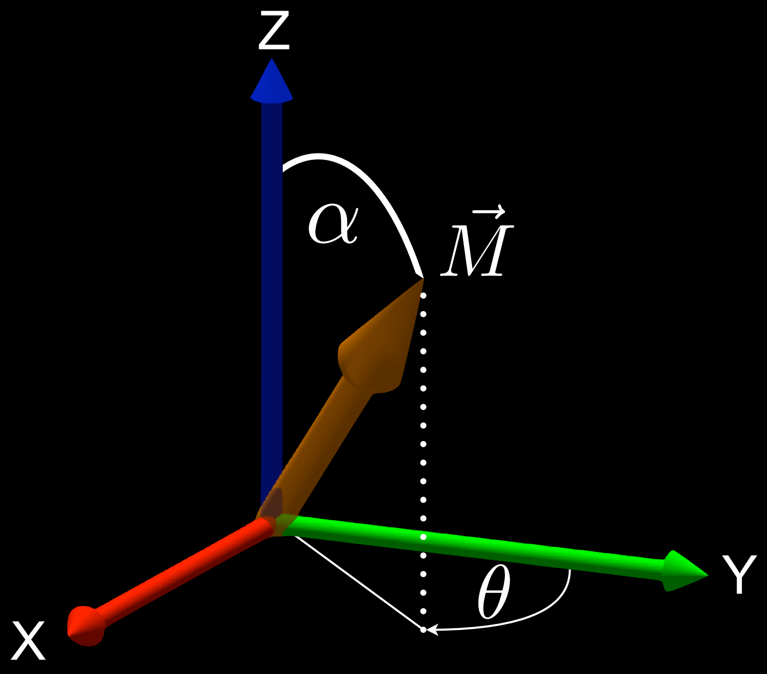
Rules for RF Pulses

- RF fields induce left-hand rotations
- Phase of 0° is about the x-axis
- Phase of 90° is about the y-axis



Flip Angle - α

- **"Amount of rotation of the bulk magnetization vector produced by an RF pulse, with respect to the direction of the static magnetic field."**
  - Liang & Lauterbur, p. 374



$$\omega_1 = \gamma B_1$$

B-fields induce precession!

David Geffen School of Medicine

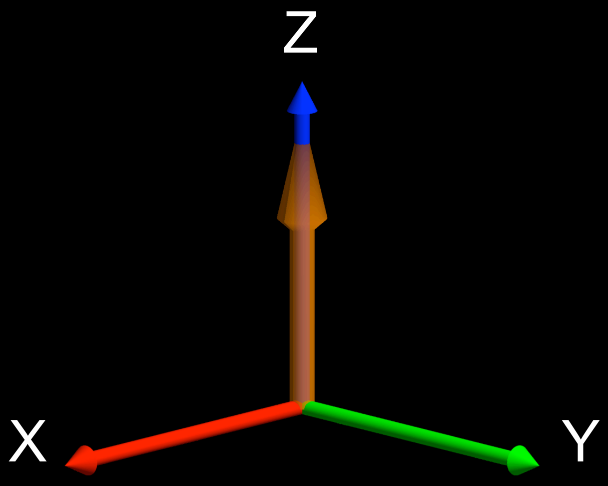
UCLA Radiology

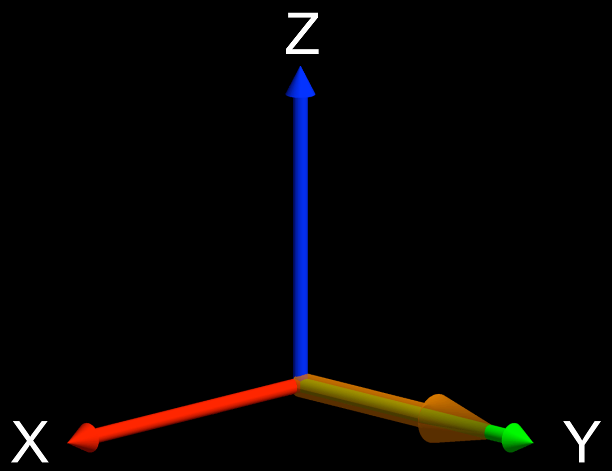
Rules for RF Pulses

$$\mathrm{R}^{\alpha}_{\theta}$$

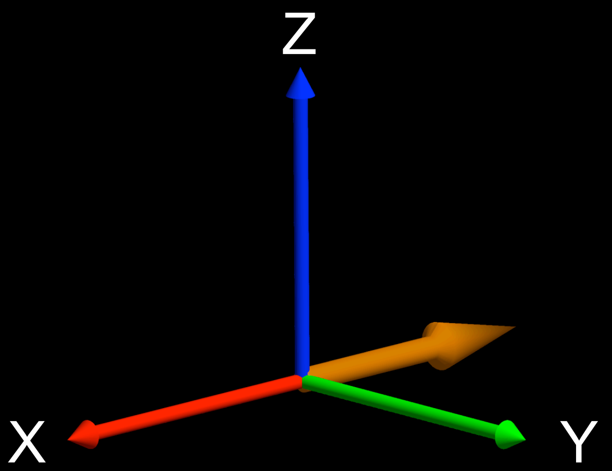
→ Flip Angle

→ Phase



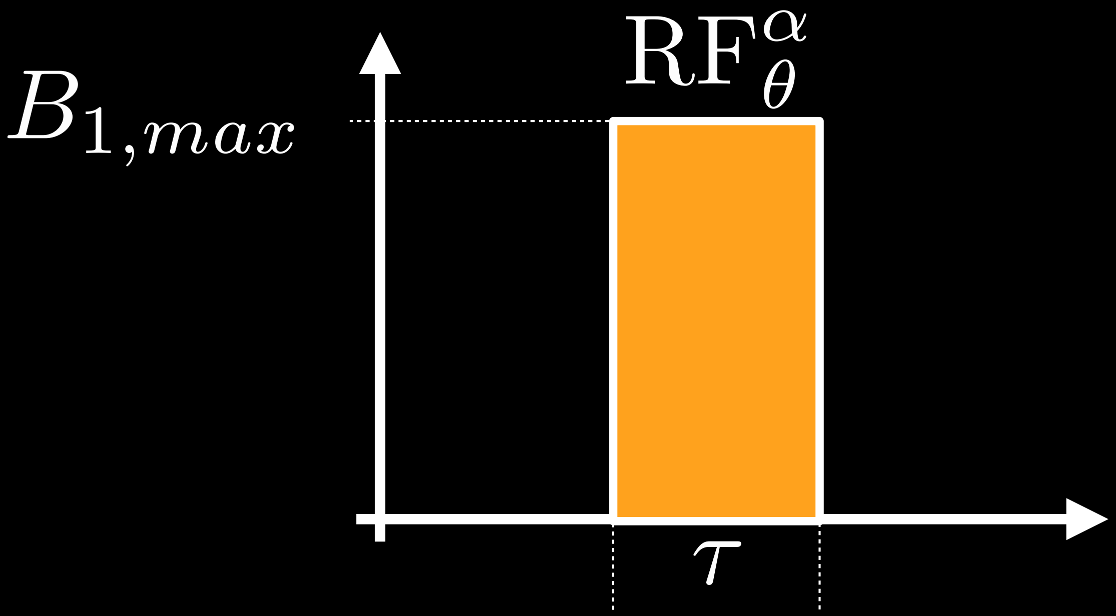


$$\mathrm{R}^{90^\circ}_{0^\circ}$$



$$\mathrm{R}^{90^\circ}_{90^\circ}$$

How to determine α?



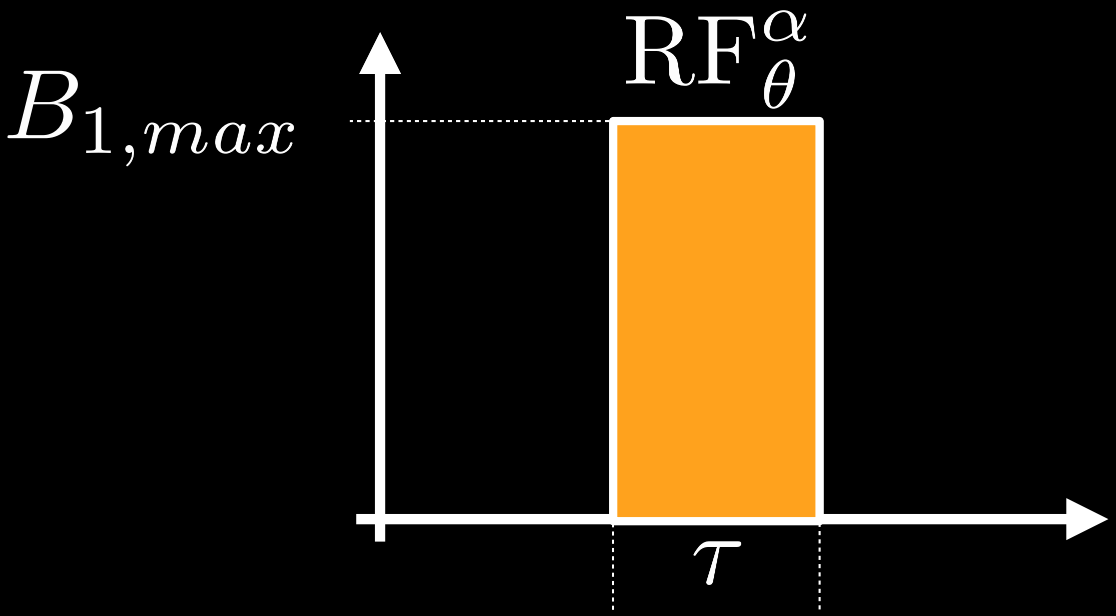
$$\alpha = \gamma \int_0^{\tau_p} B_1^e\left(t\right) dt$$

Rules: 1) Specify α

2) Use $B_{1,max}$ if we can

3) Shortest duration pulse

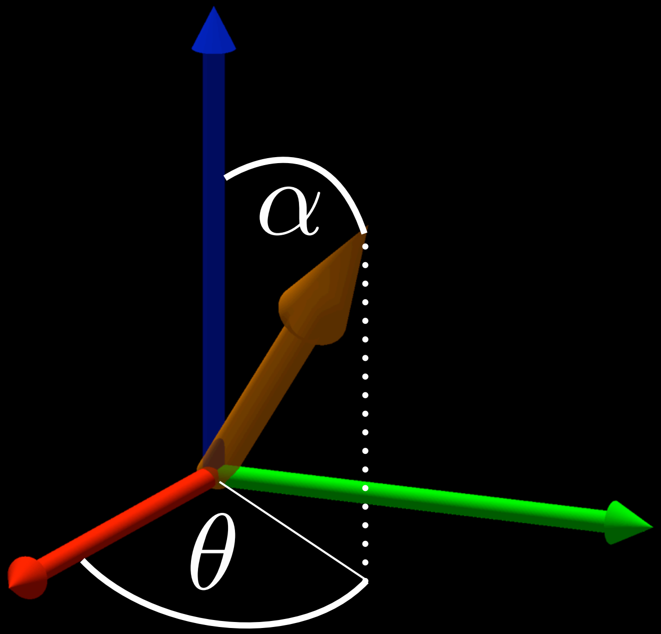
How to determine α?



$$\alpha = \gamma \int_{0}^{\tau_p} B_1^e\left(t\right) dt$$

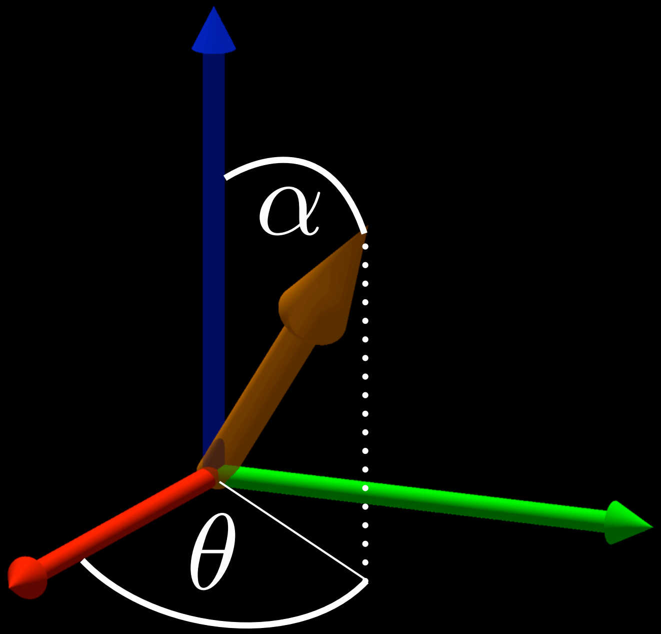
$$\tau = \frac{\alpha}{\gamma B_{1,max}} = \frac{\pi/2}{2\pi \cdot 42.57 Hz/\mu T \cdot 60\mu T} = 0.098ms$$

Change of Basis (θ)



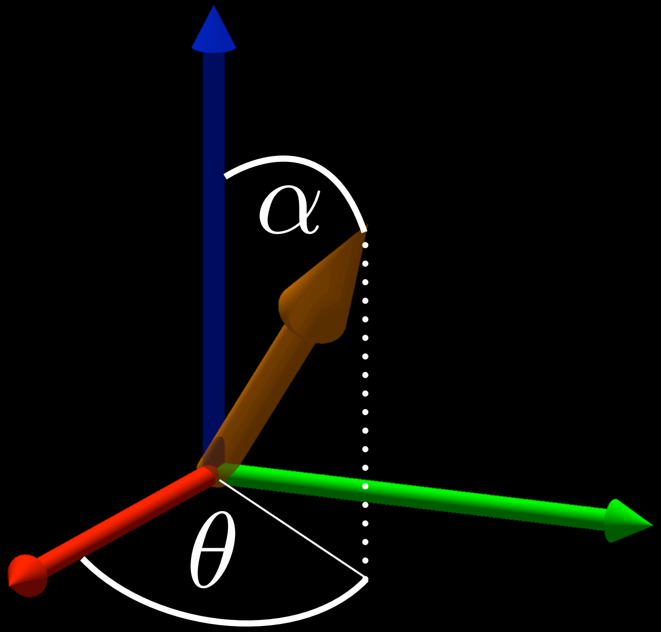
$$\mathbf{R}_Z(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation by Alpha



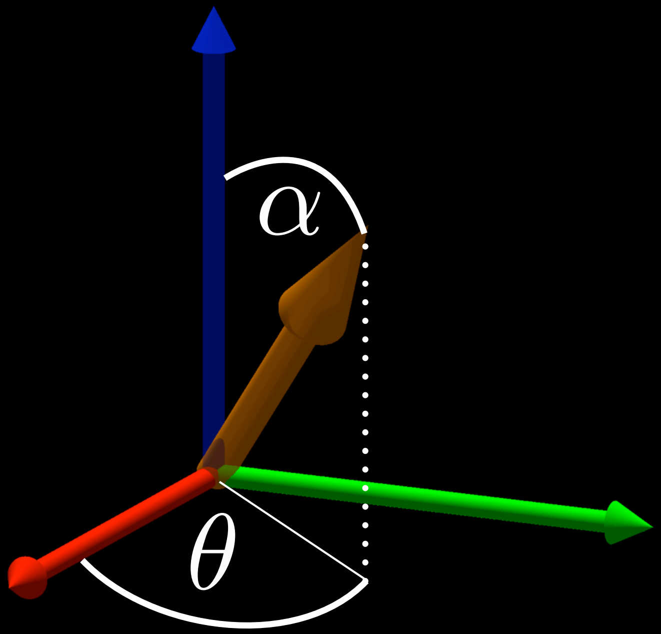
$$\mathbf{R}_X\left(\alpha\right)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha & \sin\alpha\\ 0 & -\sin\alpha & \cos\alpha\end{bmatrix}$$

Change of Basis (-θ)



$$\mathbf{R}_Z\left(-\theta\right) = \begin{bmatrix} \cos\left(-\theta\right) & \sin\left(-\theta\right) & 0 \\ -\sin\left(-\theta\right) & \cos\left(-\theta\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

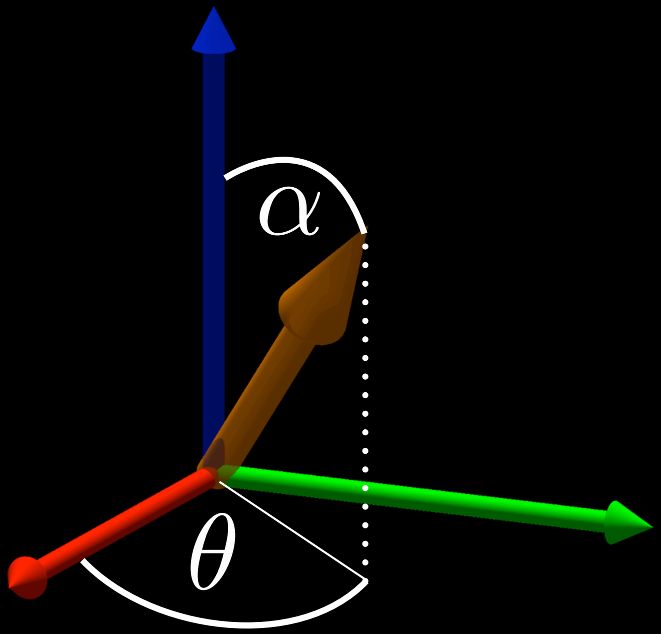
RF Pulse Operator



$$\mathbf{R}_\theta^\alpha = \mathbf{R}_Z\left(-\theta\right)\mathbf{R}_X\left(\alpha\right)\mathbf{R}_Z\left(\theta\right)$$

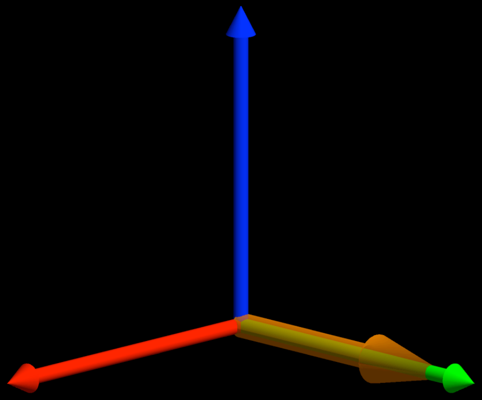
$$= \begin{bmatrix} \mathrm{c}^2\theta + \mathrm{s}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & -\mathrm{s}\theta\mathrm{s}\alpha \\ \mathrm{c}\theta\mathrm{s}\theta - \mathrm{c}\theta\mathrm{s}\theta\mathrm{c}\alpha & \mathrm{s}^2\theta + \mathrm{c}^2\theta\mathrm{c}\alpha & \mathrm{c}\theta\mathrm{s}\alpha \\ \mathrm{s}\theta\mathrm{s}\alpha & -\mathrm{c}\theta\mathrm{s}\alpha & \mathrm{c}\alpha \end{bmatrix}$$

RF Pulse Operator

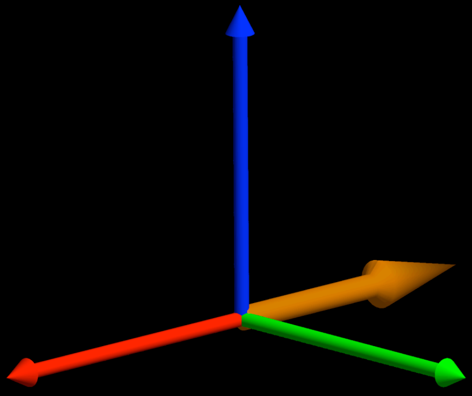


$$\vec{\mathrm{M}}\left(0_{+}\right)=\mathrm{RF}_{\theta}^{\alpha}\vec{\mathrm{M}}\left(0_{-}\right)$$

Hard RF Pulses



$\mathrm{R}_{0^\circ}^{90^\circ}$



$\mathrm{R}_{90^\circ}^{90^\circ}$

$$\mathbf{R}_{0^\circ}^{90^\circ} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{R}_{90^\circ}^{90^\circ} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Questions?

- Related reading materials
  - Liang/Lauterbur - Chap 3.2

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Types of RF Pulses

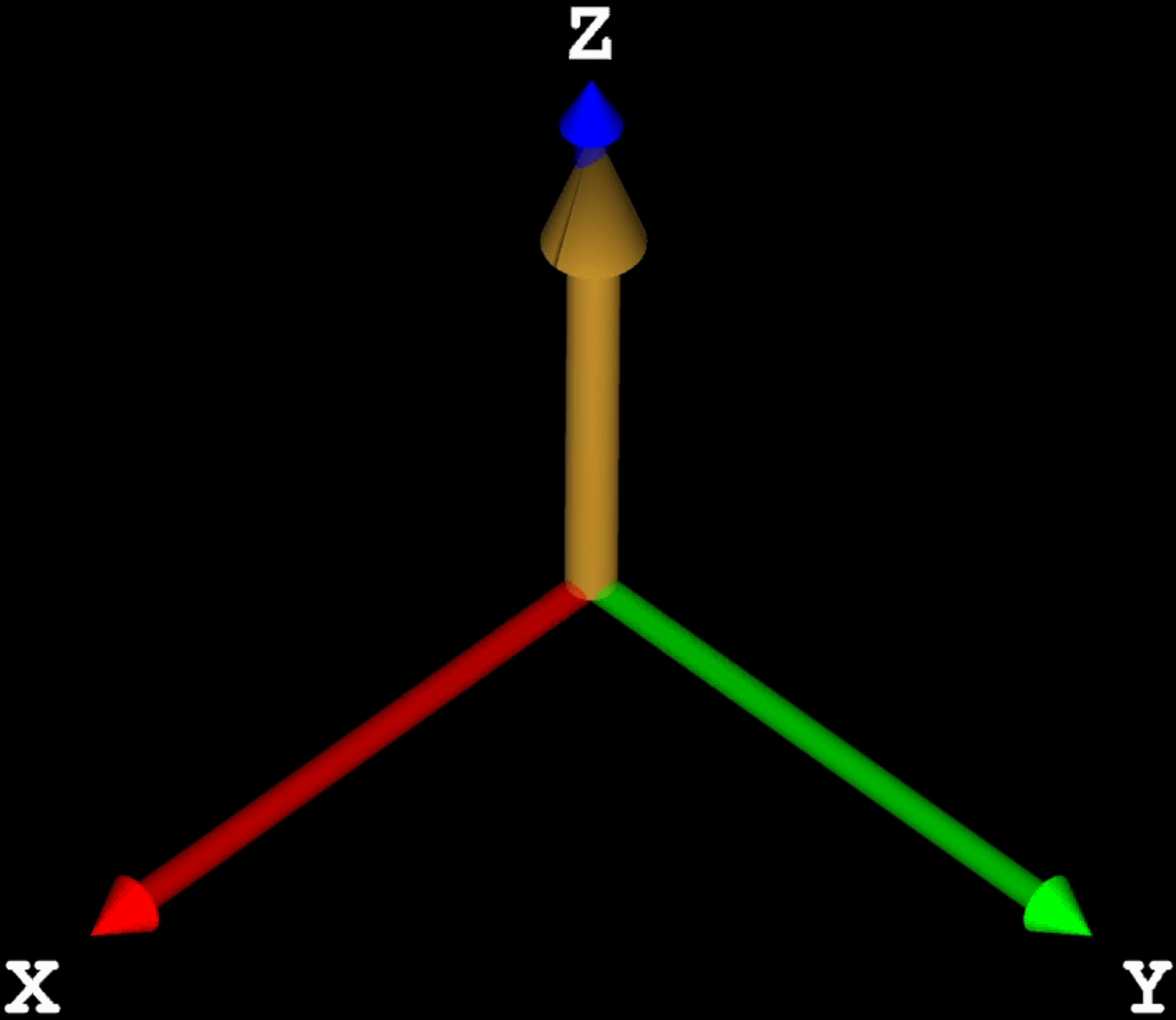
- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses

Excitation Pulses

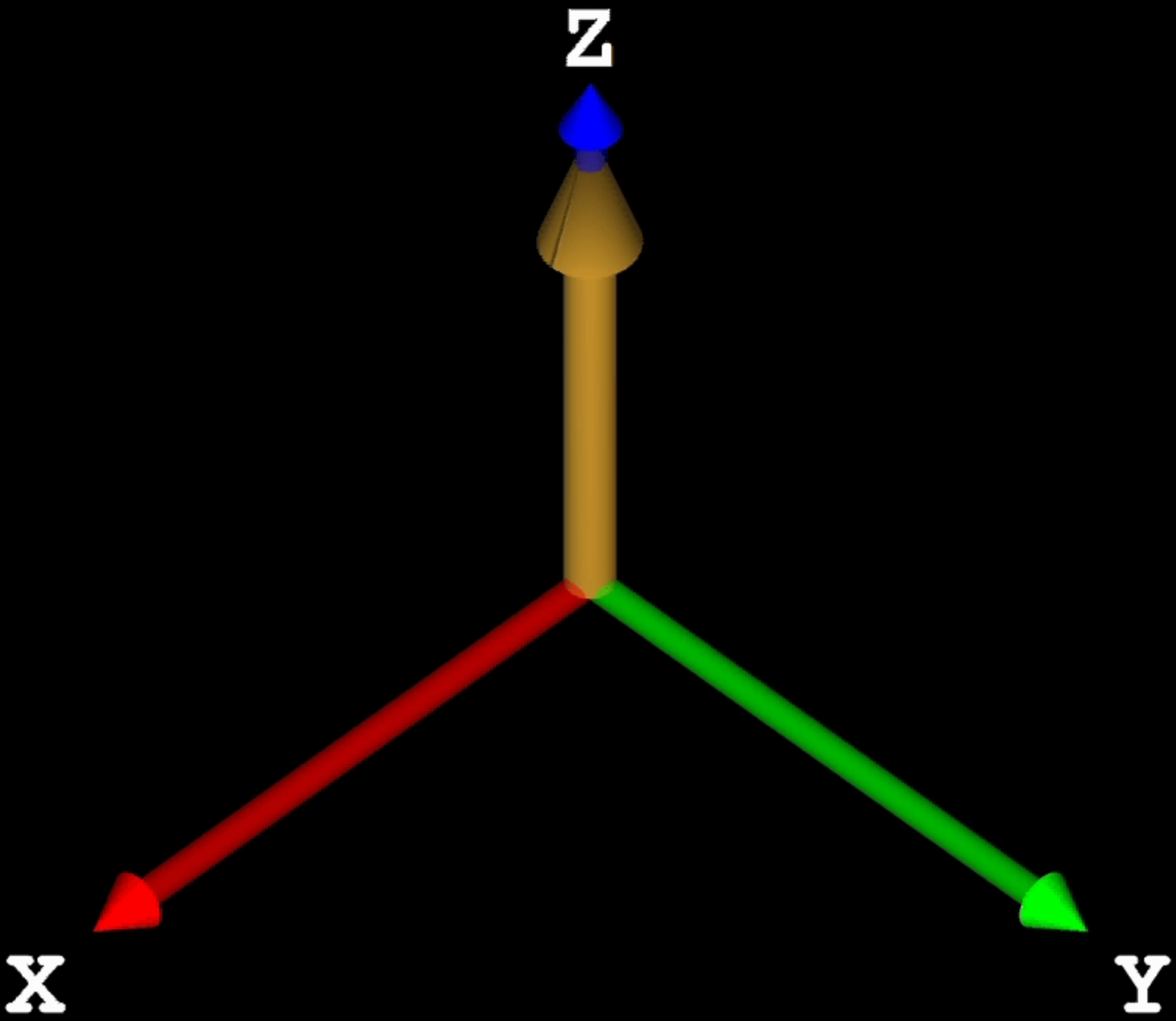
Excitation Pulses

- Tip M_z into the transverse plane
- Typically 200μs to 5ms
- Non-uniform across slice thickness
  - Imperfect slice profile
- Non-uniform within slice
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV

90° Excitation Pulse



Small Flip Angle Excitation



Excitation Pulses - Applications

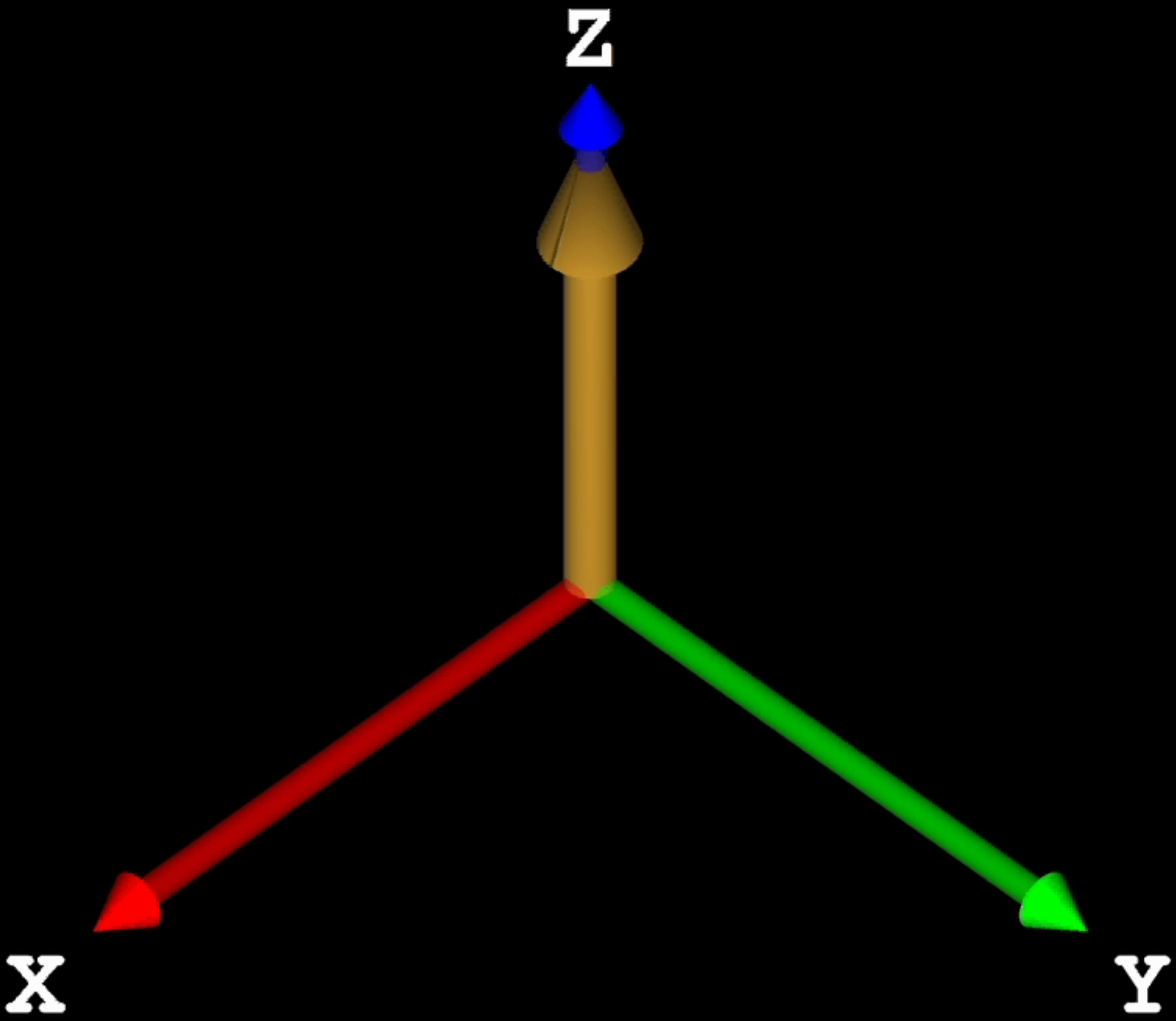
- 90° RF Pulse
  - Spin Echo
  - Saturation Recovery
- Small Flip Angle (<~20°)
  - FLASH (Fast Low Angle Shot)
    - AKA SPGR
- Moderate Flip Angle (30°-90°)
  - TrueFISP

Inversion Pulses

Inversion Pulses

- Typically, 180° RF Pulse
  - non-180° that still results in $-M_Z$
- Invert M_Z to $-M_Z$
  - Ideally produces no M_{XY}
- Hard Pulse
  - Constant RF amplitude
  - Typically non-selective
- Soft (Amplitude Modulated) Pulse

Inversion Pulses



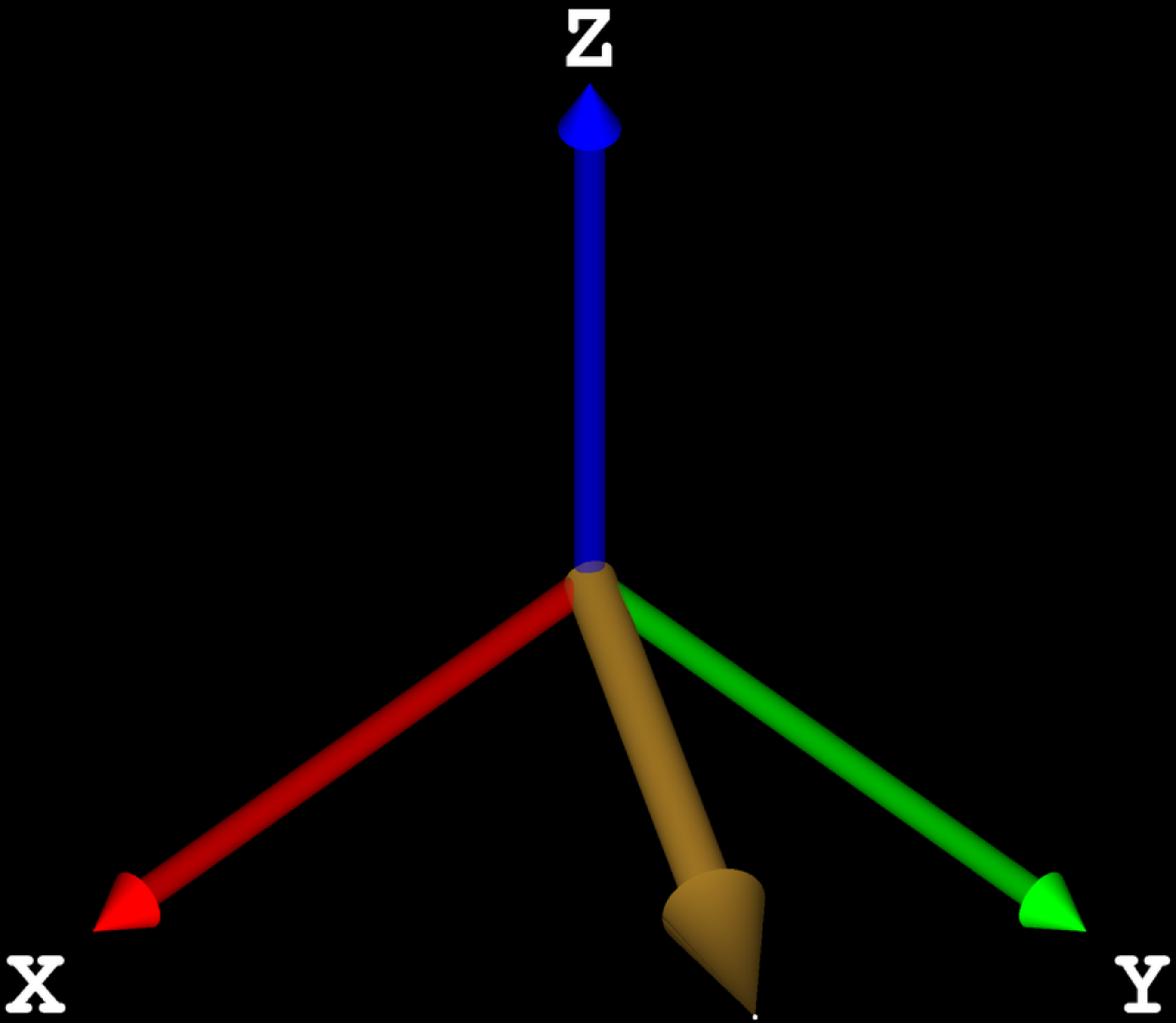
Inversion Pulse - Applications

- T1 species nulling/attenuation
  - STIR (Short Tau Inversion Recovery)
    - Suppress specific tissue-T1
  - SPECIAL (Spectral Inversion at Lipids)
    - Suppress lipid signals (short T1)
  - FLAIR (Fluid Attenuated Inversion Recovery)

Refocusing Pulses

- Typically, 180° RF Pulse
  - Provides optimally refocused M_{XY}
  - Largest spin echo signal
- non-180°
  - Partial refocusing
  - Lower SAR
  - Multiple non-180° produce stimulated echoes

Refocusing Pulses



Refocusing Pulses - Applications

- Spin Echo imaging
- RARE
  - Rapid Acquisition with Relaxation Enhancement
  - RF Excitation followed by 180° train
  - Reduce acquisition time by N-echoes
  - Common for T2-weighted imaging
  - AKA Fast Spin Echo
- Spin-Echo EPI