Spatial Localization II

M219 - Principles and Applications of MRI

Kyung Sung, Ph.D.

2/2/2022

Course Overview

- Course website
  - https://mrrl.ucla.edu/pages/m219
- Course schedule
  - https://mrrl.ucla.edu/pages/m219_2022
- Assignments
  - Homework #2 due on 2/14 by 5pm

Course Overview

- Office Hours
  - TA (Ran Yan) - Tuesday 4-5pm
    https://uclahs.zoom.us/j/96870184581?pwd=VkczL0lyRkxsQ3FHcnIxQ1M2U3hPdz09

    Password: 900645

  - Instructor (Kyung Sung) - Friday 2-3pm
    https://uclahs.zoom.us/j/94058312815?pwd=TkI3ajhkamdGTnhqOVNnbk5RMnJGQT09

    Password: 888767

Summary for Last Time...

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

Non-selective vs. Selective Excitation

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \boxed{B_0 - \frac{\omega_{RF}}{\gamma}} \end{pmatrix} \quad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \boxed{B_0 - \frac{\omega_{RF}}{\gamma} + G_z z} \end{pmatrix}$$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \Delta\omega & 0 \\ -\Delta\omega & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

Summary for Last Time...

Assuming carrier frequency = resonance frequency

w = w0

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,\Big|_{f=-(\gamma/2\pi)G_z z}$$

Time Bandwidth Product

- Time bandwidth (TBW) product:
  - Pulse duration X Pulse bandwidth
  - Unitless
  - # of zero crossings
- Some numbers:
  - TBW = 4, RF = 1ms, RF bandwidth?
  - TBW = 16, RF = 1ms, RF bandwidth?

MATLAB Demo

```
%% Design of Windowed Sinc RF Pulses
tbw = 4;
samples = 512;
rf = wsinc(tbw, samples);
```

```
%% Plot RF Amplitude
flip_angle = pi/2;
rf = flip_angle*rf;

pulseduration = 1;       % in msec
dt  = pulseduration/samples;
rfs = rf/(gamma*dt);     % Scaled to Gauss

bw = tbw/pulseduration; % in kHz
gmax = bw/gamma_2pi;

b1     = [rfs zeros(1,samples/2)];                          % in Gauss
g      = [ones(1,samples) -ones(1,samples/2)]*gmax;  % in G/cm
t_all = (1:length(g))*dt;  % in msec
```

MATLAB Demo

```
%% Simulate Slice Profile using Bloch Simulation
x  = (-2:.01:2);                 % in cm
f  = 0;                          % in Hz
dt = pulseduration/samples/1e3;
t  = (1:length(b1))*dt;  % in usec

% Bloch Simulation
[mx,my,mz] = bloch(b1(:),g(:),t(:),1,.2,f(:),x(:),0);

% Transverse Magnetization
mxy_bloch = mx+1i*my;
```

```
%% Simulate Slice Profile using Small Tip Approximation
samples_st = 4096;
f_st = linspace(-0.5/dt,0.5/dt,samples_st)/1e3;
x_st = -f_st/(gamma_2pi*gmax);

rfs_zp = zeros(1,samples_st);
rfs_zp(1:samples) = rfs;

mxy_st = fftshift(fftn(fftshift(rfs_zp)))/30;
```

Topics for Today

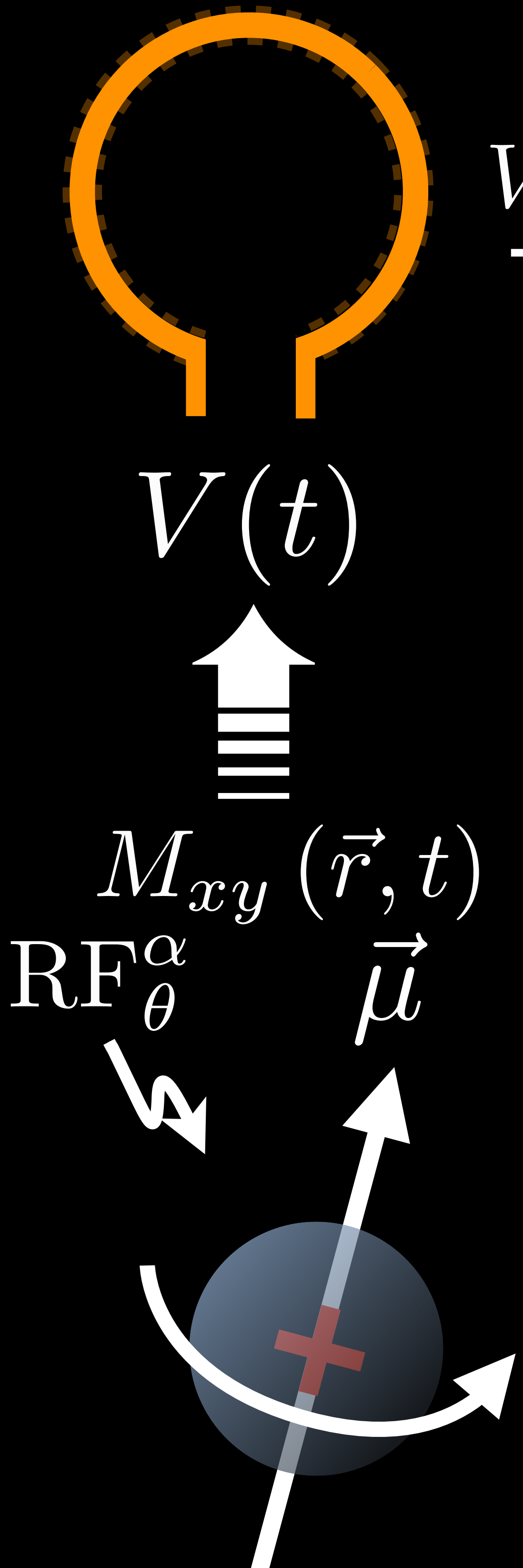
Frequency & Phase Encoding
-1D Imaging
-2D Imaging
(Cartesian Sampling)
-3D Imaging

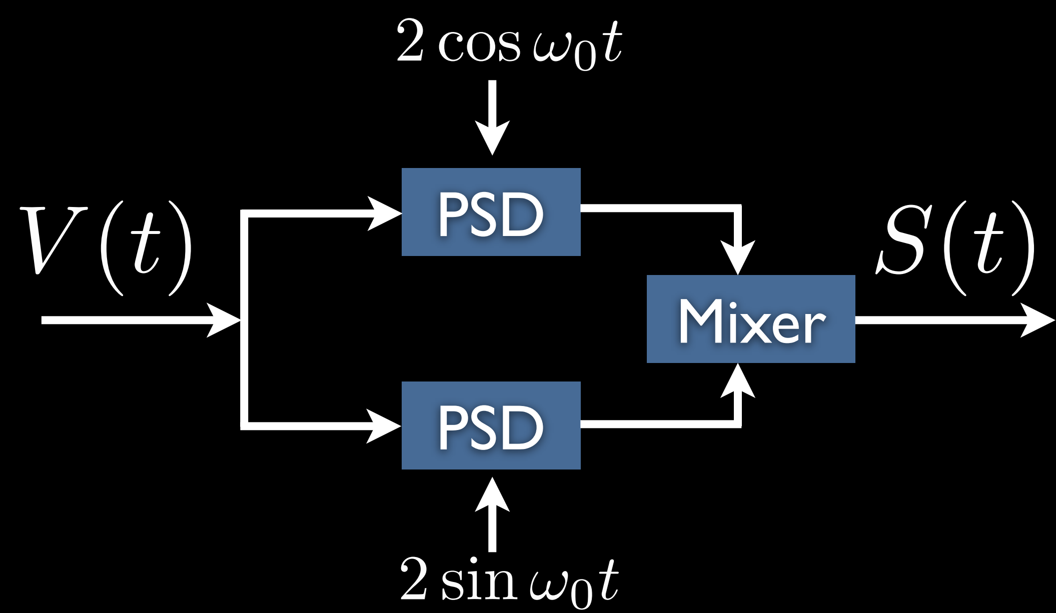
Sampling Considerations
- Field of View
- Spatial Resolution

Motion Artifacts

Phase & Frequency Encoding

So far, we have learned...





$$s(t) = \int_{object} M_{xy}(\vec{r}, 0) \cdot e^{-i\underline{\Delta\omega(\vec{r})}t}\, \mathrm{d}\vec{r}$$

How to form an image?

3 Types of Magnetic Fields

B_0 - Large static field

e.g., 1.5 Tesla or 3 Tesla

B_1 - Radiofrequency field

e.g., 0.16 G

$G_{x,y,z}$ - Gradient fields

e.g., 4 G/cm

Selective Excitation

Frequency and Phase Encoding

3 Types of Magnetic Fields

B_0 - Large static field

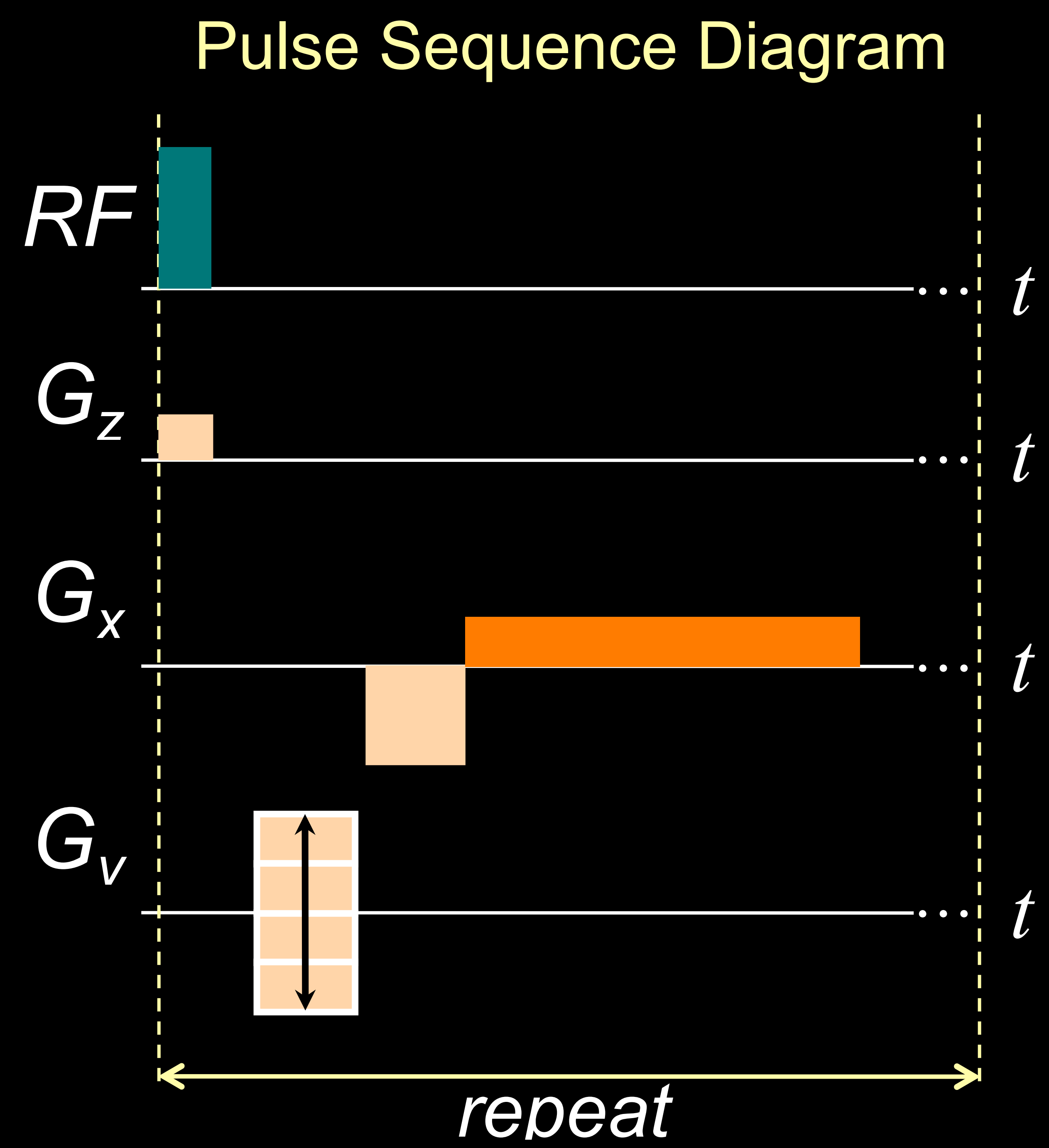
e.g., 1.5 Tesla or 3 Tesla

B_1 - Radiofrequency field

e.g., 0.16 G

$G_{x,y,z}$ - Gradient fields

e.g., 4 G/cm



Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

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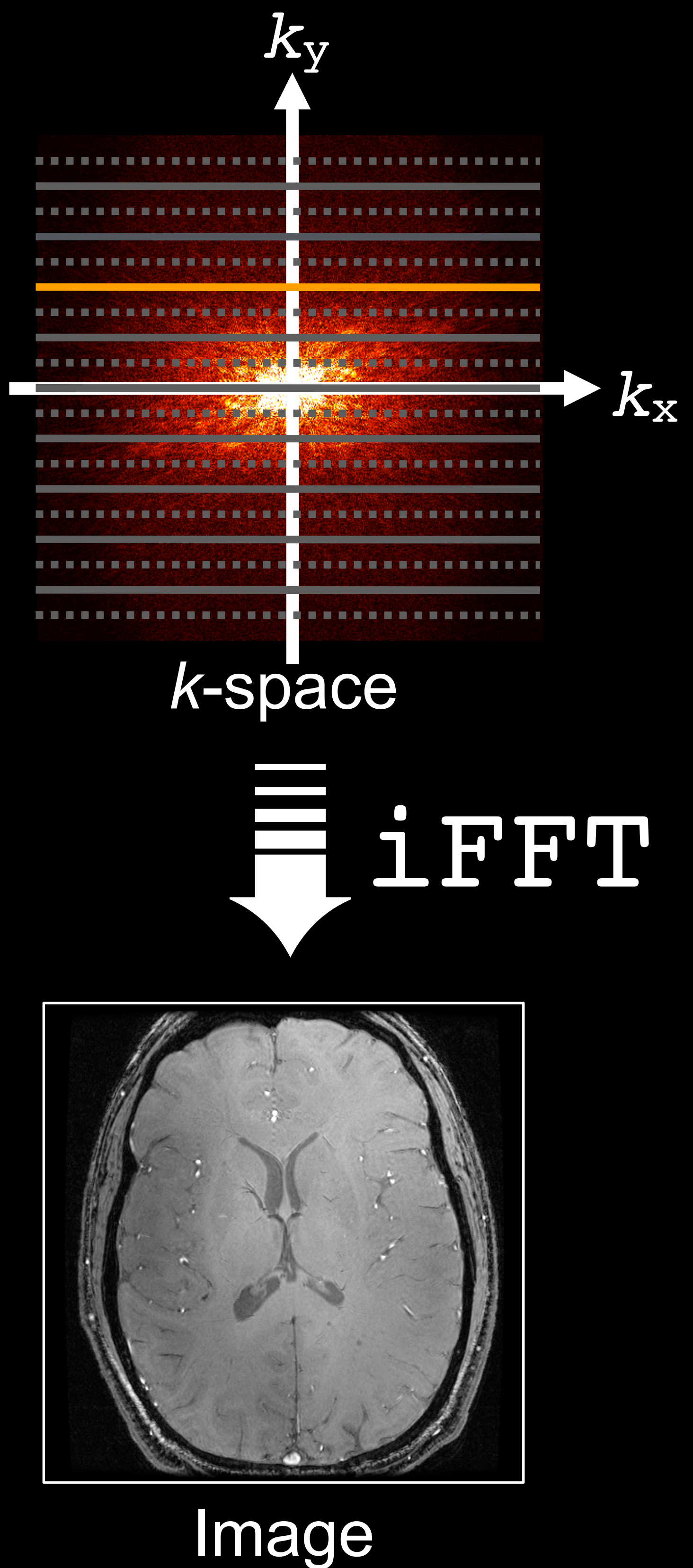
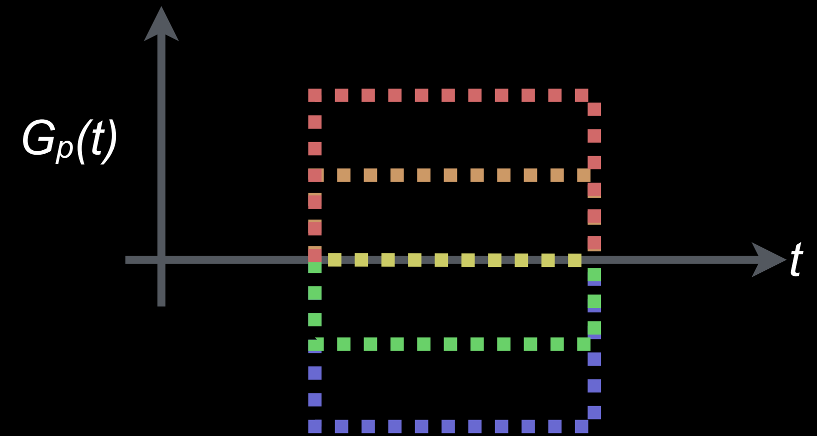
UCLA Radiology

Phase Encoding

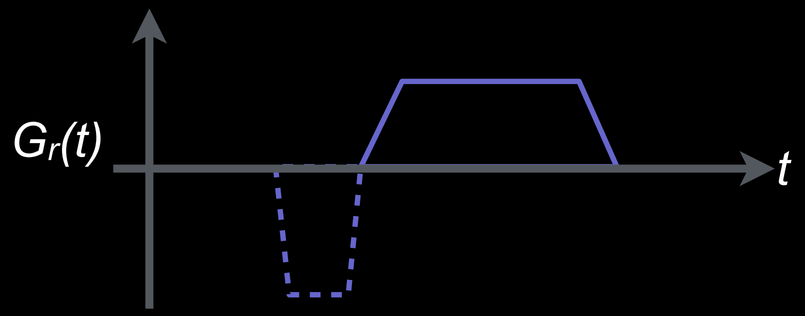
- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**



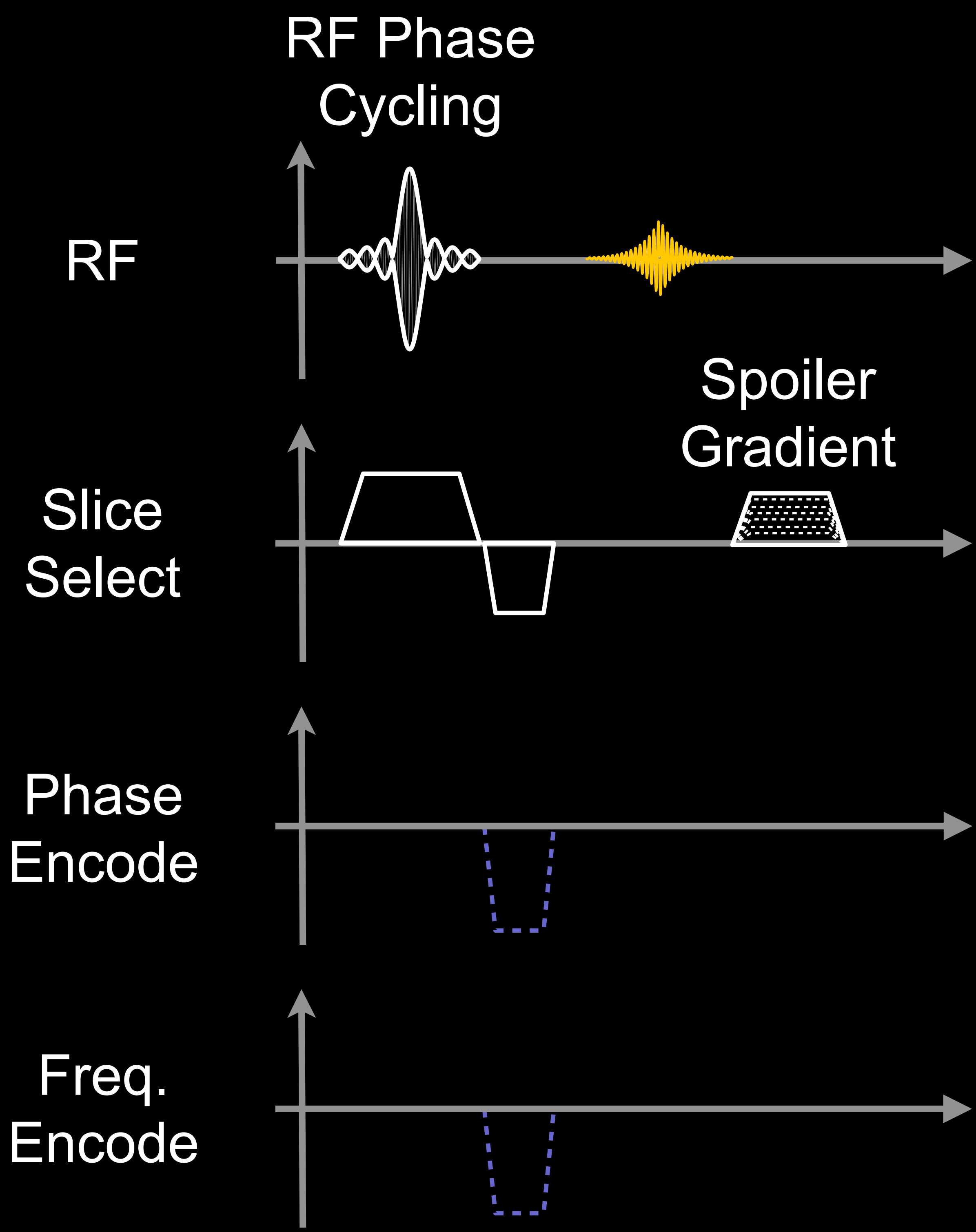


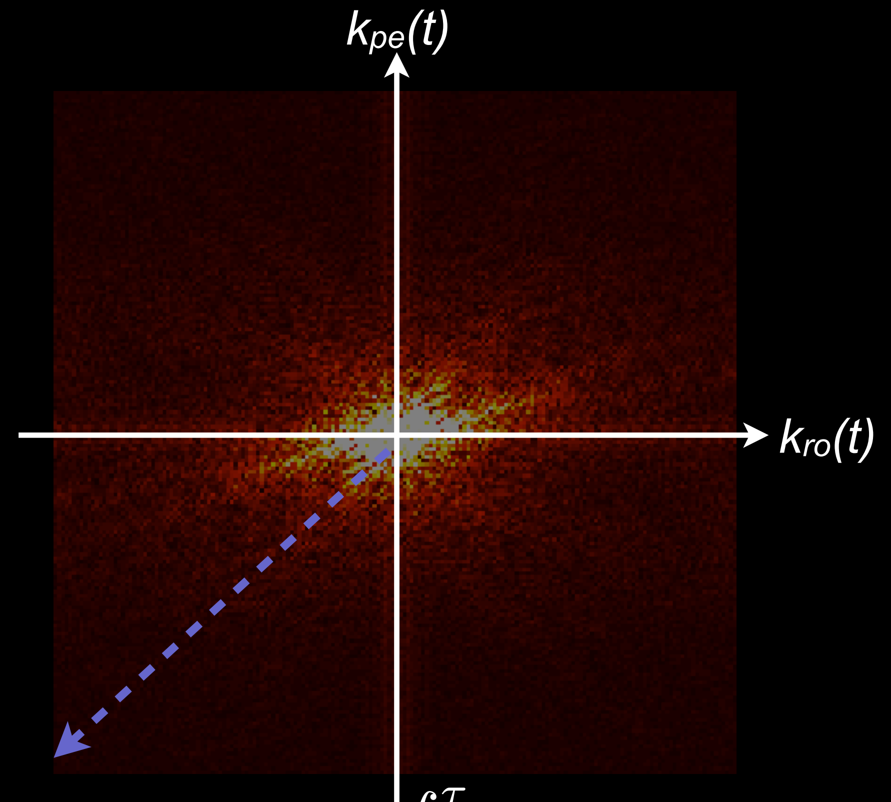
Frequency Encoding

- Consists of:
  - Frequency encoding gradient
    - Constant magnitude for Cartesian imaging
  - No simultaneous
    - RF (B_1)
    - Other gradients
      - phase encoding, slice encoding, crushers
  - Readout pre-phasing gradient
    - Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)
    - AKA “readout de-phasing gradient”
- Adds linear spatial variation of frequency
- Helps form an *echo*

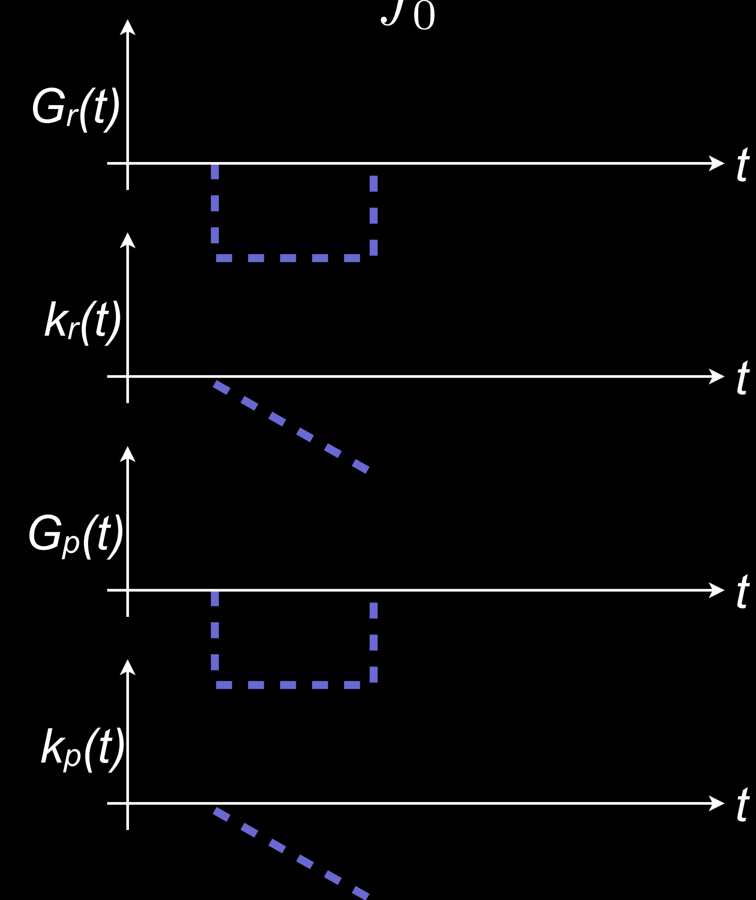


Where am I in *k*-space?



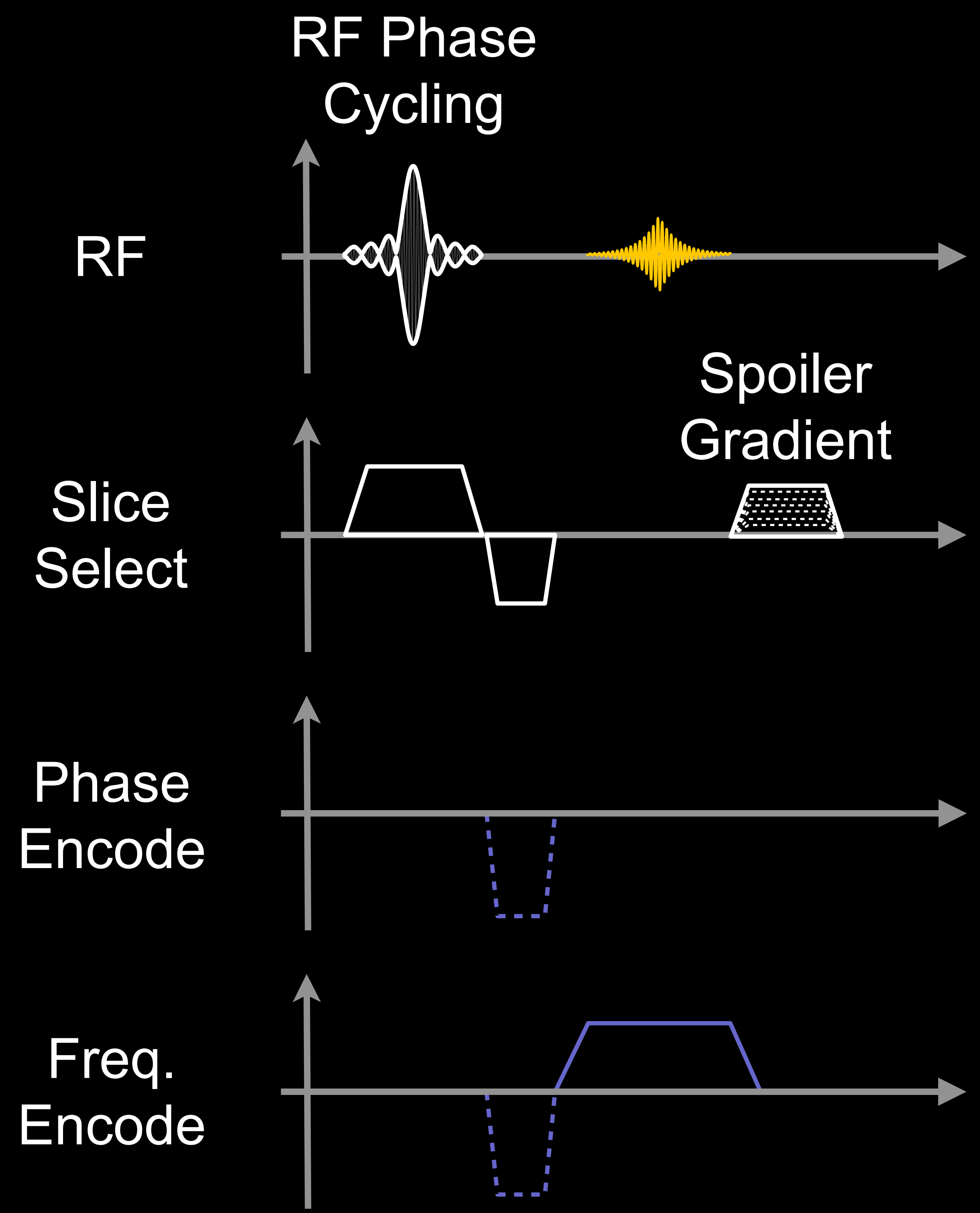


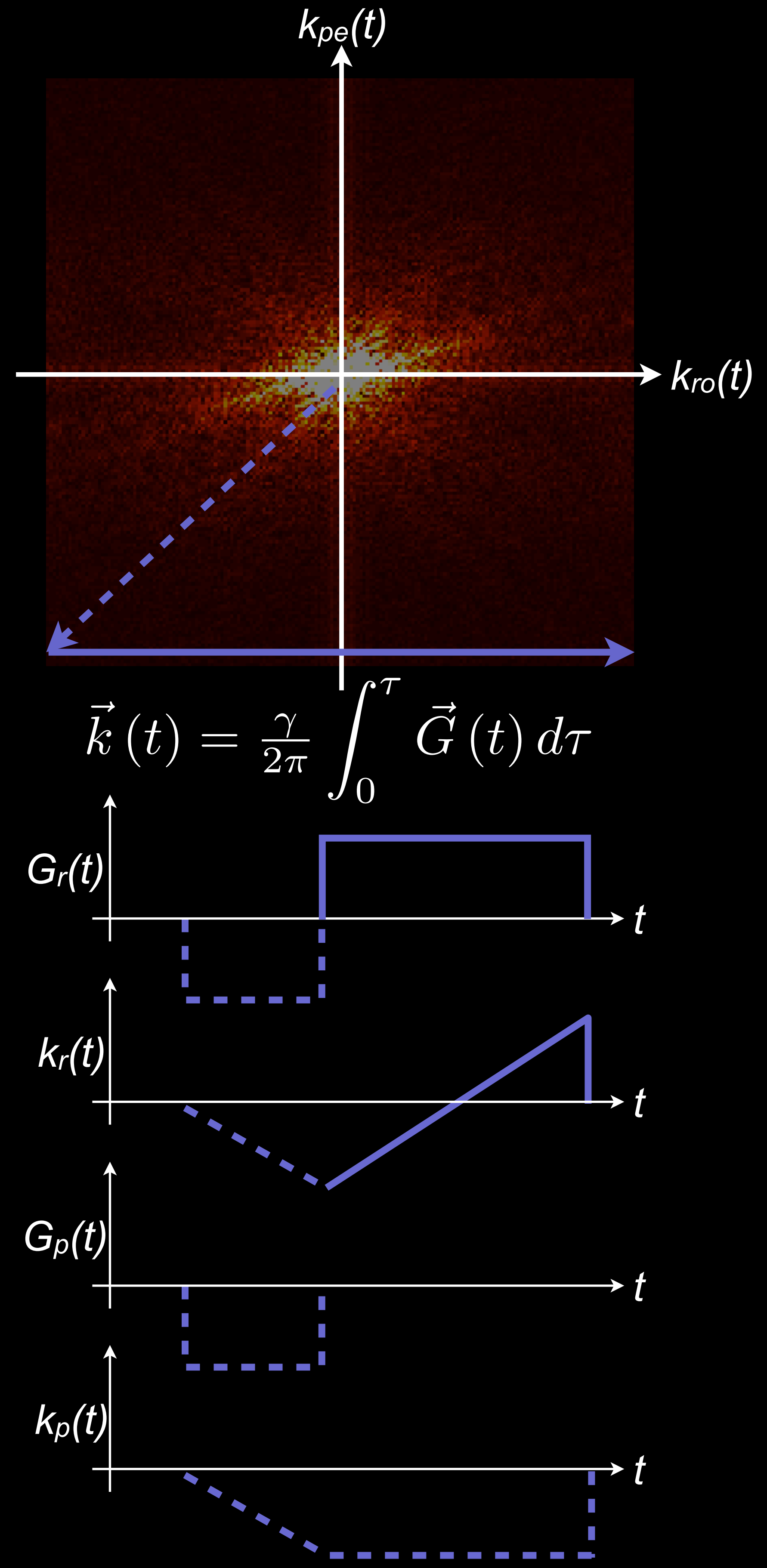
$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\, d\tau$$



One phase encoded echo is acquired per TR.

Where am I in *k*-space?

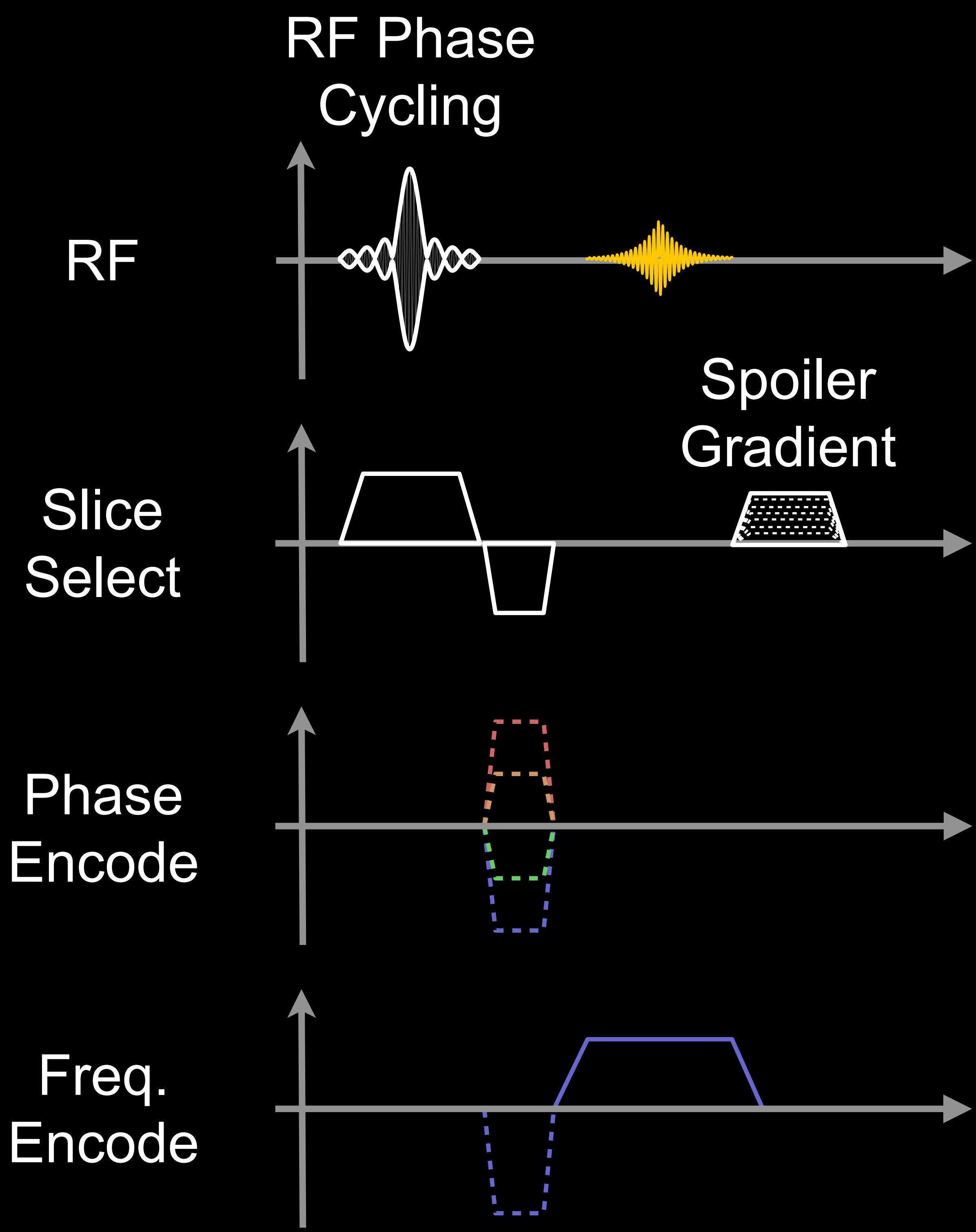


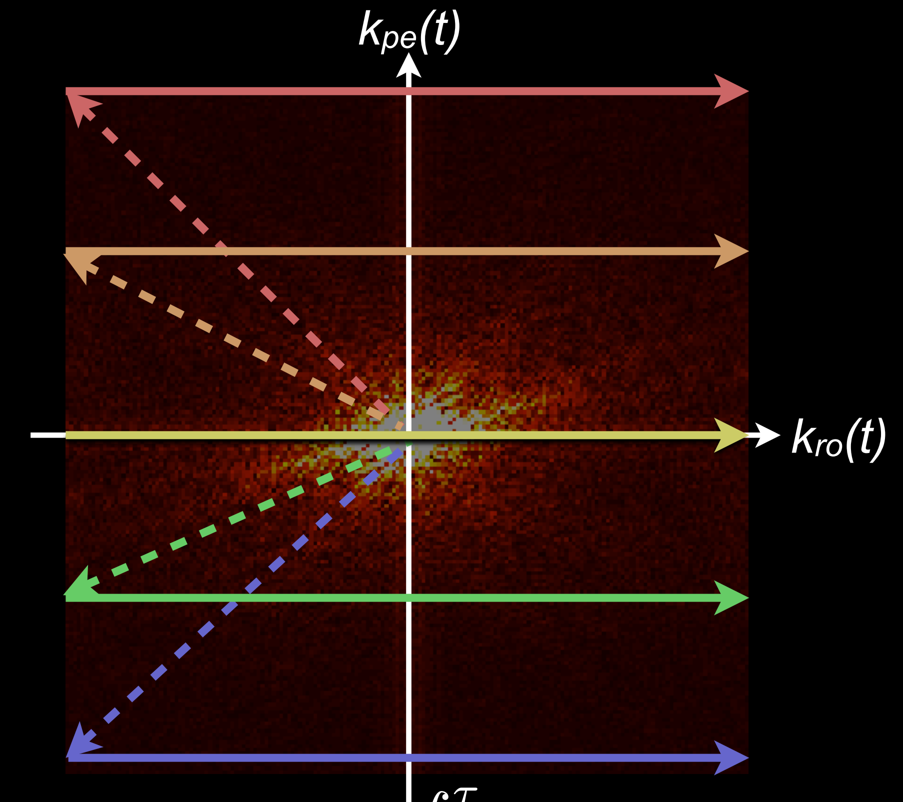


$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$

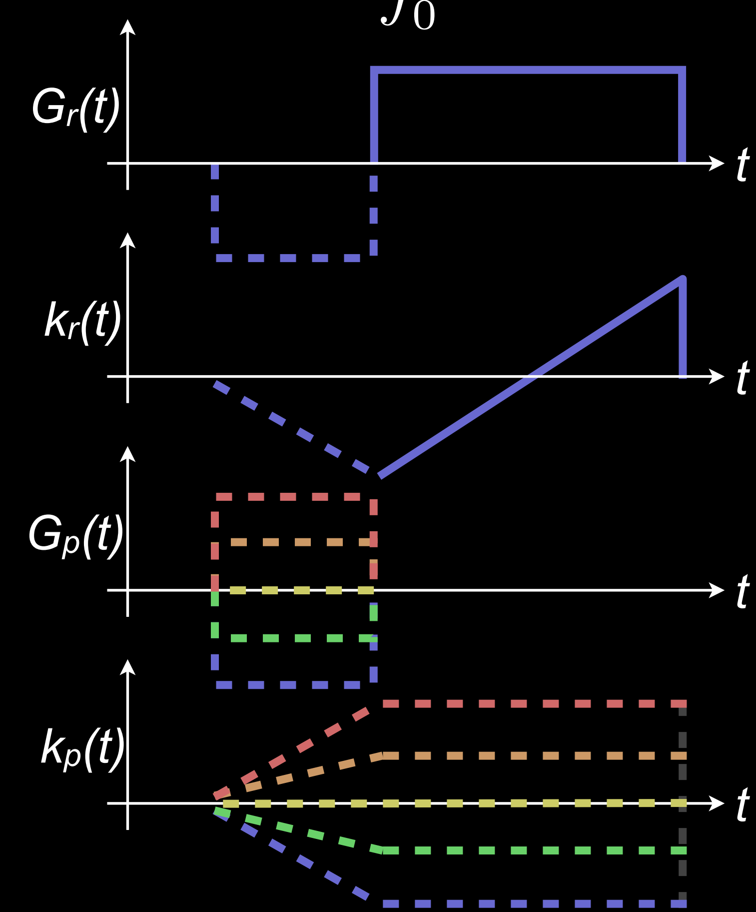
One phase encoded echo is acquired per TR.

Where am I in *k*-space?





$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$



One phase encoded echo is acquired per TR.

N-Dimensional Imaging

MR Signal Equation

$$s(t) = \int_{object} M_{xy}(\vec{r}, 0) \cdot e^{-i\Delta\omega(\vec{r})t} \, \mathrm{d}\vec{r}$$

$$s(t) = \iint_{X,Y} M(x, y) \cdot e^{-i\,\Delta\omega(x,y)\,t} \, \mathrm{d}x\mathrm{d}y$$

$$\Delta\omega(x, y) = \gamma G_x \cdot x + \gamma G_y \cdot y$$

$$s(t) = \iint_{X,Y} M(x, y) \cdot e^{-i2\pi[k_x(t)x + k_y(t)y]} \, \mathrm{d}x\mathrm{d}y$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t \qquad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

MR Signal Equation

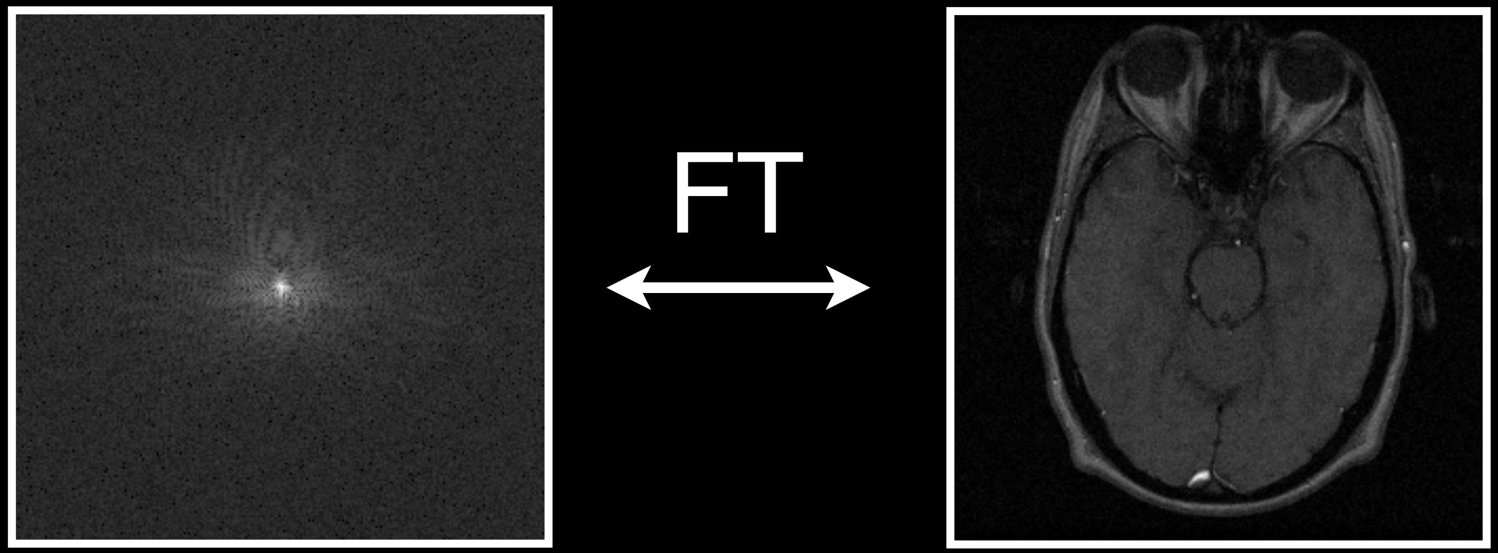
$$s(t) = \iint_{X,Y} M(x,y) \cdot e^{-i2\pi[k_x(t)x + k_y(t)y]}\, \mathrm{d}x\mathrm{d}y$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t$$

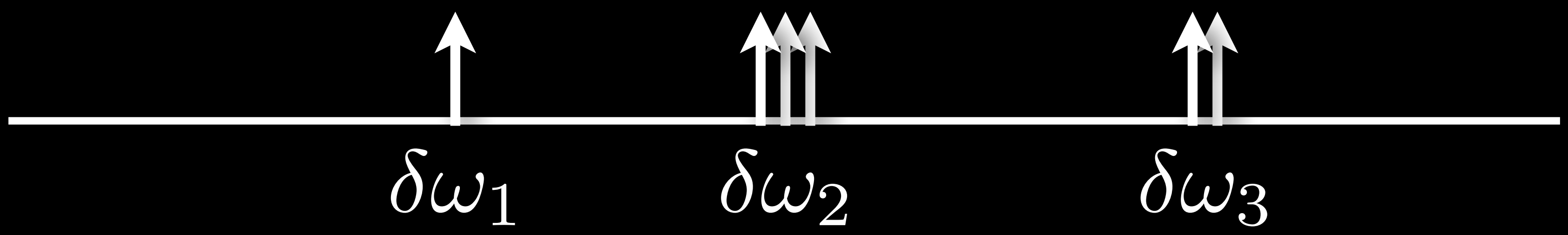
$$k_y(t) = \frac{\gamma}{2\pi} G_y t$$

$$s(t) = m(k_x(t), k_y(t))$$

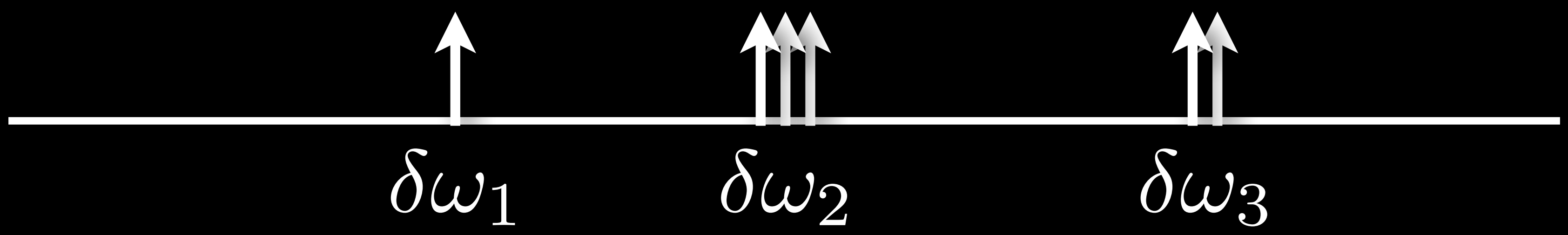
$$m = \mathcal{FT}(\, M(x,y)\,)$$



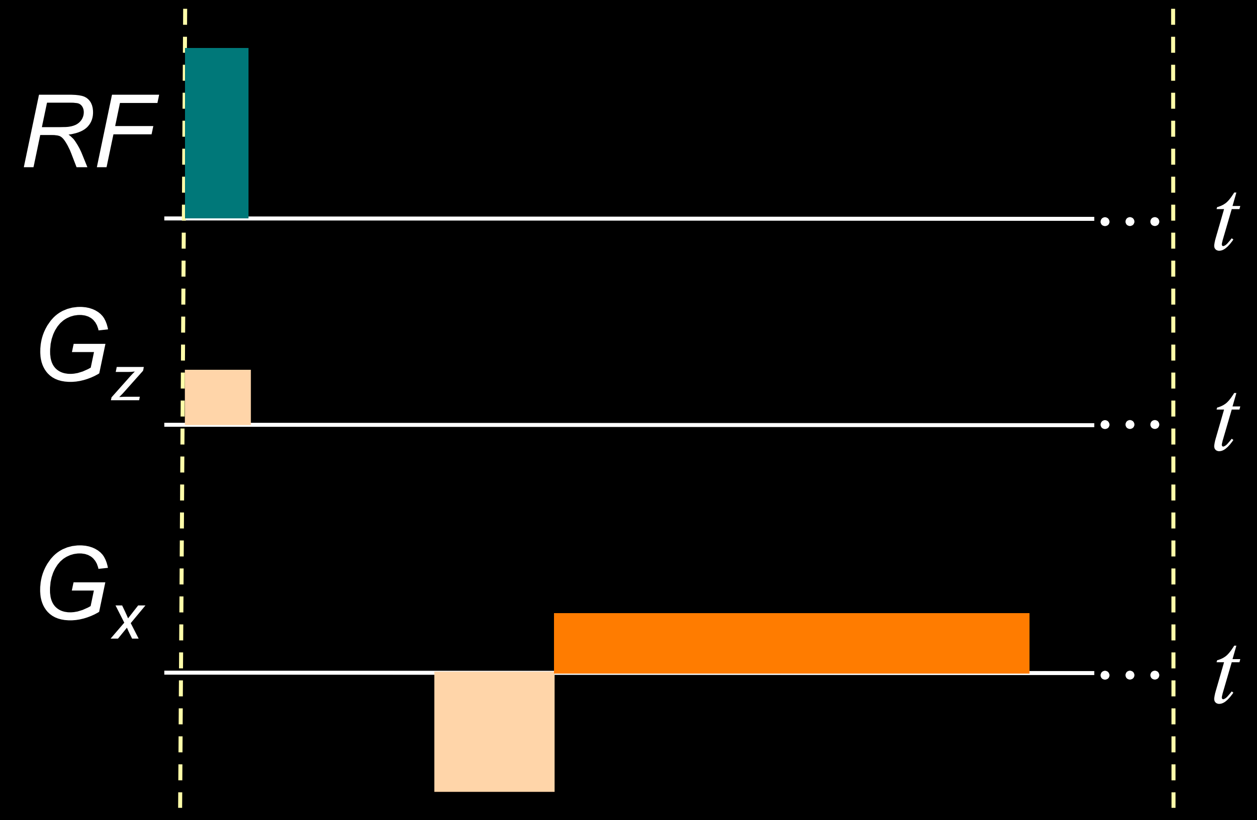
1D Imaging



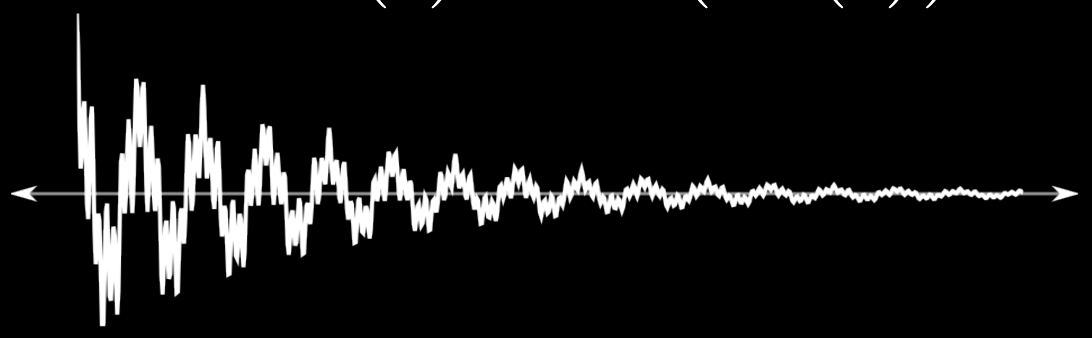
1D Imaging



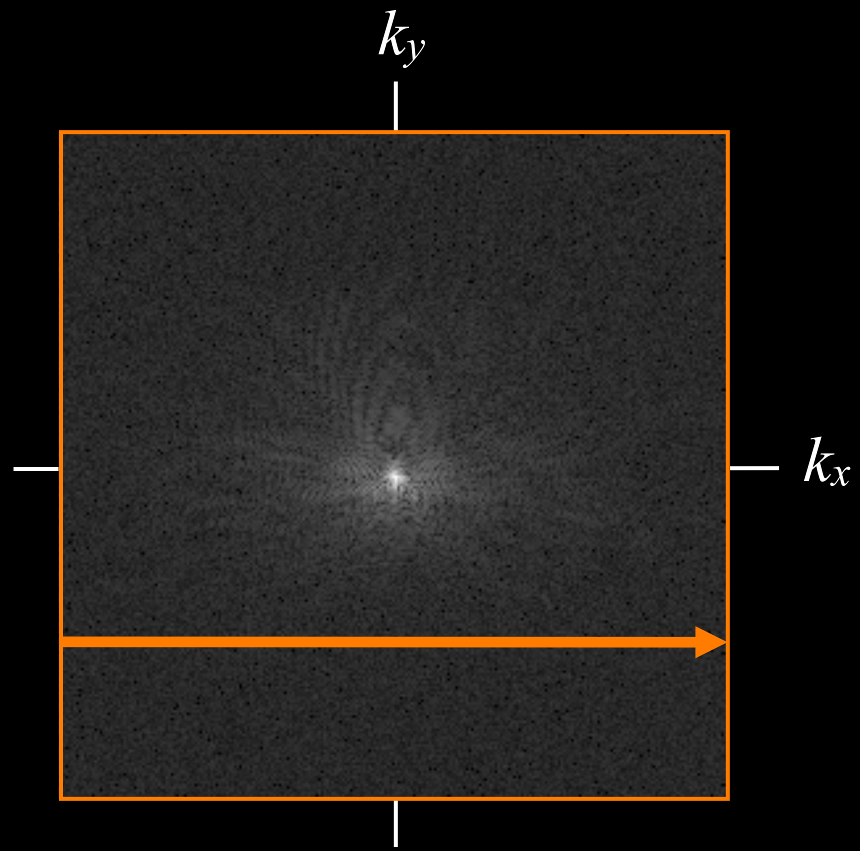
Pulse Sequence Diagram



$$s(t) = m(k_x(t))$$

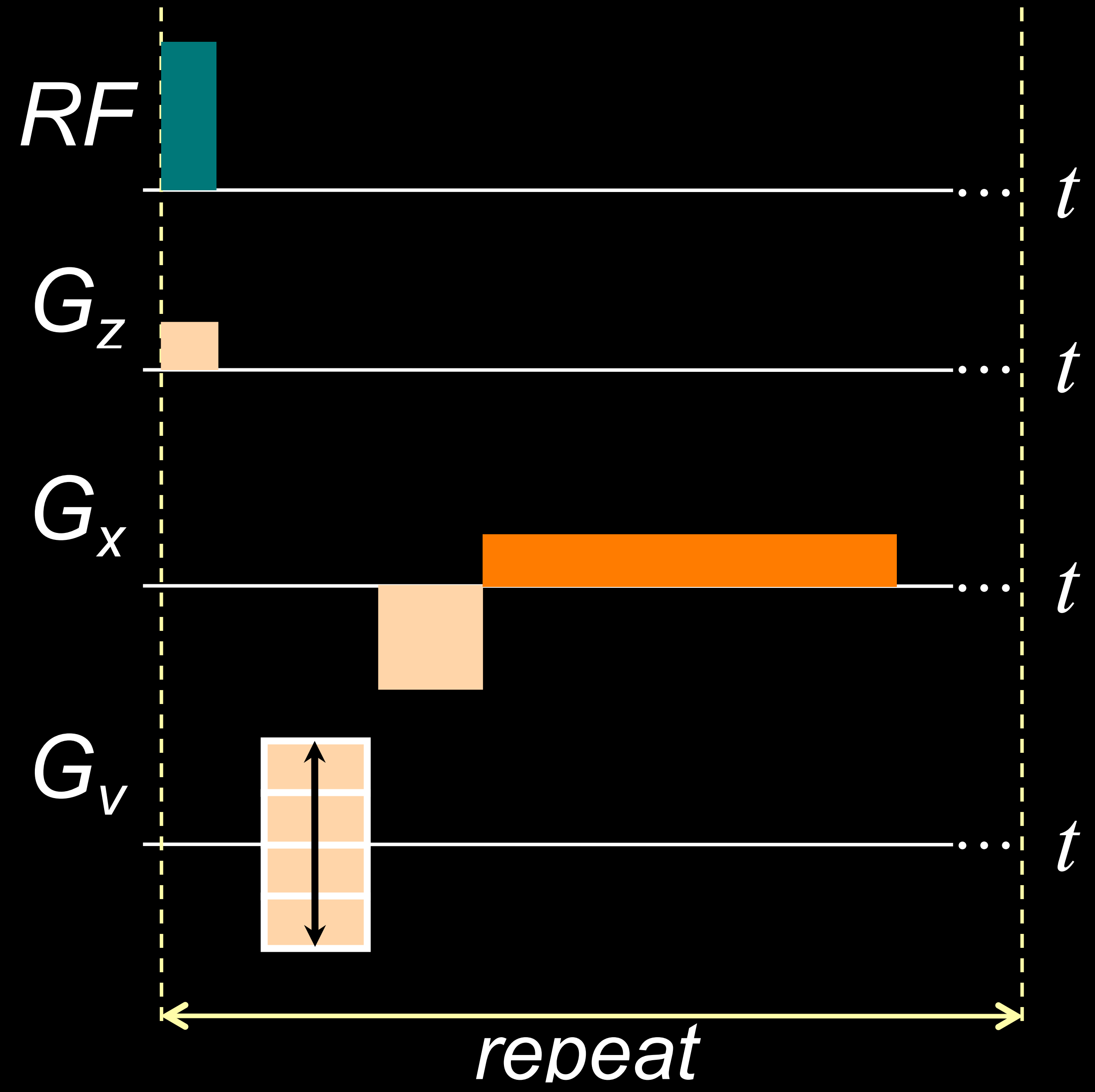


2D Imaging

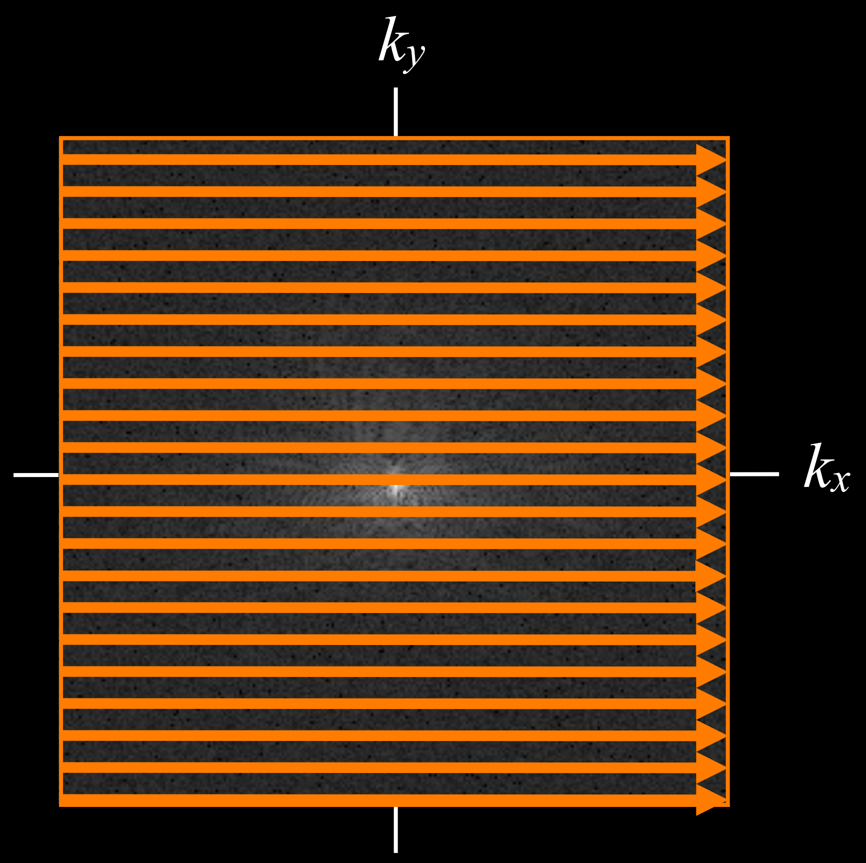


$$s(t) = m(\, k_x(t), k_y(t)\,)$$

Pulse Sequence Diagram

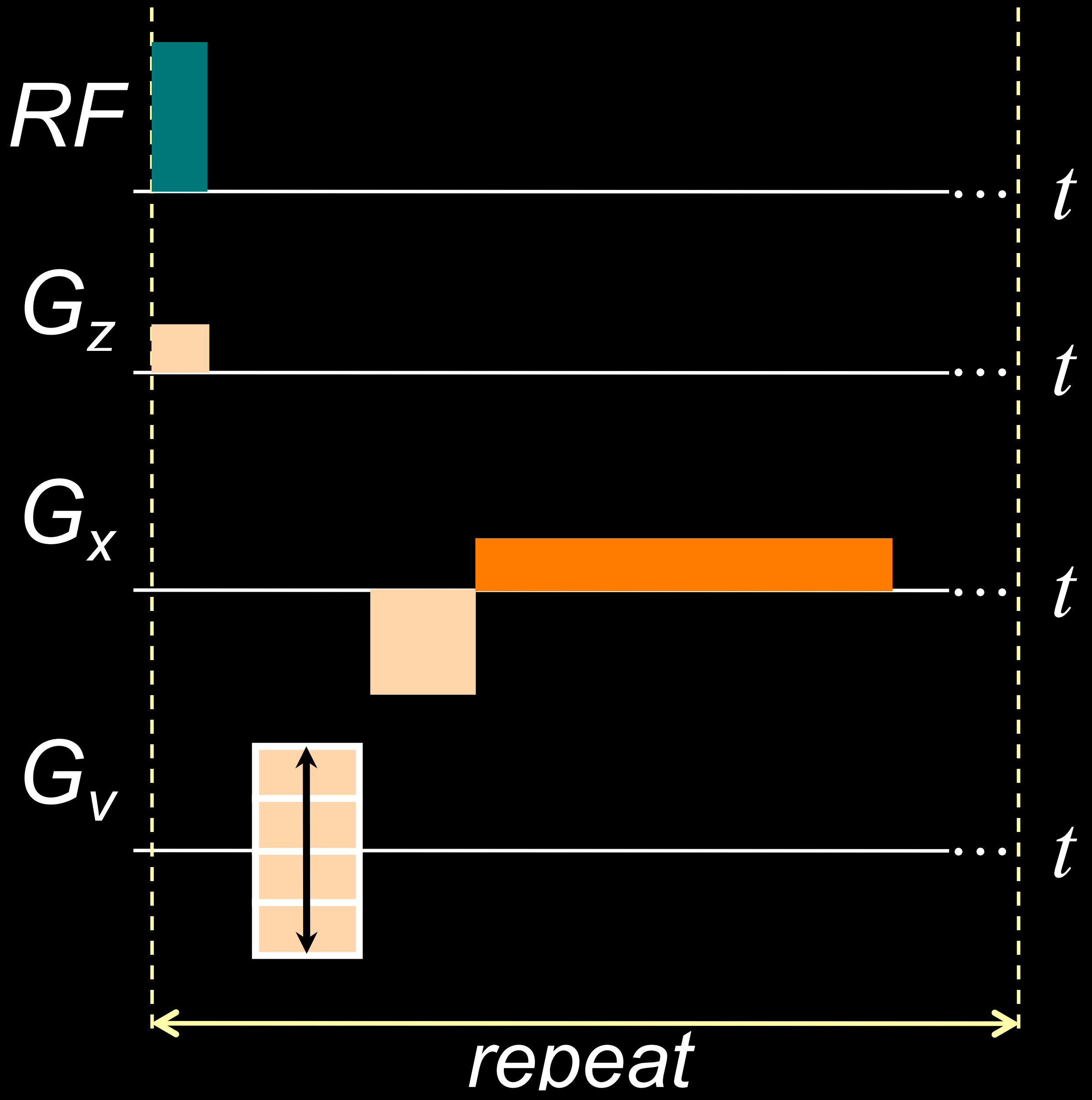


2D Imaging

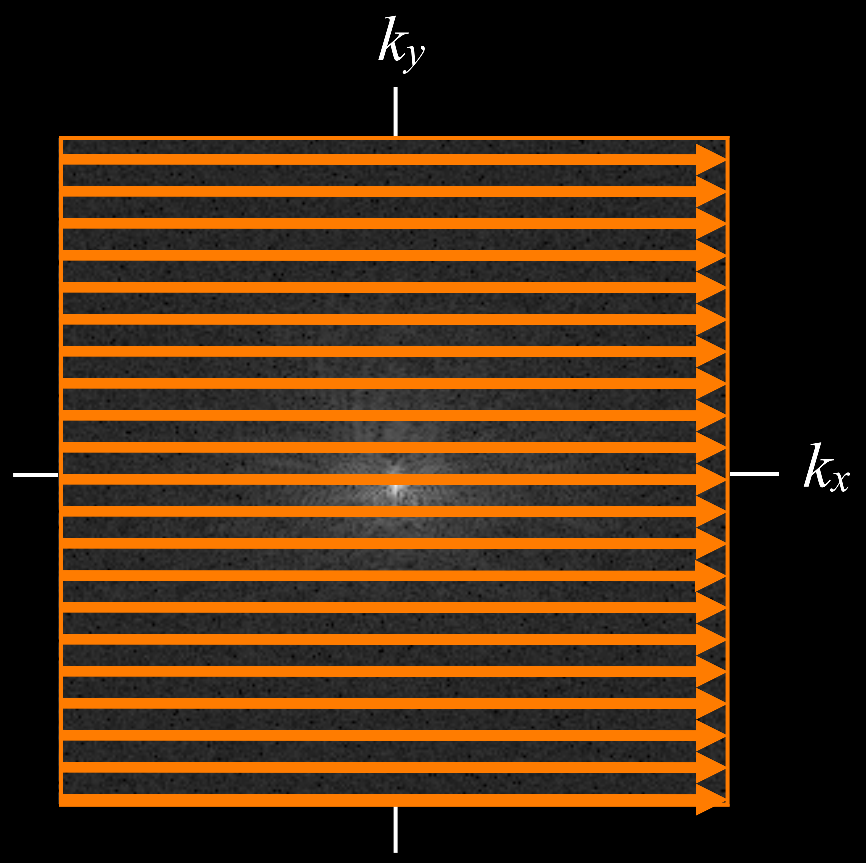


$$s(t) = m(\,k_x(t), k_y(t)\,)$$

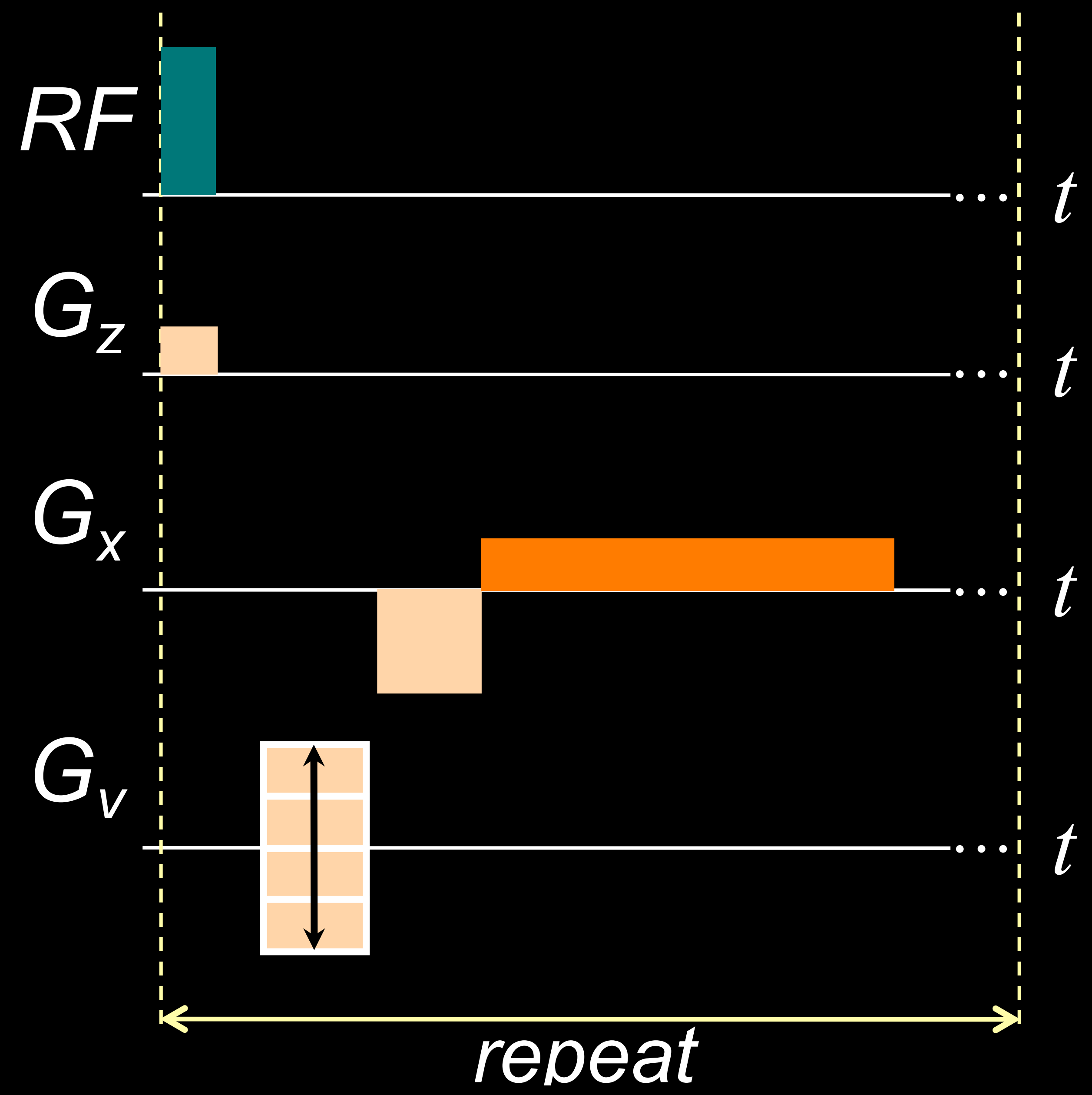
Pulse Sequence Diagram



2D Imaging

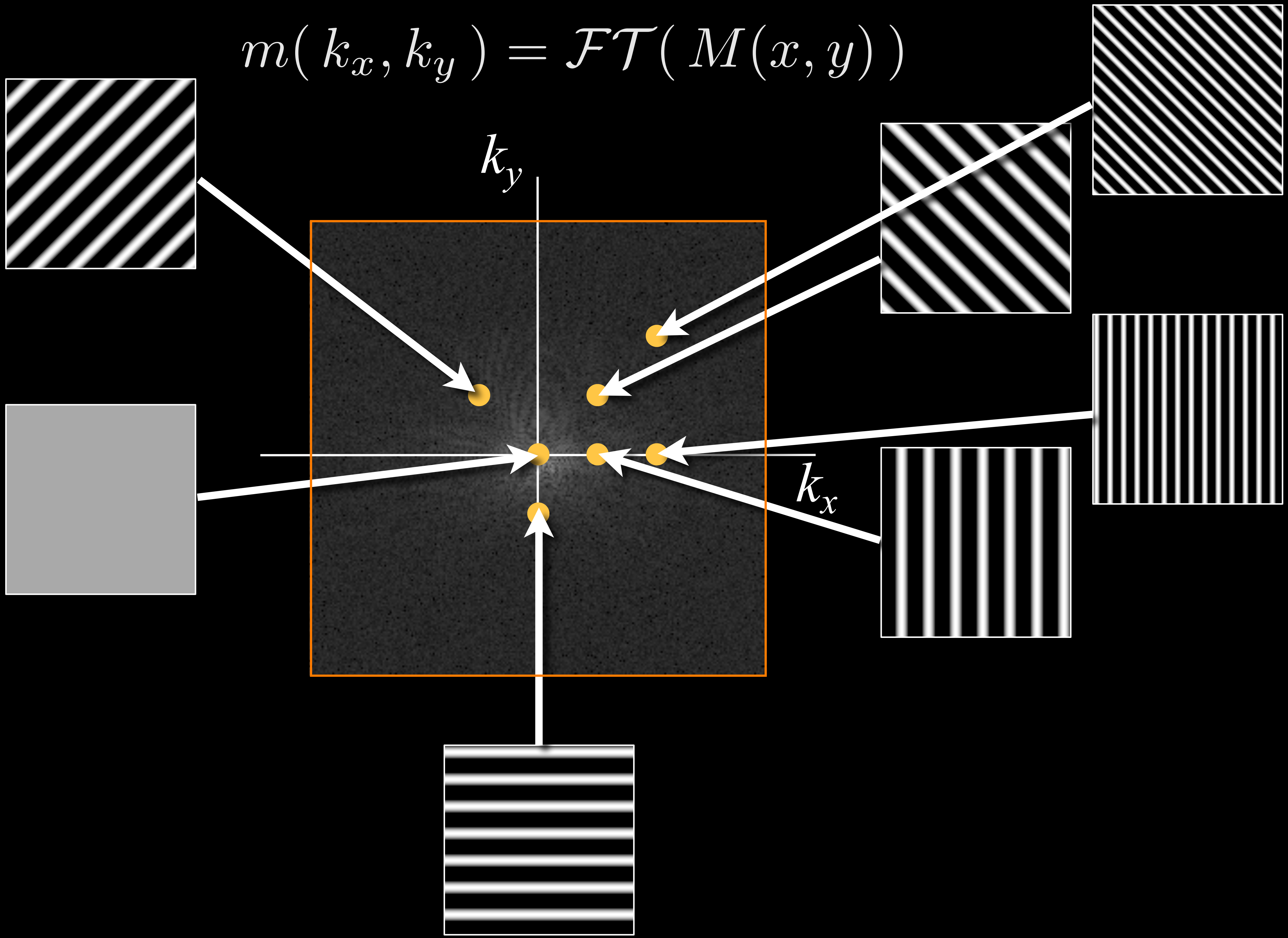


Pulse Sequence Diagram



2D k-Space: MRI Data

$$m(\,k_x, k_y\,) = \mathcal{FT}(\,M(x,y)\,)$$



3D Imaging

$$s(t) = \int_{object} M_{xy}(\vec{r}, 0) \cdot e^{-i\Delta\omega(\vec{r})t} \,\mathrm{d}\vec{r}$$

$$s(t) = \iint_{X,Y,Z} M(x,y,z) \cdot e^{-i\,\Delta\omega(x,y,z)\,t} \,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

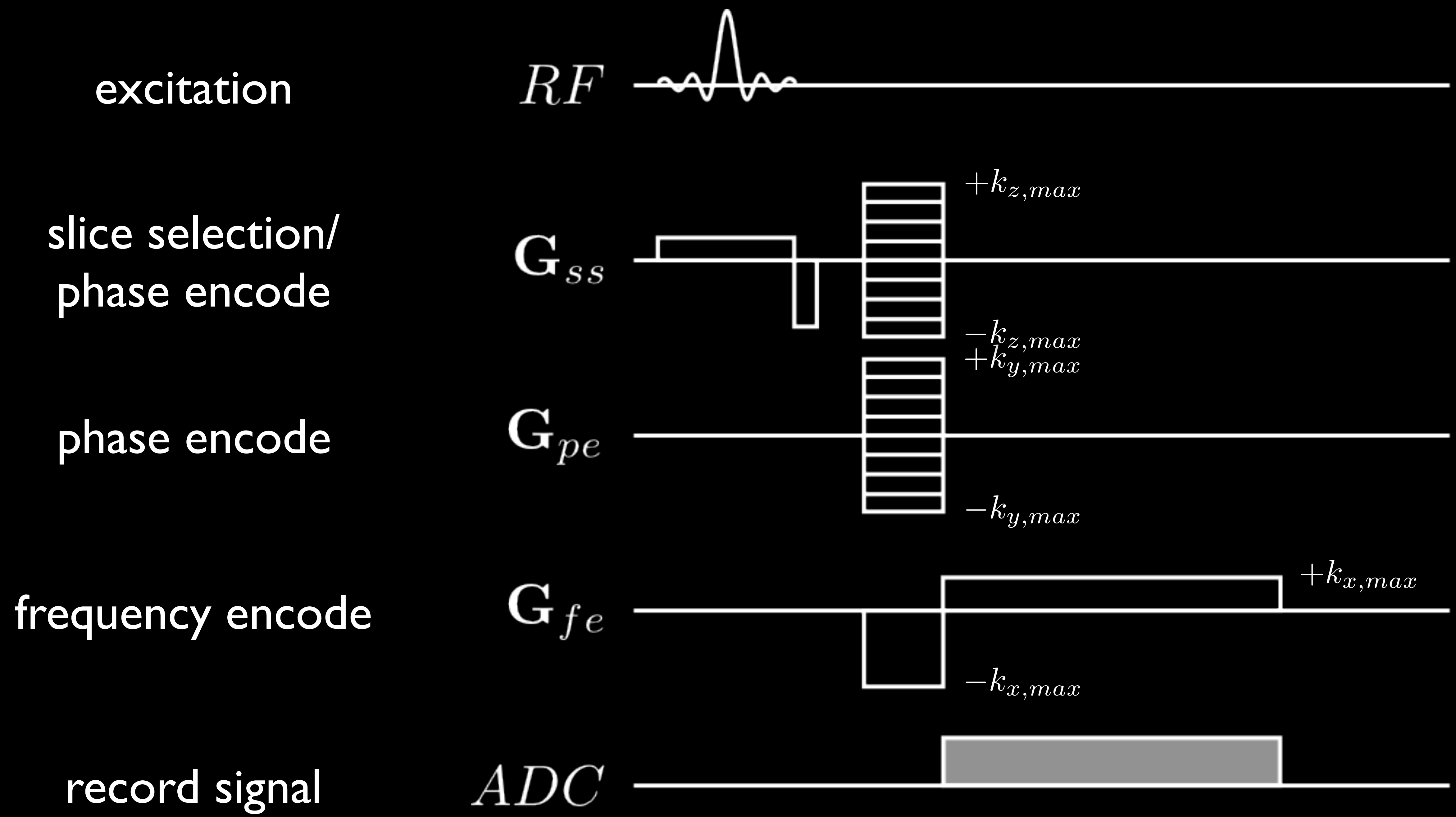
$$\Delta\omega(x,y,z) = \gamma G_x \cdot x + \gamma G_y \cdot y + \gamma G_z \cdot z$$

$$s(t) = \iint_{X,Y,Z} M(x,y,z) \cdot e^{-i2\pi[k_x(t)x + k_y(t)y + k_z(t)z]} \,\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t \qquad k_y(t) = \frac{\gamma}{2\pi} G_y t \qquad k_z(t) = \frac{\gamma}{2\pi} G_z t$$

3D Imaging

Pulse sequence



MRI Sampling Requirements

k-space Sampling

Remember that the collected data in MRI is discrete

Discrete sampling can lead to artifacts if not careful

Sampling considerations
- Field of View
- Spatial Resolution

Sampling Considerations

$$s(t) = m(\, k_x(t), k_y(t) \,)$$

$$s(n\Delta t) = M(k_x(n\Delta t), k_y(n\Delta t))$$

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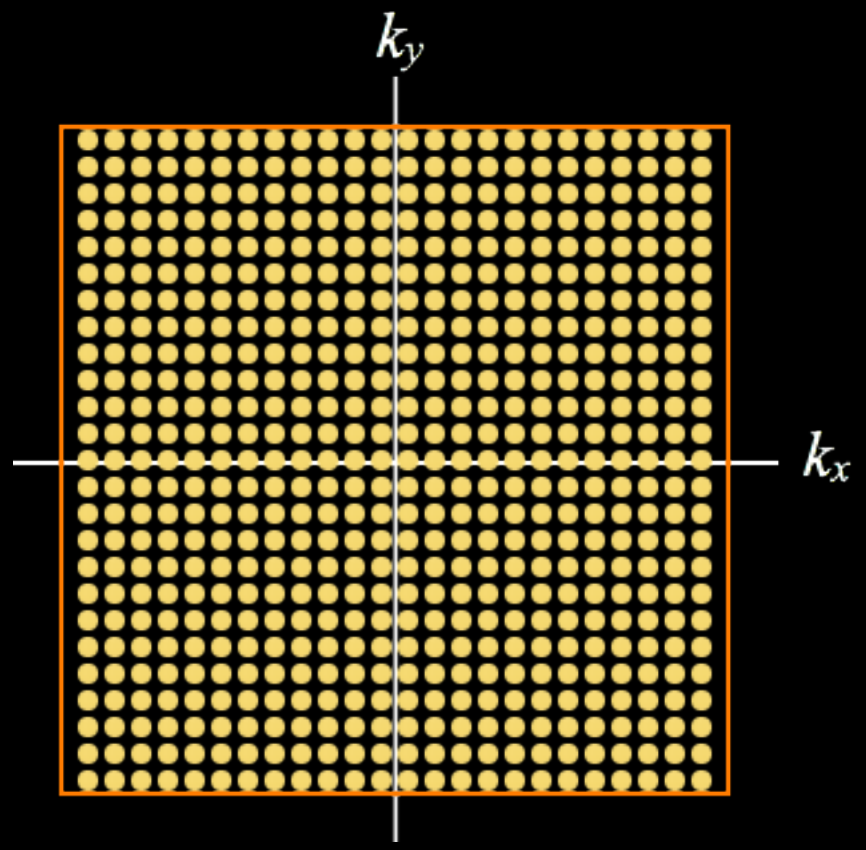
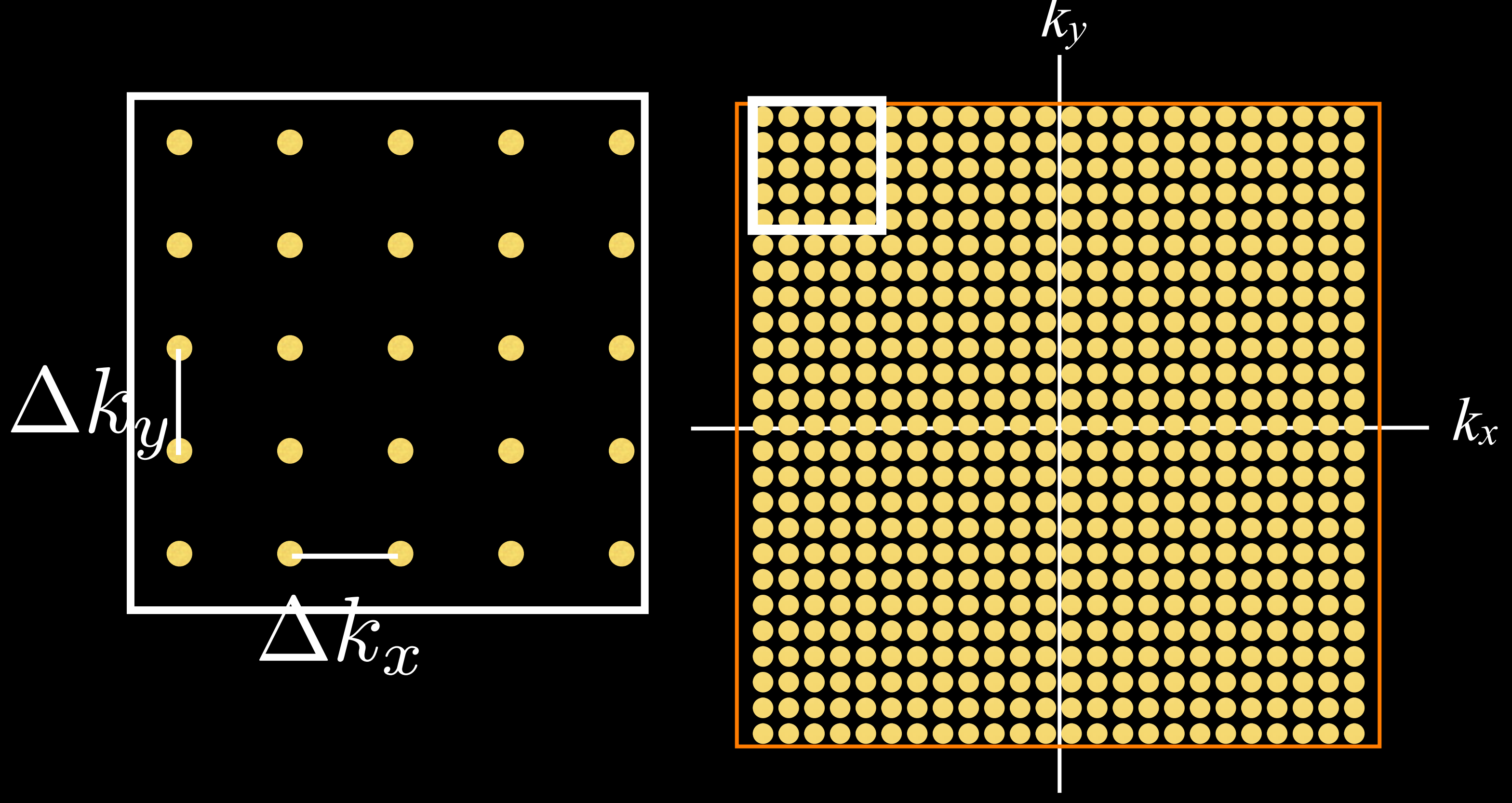
Sampling period

discrete sampling in spatial frequency domain



Sampling Considerations

discrete sampling in spatial frequency domain



$$w_{k_x} = N_{read} \times \Delta k_x$$

$$w_{k_y} = N_{PE} \times \Delta k_y$$

Review: Properties of DFT

Convolution

$$f(x) * h(x) \longleftrightarrow F(k_x)\,H(k_x)$$

Similarity (scaling)

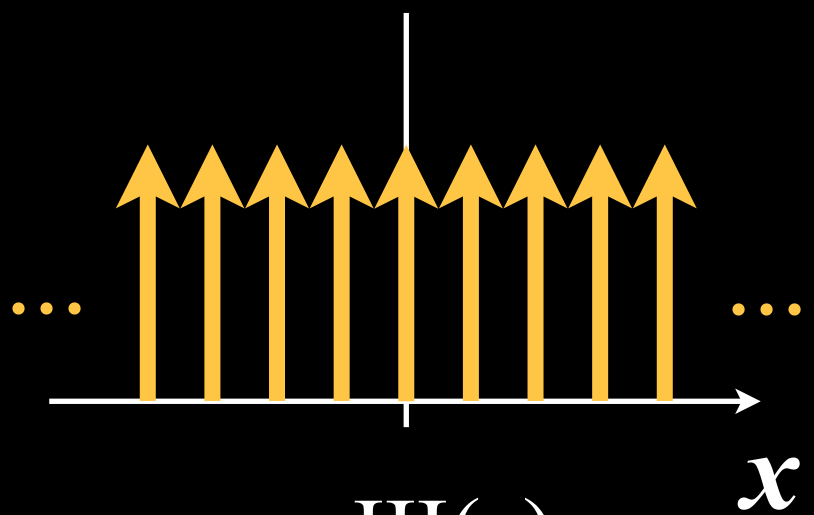
$$f(ax) \longleftrightarrow \frac{1}{|a|}F(\frac{k_x}{a})$$

Shift

$$f(x-a) \longleftrightarrow \exp(-i2\pi(ak_x)) \cdot F(k_x)$$

Review: Properties of DFT

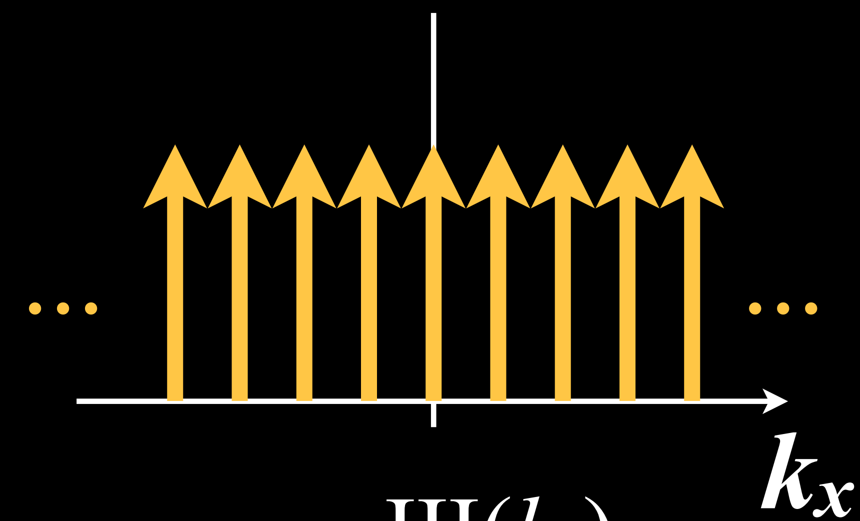
comb or “Shah”



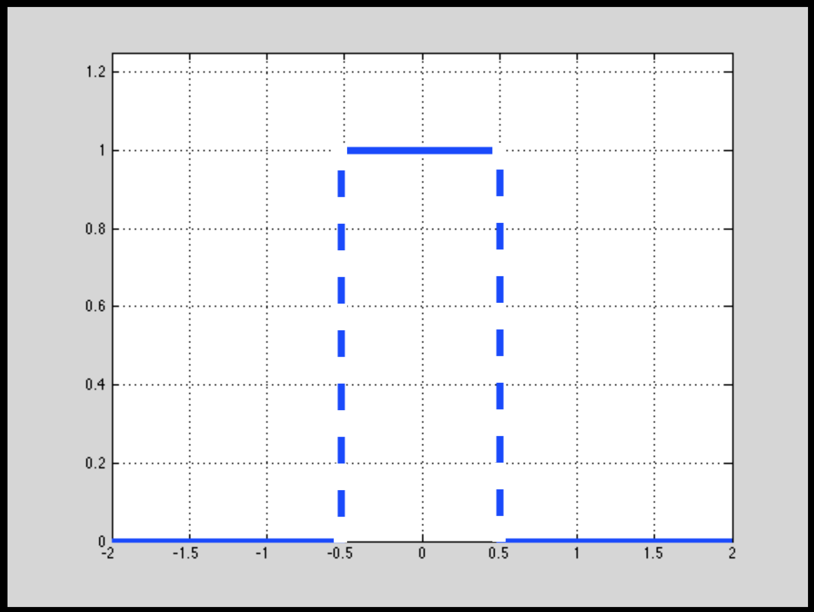
$\mathrm{III}(x)$

FT

$\longleftrightarrow$



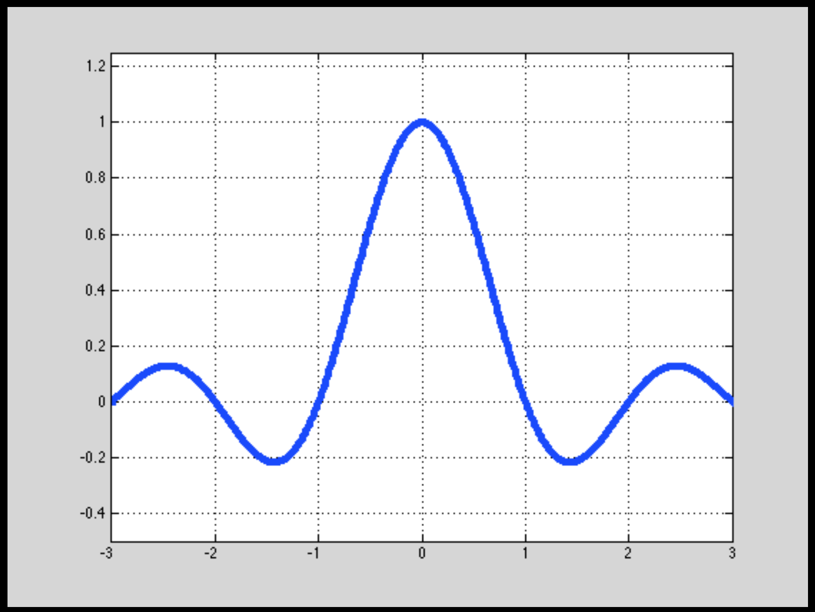
$\mathrm{III}(k_x)$



rect

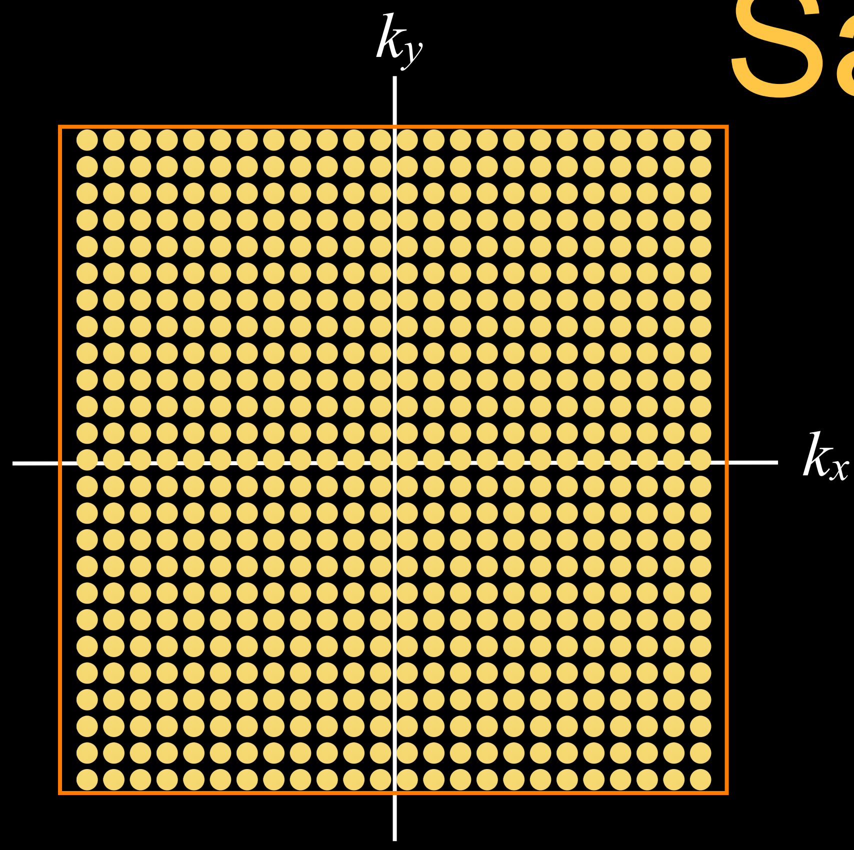
FT

$\longleftrightarrow$



$$\mathrm{sinc}(k_x) = \frac{\sin(\pi k_x)}{\pi k_x}$$

Sampling Model



Sampled Version

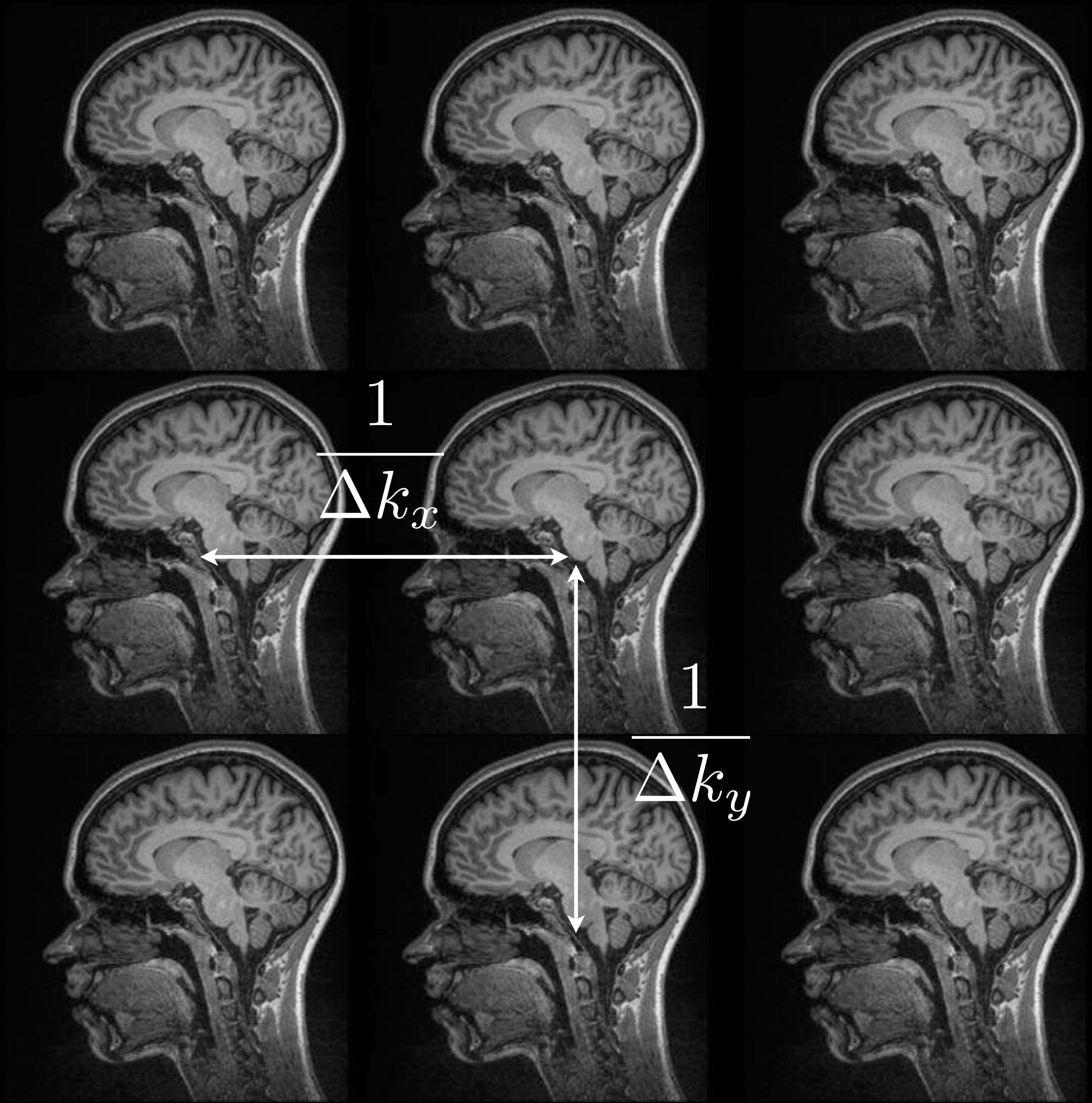
$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \underbrace{\text{III}\left(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}\right)\frac{1}{\Delta k_x \Delta k_y}}_{\text{Sampling}} \underbrace{\sqcap\left(\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}}\right)}_{\text{Extent}}$$

FT ↕

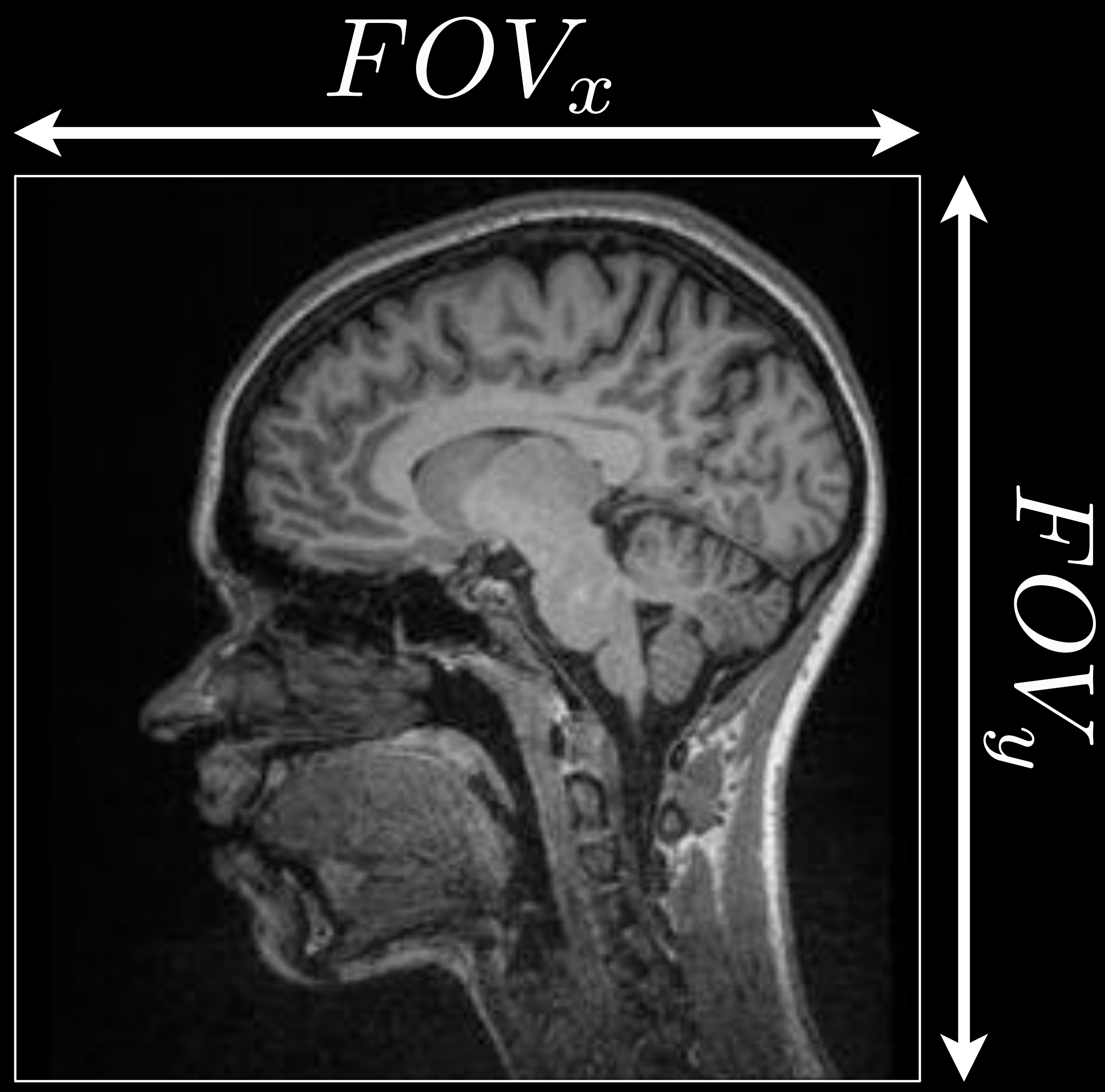
$$\hat{m}(x, y) = m(x, y) * \underbrace{\text{III}(\Delta k_x x, \Delta k_y y)}_{\text{Field of View}} * \underbrace{sinc(w_{k_x} x) sinc(w_{k_y} y)}_{\text{Spatial Resolution}}$$

Field of View

$$m(x,y) * \text{III}(\Delta k_x x, \Delta k_y y)$$



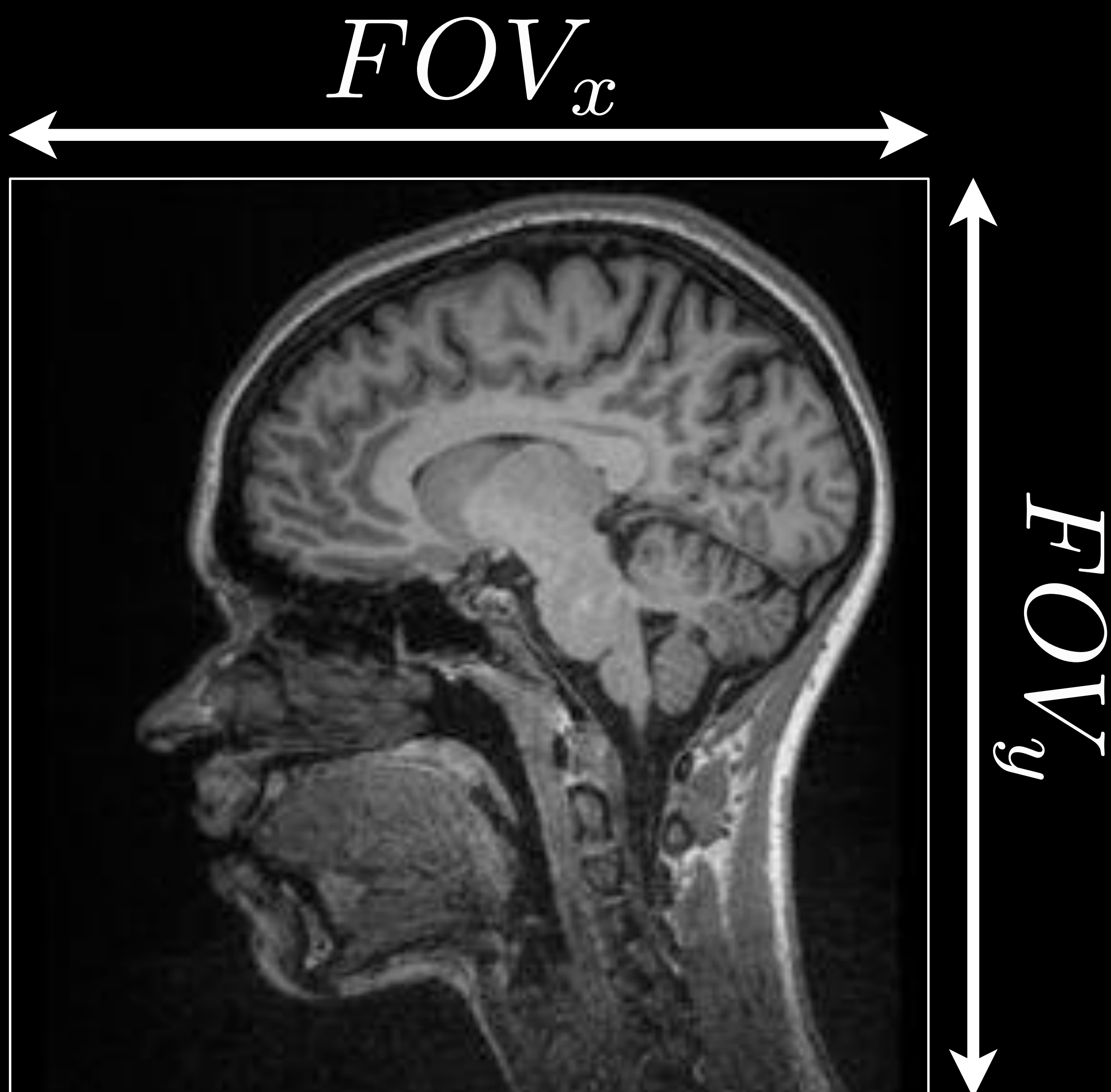
Field of View



$$\Delta k_x = \frac{1}{\mathrm{FOV}_x} = \gamma |\mathbf{G}_x| \Delta t$$

$$\Delta k_y = \frac{1}{\mathrm{FOV}_y} = \gamma \Delta \mathbf{G}_y T_{pe}$$

Field of View



$$\Delta k_x = \frac{1}{\mathrm{FOV}_x} = \gamma |\mathbf{G}_x| \Delta t$$

$$\Delta k_y = \frac{1}{\mathrm{FOV}_y} = \gamma \Delta \mathbf{G}_y T_{pe}$$

$$\Delta t = \frac{1}{\gamma |\mathbf{G}_x| \mathrm{FOV}_x}$$

$$\Delta \mathbf{G}_y = \frac{1}{\gamma T_{pe} \mathrm{FOV}_y}$$

Eqn. 5.124

Field of View

To avoid any aliasing artifacts:

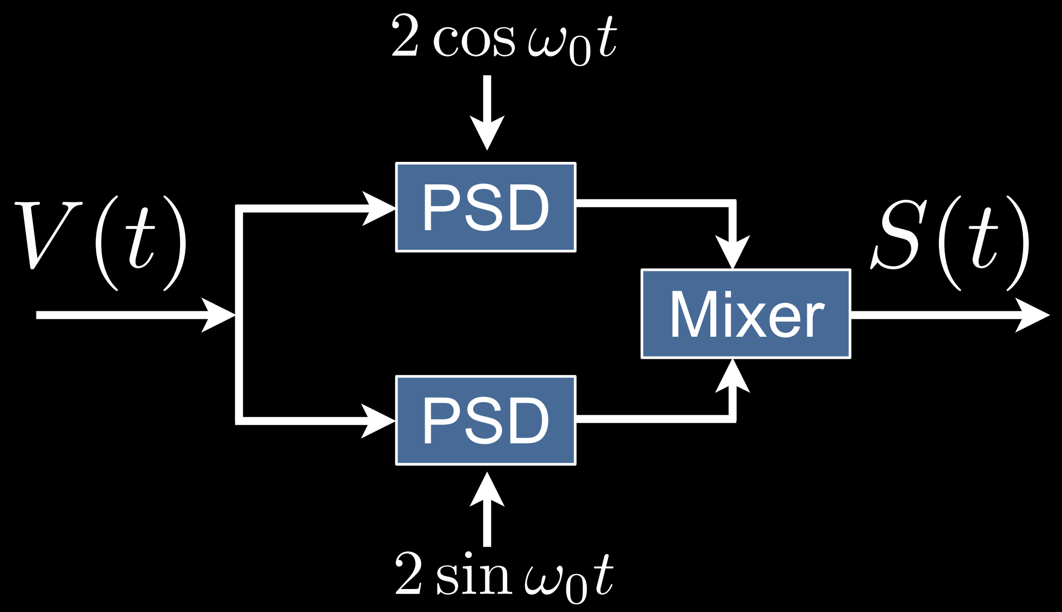
In phase encoding,
- Reduce Δk_y

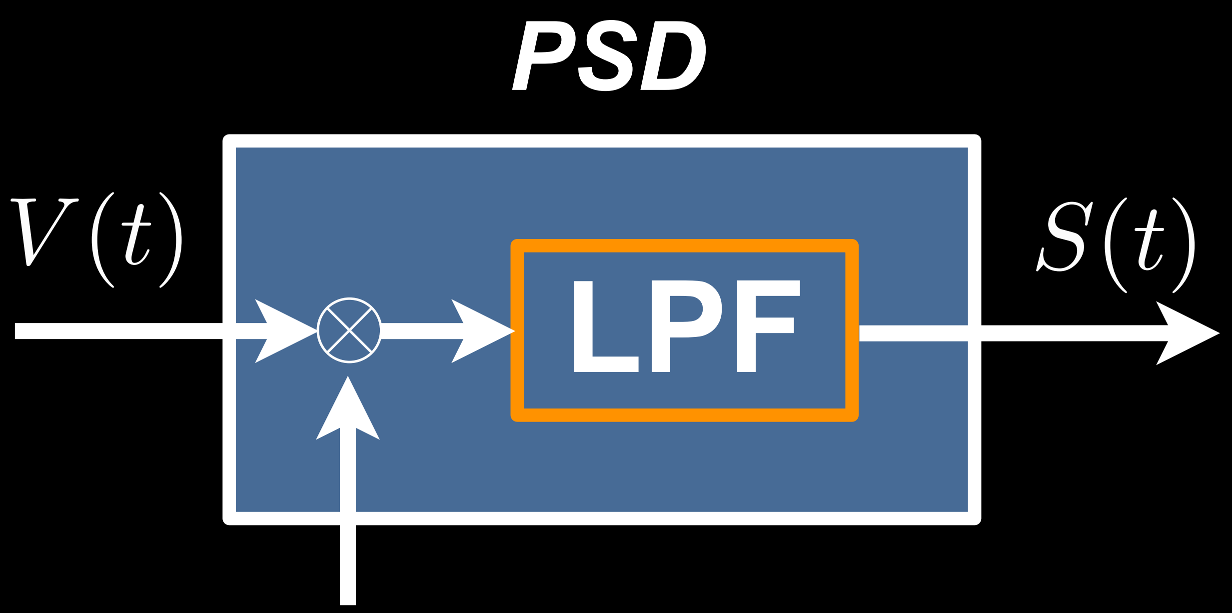
Either lose spatial resolution
or
increase scan time

Field of View

To avoid any aliasing artifacts:

In frequency encoding,
- Reduce Δk_x
- Utilize LPF (low pass filter)

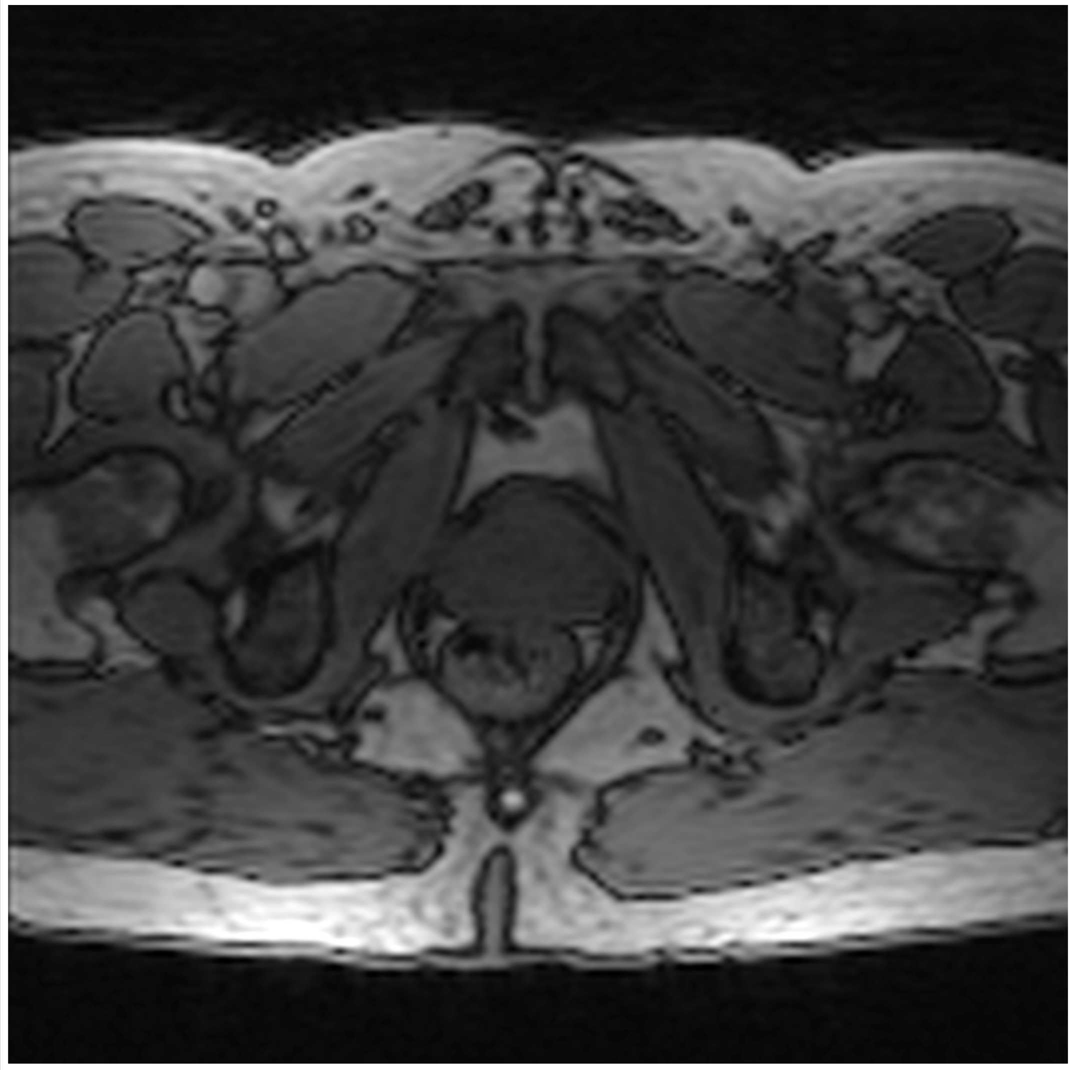




Typically, put long axis of object in readout direction

Field of View

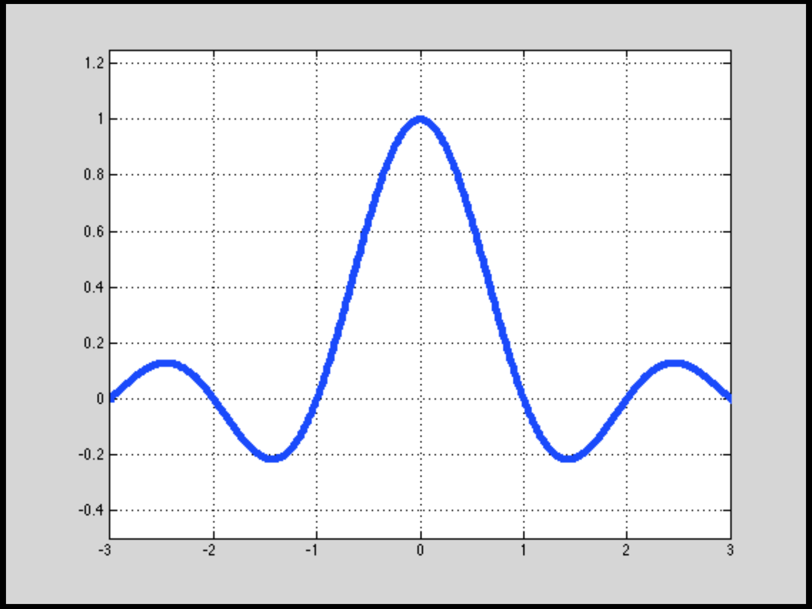
Prostate Imaging Example



Which direction will be readout direction?

Spatial Resolution

$$m(x,y) * sinc(w_{k_x} x) sinc(w_{k_y} y) w_{k_x} w_{k_y}$$



Main lobe causes blurring!
(spatial resolution)

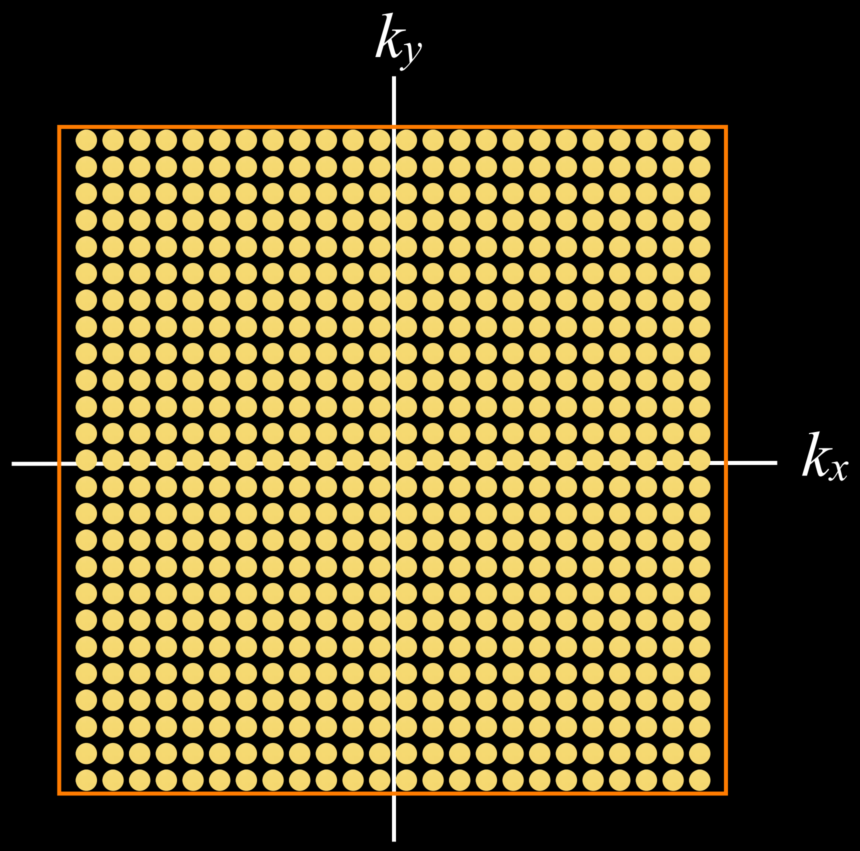
Spatial resolution: δ_x, δ_y

$$\delta_x = \frac{1}{w_{k_x}} \qquad \delta_y = \frac{1}{w_{k_y}}$$

Spatial Resolution

Point Spread Function (PSF)

$$\hat{M}(k_x, k_y) = M(k_x, k_y) \cdot \text{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y}) \frac{1}{\Delta k_x \Delta k_y} \sqcap (\frac{k_x}{w_{k_x}}, \frac{k_y}{w_{k_y}})$$



Sampled Version

$$\hat{M}'(k_x, k_y) = \hat{M}(k_x, k_y) \cdot \text{window}$$

$$\text{PSF} = \text{FT}(\text{window})$$

Point spread function can show the extent of blurring of the image

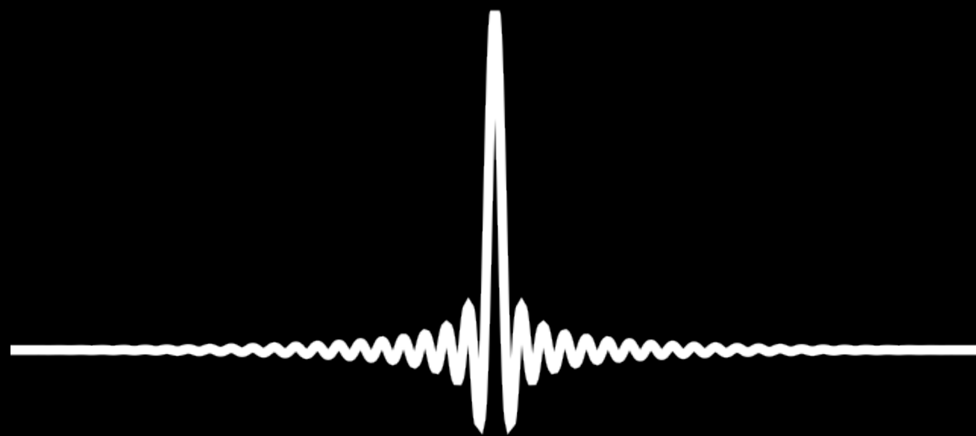
Spatial Resolution

- Spatial resolution of an imaging system is the smallest separation δx of two point sources necessary for them to remain resolvable in the resultant image.

$$\hat{I}(x) = I(x) * h(x)$$

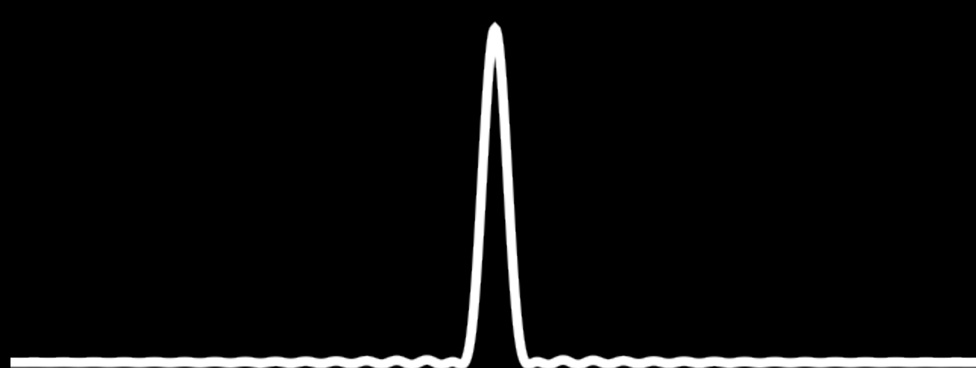
Image — Object — Point Spread Function

PSFs



$\mathcal{F}\{\Pi(t)\}$

Narrower central peak, but lots of ringing



$\mathcal{F}\{H(t)\}$

Reduced ringing, but broader central peak

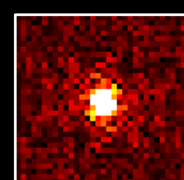
PSFs

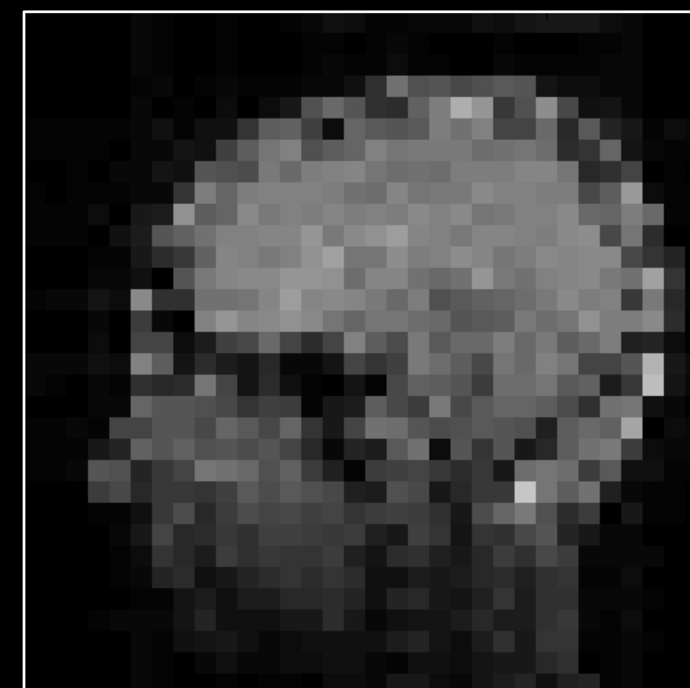
Filters can be used to reduce ringing artifacts but often at the expense of spatial resolution

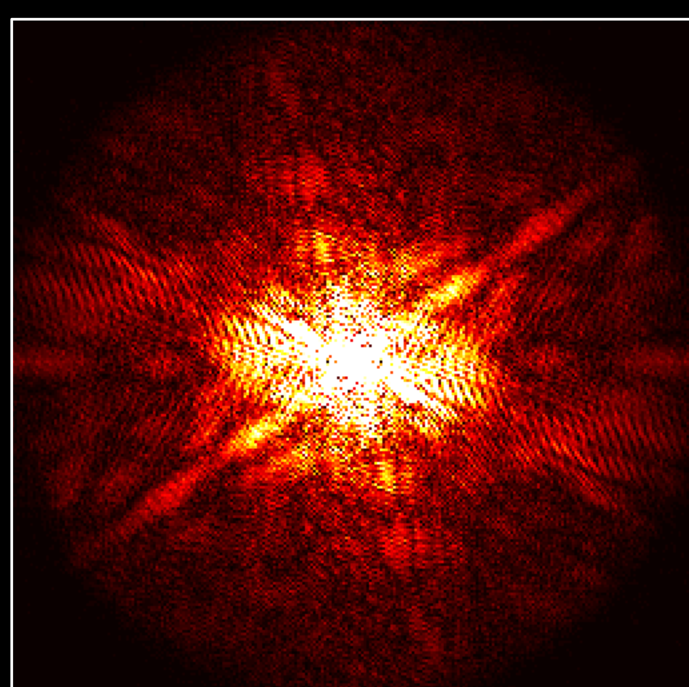
Hamming window seems to have good balance in reducing ringing

Finite Sampling

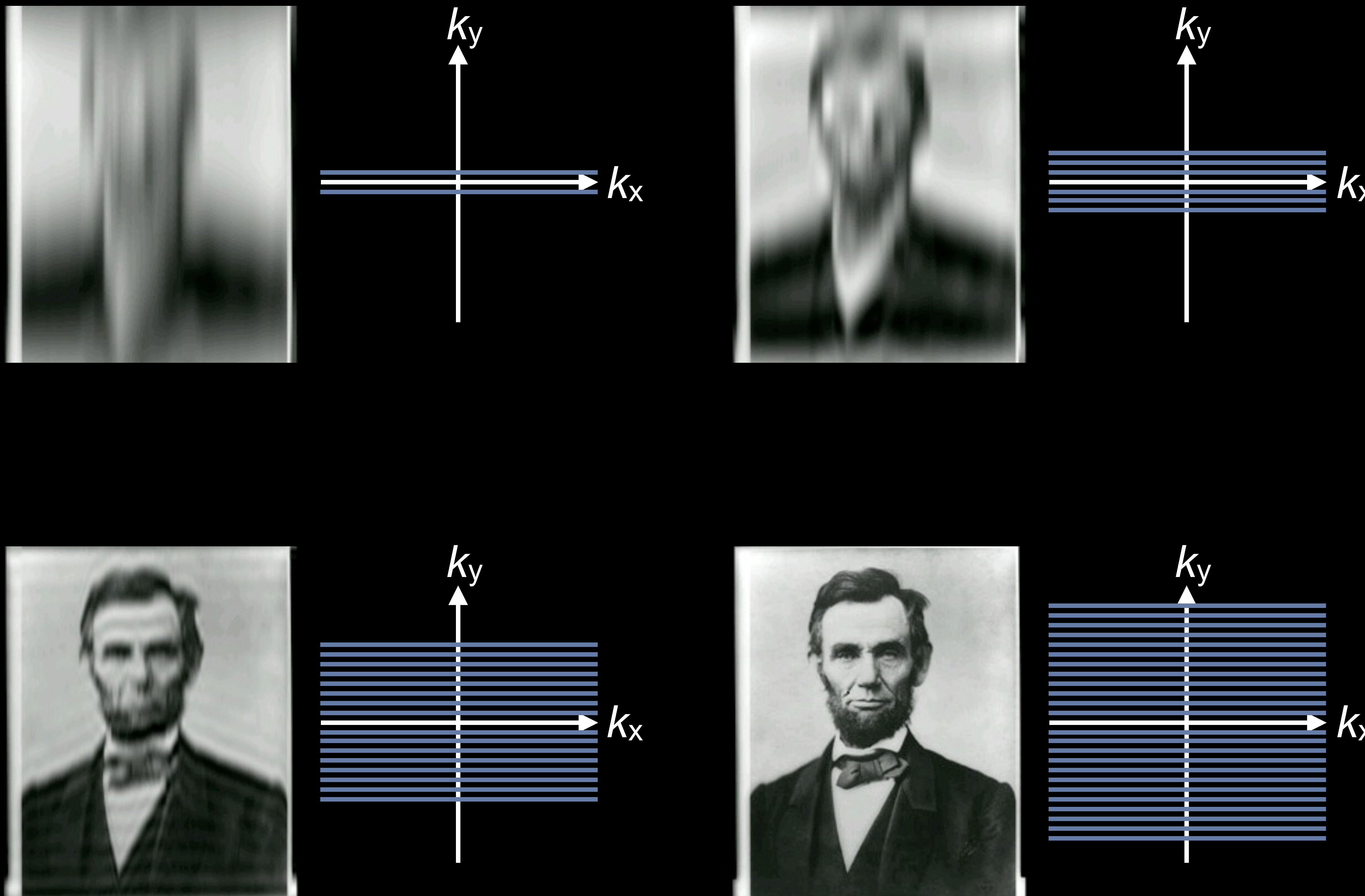
$$W_h = \frac{1}{N\Delta k} = \frac{FOV}{N}$$





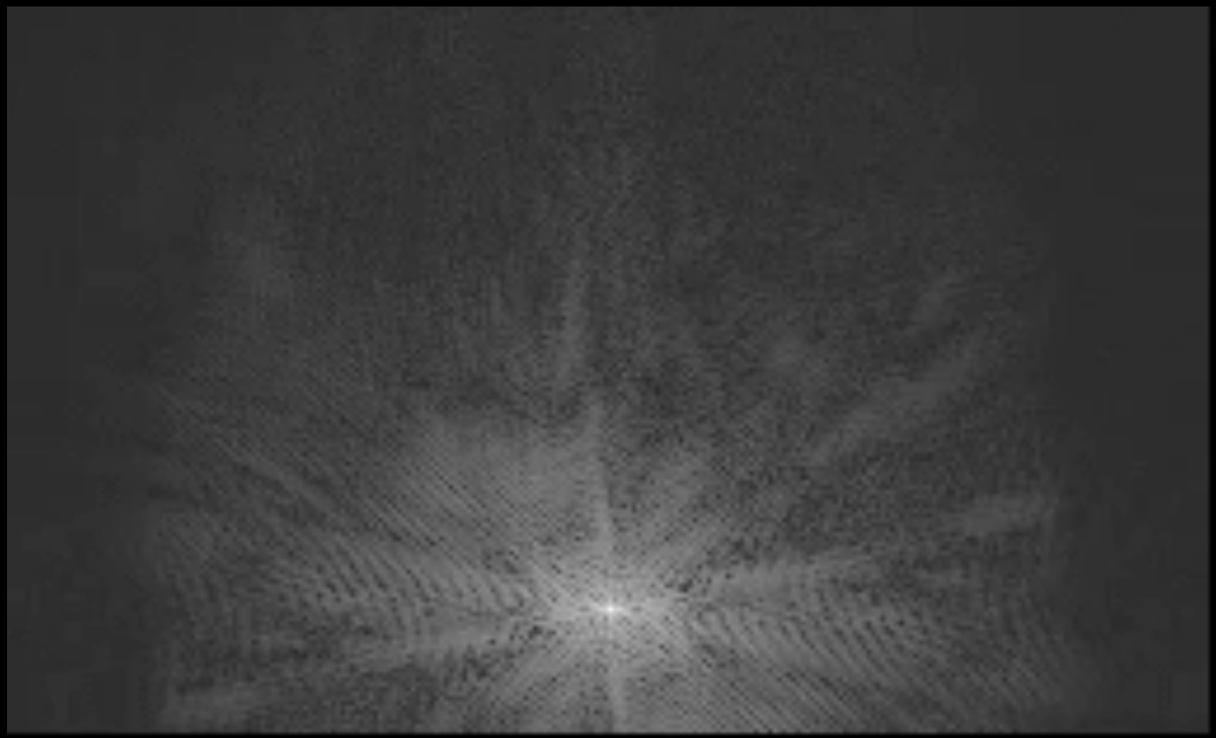


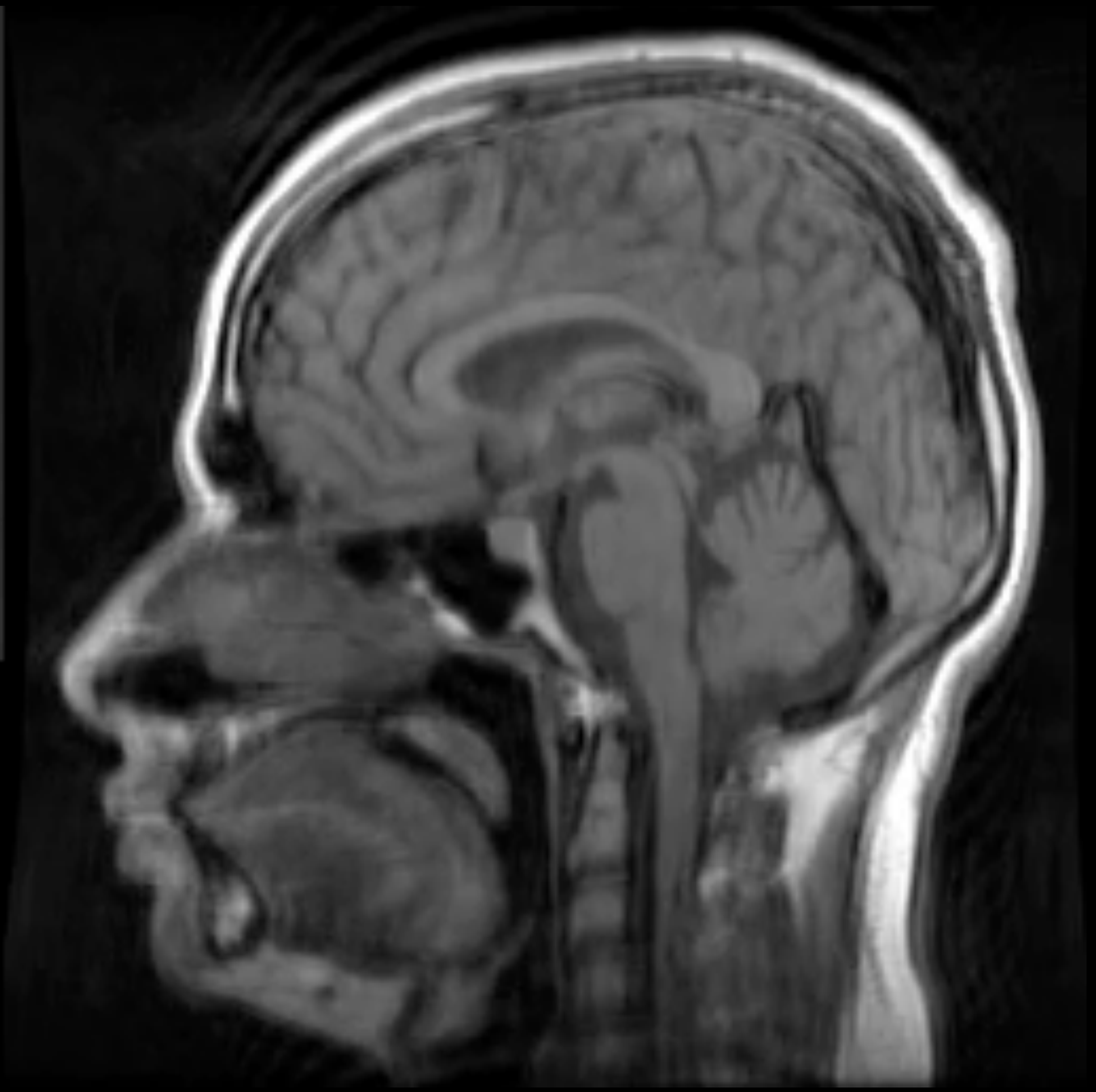
k-space Sampling



example from Dan

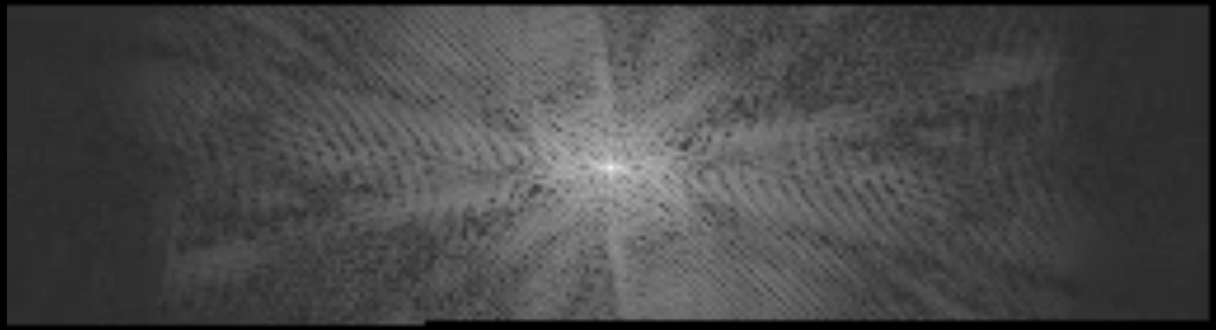
k-space Sampling

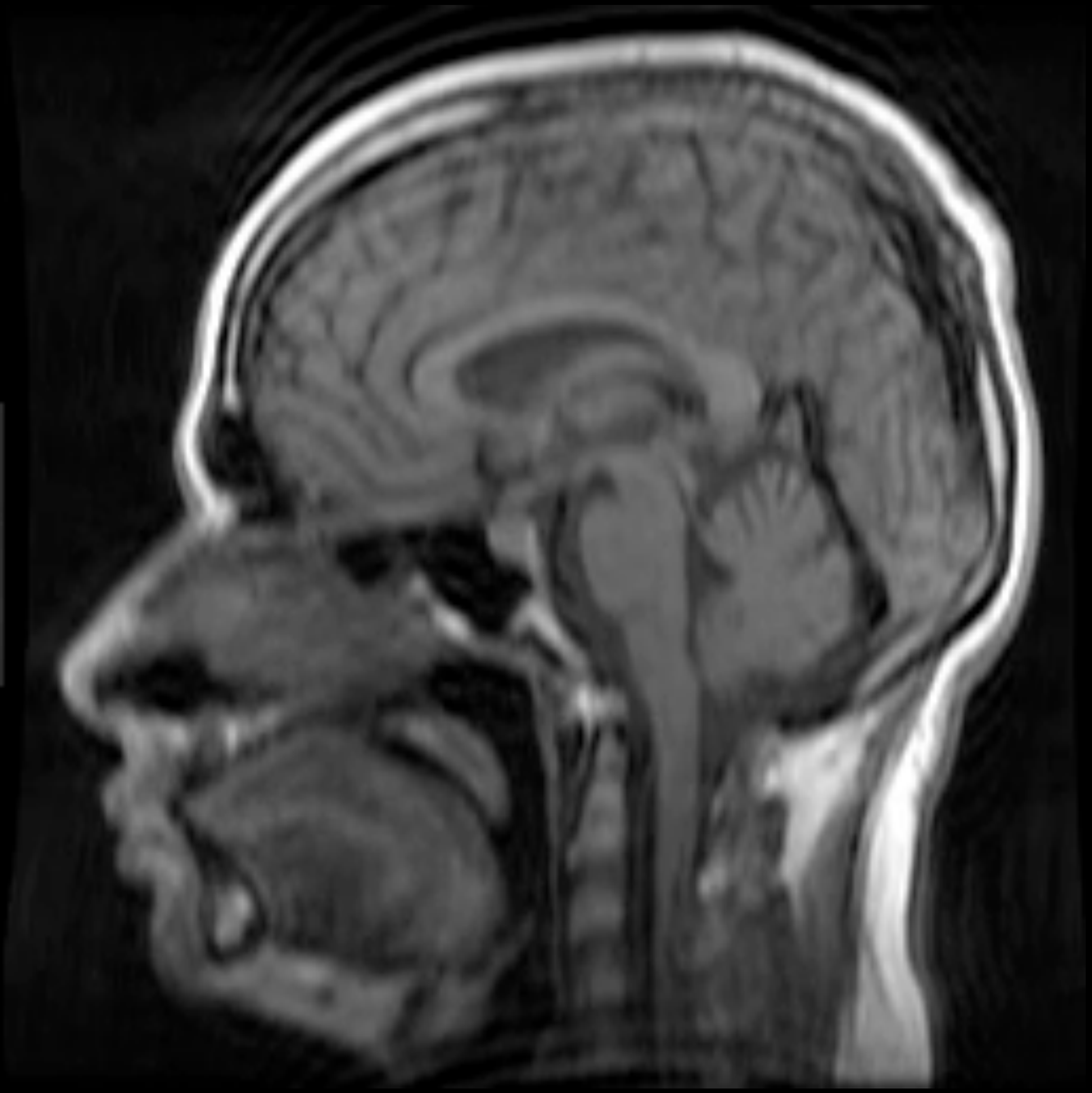




example from Dan

k-space Sampling

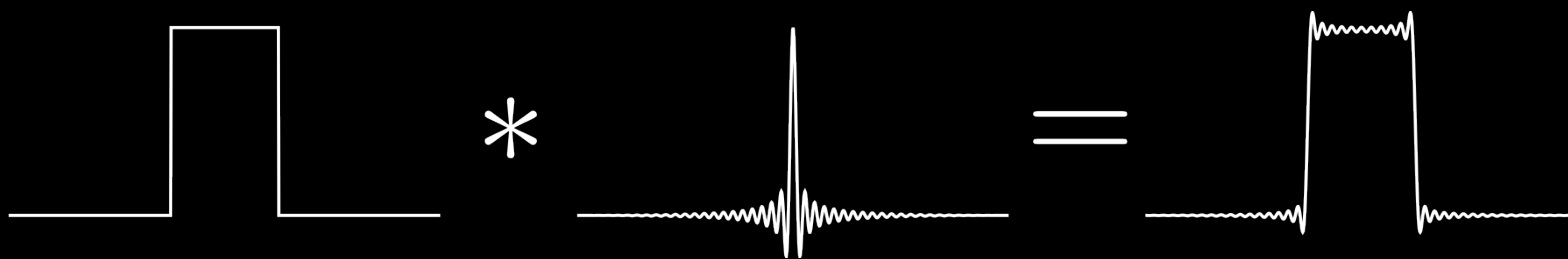




example from Dan

Gibb's Ringing

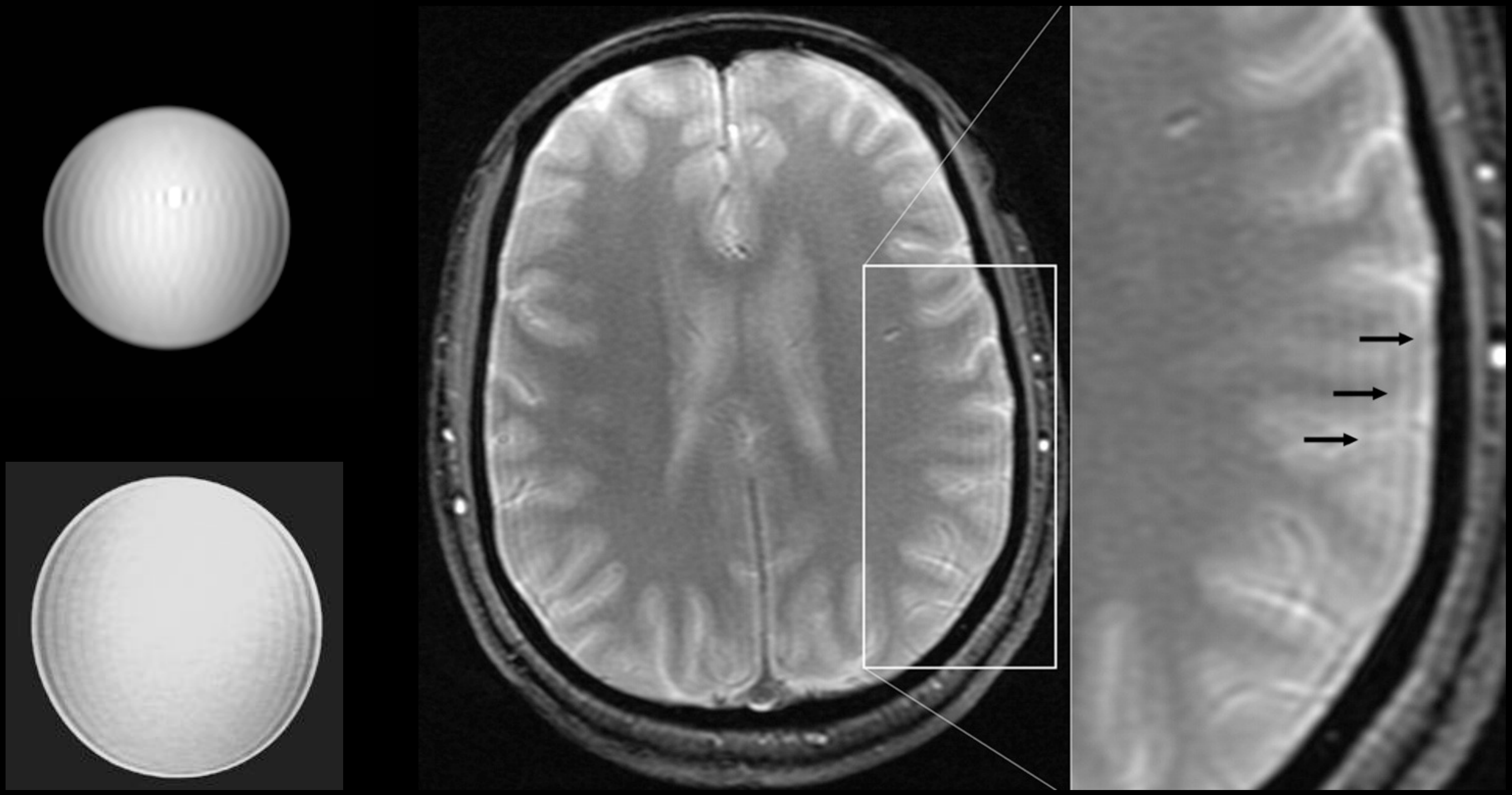
Distortions in the profile arising from the finite sampling of the data



$$\rho_{true}(\omega) \quad * \quad \mathrm{sinc}(\omega) \quad = \quad \rho_{meas}(\omega)$$

This type of distortion is most commonly referred to as Gibb's ringing

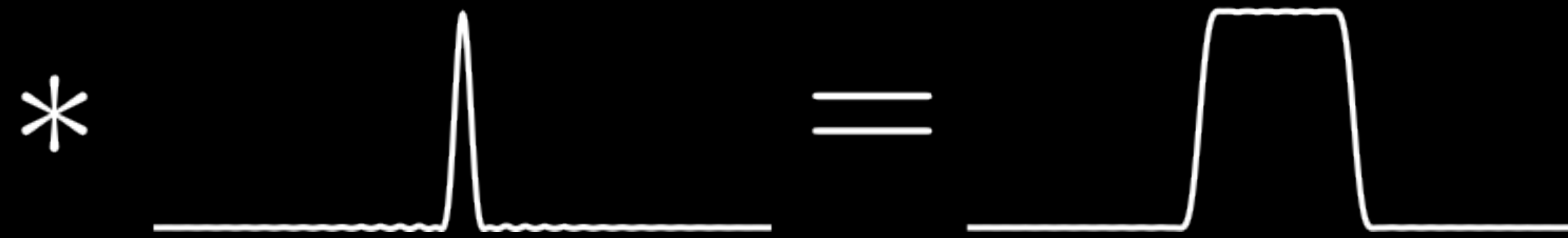
Examples of Gibb's Ringing



Gibb's Ringing

how to reduce ringing

$\mathcal{F}^{-1}\{\Pi(t)\}$



$\mathcal{F}^{-1}\{H(t)\}$

Hamming window can be used to reduce ringing

Questions?

- Related reading materials
  - Liang/Lauterbur - Chap 5.2, 5.3
  - Nishimura - Chap 5.2, 5.4, 5.5, 5.6, 5.7

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