

**M219 Principles and Applications of MRI (Winter 2023)**  
**Homework Assignment #1 (20 points)**

Assigned: 1/16/2023, Due: 1/30/2023 at 5 pm by email

E-mail a PDF (entitled M219\_HW01\_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

**Problem #1 (4 points) – B<sub>0</sub> vs. B<sub>1</sub> fields**

For a 1ms <sup>1</sup>H RF hard pulse with an  $\alpha=\pi/2$  flip angle at  $B_0 = 3T$ :

- A. Find  $\omega_1$ , the precession frequency for the B<sub>1</sub> field. (1 point)
- B. Plot the M<sub>x</sub> and M<sub>y</sub> components for Part-A (above) from 0 to 1 ms (the RF pulse duration) in the rotating frame using MATLAB. Use 1,000 points for your simulation. (1 point)
- C. Find  $\omega_0$ , the precession frequency with respect to the B<sub>0</sub> field. How many cycles of precession take place during the duration of the RF pulse? How does it compare to the B<sub>1</sub> precession frequency? (1 point)
- D. Plot M<sub>x</sub> and M<sub>y</sub> components from Part-C (above) in the laboratory frame using Matlab. Note that you'll need to change the limits of your graph and number of simulation points compared to part B. to see the precession behavior clearly. Why is this necessary? (1 point)
- E. **Extra Credit:** How does the amplitude of this B<sub>1</sub> pulse compare to the amplitude of the B<sub>0</sub> field? Explain how B<sub>1</sub> can be effective at perturbing the spin system when B<sub>0</sub> is so different in magnitude. (1 point)

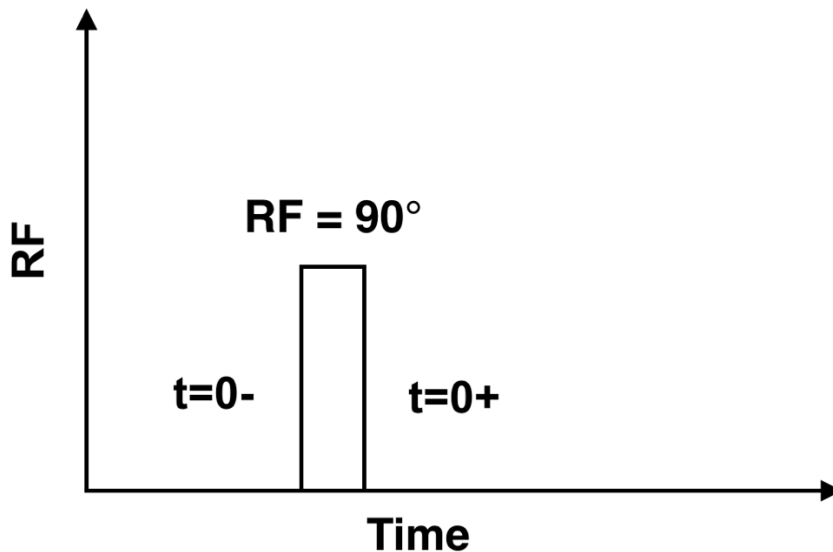
**Problem #2 (4 points) – Free Precession in the Laboratory Frame Without Relaxation**

- A. Write an expression for each component M<sub>x</sub>(0-), M<sub>y</sub>(0-), and M<sub>z</sub>(0-) immediately prior to a 90 degree RF pulse as shown in the included figure. (1 point)
- B. Write an expression for each component M<sub>x</sub>(0+), M<sub>y</sub>(0+), and M<sub>z</sub>(0+) immediately after a 90 degree RF pulse. Explain what RF phase you used. (1 point)

C. Write a general expression (include the RF phase!) as a function of time for each component  $M_x(t)$ ,  $M_y(t)$ , and  $M_z(t)$  beginning immediately after the  $90^\circ$  RF pulse. (1 point)

D. Plot all three components of your answer from (C) in MATLAB. Assume  $B_0 = 1.5T$ . Remember to make your simulation time-steps sufficiently small to observe the time varying magnetization. (1 point)

### Free Precession after a $90^\circ$ RF Pulse



### Problem #3 (4 points) – RF Pulses

Design the shortest possible inversion RF half sin-pulse (i.e.  $\sin$  on  $[0, \pi]$ ) for  $^{31}\text{P}$  at  $1.5T$  that does not exceed the RF amplifier's ability to output  $B_{1,\text{max}} = 25\mu\text{T}$ . In doing so, fully specify the variables in the following equation:

$$B_1(t) = B_1^e(t)e^{-i\omega_{RF}t}$$

Plot the components of the resultant  $B_1$  pulse amplitude in the laboratory and rotating frames as a function of time. (2 point)

Now design the shortest possible saturation RF half sin-pulse (i.e.  $\sin$  on  $[0, \pi]$ ) for  $^{19}\text{F}$  at  $3.0T$  that does not exceed the same RF amplifier's ability to output  $B_{1,\text{max}} = 25\mu\text{T}$ . Plot the components of the resultant  $B_1$  pulse amplitude in the laboratory frame as a function of time. (2 point)

#### Problem #4 (8 points) – Forced Precession in the Rotating Frame

The goal is to derive the solution for the individual magnetization components for Forced Precession in the Rotating Frame without Relaxation when the RF phase is non-zero. To do so, consider the following steps.

A. First, define  $\vec{B}_{1,rot}(t)$  from the general form of the RF pulse in the laboratory frame: (1 point)

$$\vec{B}_1(t) = B_1^E(t)[\cos(\omega_{RF}t + \theta)\hat{i} - \sin(\omega_{RF}t + \theta)\hat{j}]$$

B. Then, show the steps required to define the system of differential equations in the rotating frame in the presence of  $B_0\hat{k}$  and  $\vec{B}_{1,rot}$ . You'll want to define  $\vec{B}_{eff}$  first. (1 point)

C. Then, write the general solution for this system of differential equations. (1 point)

D. Next, write a specific solution when  $B_1^E(t) = B_1 \Pi\left(\frac{t-\tau_p/2}{\tau_p}\right)$ . (1 point)

E. Finally, use homogeneous coordinate operators to simulate the action of this hard RF pulse for  $\alpha=90^\circ$  RF pulse ( $\theta=\pi/4$ ) with  $B_{1,max}=1\mu\text{T}$  and  $B_0=0.5$  Gauss in the rotating frame. Plot the components of the bulk magnetization as a function of time. (2 points)

G. Add relaxation to your simulation in the rotating frame. Use  $T_1=1000$  ms and  $T_2=100$  ms. Plot the components of the bulk magnetization during the application of the same RF pulse. Overlay a plot when  $T_1=250$  ms and  $T_2=25$  ms. (2 points)