M219 Principles and Applications of MRI (Winter 2023)
Homework Assignment #3 (20 points)
Assigned: 2/15/2023, Due: 3/8/2023 at 5 pm by email

E-mail a PDF (entitled M219_HW03_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

Problem #1. Gradient Echo vs. Spin Echo Contrast (2 points)

The equations that describe the echo amplitude for a gradient echo and spin echo sequence are as follows:

$$A_{GRE} = \frac{\rho\left(1 - e^{-\frac{TR}{T1}}\right)}{1 - \cos(a)e^{-\frac{TR}{T1}}} \sin\alpha\, e^{-TE/T_2^*}$$

$$A_{SE} = \rho\left(1 - e^{-\frac{TR}{T1}}\right) e^{-TE/T2}$$

a) Using the above equations in MATLAB to determine the TE and TR needed to generate the maximum T_1 contrast between Tissue A (ρ = 1.0, T_1=2000ms, T_2=40ms, T_2^* =25ms) and Tissue B (ρ = 1:0, T_1=500ms, T_2=40ms, T_2^* =25ms) for both a gradient echo sequence and a spin echo sequence. Assume the pulse sequences are both limited by: 5ms<TE<100ms, 10ms<TR<10,000ms. Assume α=30 for GRE and α=90 for SE. This can be done by simulating the signal amplitude for a range of TE and TR. (1 point)

b) Is it preferable to use a gradient echo or a spin echo sequence for T_1 contrast? Why? (1 point)

Problem #2. Slice Selection (3 points)

a) If a gradient of G_z = 8 G/cm is applied in order to excite a slice that is 3mm thick at isocenter, what should be the center frequency (ω) and bandwidth (Δω) of the RF pulse for ^{1}H on a 3.0T scanner? (1 point)

b) Define ω and Δω for a slice that is +30mm from isocenter in the z-direction and 3mm thick. (1 point)

c) Redesign the RF pulse from Part A to excite 31P (γ = 17.235 MHz/T) at isocenter. What is the new center frequency ω? With the same bandwidth $\Delta\omega$ and G_z (8 G/cm), what is the new slice thickness? (1 point)

Problem #3. k-space and Image Space (5 points)

a) Import the provided image (heart.mat) into Matlab and render an image of the k-space magnitude (fft2.m). Hint: Use fftshift.m to ensure the dominant signals (low spatial frequencies) occur at the k-space center. (1 point)

b) Add a noisy spike artifact to a Fourier coefficient in the upper left quadrant of k-space and show the result in image space (ifft2.m). Describe the result and why this occurs. (1 point)

c) Remove (set to zero) all but the middle ten lines from the original k-space data (from the original FFT, without the noisy spike). Show the resulting k-space magnitude and the resultant image. Describe what you see. (1 point)

d) Rotate the image by 45º (J = imrotate(IM,-45,'bilinear','crop');). Show the resulting k-space magnitude and the resultant image. Describe what you see. (1 point)

e) Remove (set to zero) every fourth line of the k-space data from the original FFT (without the noisy spike). Show the resulting k-space magnitude and the resultant image. Describe what you see. (1 point)

Problem #4. k-space Sampling (4 points)

The bandwidth (Δf) of a rectangular readout gradient with amplitude G for an arbitrary field of view (FOV) is given by:

$$\Delta f = \gamma/2\pi \cdot G \cdot FOV$$

If the signal is read out discretely with this gradient across N-points devoting a dwell-time Δt to each point in k-space, the k-space increment is given by:

$$\Delta k = \gamma/2\pi \cdot G \cdot \Delta t$$

a) What is the spatial resolution Δx of this readout given these general expressions? (1 point)

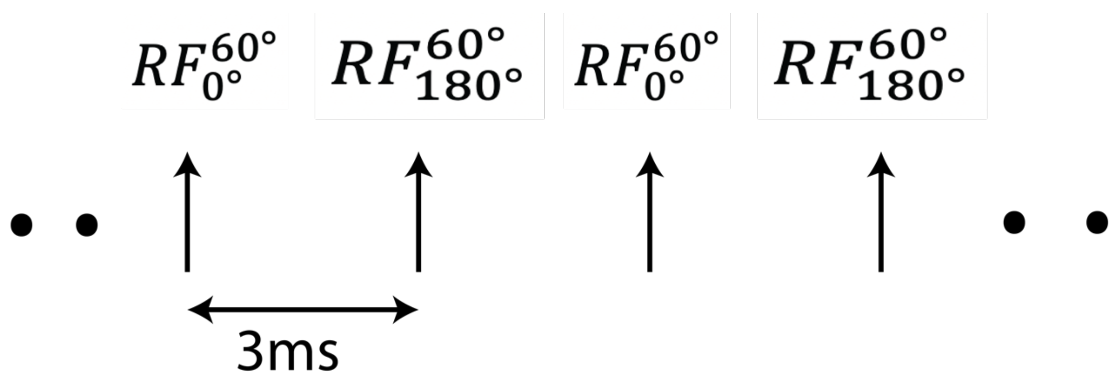
b) Combine the result of 4A with the given expressions to rewrite a simplified expression for Δf. (1 point)

c) Let N = 128, Δx = 2mm. On a 3.0T scanner, calculate Δf and Δt for G = 20 mT/m and G = 40 mT/m. Does increasing G increase or decrease Δt? (1 point)

d) How many cycles of precession are captured for each of the readout gradients from 4C within a single dwell time Δt for water at 3.0T? (1 point)

Problem #5. Steady State Signal (6 points)

The following figure shows a commonly used type of pulse sequence named "balanced steady state free precession", i.e. bSSFP. For simplicity, we only consider the RF pulses of the sequence and ignore the gradients in all three axes. The sequence uses a series of RF pulses that all have 60º flip angle, but with alternating RF phase of 0º (i.e. B1 along X' axis) and 180º. The RF pulses are equally spaced 3ms apart from each other. Assume these RF are perfect non-selective hard pulses that excite the entire volume of an object that is a perfect 30cm X 30cm X 30cm cube. The centroid of the object is at the isocenter of the MRI system. The object is a perfectly uniform object with T1=1000ms and T2=150ms.



a) Using Matlab, simulate the 3D magnetization vector M as a function of time starting from t=0ms to t=2s in 1ms increments. Assume the first RF pulse (with RF phase of 0º) is played at t=0ms and assume an initial magnetization vector of [0; 0 ;1] for the object. Make sure T1 and T2 effects are also simulated. Please plot the individual X/Y/Z components and the absolute value of M as a function of time. Describe in words the steady state motion of the M vector in 3D space during the last 10 RF pulses of the 2s pulse sequence. (3 points)

b) After you finished playing the RF displayed above, you realized that you inadvertently also included a constant gradient in the X direction with an amplitude of Gx= 0.1 mT/m. This gradient was started at t=0 ms for the entire duration of the 2s pulse sequence. Plot the X/Y/Z components and absolute value of the M vector as a function of space in the X direction in increments of 1mm (from -15cm to 15cm). What is the most characteristic pattern that can be observed from those plots? (3 points)