Bloch Equations and Relaxation II

M219 - Principles and Applications of MRI

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1/22/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #1 due on 1/29

- TA office hours, Wed 4-6pm
- Office hours, Fri 10-11am

Relationship Between Lab and Rotating Frames

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \qquad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \qquad \begin{matrix} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{matrix}$$

$$\overrightarrow{M}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{M}_{rot}(t)$$

Bulk magnetization components in the rotating frame.

$$\overrightarrow{B}_{lab}(t) = R_Z(\omega_{RF}t) \cdot \overrightarrow{B}_{rot}(t)$$

Applied B-field components in the rotating frame.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \Longrightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Nishimura - Chap 6 (Appendix I)

Bloch Equation (Rotating Frame)

$$\overrightarrow{B}(t) = B_0\hat{k} + B_1^e(t)[\cos(\omega_{RF}t+\theta)\hat{i} - \sin(\omega_{RF}t+\theta)\hat{j}]$$

$$\overrightarrow{B}_{lab}(t) = \begin{pmatrix} B_1^e(t)\cos(\omega_{RF}t+\theta) \\ -B_1^e(t)\sin(\omega_{RF}t+\theta) \\ B_0 \end{pmatrix} \qquad \overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t)\cos\theta \\ -B_1^e(t)\sin\theta \\ B_0 \end{pmatrix}$$

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

Effective B-field that *M* experiences in the rotating frame.

Fictitious field that demodulates the apparent effect of B_0.

Applied B-field in the rotating frame.

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

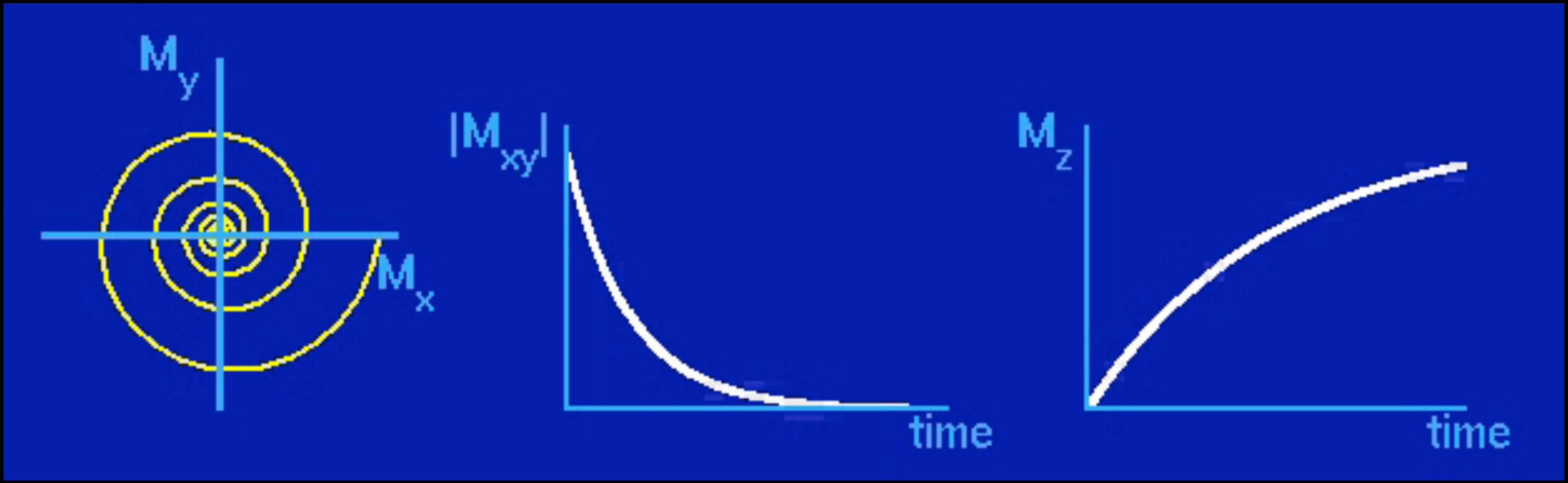
Assume no RF phase ($\theta = 0$)

$$\overrightarrow{B}_{rot}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \end{pmatrix} \qquad \overrightarrow{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

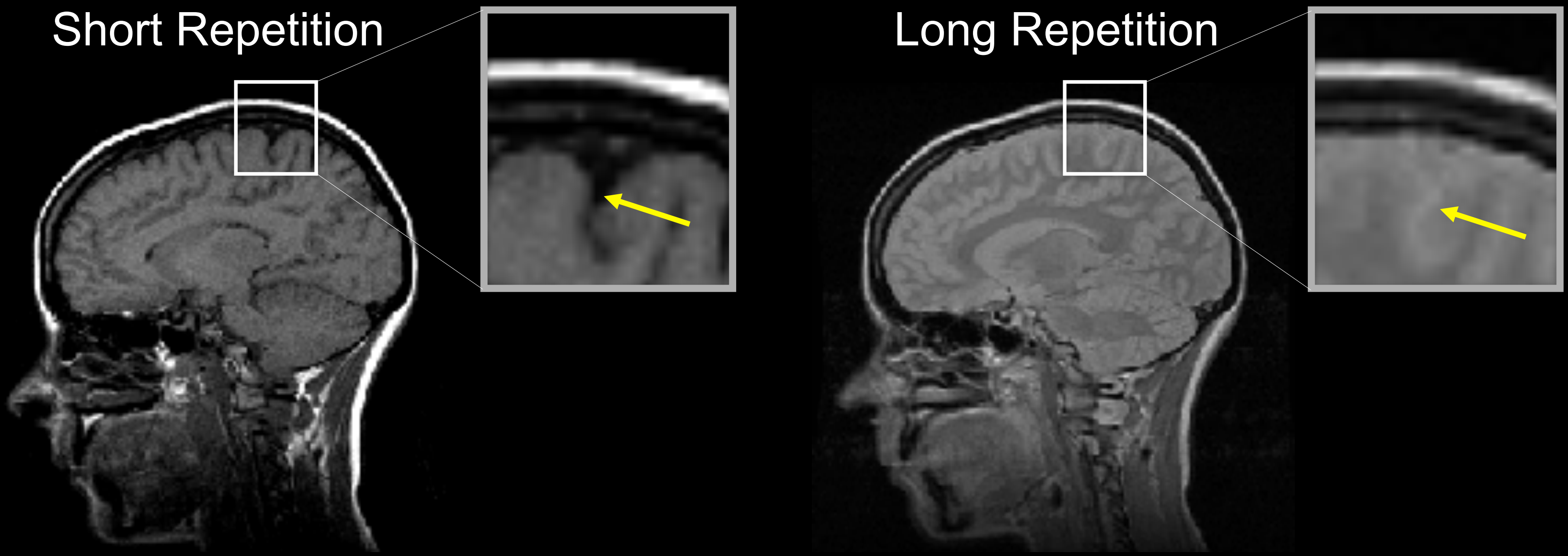
$$\overrightarrow{B}_{eff}(t) = \begin{pmatrix} B_1^e(t) \\ 0 \\ B_0 \quad \frac{\omega_{RF}}{\gamma} \end{pmatrix}$$

Relaxation

- Magnetization returns exponentially to equilibrium:
  - Longitudinal recovery time constant is T1
  - Transverse decay time constant is T2
- Relaxation and precession are independent

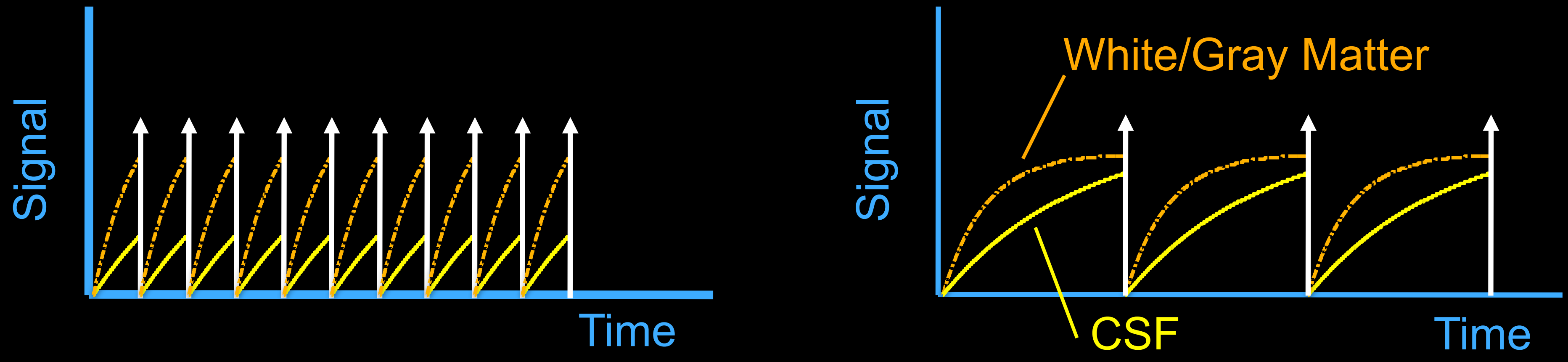


T$_1$ Contrast



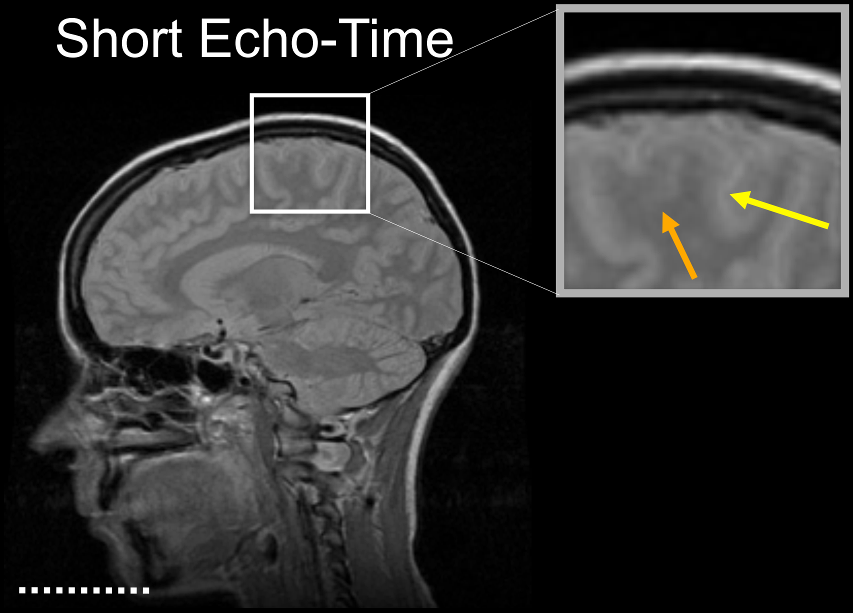
Short Repetition

Long Repetition

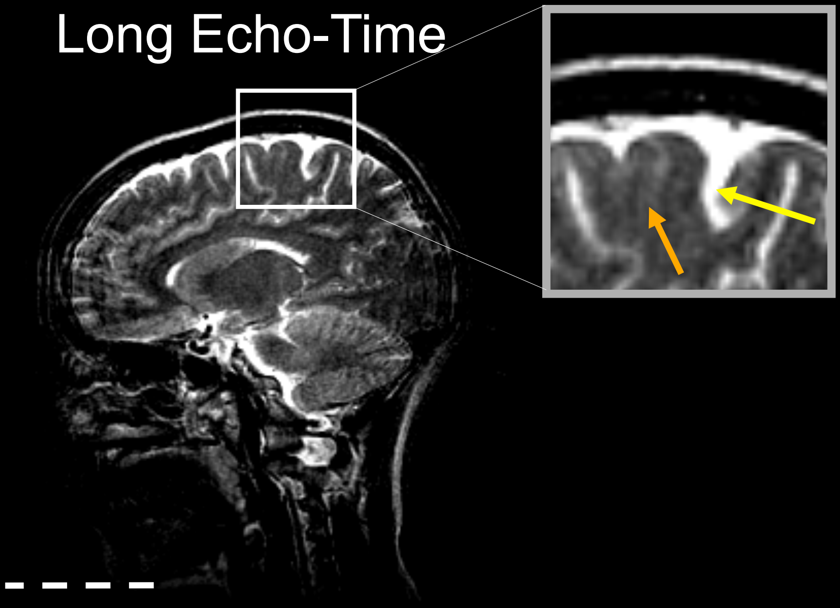


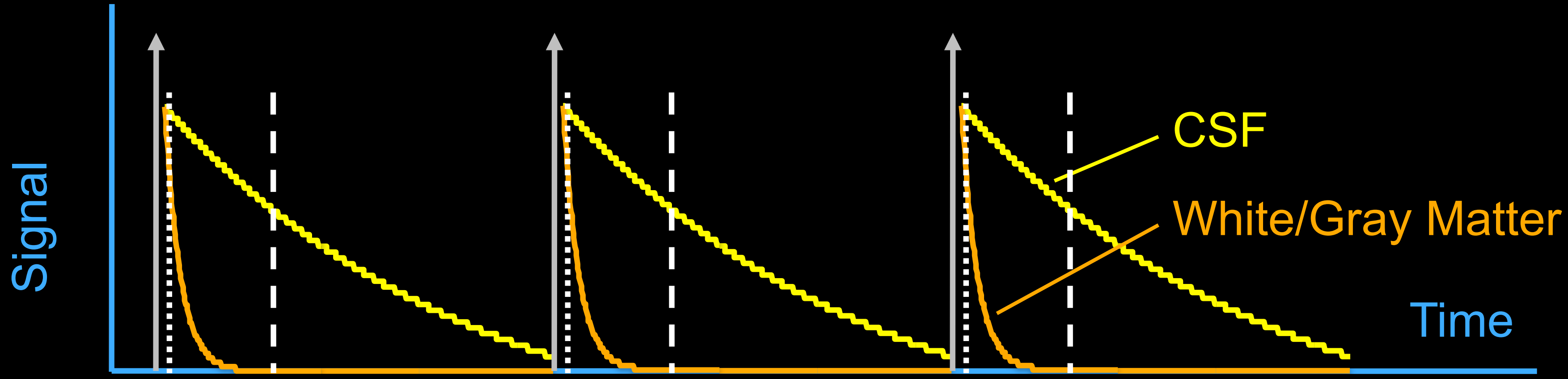
T2 Contrast

Short Echo-Time



Long Echo-Time





T_2^* Relaxation

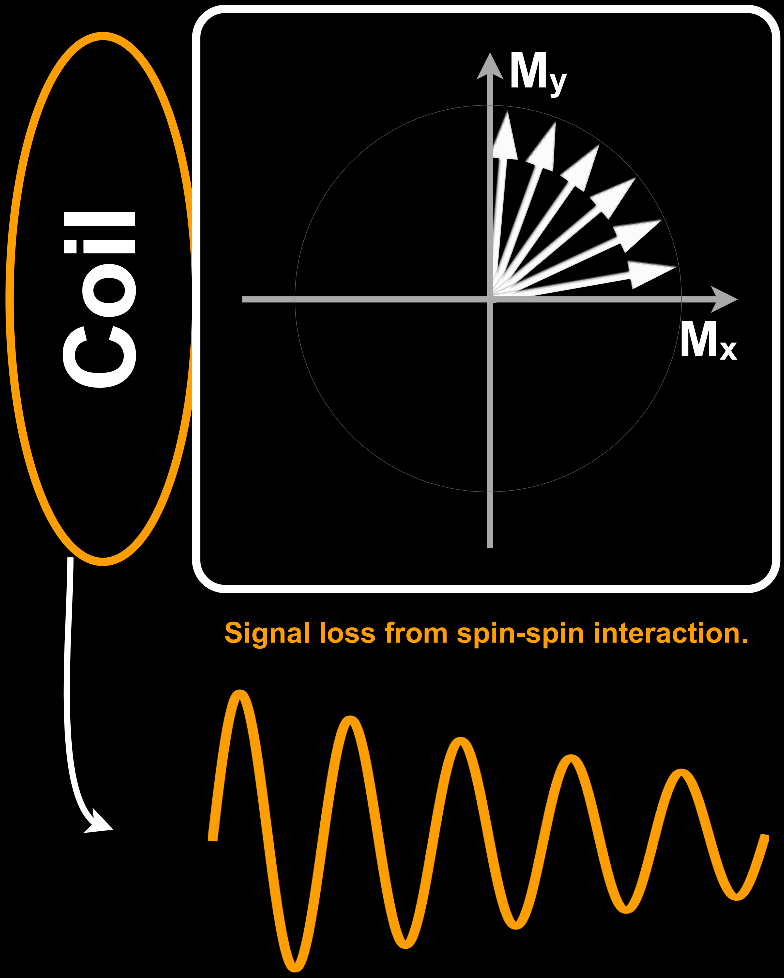
T_2^* Relaxation

$$\frac{1}{T_2^*} = \frac{1}{T_2} + \gamma \Delta B_0$$

- T_2^* is "observed" transverse relaxation time constant
- T_2^* consists of *irreversible spin-spin (T_2) dephasing* and *reversible intravoxel spin de-phasing* due to off-resonance
- Sources of off-resonance:
  - B_0 inhomogeneity
  - susceptibility differences (e.g. air spaces)

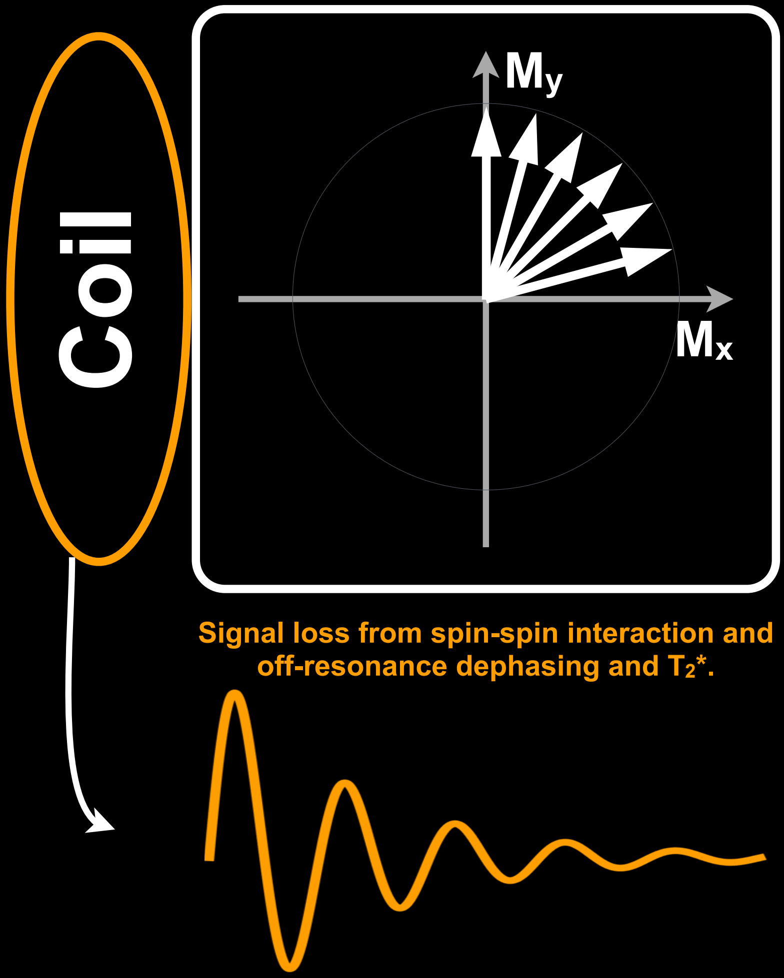
T_2 versus T_2^*

T_2 Decay



Signal loss from spin-spin interaction.

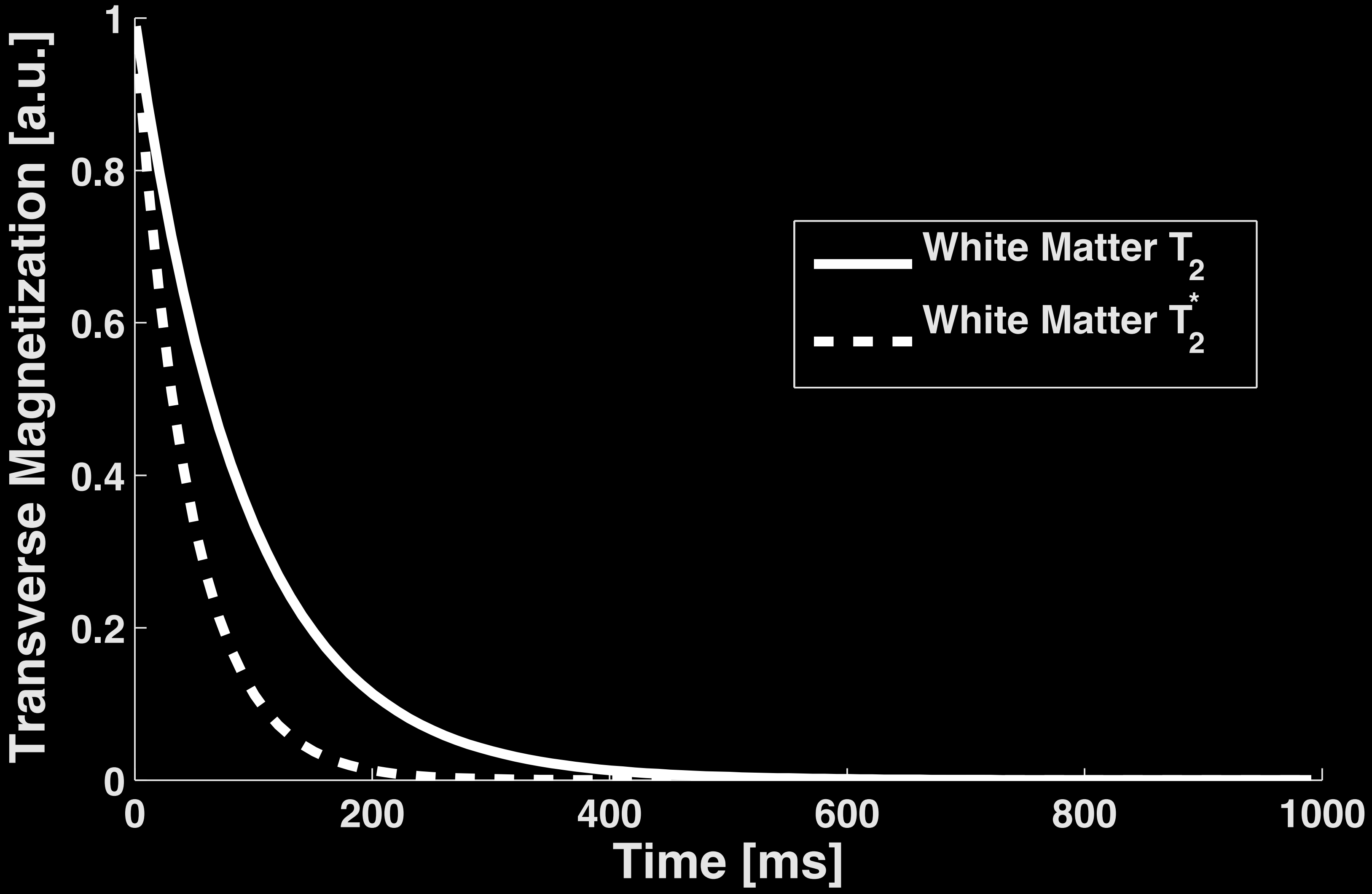
T_2^* Decay



Signal loss from spin-spin interaction and off-resonance dephasing and T_2^*.

T_2^* is signal loss from spin dephasing and T_2

T2*<T2 (always!)


Transverse Magnetization [a.u.]
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
Time [ms]
White Matter T_2
White Matter T_2^*

Bloch Equations with Relaxation

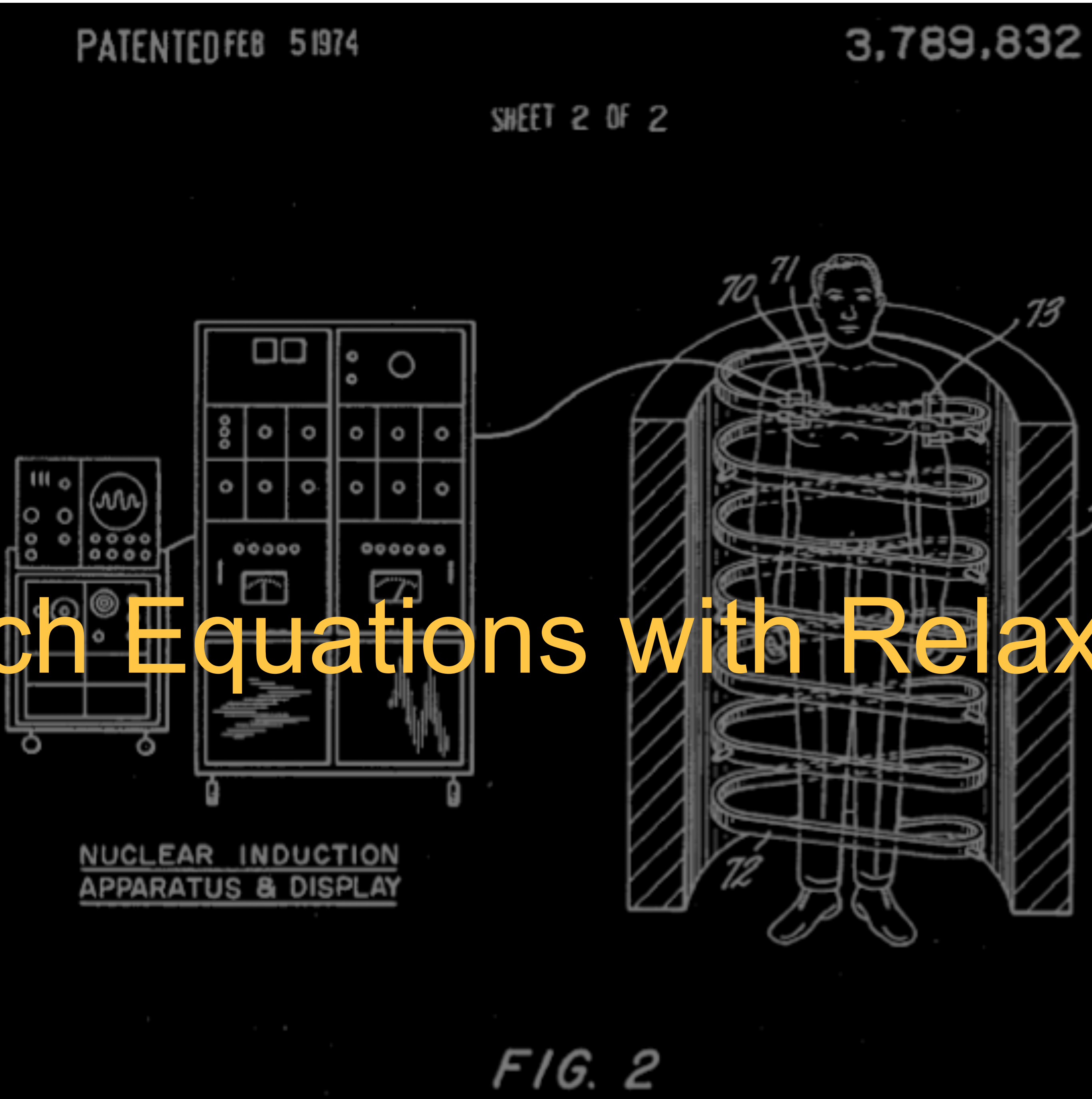


FIG. 2

Bloch Equations with Relaxation

$$\frac{d\vec{\mathrm{M}}}{dt} = \vec{\mathrm{M}} \times \gamma\vec{\mathrm{B}} - \frac{M_x\hat{\mathrm{i}} + M_y\hat{\mathrm{j}}}{T_2} - \frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}$$

- **Differential Equation**
  - Ordinary, Coupled, Non-linear
- **No analytic solution, in general.**
  - Analytic solutions for simple cases.
  - Numerical solutions for all cases.
- **Phenomenological**
  - Exponential behavior is an approximation.

Bloch Equations - Lab Frame

$$\frac{d\vec{\mathrm{M}}}{dt} = \underbrace{\vec{\mathrm{M}} \times \gamma \vec{\mathrm{B}}}_{\text{Precession}} - \underbrace{\frac{M_x \hat{\mathrm{i}} + M_y \hat{\mathrm{j}}}{T_2}}_{\substack{\text{Transverse} \\ \text{Relaxation}}} - \underbrace{\frac{(M_z - M_0)\,\hat{\mathrm{k}}}{T_1}}_{\substack{\text{Longitudinal} \\ \text{Relaxation}}}$$

- Precession
  - Magnitude of M unchanged
  - Phase (rotation) of M changes due to B
- Relaxation
  - T_1 changes are slow O(100ms)
  - T_2 changes are fast O(10ms)
  - Magnitude of M can be ZERO
- Diffusion
  - Spins are thermodynamically driven to exchange positions.
  - Bloch-Torrey Equations

Bloch Equations – Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \underbrace{\gamma \vec{M}_{rot} \times \vec{B}_{eff}}_{\text{“Precession”}} - \underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

Effective B-field that M experiences in the rotating frame

Fictitious field created by the rotating frame that demodulates the apparent effect of B_0

The applied B_0 and B_1 field in the rotating frame

Free Precession
in the Rotating Frame
with Relaxation

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{\mathrm{B}}_{rot} = \mathrm{B}_0 \hat{k}$$

$$\vec{B}_{eff} = \vec{0}$$

$$\frac{\partial \vec{M}_{rot}}{\partial t} = - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \underbrace{-\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\text{Transverse Relaxation}} \underbrace{-\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\text{Longitudinal Relaxation}}$$

- **No precession**
- **T_1 and T_2 Relaxation**
- **Drop the diffusion term**
- **System or first order, linear, separable ODEs!**

The precessional term drops out in the rotating frame.

Free Precession in the Rotating Frame

$$\frac{\partial \vec{M}_{rot}}{\partial t} = -\underbrace{\frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2}}_{\substack{\text{Transverse}\\ \text{Relaxation}}} - \underbrace{\frac{(M_{z'} - M_0)\vec{k}'}{T_1}}_{\substack{\text{Longitudinal}\\ \text{Relaxation}}}$$

Solution:

$$M_{z'}(t) = M_z^0 e^{-t/T_1} + M_0\left(1 - e^{-t/T_1}\right)$$

$$M_{x'y'}(t) = M_{x'y'}(0_+)e^{-t/T_2}$$

The precessional term drops out in the rotating frame.

Forced Precession
in the Rotating Frame
with Relaxation

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} \triangleq \frac{\vec{\omega}}{\gamma} + \vec{B}_{rot}$$

$$\vec{\omega}_{rot} = \vec{\omega} = -\gamma \mathrm{B}_0 \hat{k} \qquad \vec{B}_{rot} = B_0 \hat{k} + B_1^e(t)\hat{i}'$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

The precessional term ***does not*** drop out in the rotating frame.

Forced Precession in the Rot. Frame with Relaxation

$$\frac{\partial \vec{M}_{rot}}{\partial t} = \gamma \vec{M}_{rot} \times \vec{B}_{eff} - \frac{M_{x'}\vec{i}' + M_{y'}\vec{j}'}{T_2} - \frac{(M_{z'} - M_0)\vec{k}'}{T_1}$$

$$\vec{B}_{eff} = B_1^e(t)\hat{i}'$$

- B1 induced nutation
- T_1 and T_2 Relaxation
- Drop the diffusion term
- System or first order, linear, coupled PDEs!
- When does this equation apply?

The precessional term ***does not*** drop out in the rotating frame.

Forced Precession in the Rotating Frame with Relaxation

- RF pulses are short
  - 100μs to 5ms
- Relaxation time constants are long
  - T_1 O(100s) ms
  - T_2 O(10s) ms
- Complicated Coupling
- Best suited for simulation

Free? Forced? Relaxation?

- We've considered all combinations of:
  - Free and forced precession
  - With and without relaxation
  - Laboratory and rotating frames
- Which one's concern M219 the most?
  - Free precession in the rotating frame with relaxation
  - Forced precession in the rotating frame without relaxation.
- We can, in fact, simulate all of them…

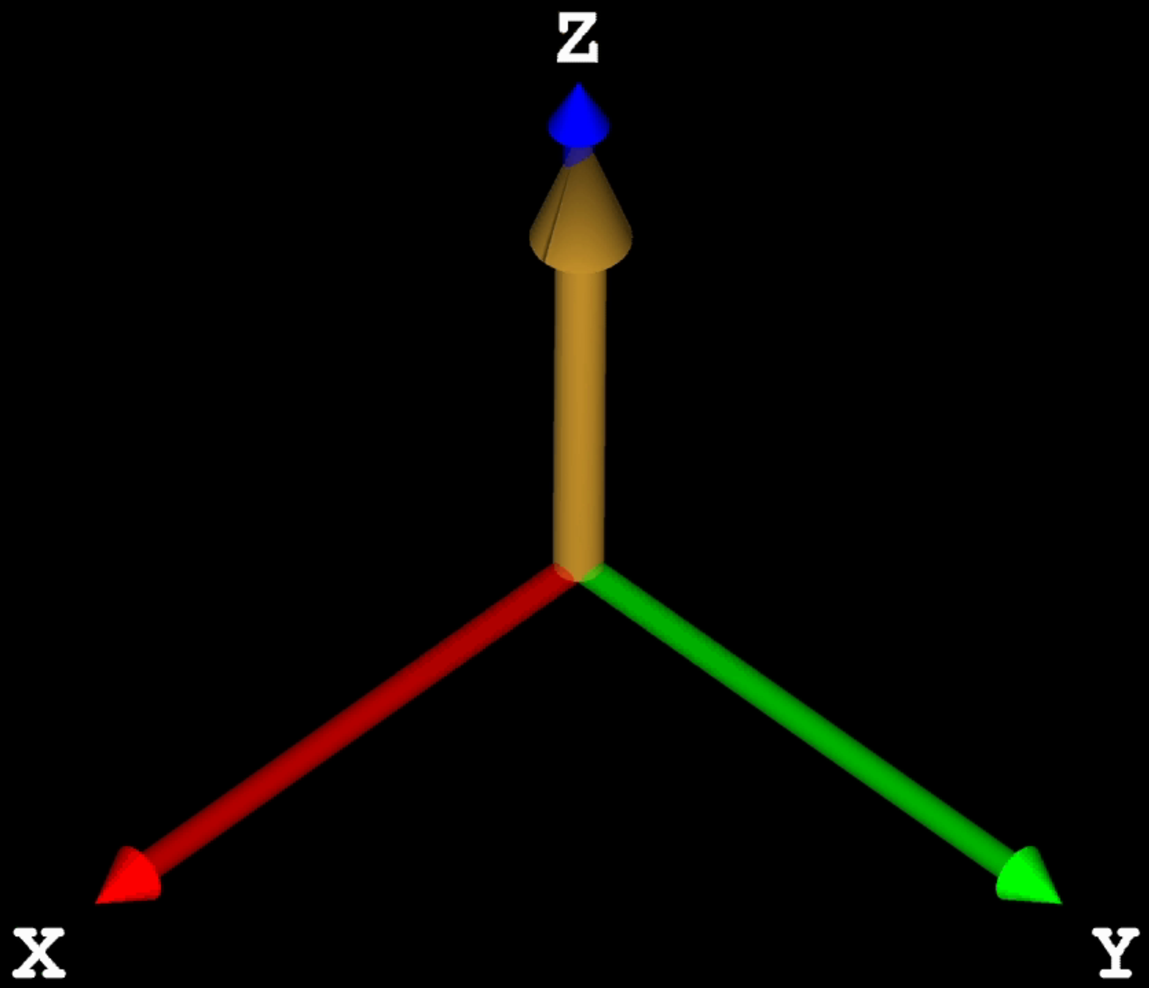
Types of RF Pulses

- Excitation Pulses
- Inversion Pulses
- Refocusing Pulses
- Saturation Pulses
- Spectrally Selective Pulses
- Spectral-spatial Pulses
- Adiabatic Pulses

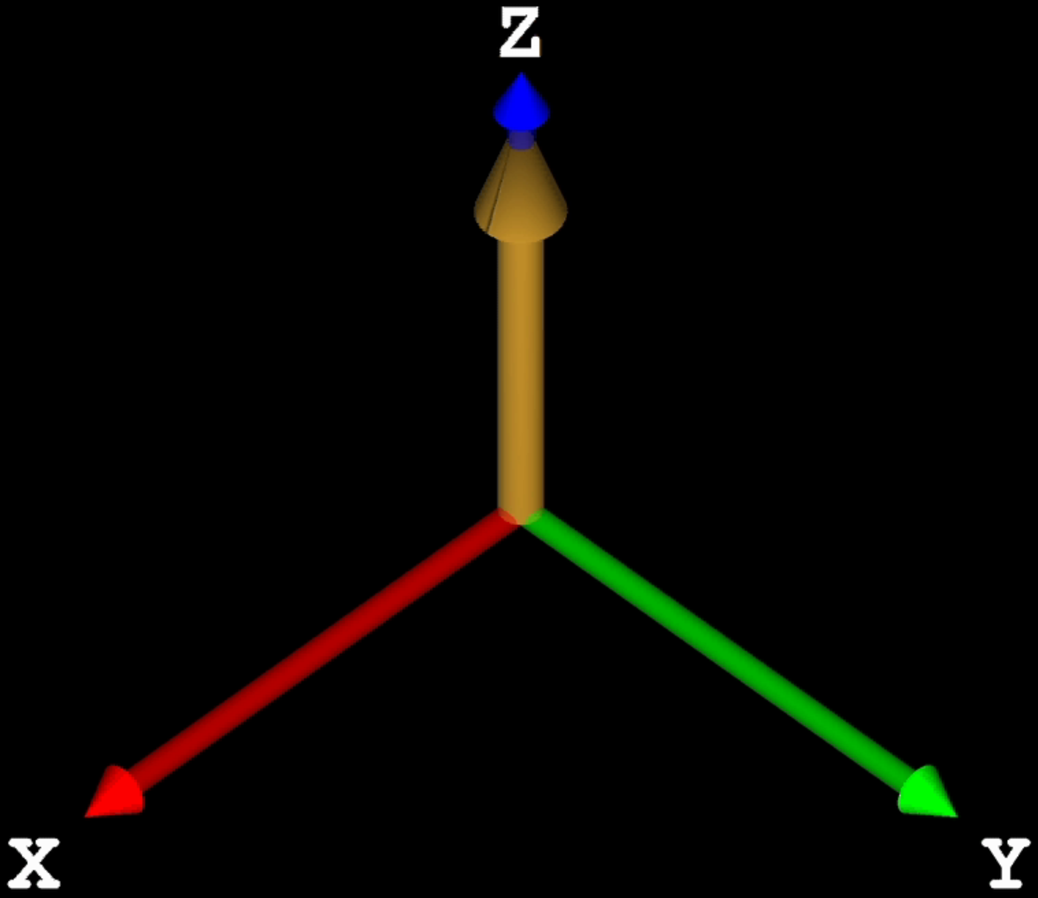
Excitation Pulses

- Tip M_z into the transverse plane
- Typically 200μs to 5ms
- Non-uniform across slice thickness
  - Imperfect slice profile
- Non-uniform within slice
  - Termed B_1 inhomogeneity
  - Non-uniform signal intensity across FOV

90° Excitation Pulse



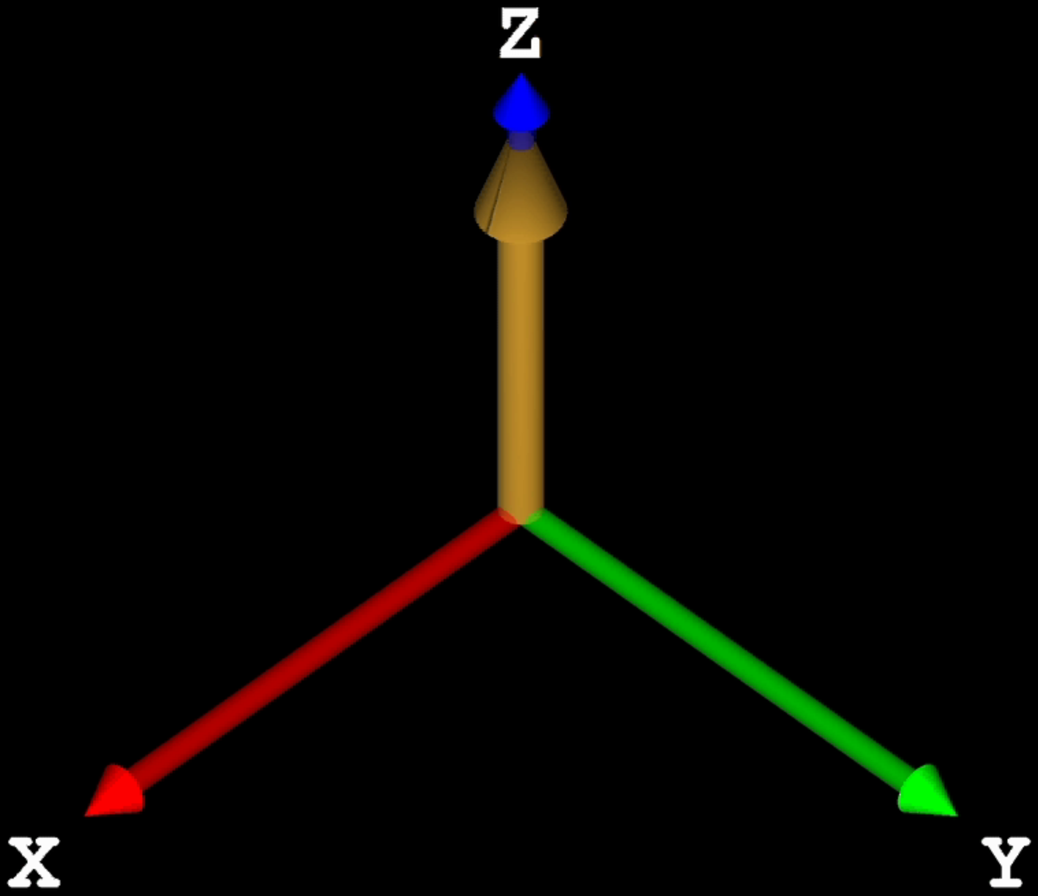
Small Flip Angle Excitation



Inversion Pulses

- Typically, 180° RF Pulse
  - non-180° that still results in $-M_Z$
- Invert M_Z to $-M_Z$
  - Ideally produces no M_{XY}
- Hard Pulse
  - Constant RF amplitude
  - Typically non-selective
- Soft (Amplitude Modulated) Pulse
  - Frequency selective
  - Spatially Selective

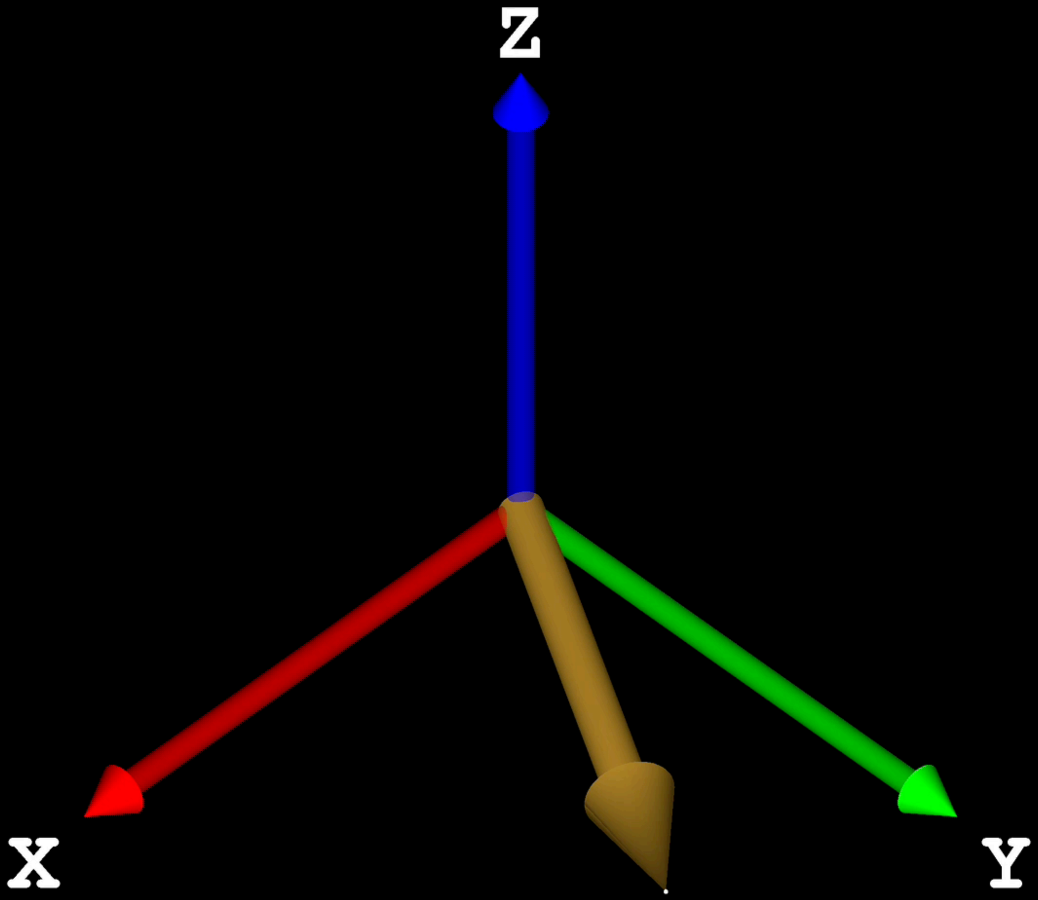
Inversion Pulses



Refocusing Pulses

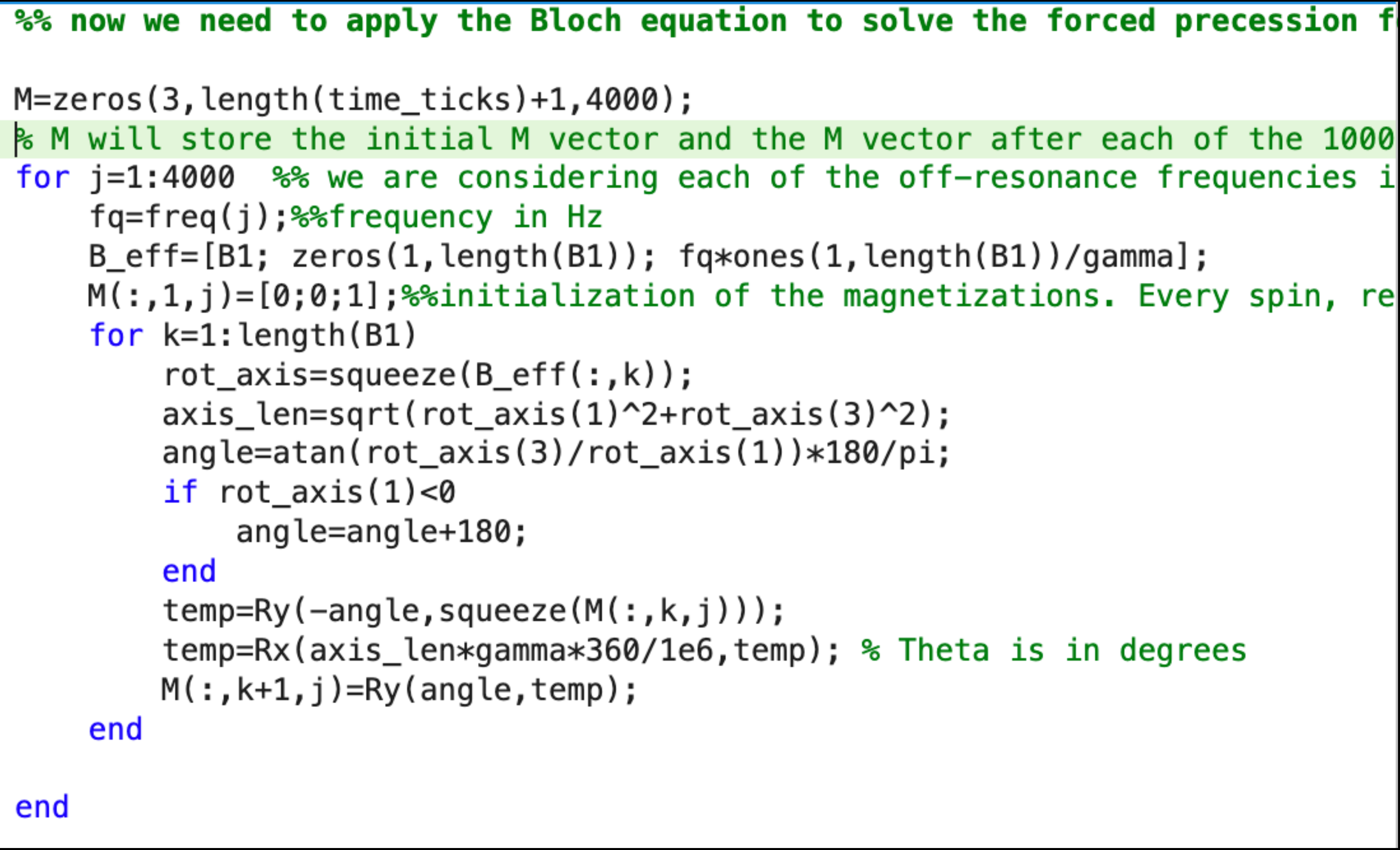
- Typically, 180° RF Pulse
  - Provides optimally refocused M_{XY}
  - Largest spin echo signal
- non-180°
  - Partial refocusing
  - Lower SAR
  - Multiple non-180° produce stimulated echoes
- Refocus spin dephasing due to
  - imaging gradients
  - local magnetic field inhomogeneity
  - magnetic susceptibility variation
  - chemical shift

Refocusing Pulses



Frequency Selectivity of RF Pulses

Matlab Demo



```
%% now we need to apply the Bloch equation to solve the forced precession f

M=zeros(3,length(time_ticks)+1,4000);
% M will store the initial M vector and the M vector after each of the 1000
for j=1:4000  %% we are considering each of the off-resonance frequencies i
    fq=freq(j);%%frequency in Hz
    B_eff=[B1; zeros(1,length(B1)); fq*ones(1,length(B1))/gamma];
    M(:,1,j)=[0;0;1];%%initialization of the magnetizations. Every spin, re
    for k=1:length(B1)
        rot_axis=squeeze(B_eff(:,k));
        axis_len=sqrt(rot_axis(1)^2+rot_axis(3)^2);
        angle=atan(rot_axis(3)/rot_axis(1))*180/pi;
        if rot_axis(1)<0
            angle=angle+180;
        end
        temp=Ry(-angle,squeeze(M(:,k,j)));
        temp=Rx(axis_len*gamma*360/1e6,temp); % Theta is in degrees
        M(:,k+1,j)=Ry(angle,temp);
    end

end
```

Questions?

- Related reading materials
  - Nishimura - Chap 4 and 5
  - Nishimura - Chap 6 (Appendix I)

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