Imaging Sequences I

M219 - Principles and Applications of MRI

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2/19/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #3 due on 3/5
- TA office hours, Mon 4-6pm
- Office hours, Fri 10-11am

RF Pulse Bandwidth and Slice Profile:

Small Tip Angle Approximation

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

When we simplify the cross product,

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

Small Tip Approximation

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

sin θ ≈ θ
cos θ ≈ 1
$M_z \approx M_0 \rightarrow$ constant

$$\frac{dM_z}{dt} = 0$$

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

$$M_{xy} = M_x + iM_y$$

First order linear differential equation. Easily solved.

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

Solving a first order linear differential equation:

$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma G_z z \cdot (t-s)} ds$$

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\mathcal{T}}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

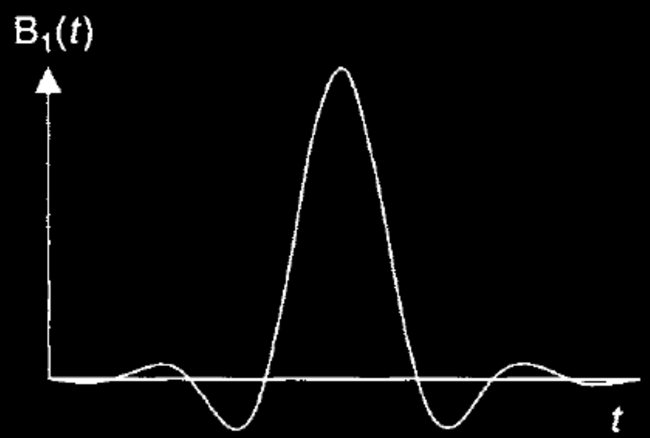
(See the note for complete derivation)

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

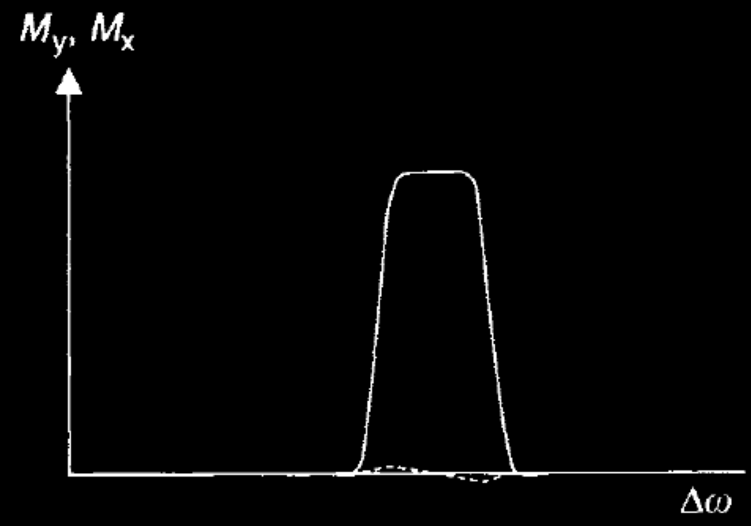
To the Board

Small Tip Approximation

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$



Fourier transform



- For small tip angles, “the slice or frequency profile is well approximated by the Fourier transform of B1(t)”
- The approximation works surprisingly well even for flip angles up to 90°

Small Tip Approximation

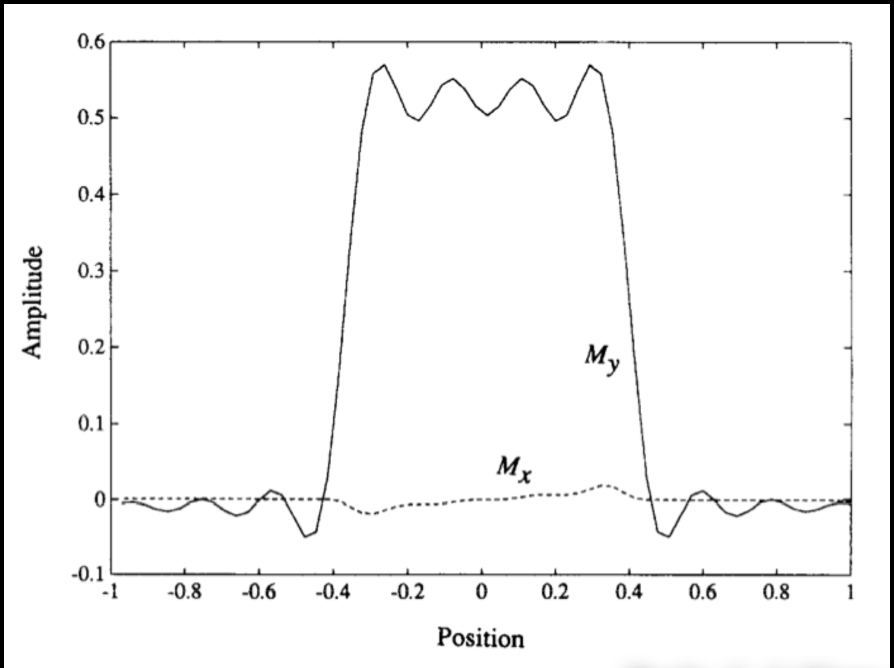
the excitation profile, within the small angle approximation, is just the Fourier transform of the pulse

remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly

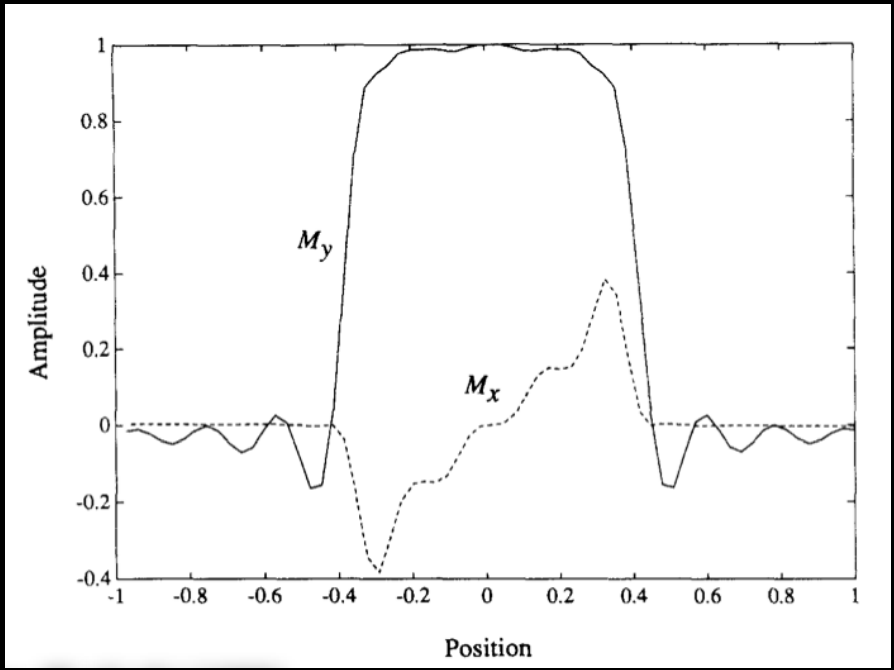
the approximation works surprisingly well even for flip angles up to 90^0

Shaped Pulses

30°



90°



Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

small-angle approximation still works reasonably well for flip angles that aren't necessarily "small"

Truncation Artifacts

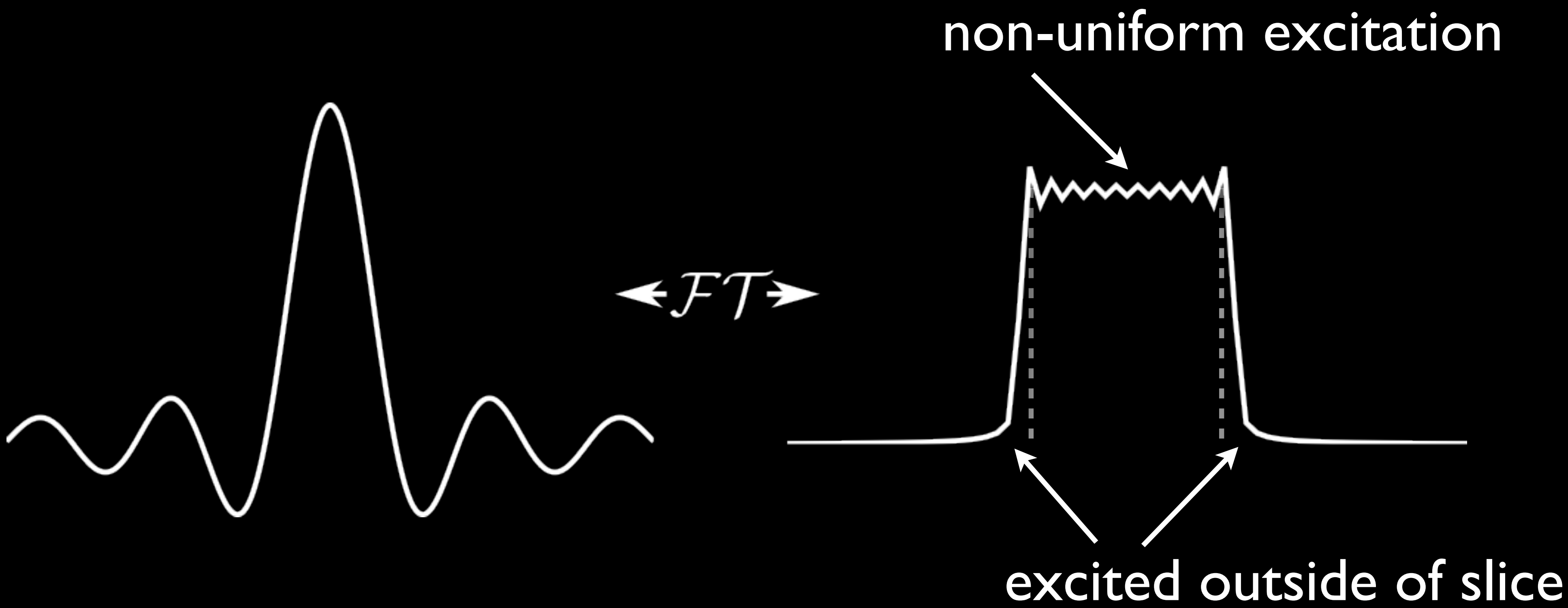
in MRI we want pulses to be as short as possible to avoid relaxation effects

the sinc function is defined over all time which is impractical in any experiment

the sinc pulse needs to be truncated to be appropriate for clinical scans

Truncation Artifacts

what happens when we truncate our pulses?



these deviations from the ideal are known as truncation artifacts

Truncation Artifacts

alternative Pulse Shapes

gaussian

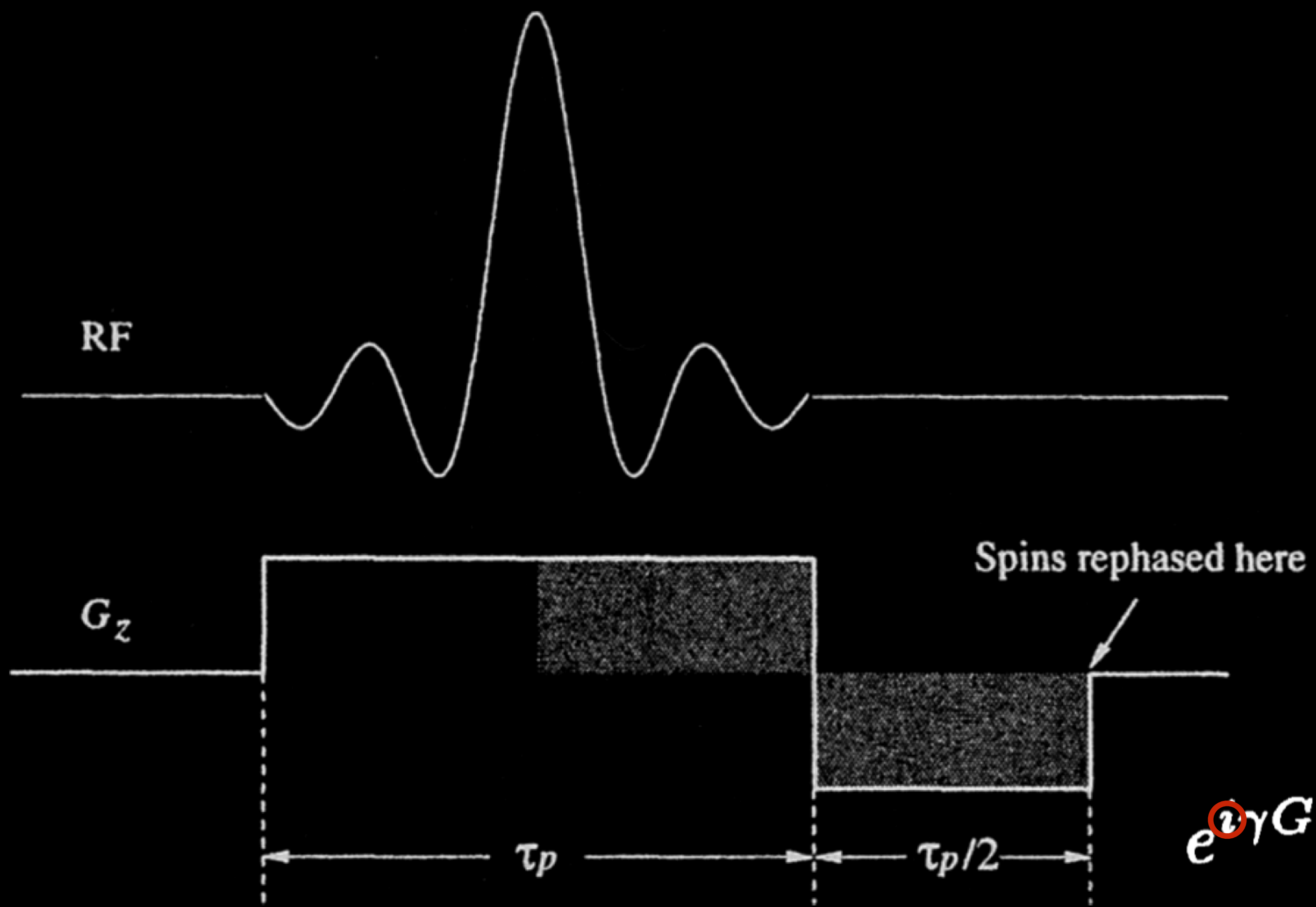
$$B_x(t) = A \exp\left[-a(t - \tau/2)^2\right]$$

reduced side-lobes, but not as flat of a profile

Window Functions

Hamming, Hanning, …

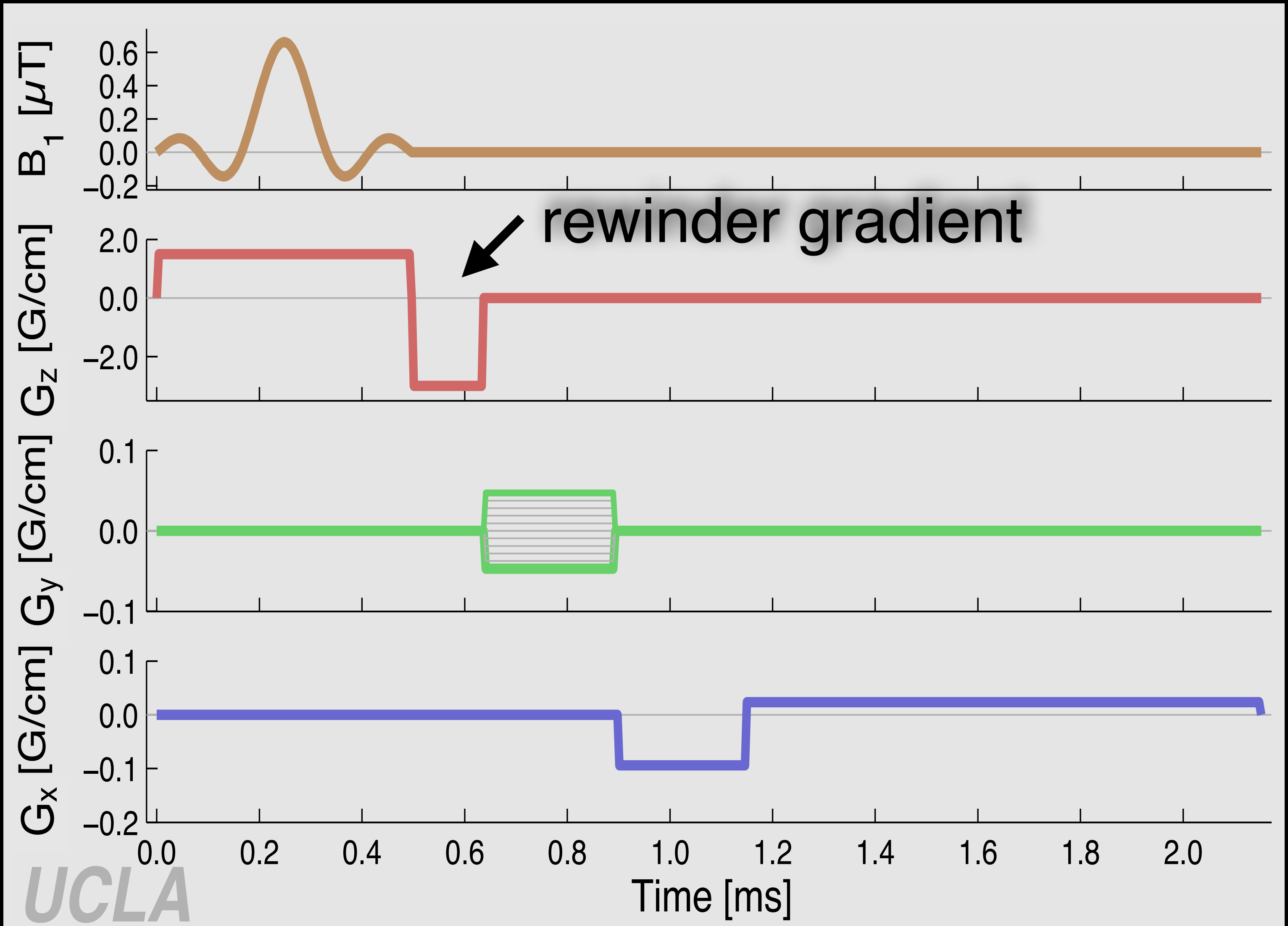
Slice Rewinder



$$e^{i\gamma G_z(z-z_0)\tau_p/2}$$

Opposite Polarity

Slice Selective Excitation Example



slice select gradient rewinder eliminates the linear phase ramp

Noise Considerations

Noise Considerations

- Signal-to-Noise Ratio (SNR)
  - A fundamental measure of image quality
  - $SNR \triangleq \dfrac{signal\ amplitude}{\sigma\ of\ noise}$
  - $SNR_{dB} = 20 \cdot log(SNR)$

Nishimura Ch. 7.5

Noise Considerations

- Effect of Acquisition Time
  - Simple 1D example (impulse in image space)
  - *N* samples in *k*-space, each with amplitude *A*
  - Noise variances add (independence)
  - $$SNR = \frac{\sum_{j=1}^{N} A}{\sqrt{\sum_{j=1}^{N} \sigma_n^2}} = \frac{NA}{\sqrt{N\sigma_n^2}} = \frac{\sqrt{N}A}{\sigma_n}$$

Nishimura Ch. 7.5

Noise Considerations

- Effect of Signal Averaging
  - Average separate measurements of the same *k*-space data samples (e.g., 2 measurements)
  - Signal amplitudes add
  - Noise variances also add (independence)
  - $$SNR_{2Ave} = \frac{\sum_{j=1}^{N} 2A}{\sqrt{\sum_{j=1}^{N} 2\sigma_n^2}} = \frac{2NA}{\sqrt{2N\sigma_n^2}} = \frac{\sqrt{2NA}}{\sigma_n}$$
  - $$SNR_{2Ave} = \sqrt{2} \cdot SNR$$

Nishimura Ch. 7.5

Noise Considerations

- Effect of Readout Time
  - Double readout duration T_{read}
  - Typically, also double sampling interval Δt to maintain k-space sampling extent
  - $\Delta f \propto 1/(\Delta t)$: halves the signal bandwidth Δf
  - Recall that $\sigma_n^2 \propto \Delta f$
  - $$SNR_{2 \cdot Tread} = \frac{NA}{\sqrt{N\sigma_n^2/2}} = \frac{\sqrt{2N}A}{\sigma_n}$$
  - $SNR_{2 \cdot Tread} = \sqrt{2} \cdot SNR$

Nishimura Ch. 7.5

Noise Considerations

- Summary of Acquisition Time Effects
  - $SNR \propto \sqrt{N_{ave} \cdot T_{read}}$
  - $SNR \propto \sqrt{measurement\ time}$

- Effect of Spatial Resolution
  - $SNR \propto (\delta_x)(\delta_y)(\delta_z)$

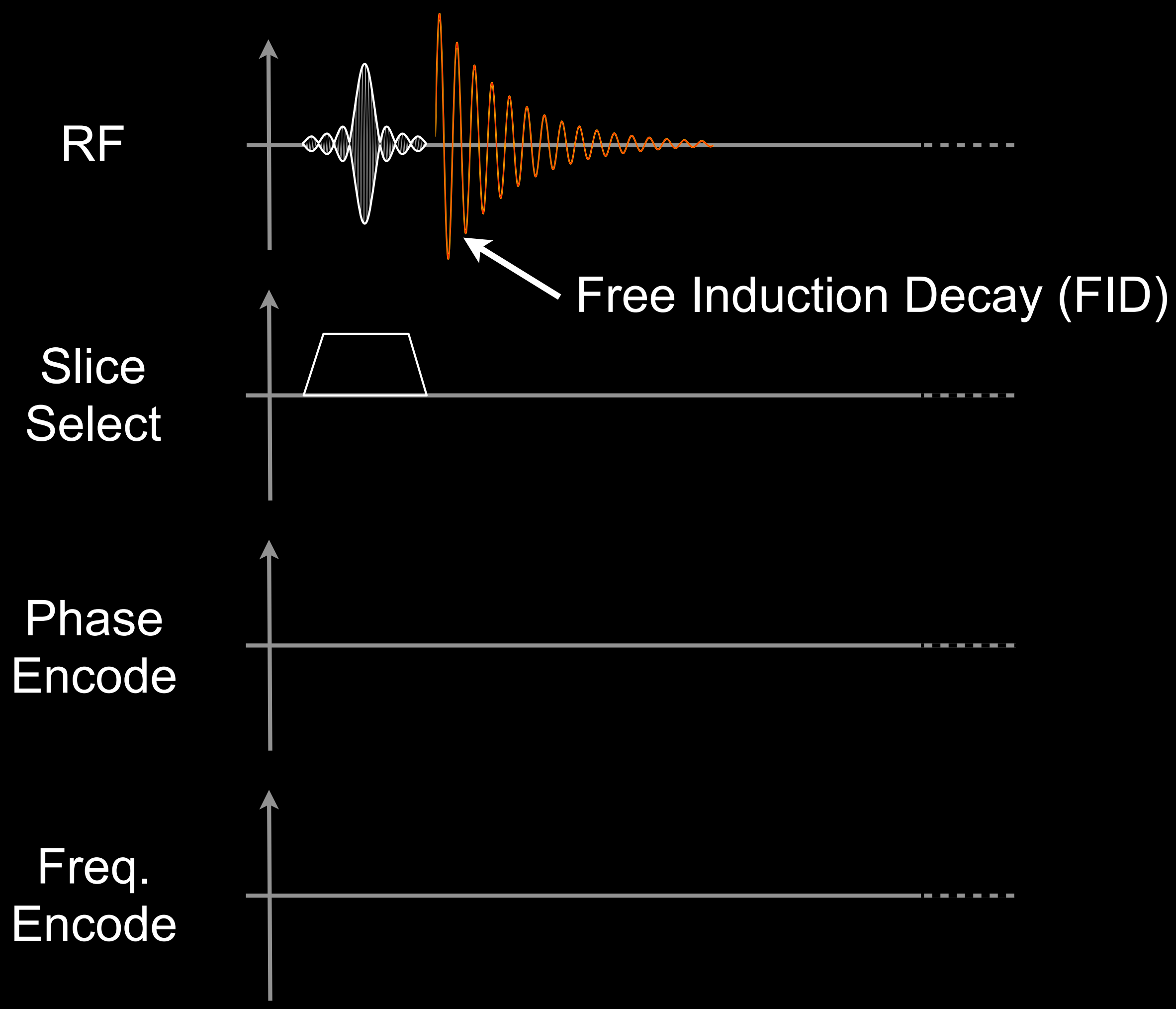
- Other factors
  - $SNR \propto f(\rho, T_1, T_2, \ldots)$

Nishimura Ch. 7.5

To the Board

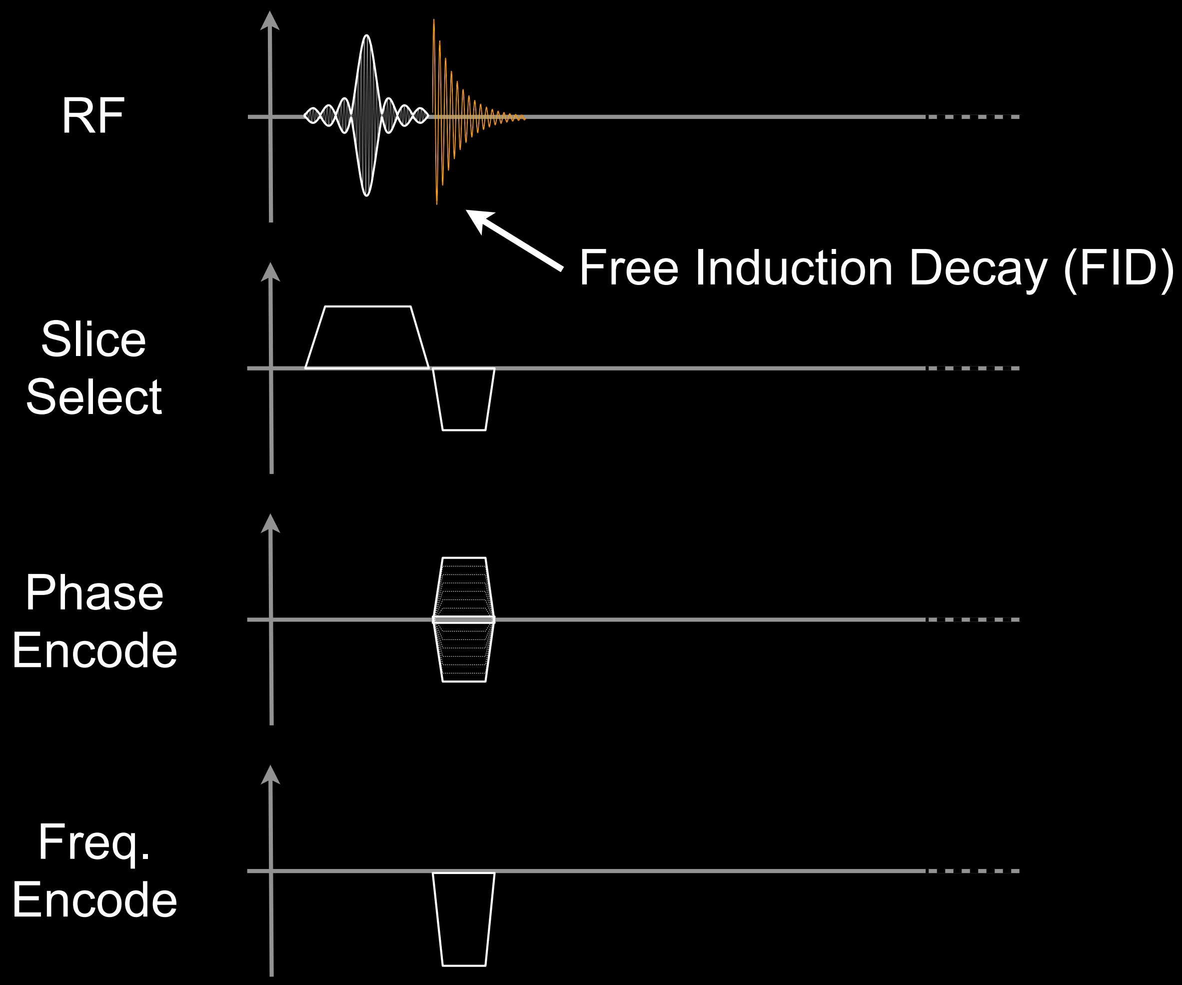
Gradient Echo Imaging

Basic Gradient Echo Sequence



- FID Decay due to
  - T2 decay
  - Spin dephasing

Basic Gradient Echo Sequence



- FID Decay due to
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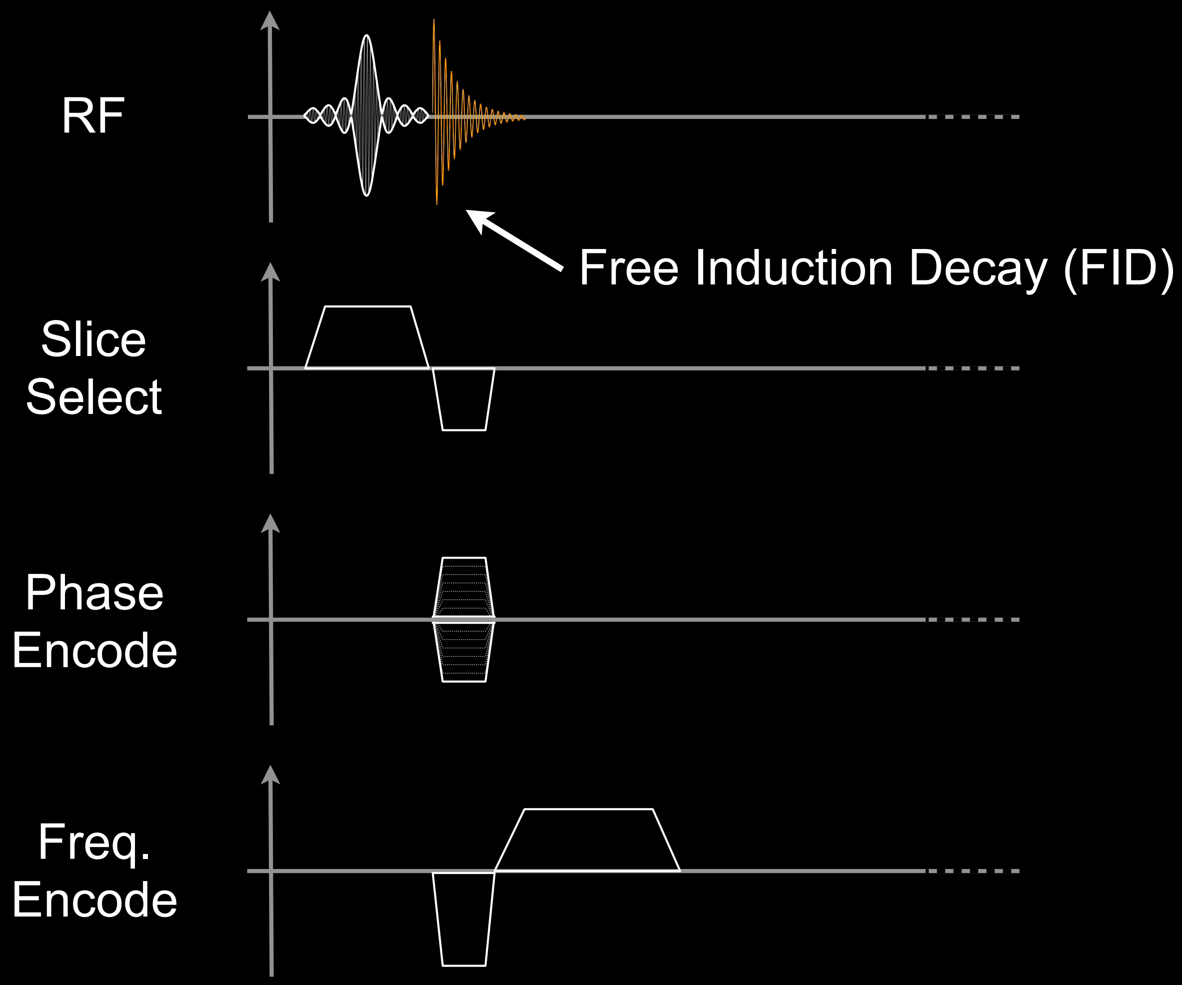
  - Spin dephasing
- Gradients accelerate spin dephasing

Basic Gradient Echo Sequence



- FID Decay due to
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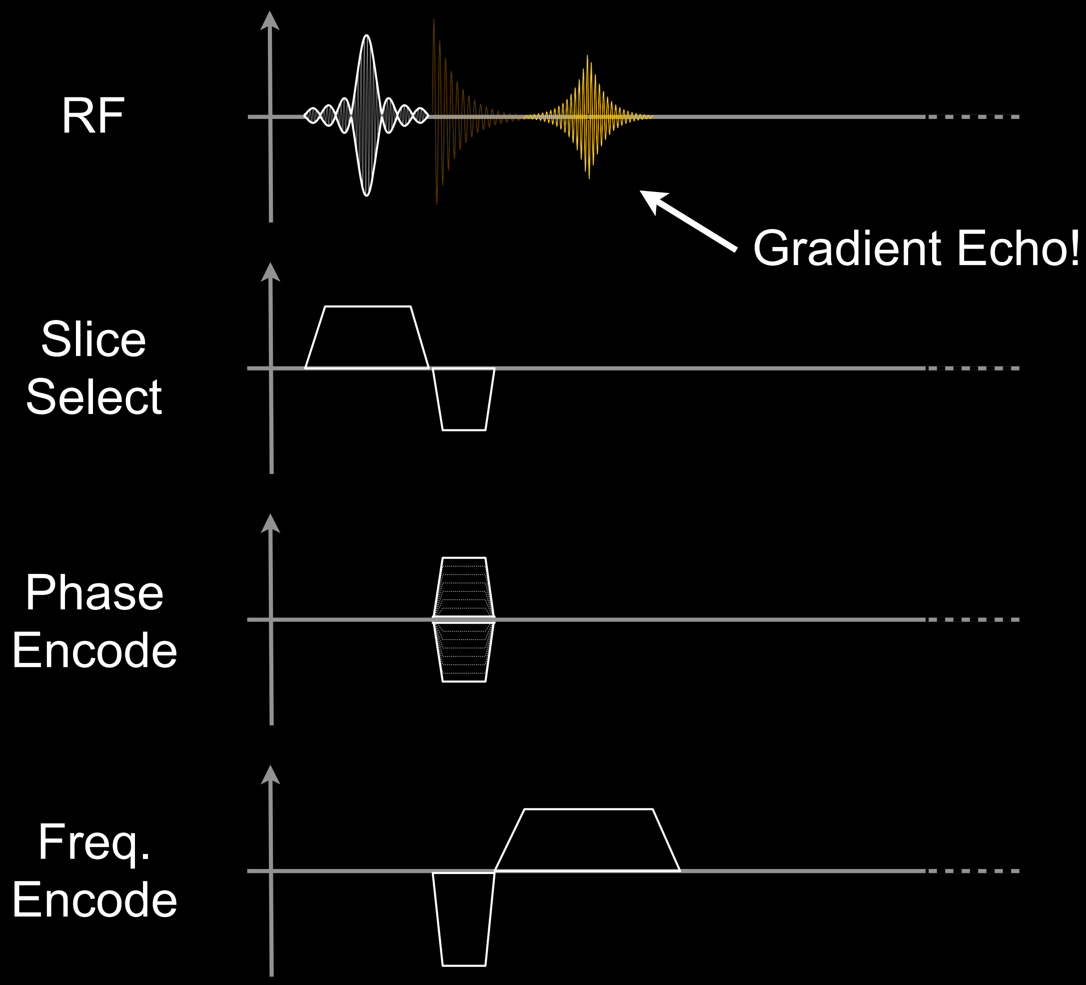
- Gradients accelerate spin dephasing
- Gradients can undo gradient induced spin dephasing

Basic Gradient Echo Sequence



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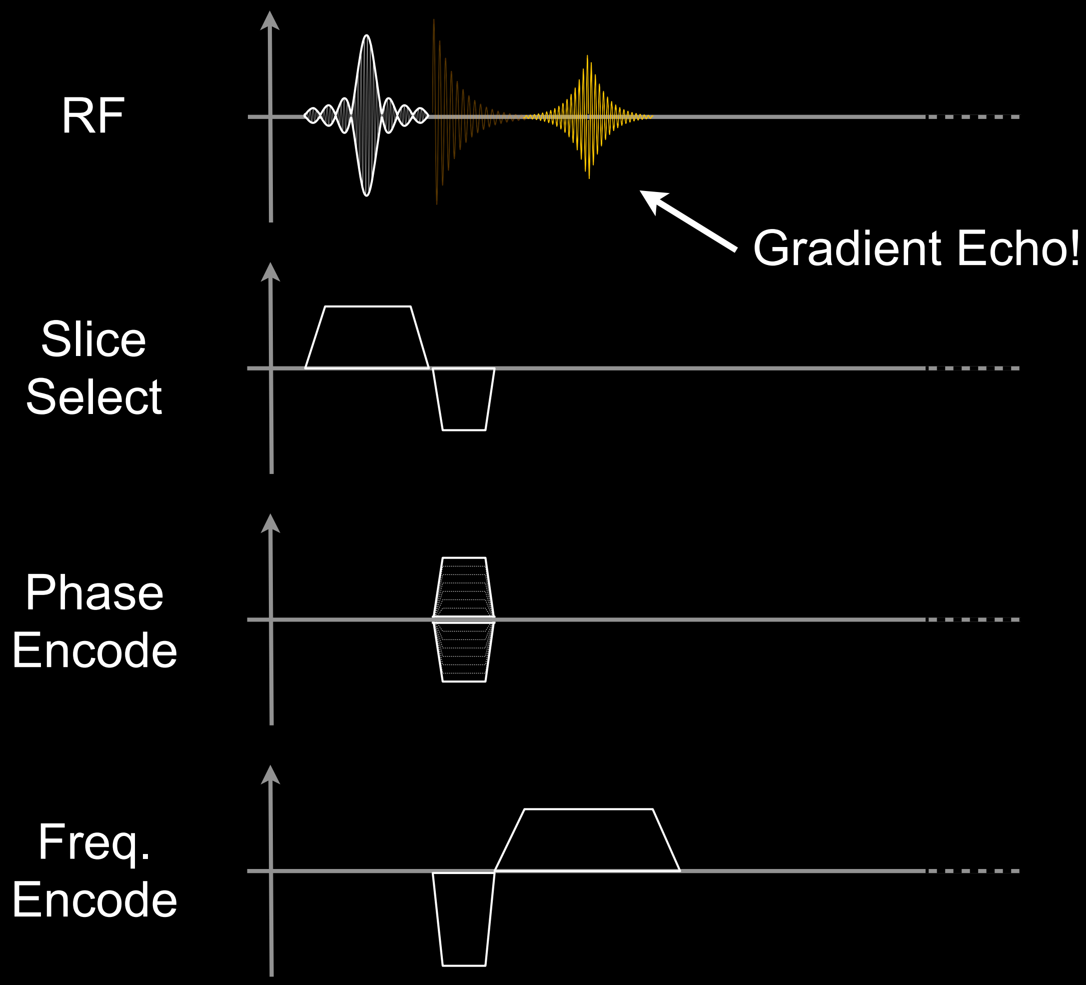

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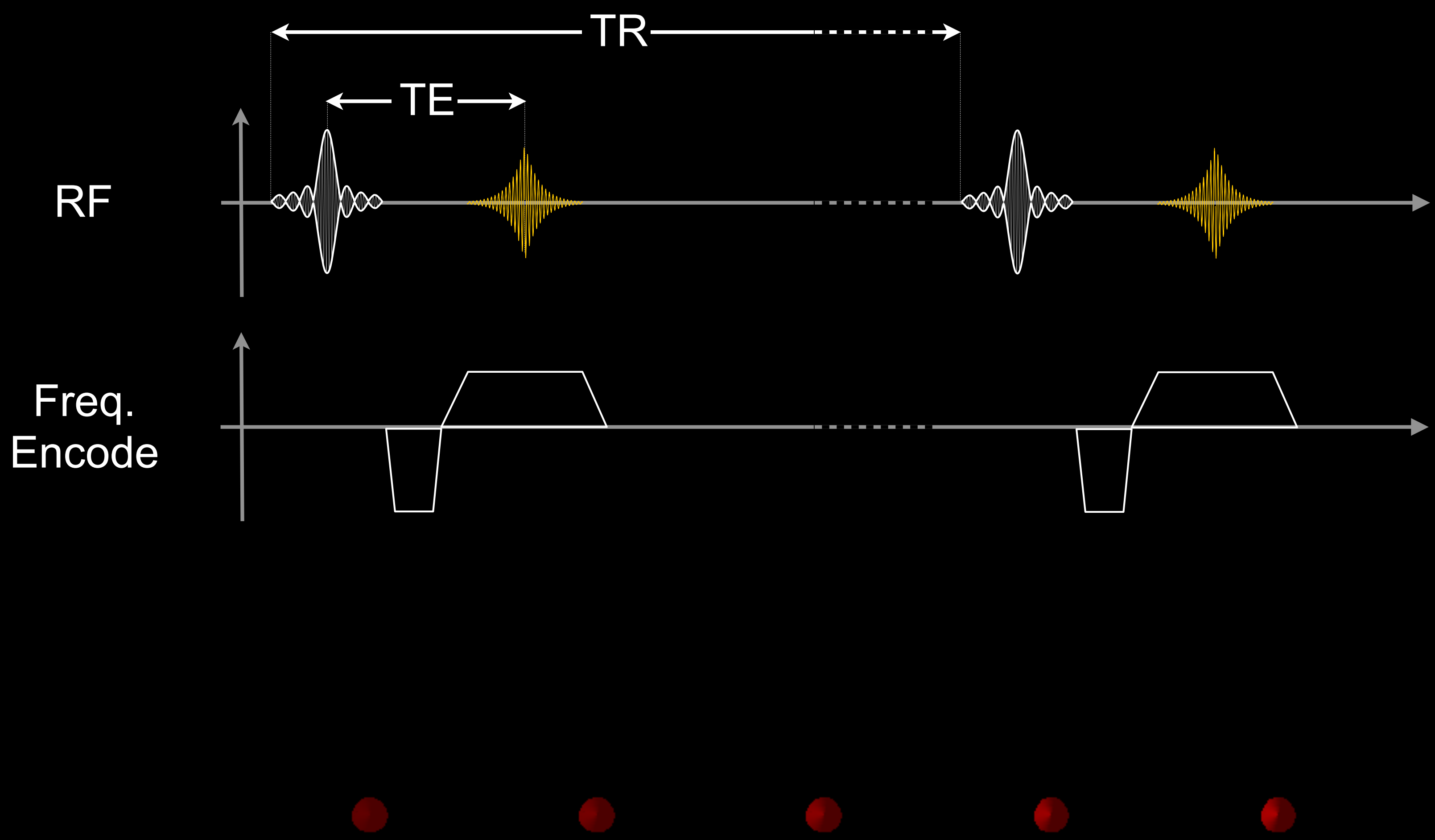

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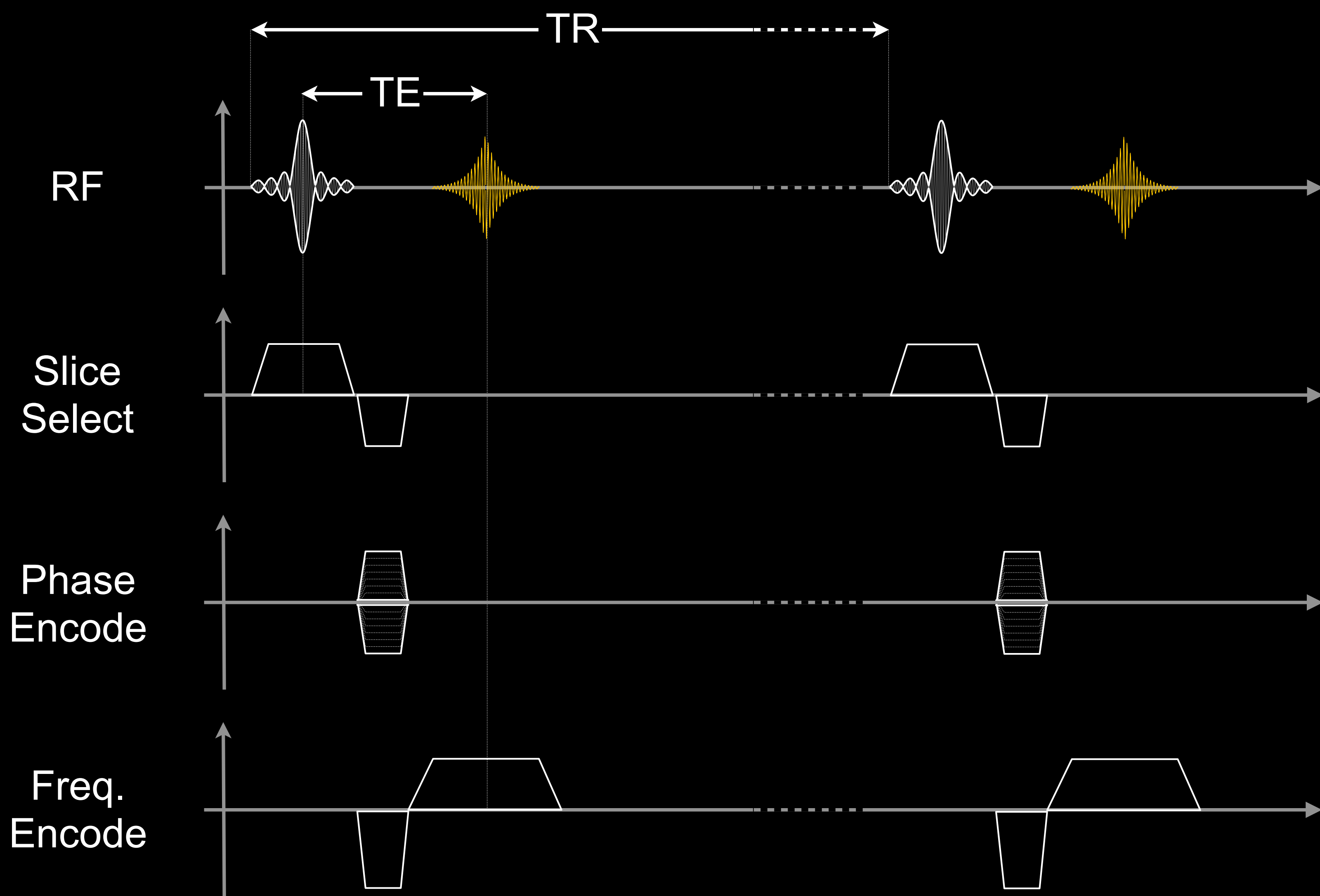
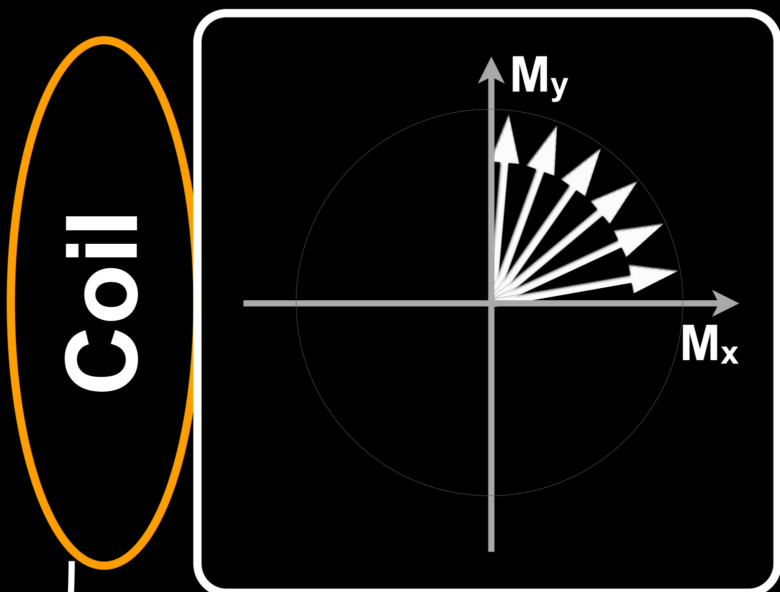


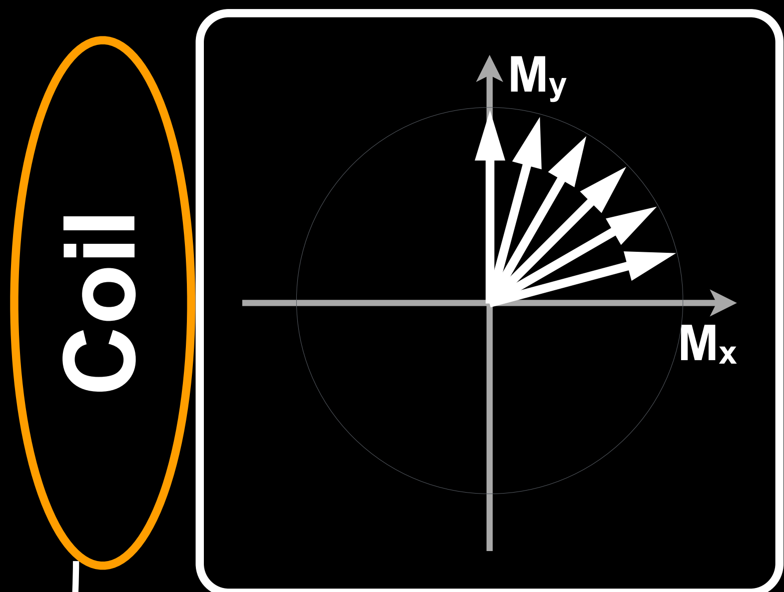
T_2 versus T_2*

T_2 Decay



Signal loss from spin-spin interaction.

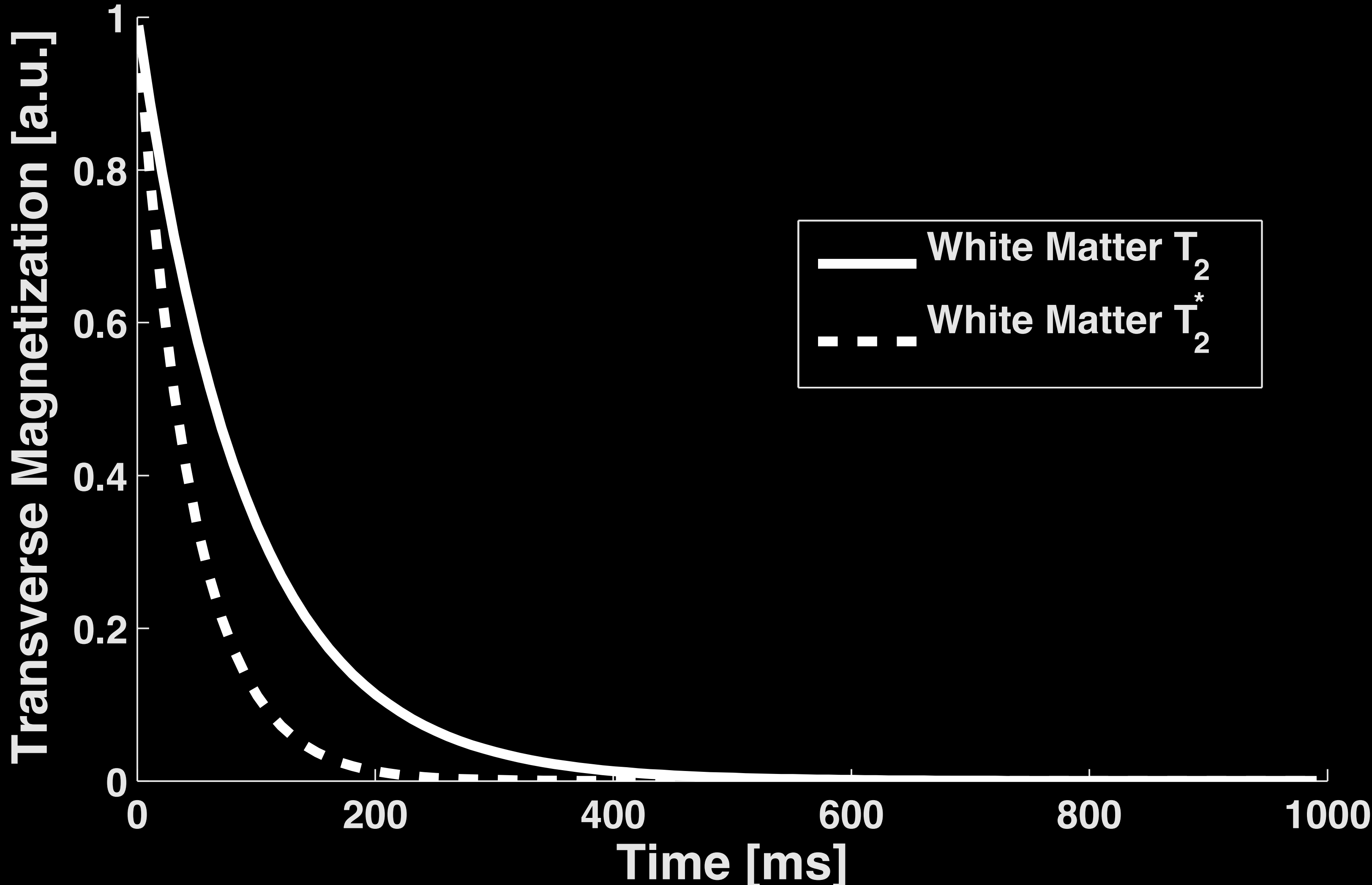
T_2* Decay



Signal loss from spin-spin interaction and off-resonance dephasing and T_2*.

T_2* is signal loss from spin dephasing and T_2

T2*<T2 (always!)

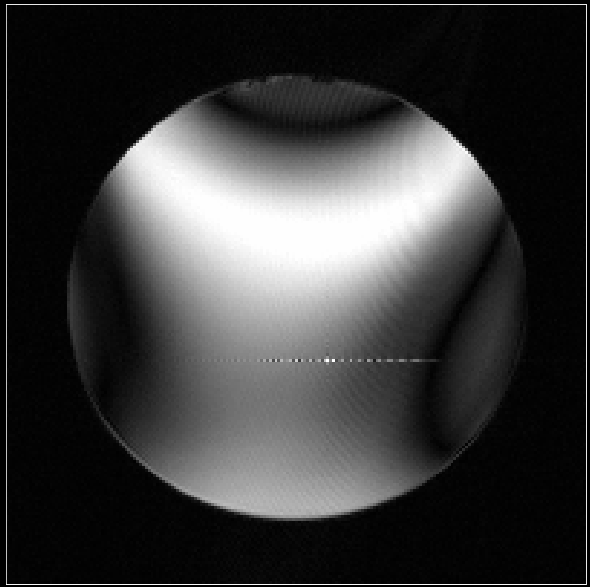


Transverse Magnetization [a.u.]
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
Time [ms]
White Matter T_2
White Matter T_2^*

SE vs. GRE: B_0 Inhomogeneity

- Images acquired with a bad shim
  - Poor B_0 homogeneity (lots of off-resonance)

Spin Echo



Gradient Echo

Images Courtesy of http://chickscope.beckman.uiuc.edu/roosts/carl/artifacts.html

Questions?

- Related reading materials
  - Nishimura - Chap 6 and 7

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