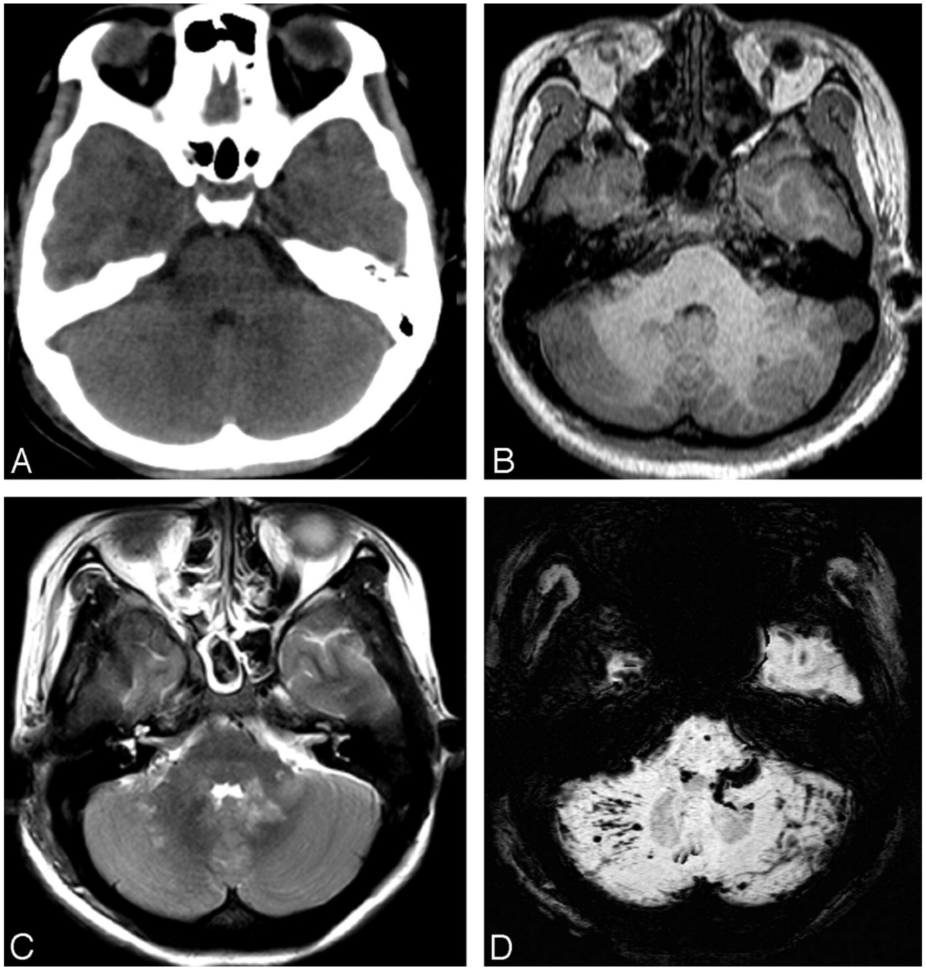
Susceptibility MRI

Jingwen Yao, Ph.D.
May 29, 2025

UCLA David Geffen School of Medicine

UCLA Samueli School of Engineering

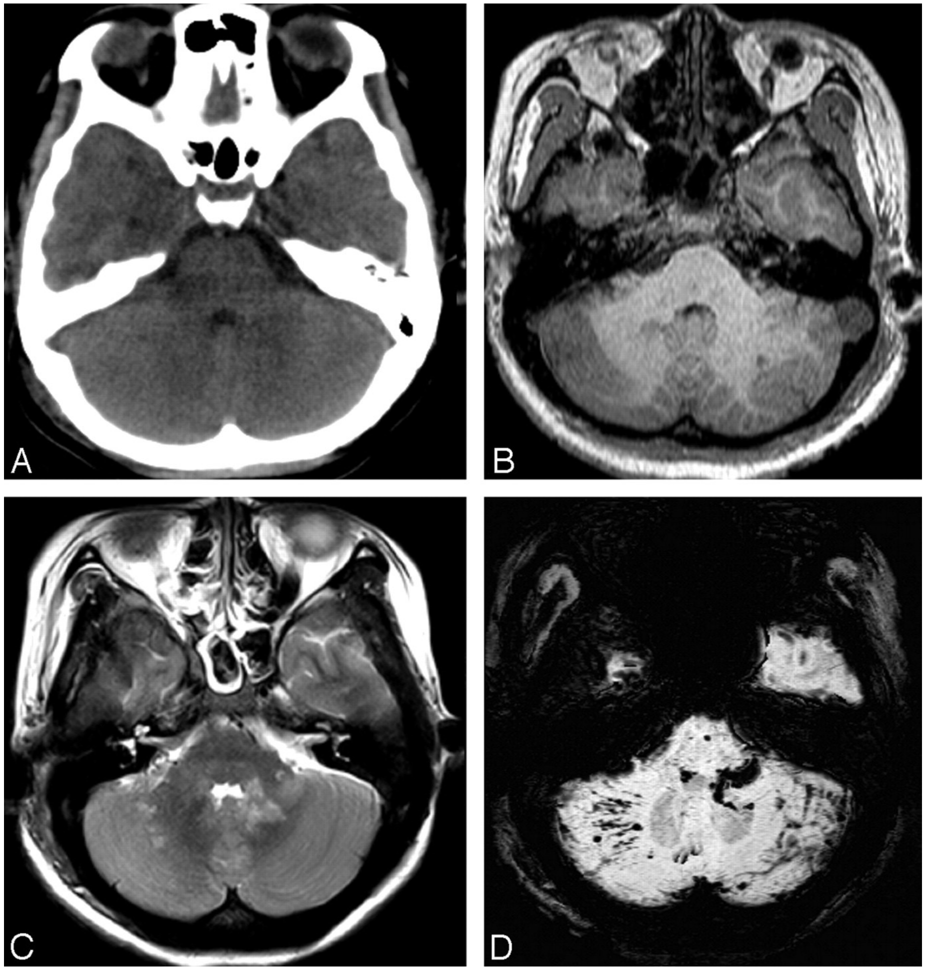


- 38 years old woman
- Involved in a **motor vehicle crash** and sustained severe head injuries resulting in a **coma**
- CT and MR images obtained on the second day of hospitalization

SWI

- Striking hypointense lesions in the left brachium pontis and both hemispheres of the cerebellum
- Greater number and larger size of lesions compared with other conventional sequences

Mittal S et al. AJNR, 2009



- 38 years old woman
- Involved in a **motor vehicle crash** and sustained severe head injuries resulting in a **coma**
- CT and MR images obtained on the second day of hospitalization

Diagnosis: Diffuse axonal injury

What is the origin of the hypointense lesions on SWI

Paramagnetic deoxygenated blood products

Mittal S et al., AJNR, 2009

Outline

Phase in MRI

Susceptibility MRI Contrast

Susceptibility MRI Processing

- Susceptibility-weighted imaging (SWI)
- Quantitative susceptibility mapping (QSM)

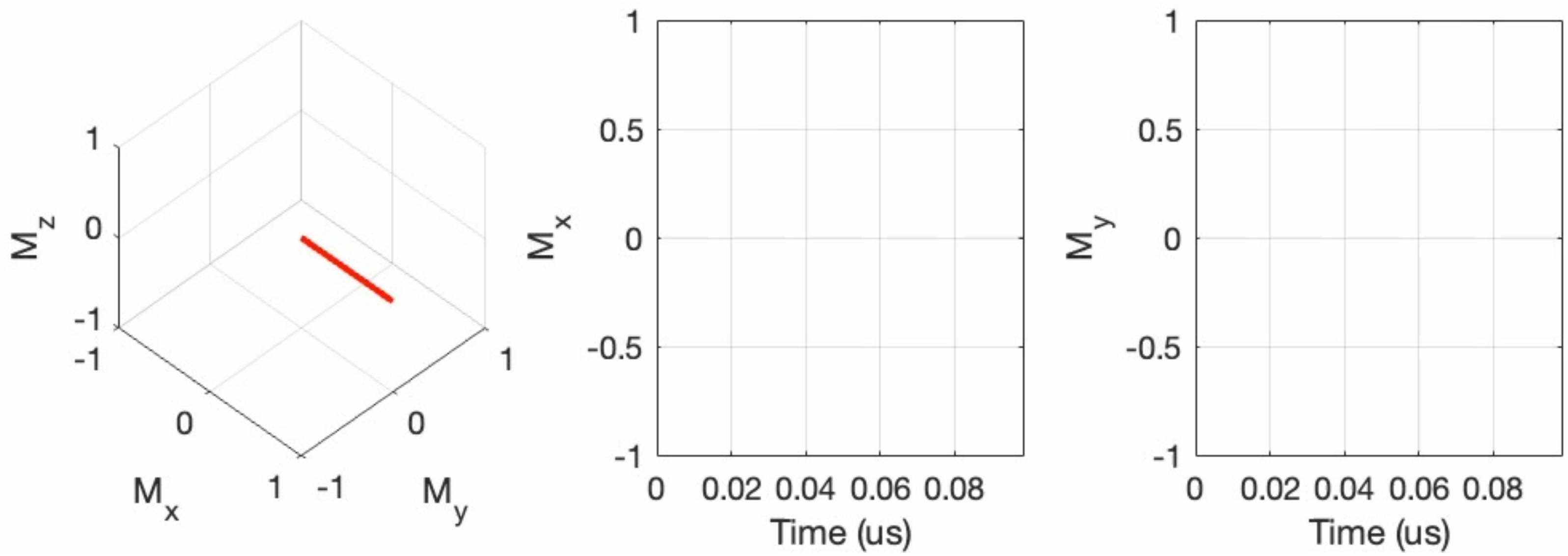
Susceptibility MRI Applications

Learning Objectives

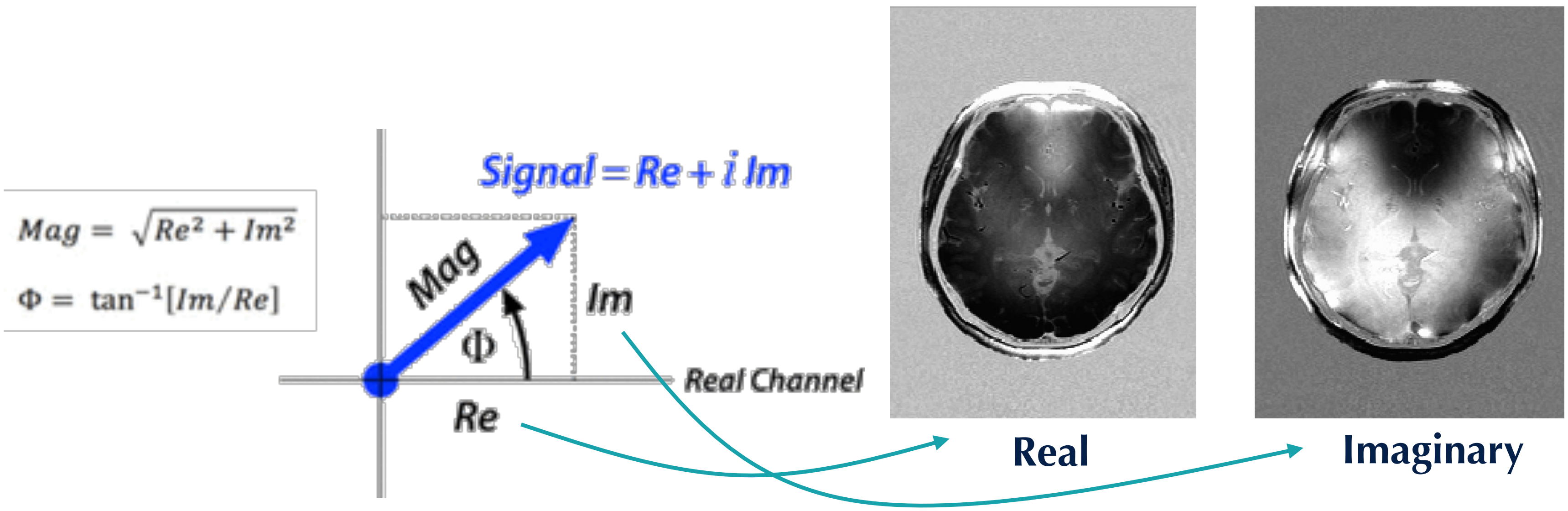
- Explain the biological and physical basis of susceptibility effects in MRI.
- Interpret and process phase images for SWI.
- Understand the procedure of processing QSM.
- Recognize common applications of susceptibility imaging in neuroimaging.
- Appreciate the challenges and assumptions in going from phase to susceptibility maps.

Phase in MRI

Precession of Magnetization **Real** **Imaginary**



Phase in MRI

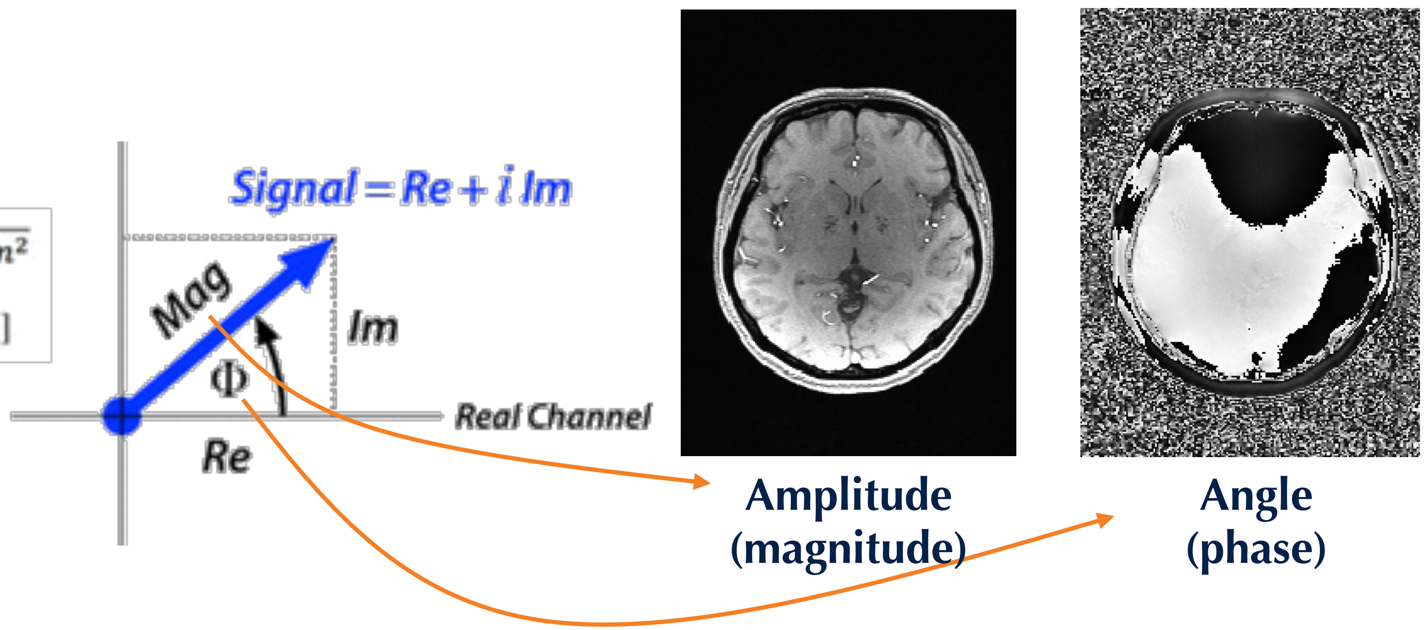


Mag = √(Re² + Im²)
Φ = tan⁻¹[Im/Re]
Signal = Re + i Im
Mag
Im
Φ
Real Channel
Re
Real
Imaginary

Phase in MRI

$$Mag = \sqrt{Re^2 + Im^2}$$

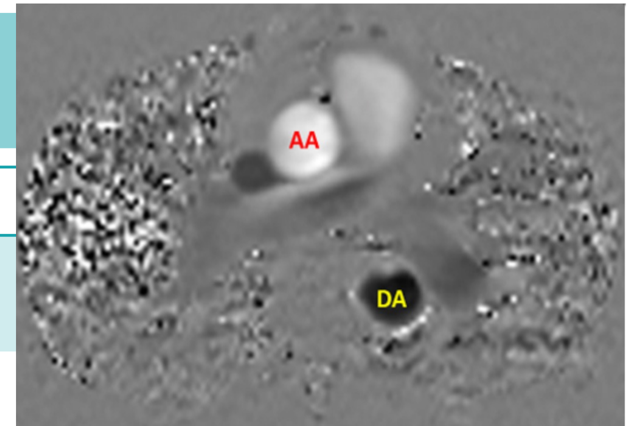
$$\Phi = \tan^{-1}[Im/Re]$$



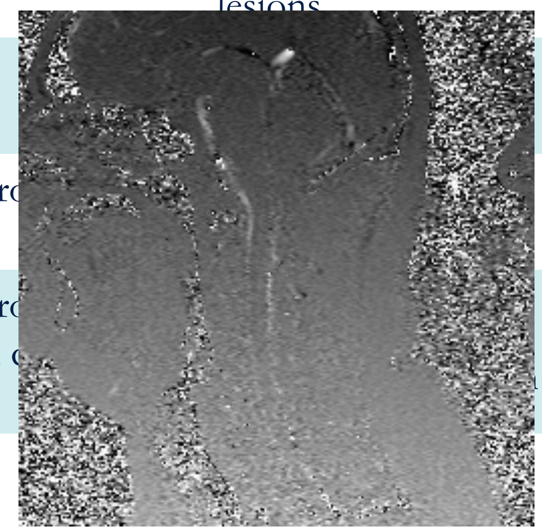
Phase MRI

	Encoding	Data	Applications
Susceptibility imaging	None	Raw phase	Iron, calcium, myelin imaging
Conductivity imaging	None / External current	Raw phase (B_1)	Tumors, ischemic lesions
MR thermometry	None	Phase shift	MR-guided procedures
Flow imaging	Velocity-encoding bipolar gradient	Subtracted phase data from opposite encodings	Cardiac flow, CSF flow
Phase contrast angiography	Bipolar gradients applied along the x, y, and z axes sequentially	Subtracted phase data from opposite encodings and combined across three directions	Angiogram, venogram, aneurysm
Elastography	Motion-encoding gradients	Phase differences	Liver fibrosis, brain

Phase MRI

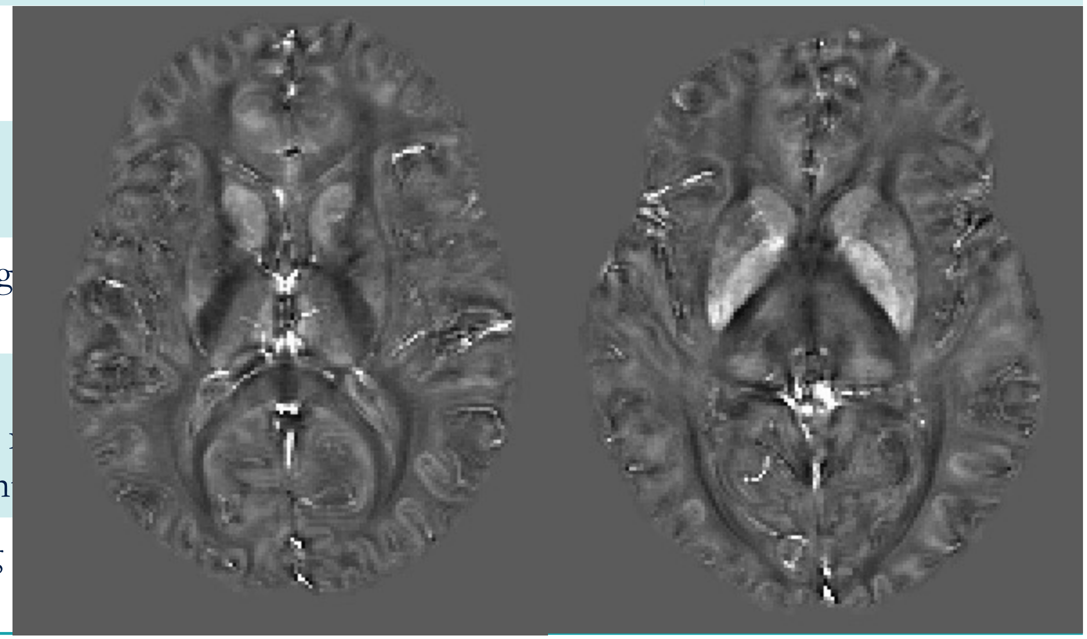


	Encoding	Data	
Susceptibility imaging	None	Raw phase	
Conductivity imaging	None / External current	Raw phase (B_1)	lesions
MR thermometry	None	Phase shift	
Flow imaging	Velocity-encoding bipolar gradient	Subtracted phase data fro opposite encodings	
Phase contrast angiography	Bipolar gradients applied along the x, y, and z axes sequentially	Subtracted phase data fr opposite encodings and c across three directions	
Elastography	Motion-encoding gradients	Phase differences	



Phase MRI

	Encoding	Data	Applications
Susceptibility imaging	None	Raw phase	Iron, calcium, myelin imaging
Conductivity imaging	None / External current		
MR thermometry	None		
Flow imaging	Velocity-encoding bipolar gradient		
Phase contrast angiography	Bipolar gradients applied along the and z axes sequen		
Elastography	Motion-encoding gradients		



Susceptibility MRI – source of contrast

Magnetic Field H	**Magnetic Flux Density B** (also referred to as magnetic field in MRI)
Externally applied "magnetizing force"	Actual magnetic field induced
Independent of medium	Dependent of medium (M/J: intrinsic magnetization)
A/m	Tesla

$$B = \mu_0(H + M) = \mu_0 H + J$$

μ_0: $4 \cdot \pi \cdot 10^{-7}$ (H/m) is the permeability of vacuum

Susceptibility MRI – source of contrast

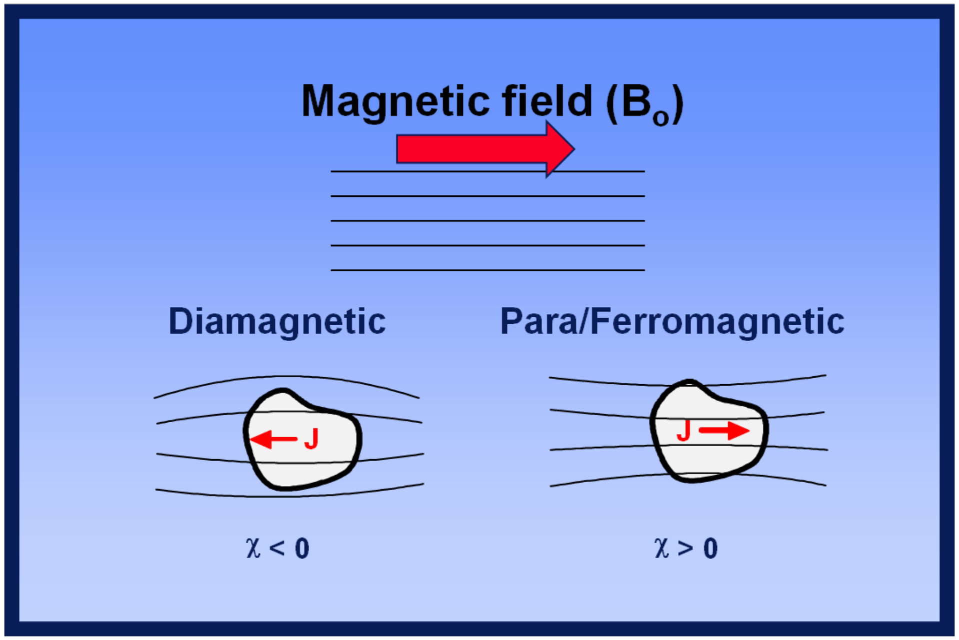
Which of the statements about H and B are correct?

A. In an empty scanner, H and B are equivalent.
B. The difference between B and H is due to the material’s magnetization.
C. H and B always point in the same direction.
D. B is always stronger than H.

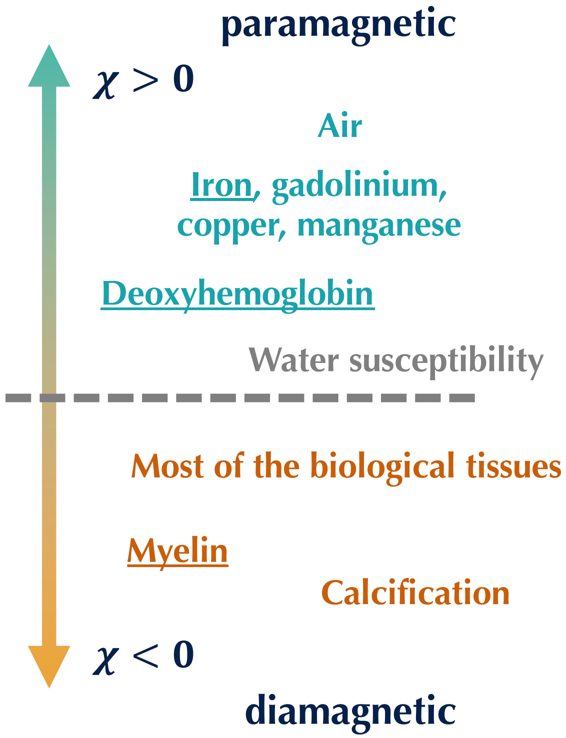
PollEv.com/smoothdove577

Susceptibility MRI – source of contrast

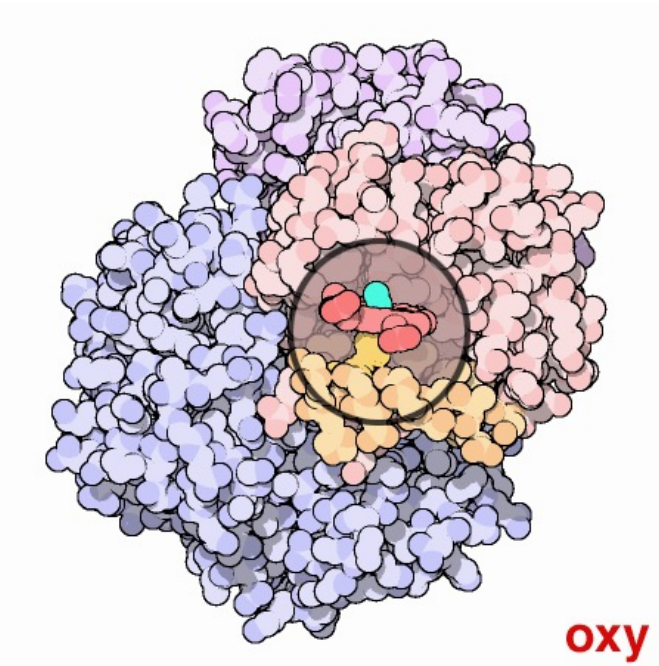
$$B = \mu_0(H + M) = \mu_0 H + J = B_0 + J$$

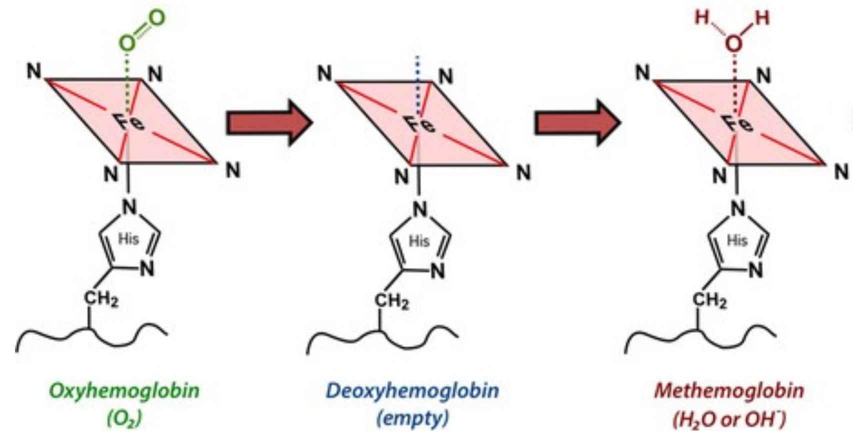


$$J = \chi B_0$$



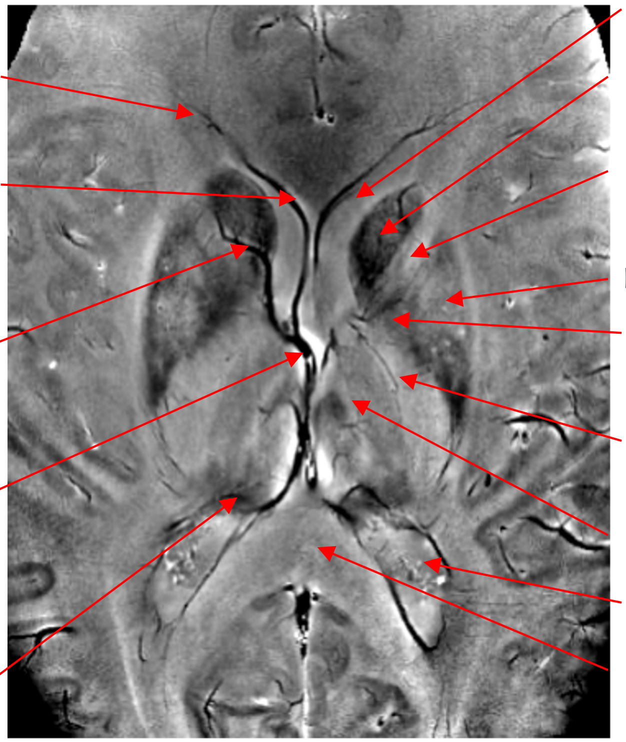
Susceptibility MRI – source of contrast



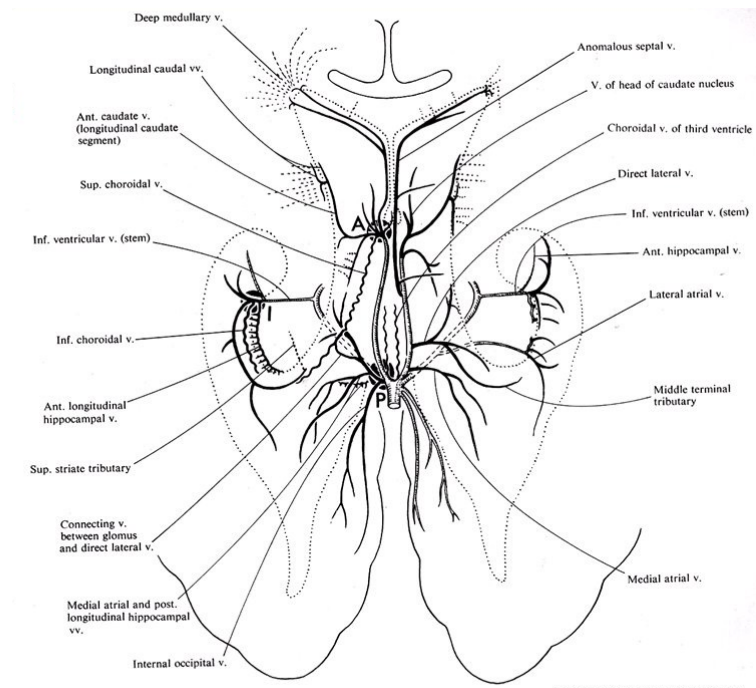


# Unpaired electrons	0	4	5
Magnetic susceptibility	Diamagnetic $\chi < 0$	Paramagnetic $\chi > 0$	Paramagnetic $\chi > 0$

Susceptibility MRI – Deoxyhemoglobin

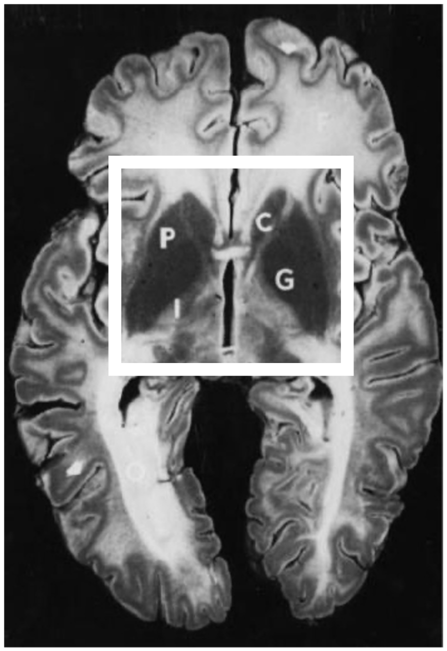


GRE Phase Image

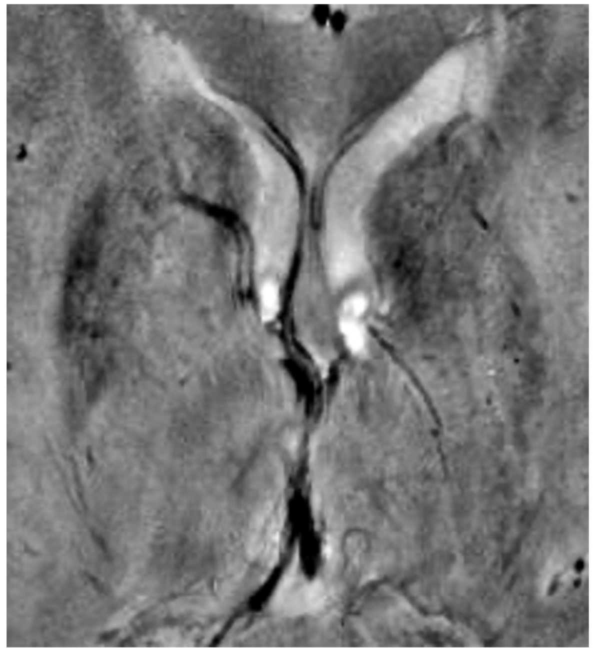


from G.Salamon.Y.P. Huang Radiologic Anatomy of the brain. Springer verlag 1976

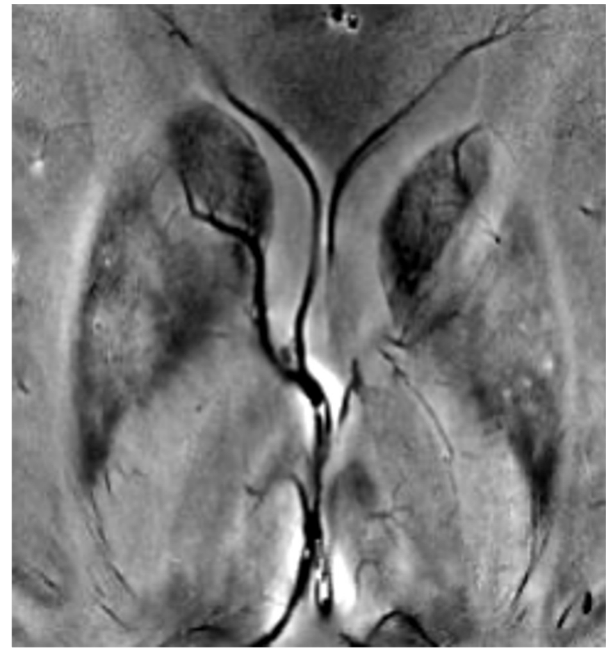
Susceptibility MRI – Iron



Iron Perl's Stain



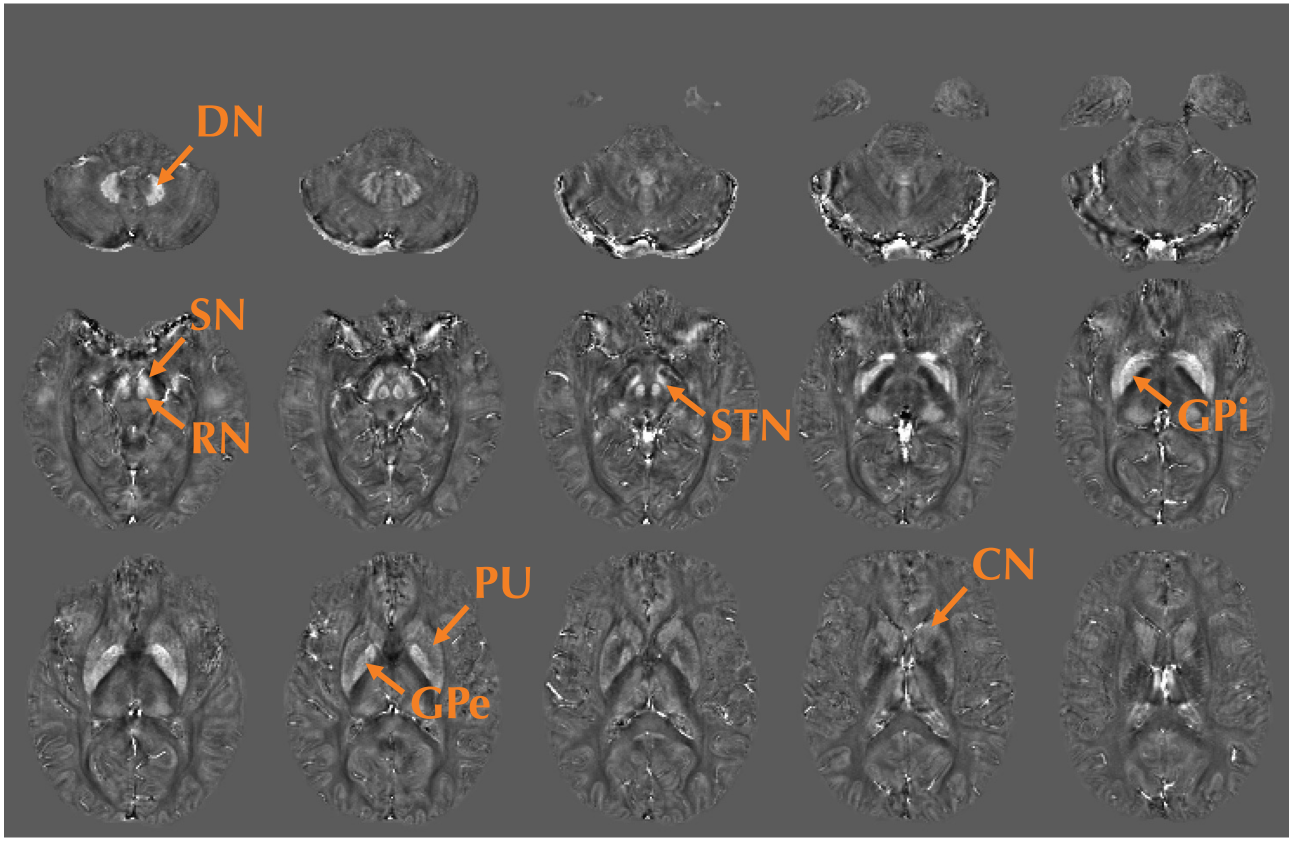
GRE Magnitude Image



GRE Phase Image

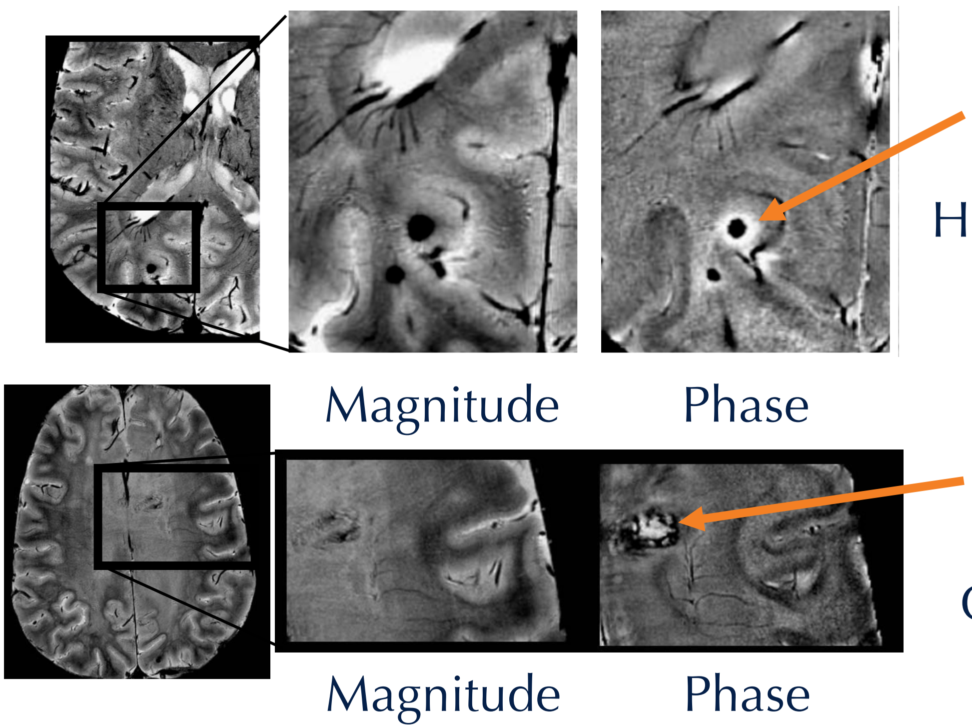
Drayer Am J Roentgenol 1986

Susceptibility MRI – Iron



Drayer Am J Roentgenol 1986

Susceptibility MRI – Calcification

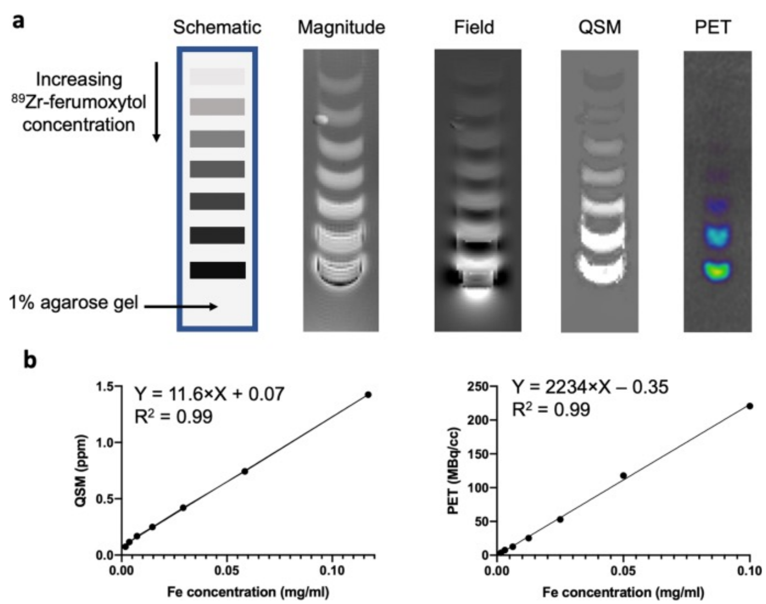


Paramagnetic
⇩
Hemorrhage from trauma

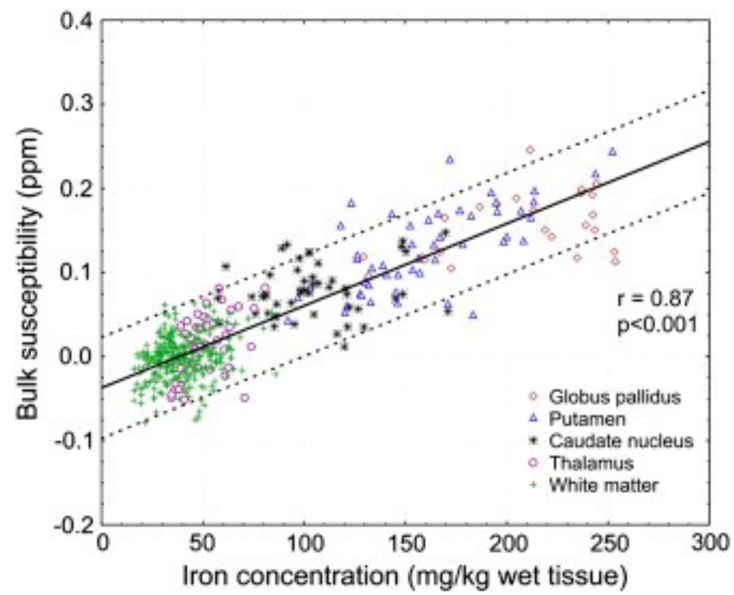
Diamagnetic
⇩
Calcification in glioma

Susceptibility MRI – Iron

Phantom validation

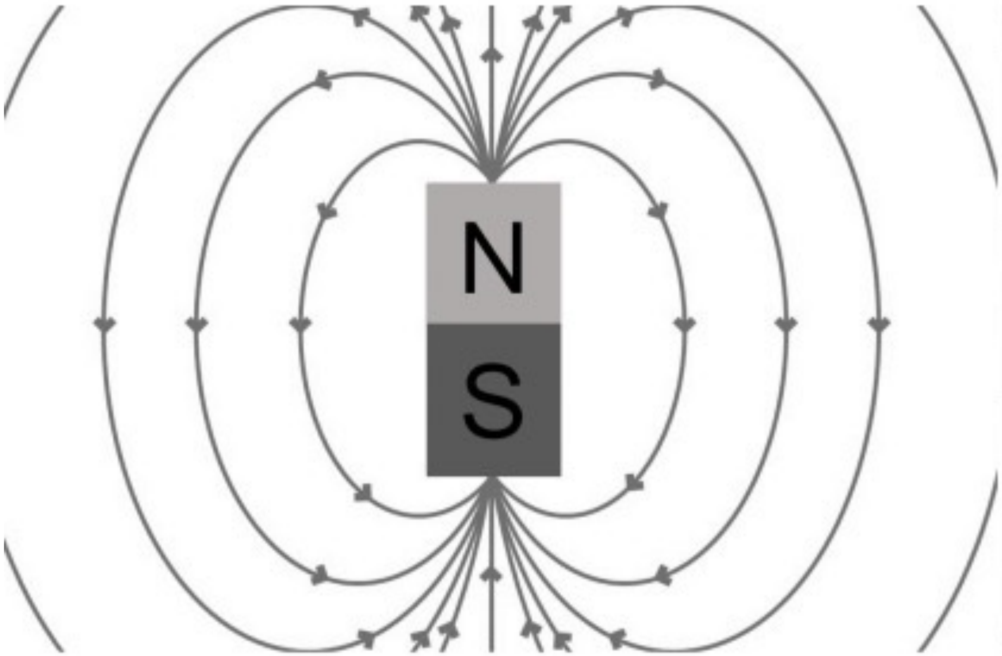


Tissue validation

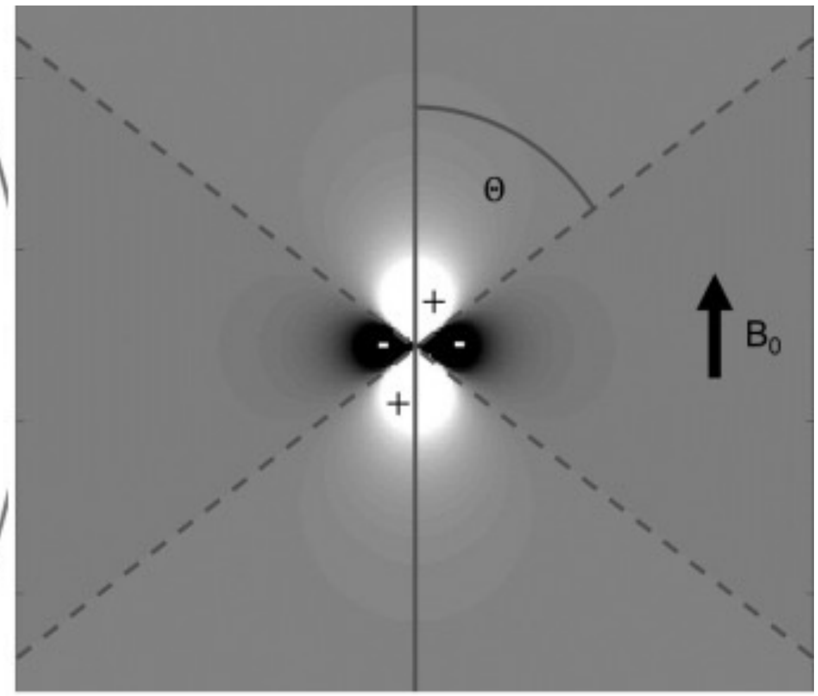


Deh K et al. Sci Rep 2020
Lankammer C et al. NeuroImage 2012

Susceptibility MRI – Signal model



a)



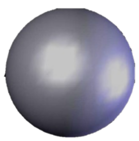
b)

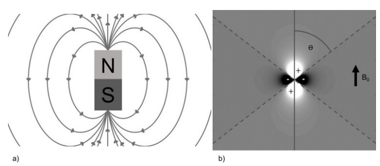
Ruetten P et al. BJR 2019

Susceptibility MRI – MR signal

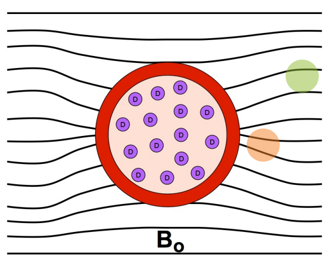
B_0

Susceptibility source (Iron)

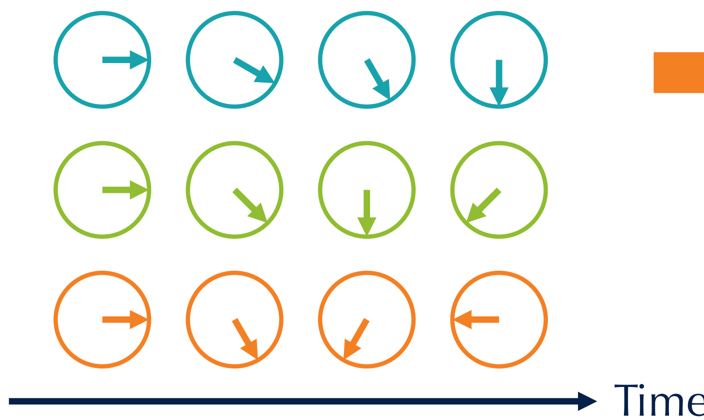


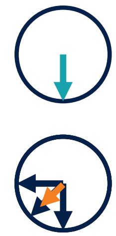


B0 field perturbation

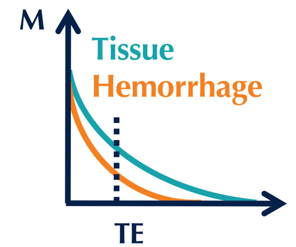


Shift in precession frequency

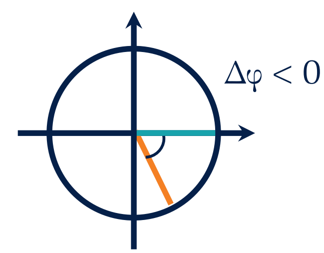




Dephasing
→ T_2^* shortening



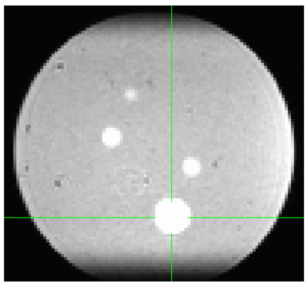
Phase accumulation

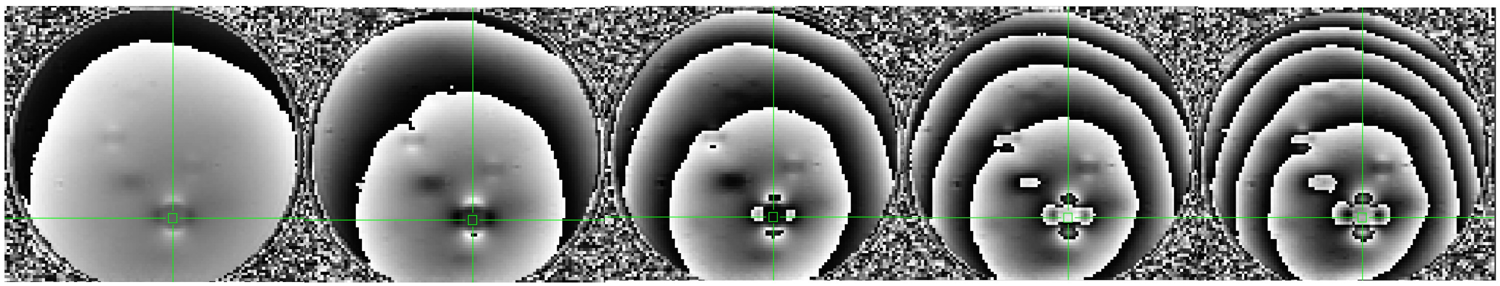


Barnes J et al. Magn Reson Imaging Clin N Am 2010

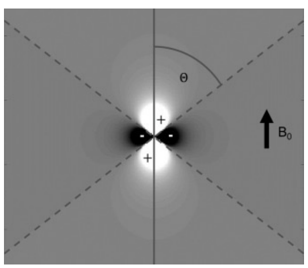
Susceptibility MRI – Signal model

Ferumoxytol phantom





5.84 ms → 40.56 ms

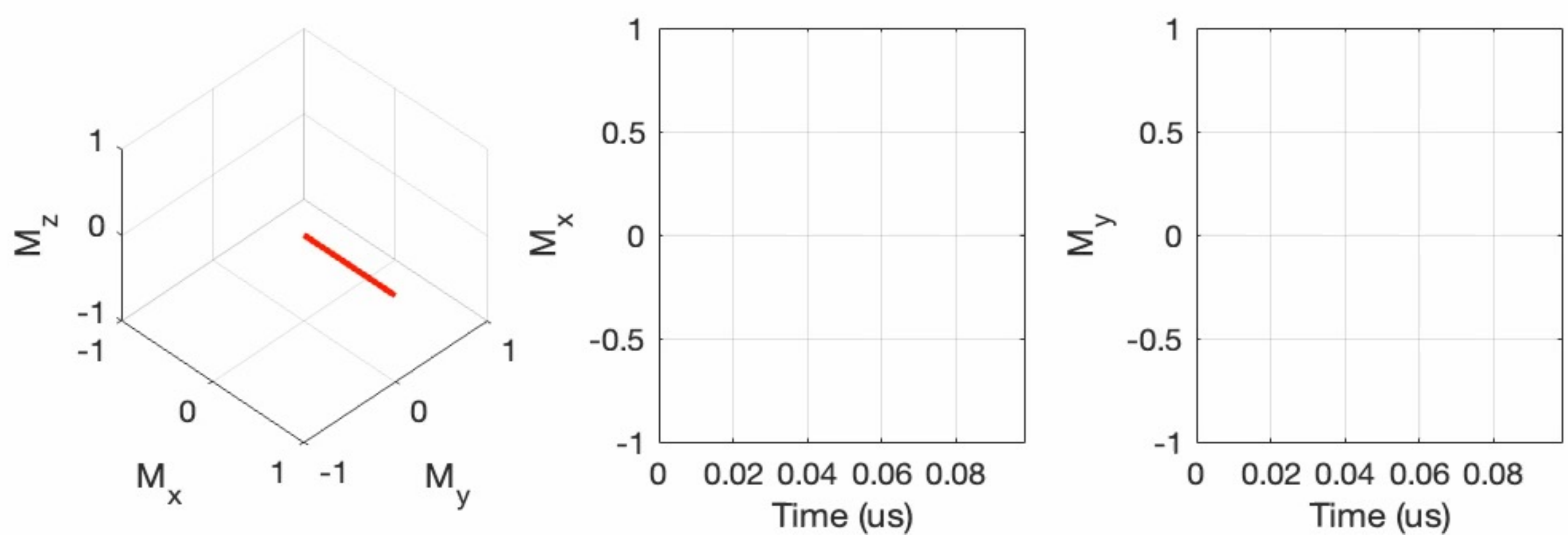


Where does the phase discontinuity come from?

[*PollEv.com/smoothdove577*](PollEv.com/smoothdove577)

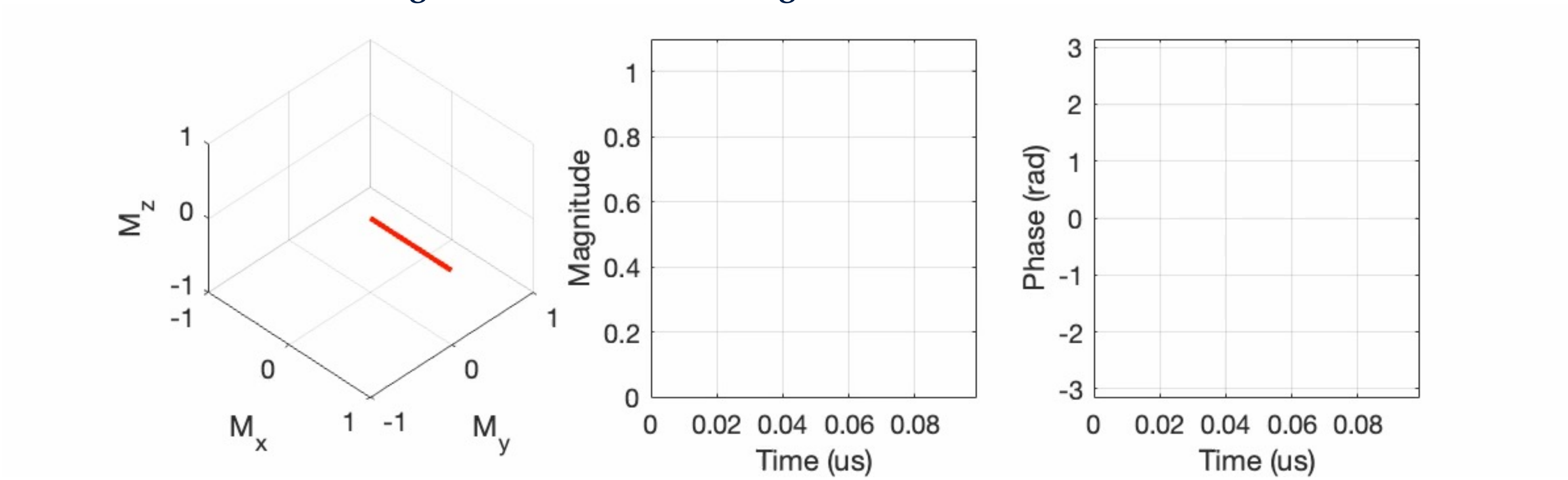
Susceptibility MRI – Signal model

Precession of Magnetization **Real** **Imaginary**



Susceptibility MRI – Signal model

Precession of Magnetization | **Magnitude** | **Phase**



Susceptibility MRI – Signal model

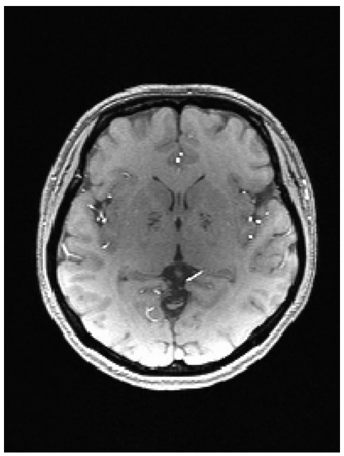
T_2*-weighted sequence

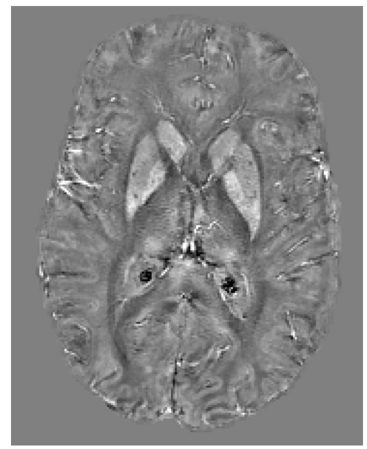
T_2*-weighted magnitude

Phase

Susceptibility-weighted imaging

Quantitative susceptibility mapping



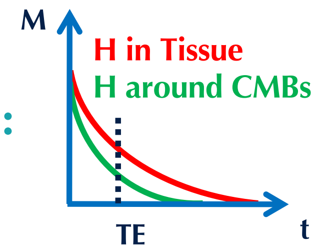


Barnes J et al. Magn Reson Imaging Clin N Am 2010

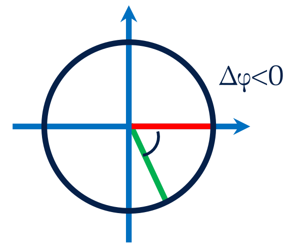
Susceptibility MRI – Signal model

Contrast: Magnitude | Phase | SWI

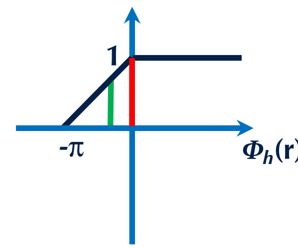
Effect of Δχ:



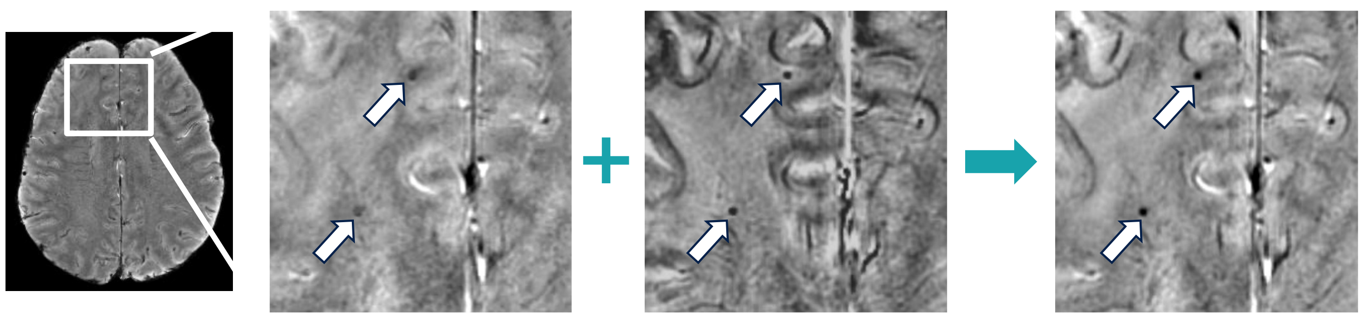
Shortens T2*



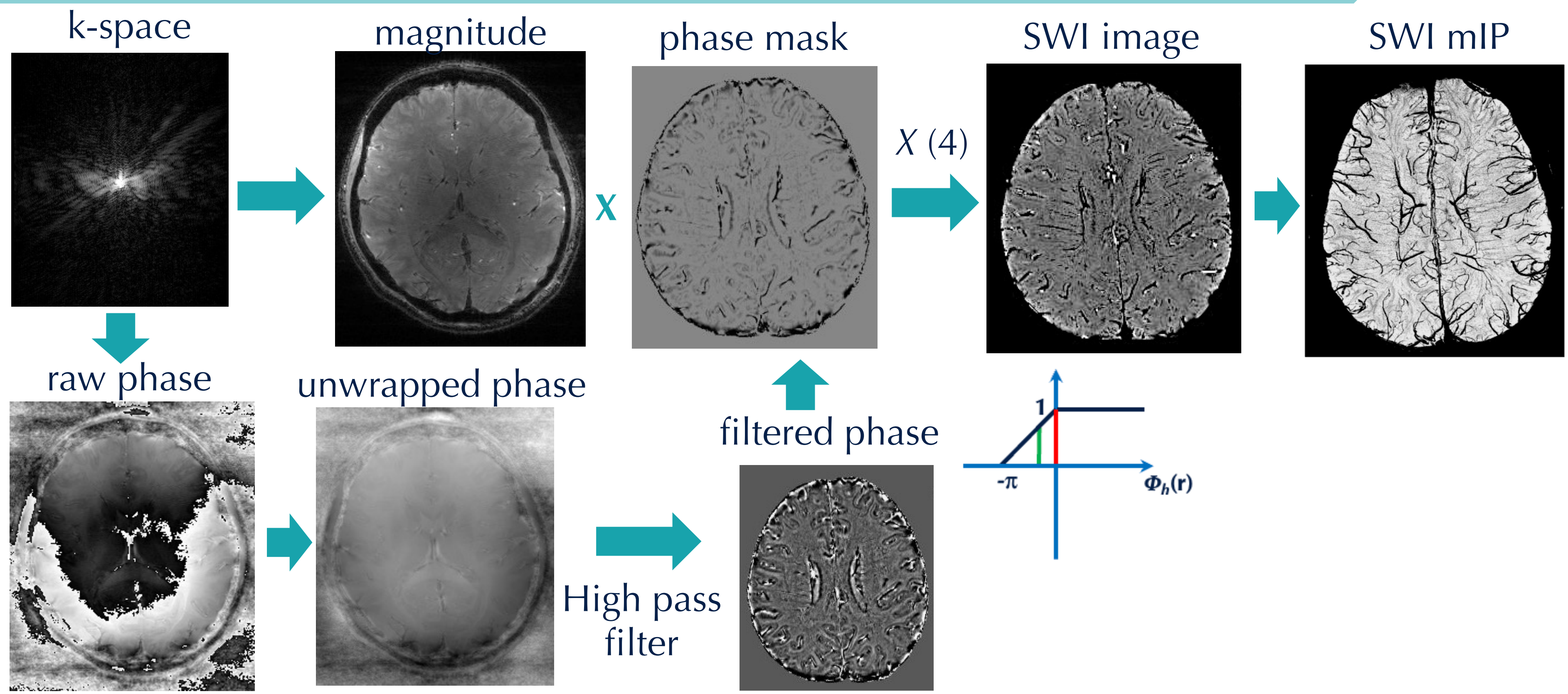
Accumulates phase



Enhances T2* shortening

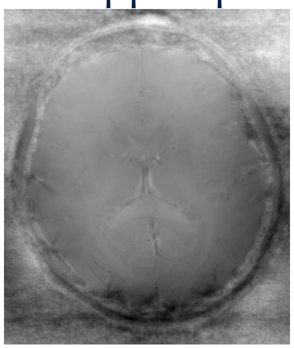


SWI – Processing



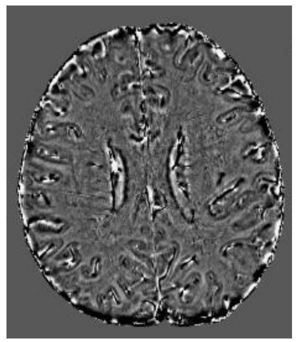
SWI – Processing

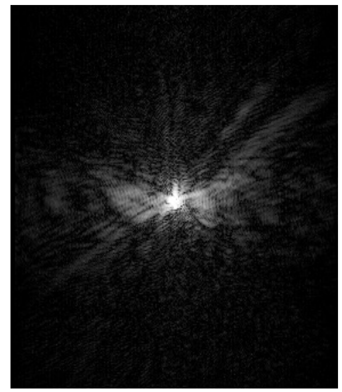
unwrapped phase



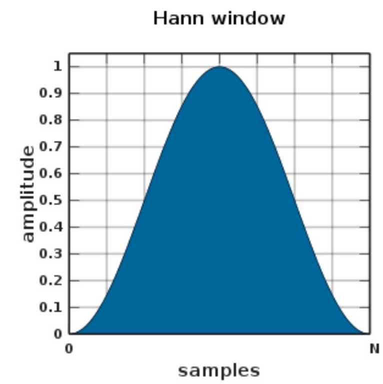
High pass filter

filtered phase





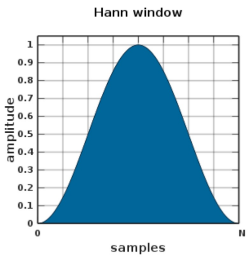
X

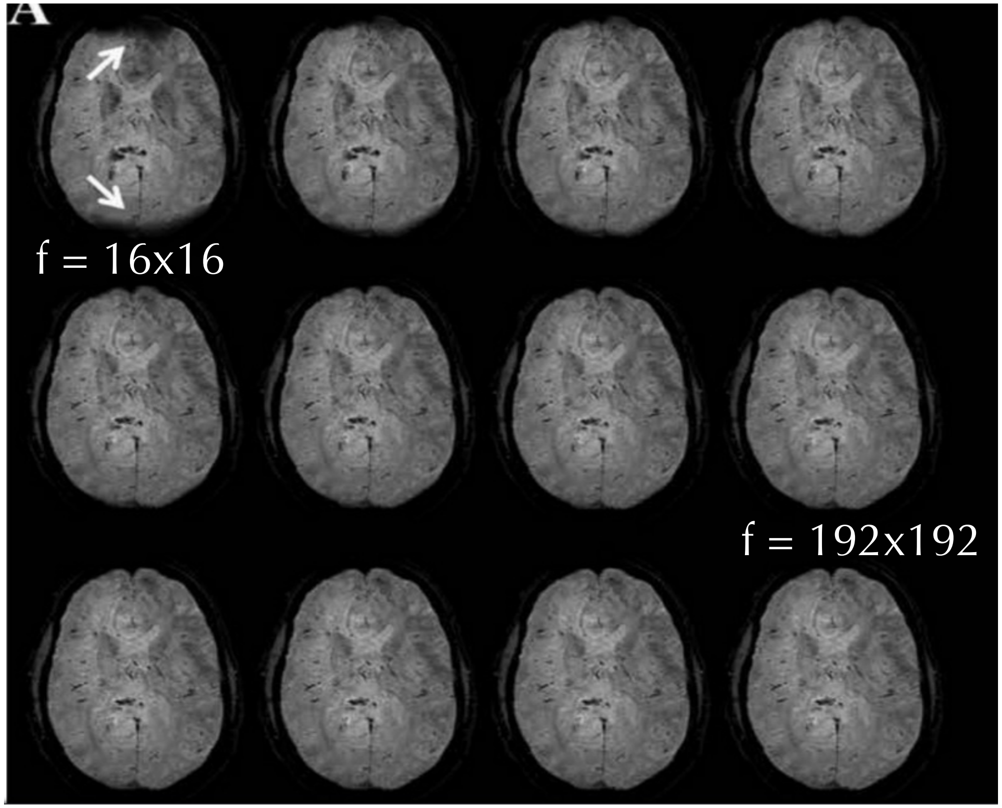


Low pass filtered phase

Complex division

SWI – Processing



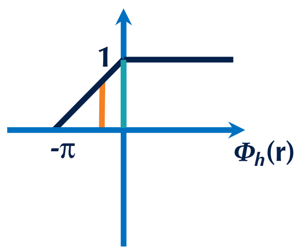


Higher contrast,
more artifacts

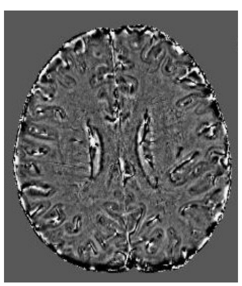
lower contrast,
less artifacts

SWI – Processing

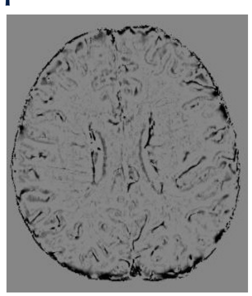
SWI phase masking

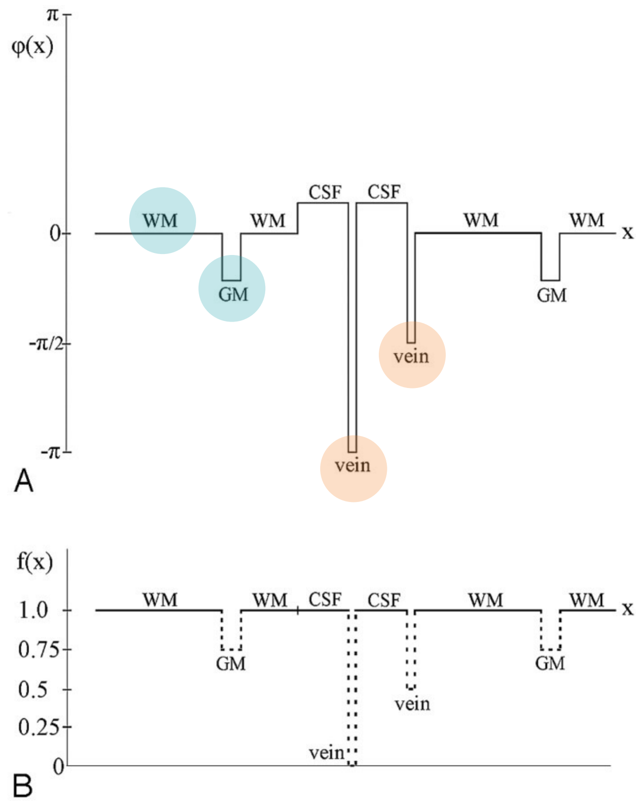


filtered phase



phase mask



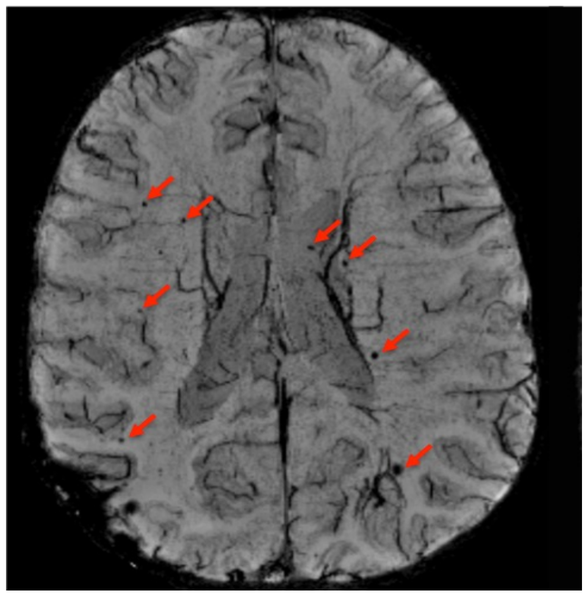


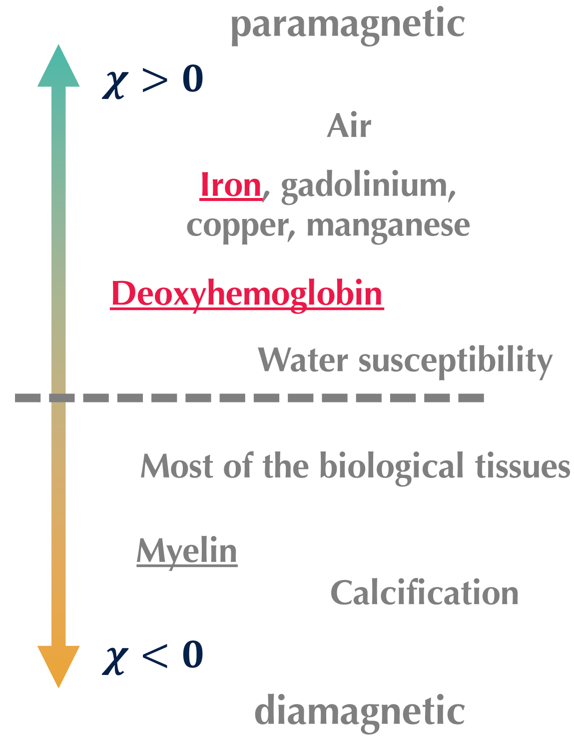
	WM	GM	Vein (small)	Vein (Large)
Phase	**0**	**-0.25π**	**-0.5π**	**-π**
$f(x)^1$	1	0.75	0.5	0
$f(x)^2$	1	0.56	0.25	0
$f(x)^3$	1	0.42	0.13	0
$f(x)^4$	**1**	**0.32**	**0.06**	**0**

Haacke EM et al. AJNR 2009

SWI – Example applications

phase-weighted magnitude imaging

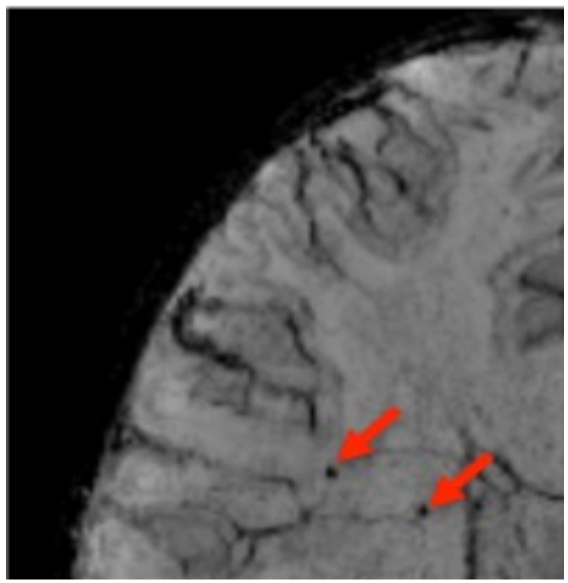


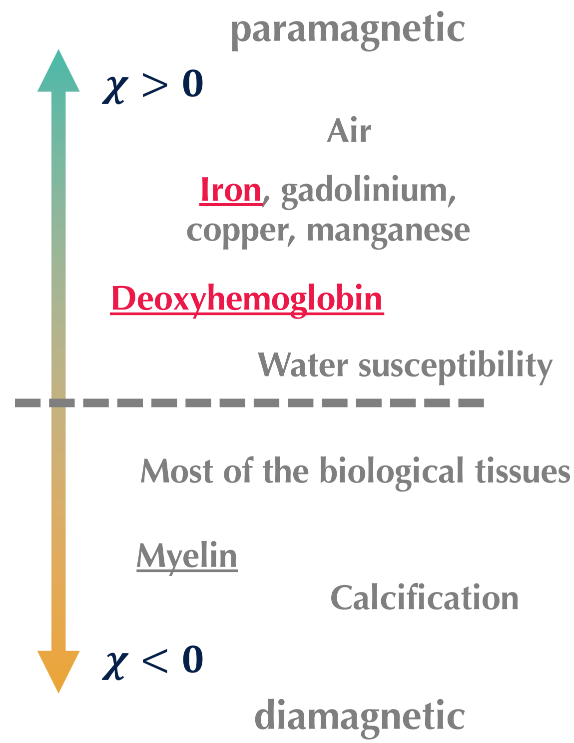


Barnes J et al. Magn Reson Imaging Clin N Am 2010

SWI – Example applications

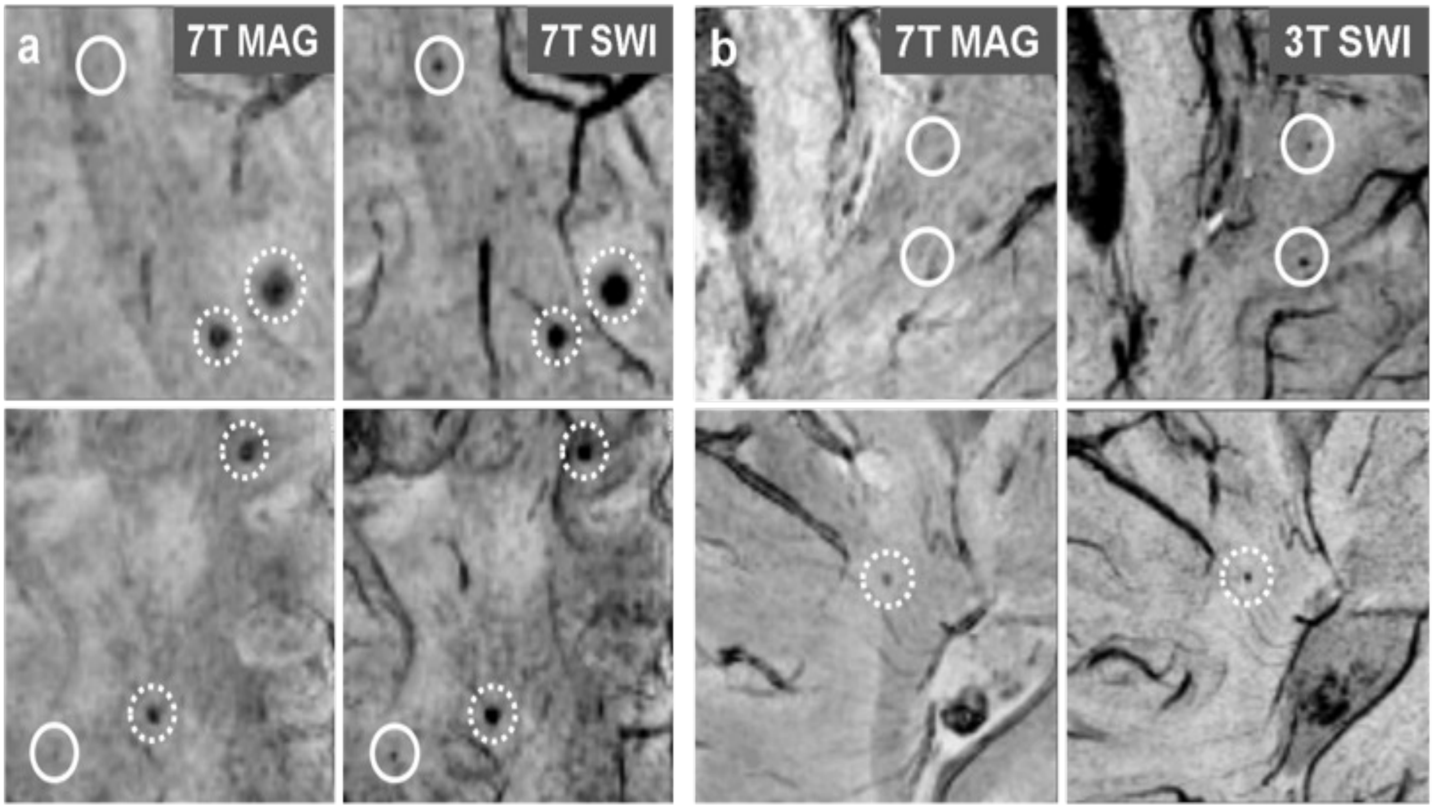
phase-weighted magnitude imaging





Barnes J et al. Magn Reson Imaging Clin N Am 2010

SWI – Example applications



Susceptibility MRI – source of contrast

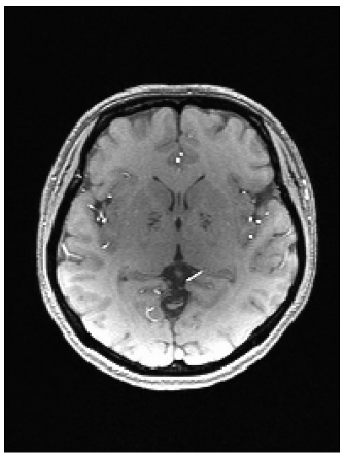
T_2*-weighted sequence

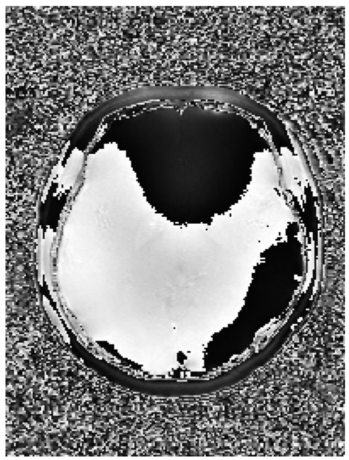
T_2*-weighted magnitude

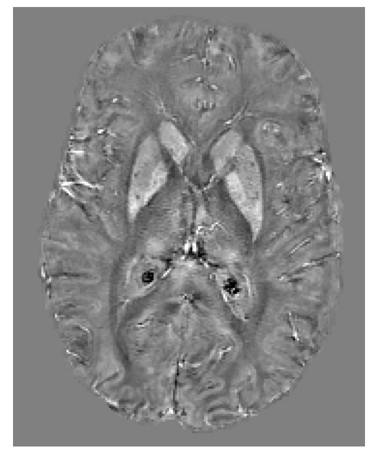
Phase

Susceptibility-weighted imaging

Quantitative susceptibility mapping

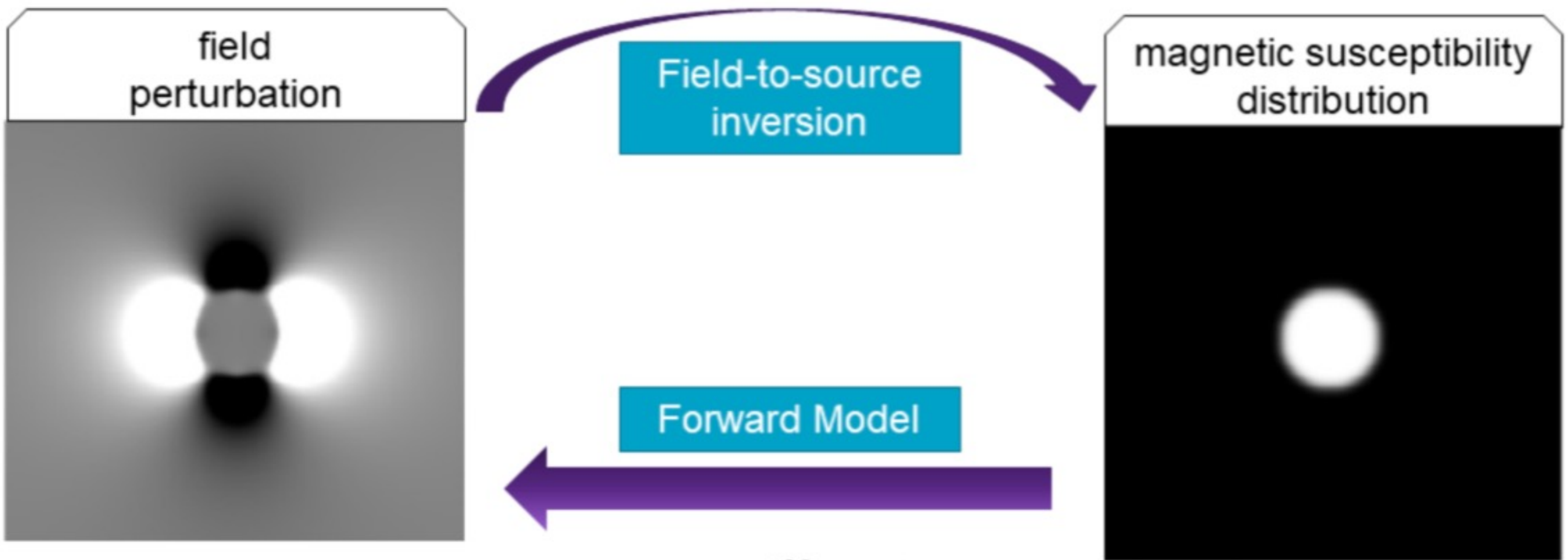






Barnes J et al. Magn Reson Imaging Clin N Am 2010

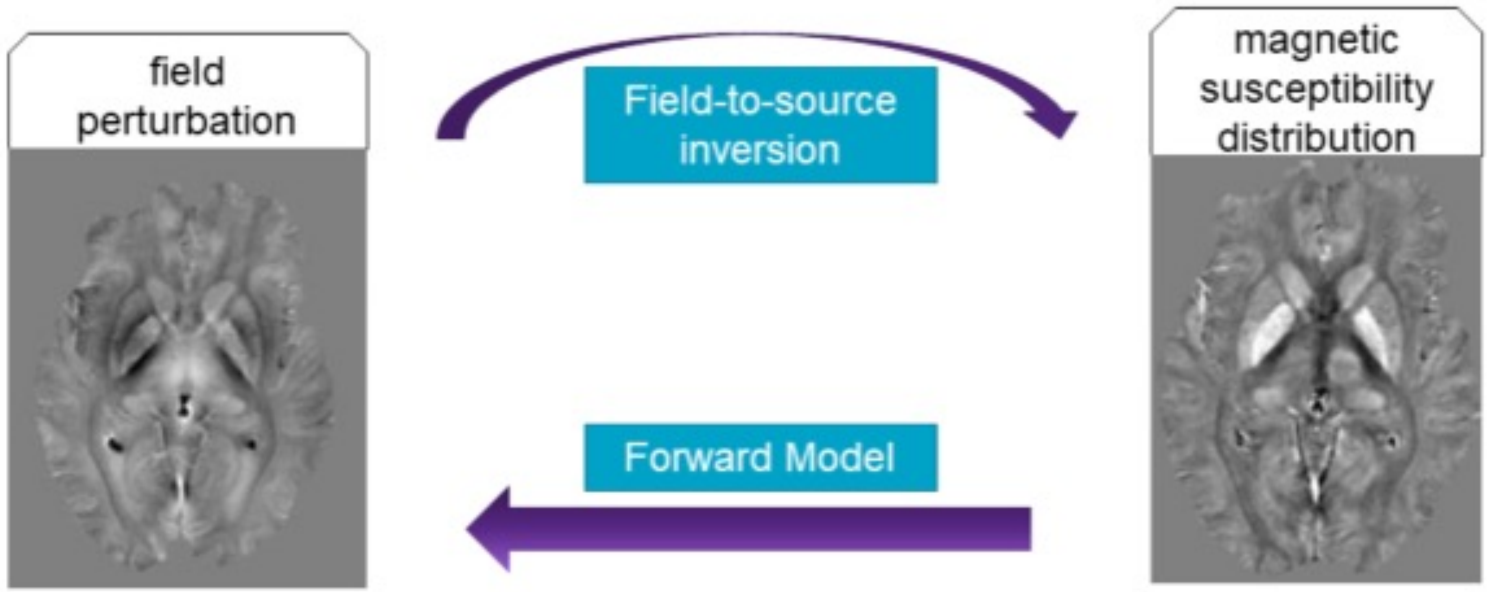
Susceptibility MRI – Signal model



$$\Delta B_{\text{int}}(\vec{r}) = B_0 \cdot \int_{-\infty}^{\infty} \tilde{\chi}(\vec{r'}) \cdot d_z(\vec{r} - \vec{r'}) d^3 \vec{r'}$$

Review: Deistung et al. NMR Biomed 2017; Schweser et al. Z Med Phys 2016

Susceptibility MRI – Signal model



$$\Delta B_{\text{int}}(\vec{r}) = B_0 \cdot \int_{-\infty}^{\infty} \tilde{\chi}(\vec{r'}) \cdot d_z(\vec{r} - \vec{r'}) d^3\vec{r'}$$

What is it called?

Review: Deistung et al. NMR Biomed 2017; Schweser et al. Z Med Phys 2016

PollEv.com/smoothdove577

Susceptibility MRI – Signal model

$$\Delta B(\vec{r}) = B_0 \int_{-\infty}^{\infty} \chi(\overrightarrow{r'})d(\vec{r} - \overrightarrow{r'})d^3\overrightarrow{r'}$$

$$d(\vec{r}) = \frac{1}{4\pi}\frac{3\cos^2(\theta) - 1}{|\vec{r}|^3}$$

$$\Delta B = B_0(\chi \otimes d)$$

Known

Measured

Unknown

FT

$$\Delta B(\vec{k}) = B_0[\chi(\vec{k})d(\vec{k})]$$

$$d(\vec{k}) = \frac{1}{3} - \frac{k_z^2}{|\vec{k}|^2}$$

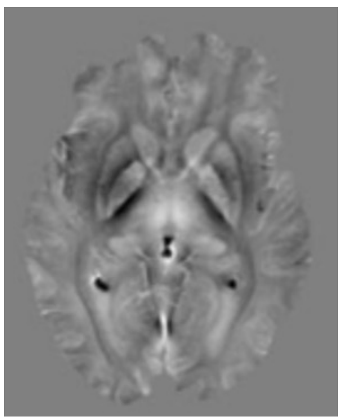
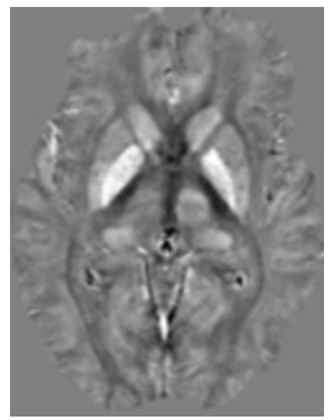
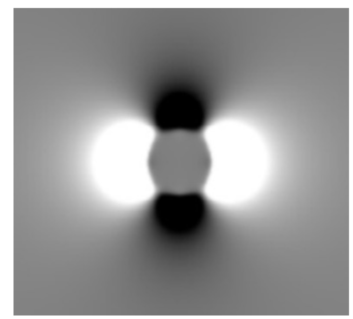
What is the FT of convolution?

PollEv.com/smoothdove577

Susceptibility MRI – Signal model

$$\Delta B(\vec{k}) = B_0[\chi(\vec{k})d(\vec{k})]$$

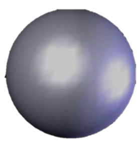
$$d(\vec{k}) = \frac{1}{3} - \frac{k_z^2}{|\vec{k}|^2}$$

$FT($  $) = B_0[FT($  $) \cdot$  $]$

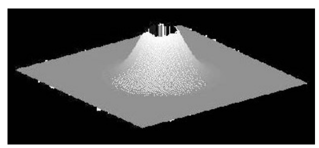
Susceptibility MRI – Signal model

$\mathbf{B_0}$

Susceptibility source (Iron)



B0 field perturbation



Frequency shift

$$\Delta\omega = \gamma \Delta B_0$$

Phase shift from tissue susceptibility

$$\Delta\phi = \Delta\omega \cdot TE$$

Phase shift from local magnetic field

$$\Delta\phi_{total} = \Delta\phi_{tissue} + \Delta\omega_{background} \cdot TE$$

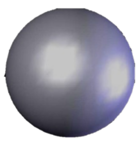
Measured phase (single coil)

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

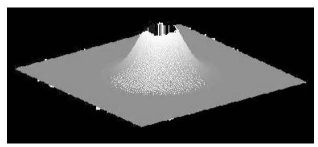
Susceptibility MRI – Processing

$\mathbf{B_0}$

Susceptibility source (Iron)



B0 field perturbation



Frequency shift

$$\Delta\omega = \gamma \Delta B_0$$

Phase shift from tissue susceptibility

$$\Delta\phi = \Delta\omega \cdot TE$$

Measured phase (single coil)

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

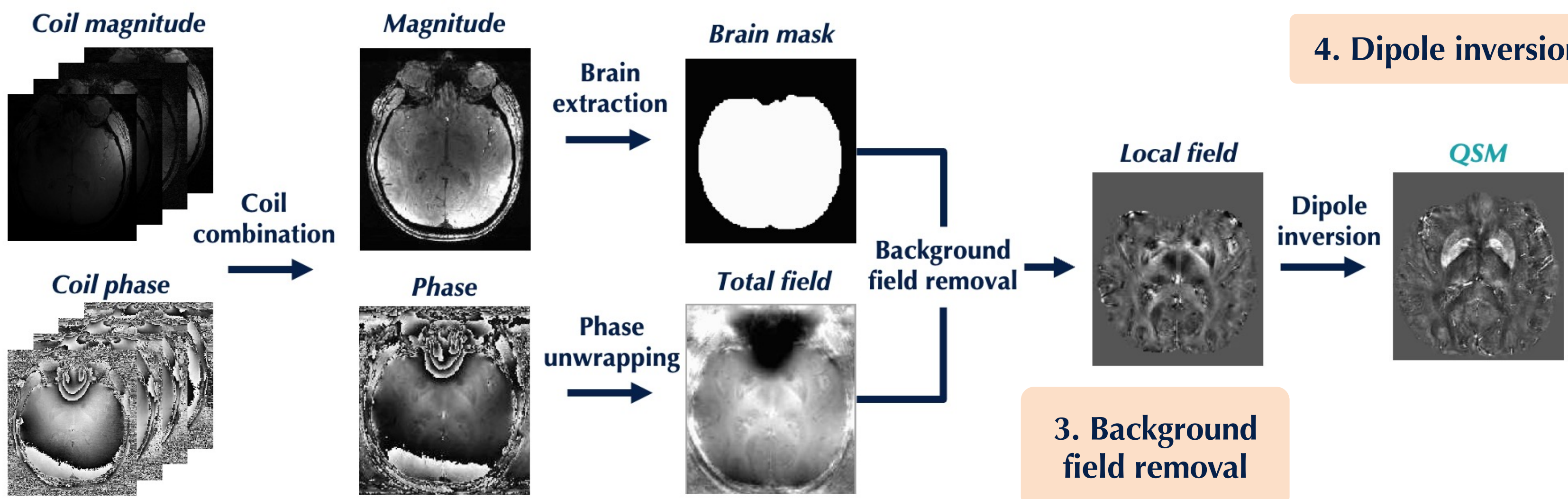
Phase shift from local magnetic field

$$\Delta\phi_{total} = \Delta\phi_{tissue} + \Delta\omega_{background} \cdot TE$$

QSM - Processing

1. Coil combination

T_2*-weighted sequence



4. Dipole inversion

3. Background field removal

2. Phase unwrapping

QSM - Processing

1. Coil combination

T_2*-weighted sequence

Coil magnitude

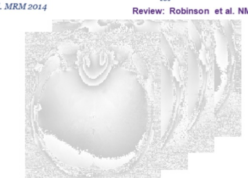
Magnitude

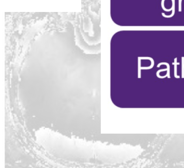
Phase combination approaches

1 echo, No reference scan	• Scalar phase matching [1] • Adaptive combine [2] • Virtual Reference Coil [3]
1 echo, Reference scan	• Roemer/SENSE [4] • COMPOSER [5]
Multiple echoes	• SVD [6] • Solve for ΔB_0 via phase difference [7] • Solve for φ_0^C: ASPIRE [8]

[1] Hammond et al. NI 2008
[2] Walsh et al. MRM 2000
[3] Parker et al. MRM 2014
[4] Roemer et al. NI 1990
[5] Robinson et al. MRM 2017
[6] Khabipova et al. NI 2015
[7] Bernstein et al. MRM 1994
[8] Eckstein et al. MRM 2018

Review: Robinson et al. NMR Biomed 2017





Background field correction methods & assumptions

no sources close to boundaries	• SHARP (Schweser et al. NI 2011) • V-SHARP (Li et al. NI 2011)
no harmonic internal and boundary fields	• LBV (Zhou et al. NMR Biomed 2014)
no implicit boundary assumption	• RESHARP (Sun et al. MRM 2014) • SHARQnet (Bollmann et al. Z Med Phys 2018)

Review: Schweser et al. NMR Biomed 2017

Unwrapping techniques

Laplacian	• differentiable operator applied to the unwrapped phase can produce the same result on the wrapped phase -> Laplacian (Schofield and Zhu, Opt. Lett. 2003) • + fast & robust • - introduces background phase
Region-growing	• Identify discontinuities between regions • PRELUDE (Jenkinson MRM 2003) can take a while to compute for highly wrapped data • SEGUE (Karsa et al. TMI 2019) similar accuracy to PRELUDE, but faster
Path Based	• 3D voxel-by-voxel unwrapping guided by the quality of voxel connections • BEST PATH (Abdul-Rahman et al. AO 2007) • ROMEO (Dymerska et al. MRM 2021)

Review: Robinson et al., NMR Biomed 2017 29

4. Dipole inversion

Dipole inversion methods & assumptions

multiple orientations	• COSMOS (Liu et al. MRM 2009) • STI (Liu MRM 2010) • analytical solutions, but not practical
inverse filtering	• TKD (Shmueli et al. MRM 2009) • fast, but need parameter tweaking
iterative methods	• LSQR (Li et al. NI 2015) • MEDI (Liu et al. MRM 2013) • slow, need parameter tweaking
agnostic deep learning	• QSMnet (Yoon et al. NI 2018) • DeepQSM (Bollmann et al. NI 2019) • fast, but fragile
hybrid methods	• FINE (Zhang et al. NI 2020) • Variational Networks (Lai et al. arXiv 2020) • Deep learning priors + data consistency constraints

Reviews: Schweser et al. NMR Biomed 2017; Jung et al. NMR Biomed 2020 44

3. Background field removal

2. Phase unwrapping

QSM - Coil Combination

Special issue review article

NMR IN BIOMEDICINE

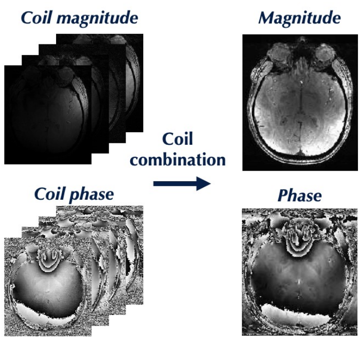
Received: 11 November 2015, Revised: 14 June 2016, Accepted: 18 July 2016, Published online in Wiley Online Library: 13 September 2016

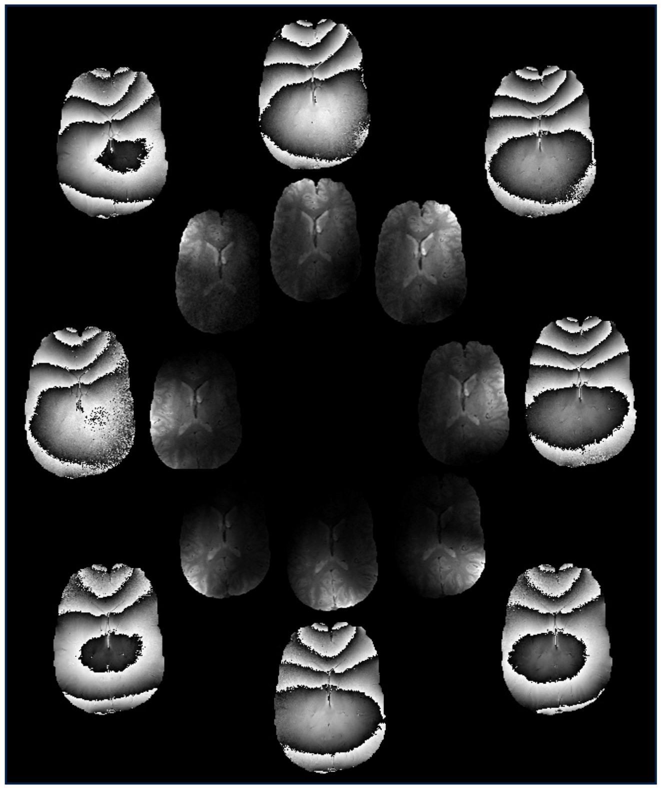
(wileyonlinelibrary.com) DOI: 10.1002/nbm.3601

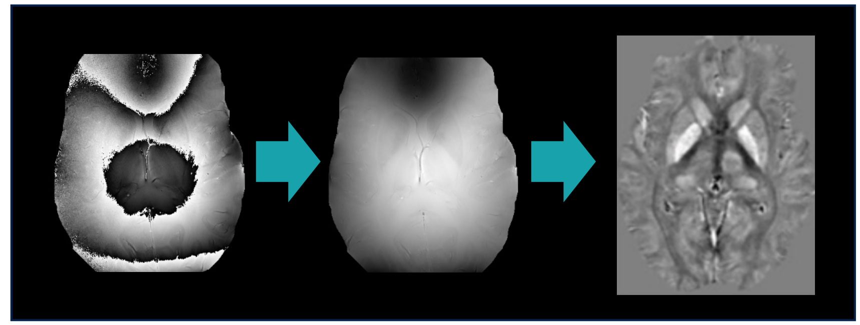
An illustrated comparison of processing methods for MR phase imaging and QSM: combining array coil signals and phase unwrapping

Simon Daniel Robinson[a]*, Kristian Bredies[b], Diana Khabipova[c,d], Barbara Dymerska[a], José P. Marques[c,d] and Ferdinand Schweser[e,f]

QSM - Coil Combination



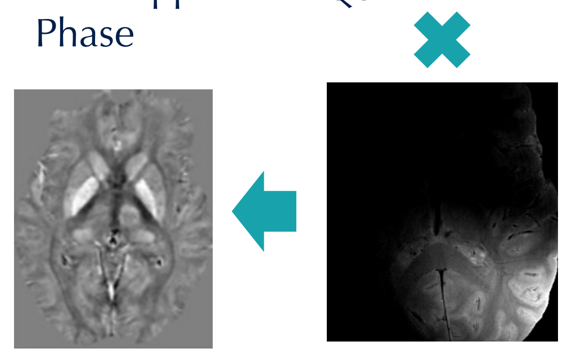




Single Channel Phase

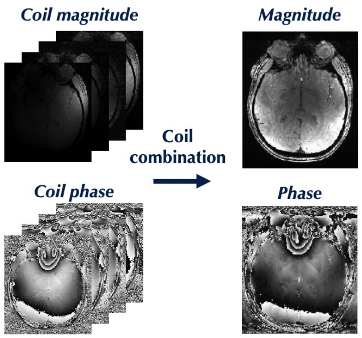
Single Channel Unwrapped Phase

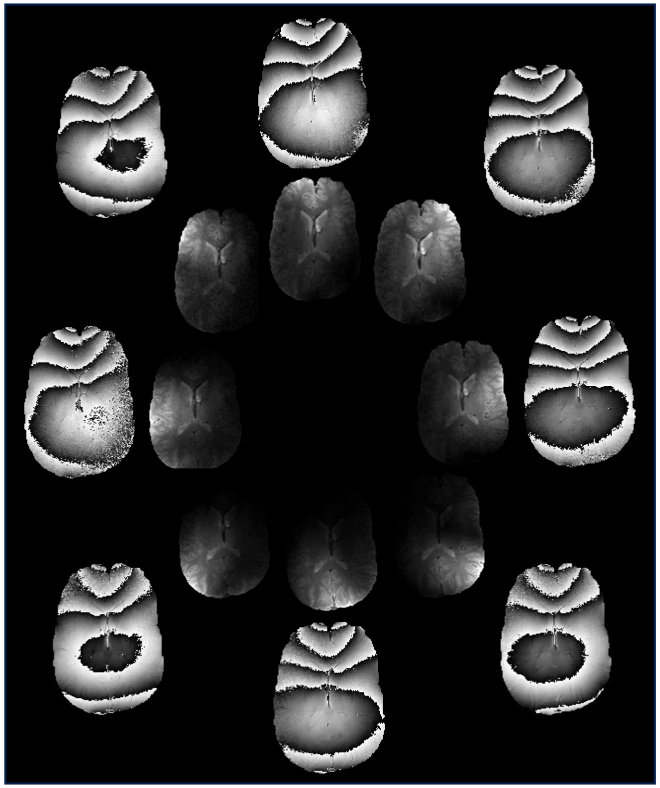
Single Channel QSM

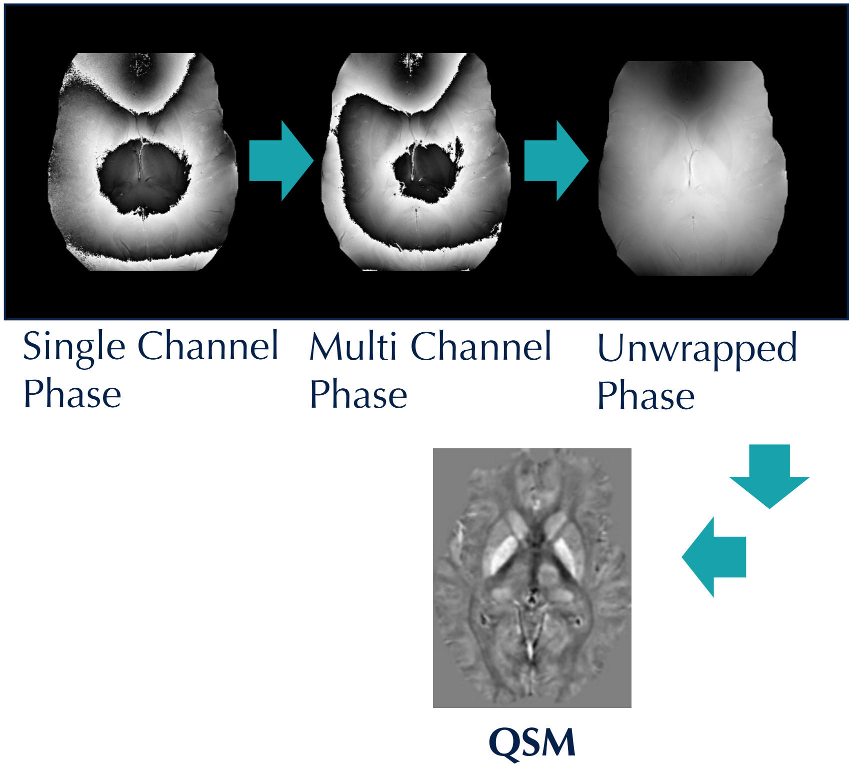


QSM

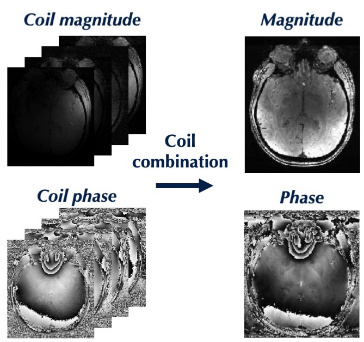
QSM - Coil Combination







QSM - Coil Combination

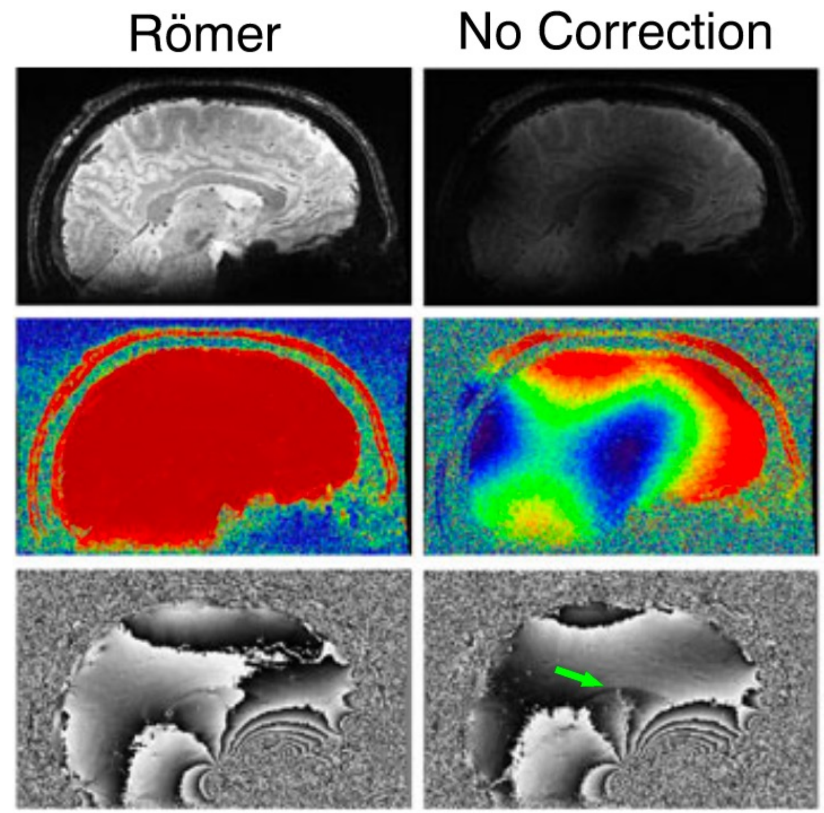


Measured phase
(single coil)

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

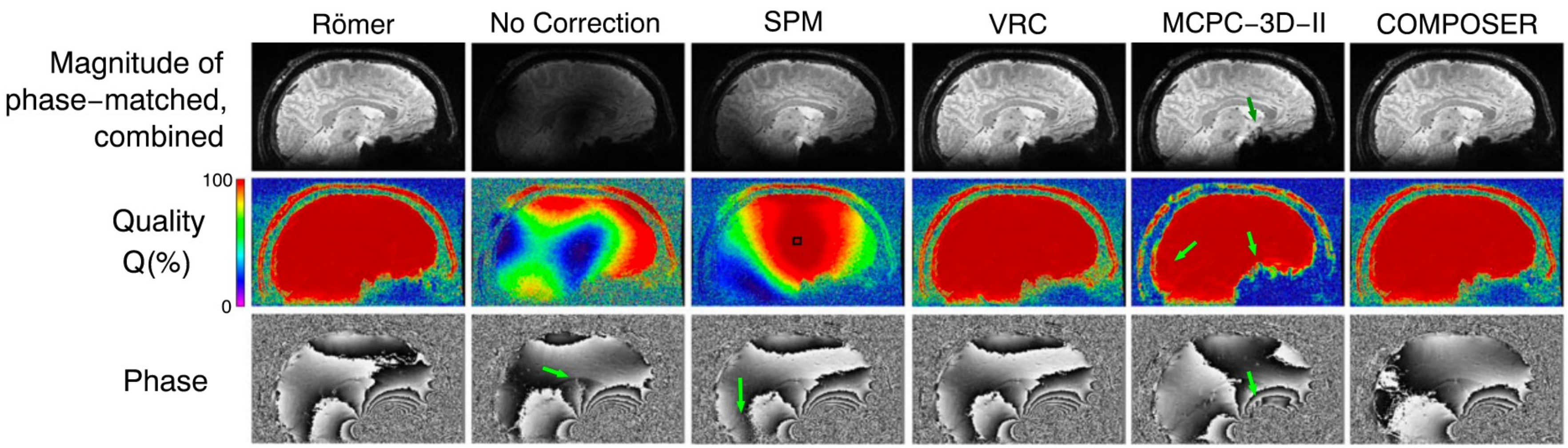
Transceiver phase

spatially varying phase offsets exist between receive coils



Robinson S et al., NMR Biomed, 2015

QSM - Coil Combination



With reference coil

Scalar phase matching (MCPC-C)

Virtual reference coil

3D phase offsets derived from a dual echo scan

Phase offsets from a short echo-time reference

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

$$\varphi_j^0(\vec{r}) = \frac{\varphi_j(\vec{r}, TE_2)TE_1 - \varphi_j(\vec{r}, TE_1)TE_2}{TE_1 - TE_2}.$$

Robinson S et al., NMR Biomed, 2015

QSM – MCPC-3D

Steps:

$$\varphi_j^0(\vec{r}) = \frac{\varphi_j(\vec{r}, TE_2)TE_1 - \varphi_j(\vec{r}, TE_1)TE_2}{TE_1 - TE_2}.$$

- unwrap each echo phase
- create 3D phase offset map for each coil using each unwrapped echo
- smooth with 5x5 median filter (complex smoothing)
- subtract 3D phase offset map from phase image of each channel
- weighted mean

Advantages:

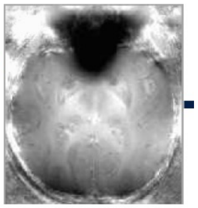
- works where there is no signal overlap between receivers
- no need for reference coil
- also works using a separate low-resolution scan

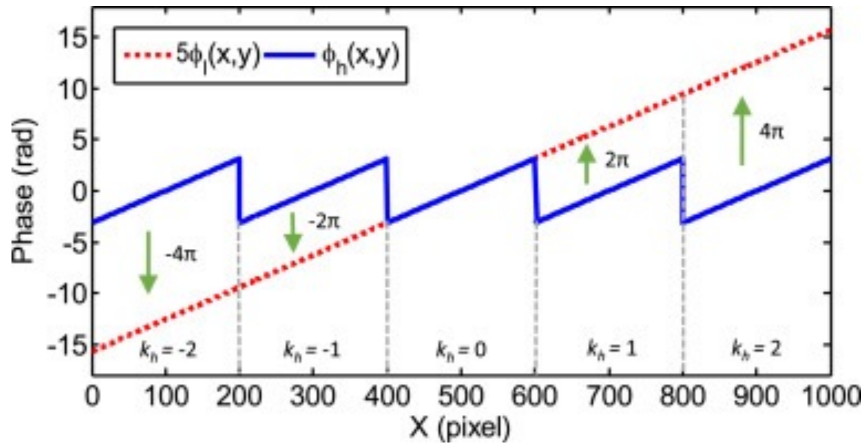
QSM - Phase unwrapping

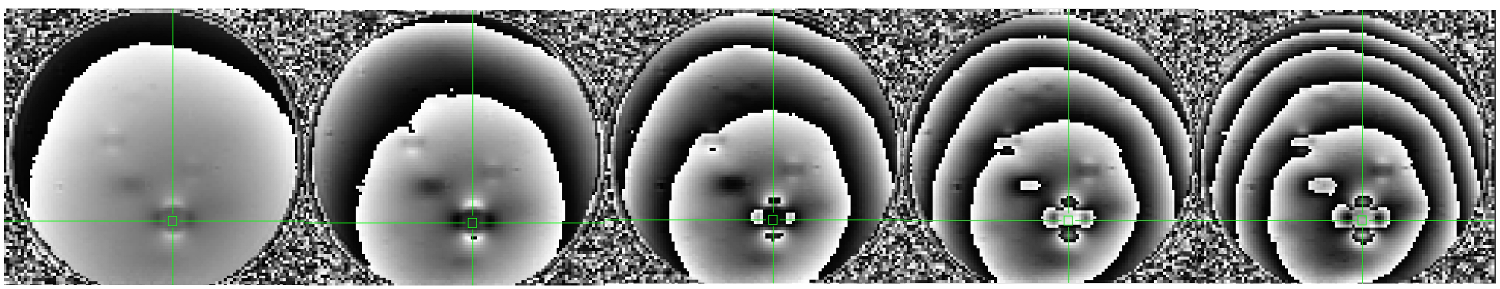
Phase

Phase unwrapping

Total field

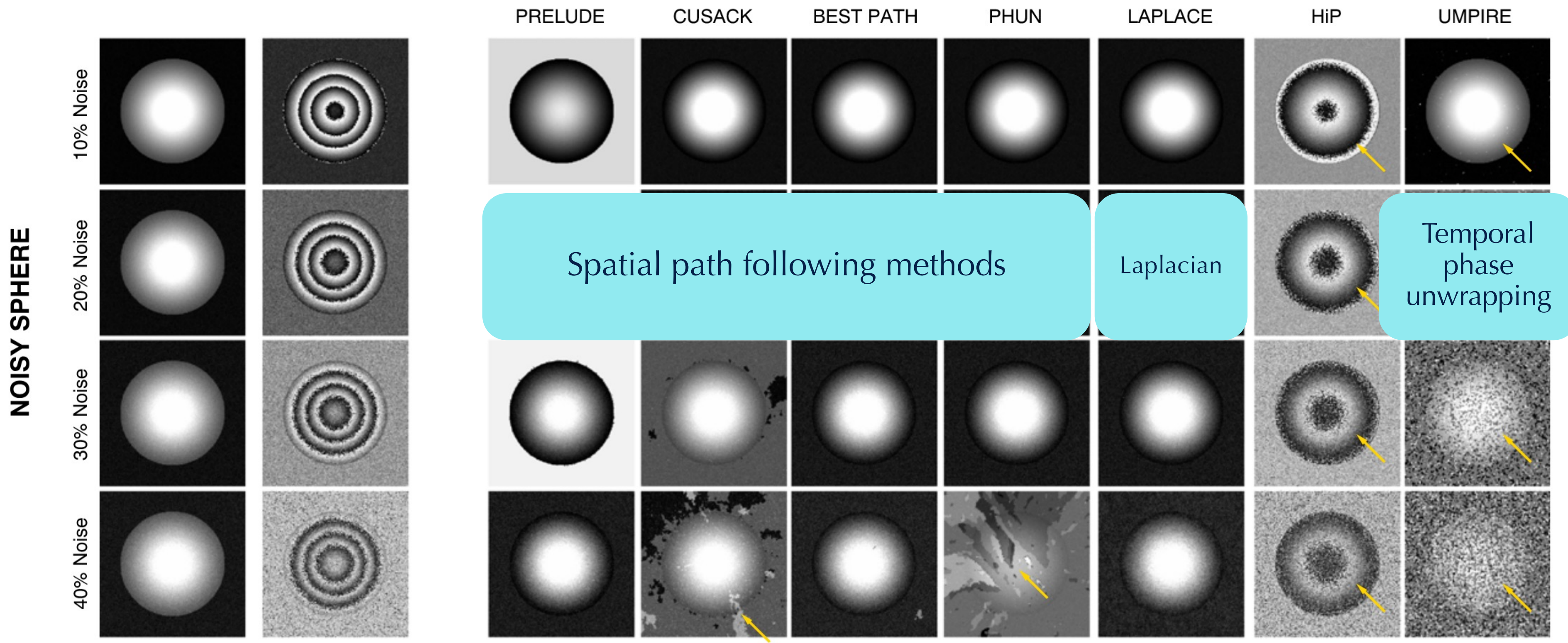






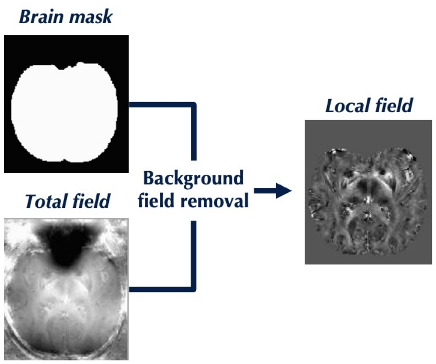
5.84 ms → 40.56 ms

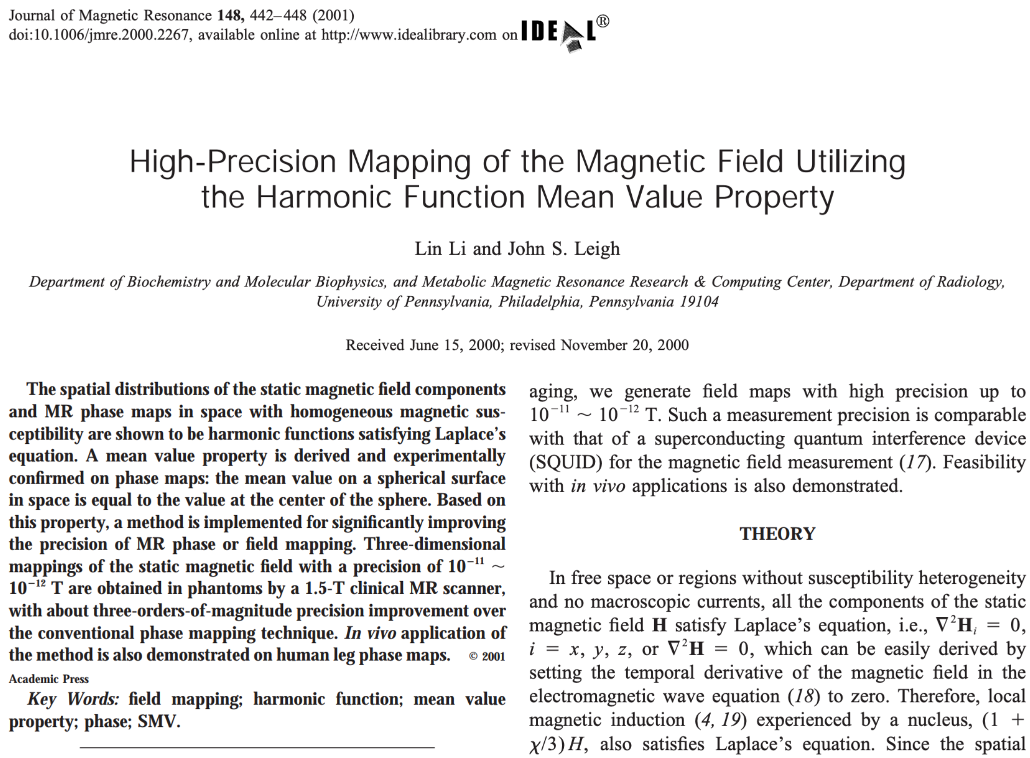
QSM - Phase unwrapping



Robinson S et al., NMR Biomed, 2015

QSM - Background field removal





Journal of Magnetic Resonance **148,** 442–448 (2001)
doi:10.1006/jmre.2000.2267, available online at http://www.idealibrary.com on IDEAL®

High-Precision Mapping of the Magnetic Field Utilizing the Harmonic Function Mean Value Property

Lin Li and John S. Leigh

Department of Biochemistry and Molecular Biophysics, and Metabolic Magnetic Resonance Research & Computing Center, Department of Radiology, University of Pennsylvania, Philadelphia, Pennsylvania 19104

Received June 15, 2000; revised November 20, 2000

The spatial distributions of the static magnetic field components and MR phase maps in space with homogeneous magnetic susceptibility are shown to be harmonic functions satisfying Laplace's equation. A mean value property is derived and experimentally confirmed on phase maps: the mean value on a spherical surface in space is equal to the value at the center of the sphere. Based on this property, a method is implemented for significantly improving the precision of MR phase or field mapping. Three-dimensional mappings of the static magnetic field with a precision of $10^{-11} \sim 10^{-12}$ T are obtained in phantoms by a 1.5-T clinical MR scanner, with about three-orders-of-magnitude precision improvement over the conventional phase mapping technique. ***In vivo*** **application of the method is also demonstrated on human leg phase maps.**

Key Words: field mapping; harmonic function; mean value property; phase; SMV.

aging, we generate field maps with high precision up to $10^{-11} \sim 10^{-12}$ T. Such a measurement precision is comparable with that of a superconducting quantum interference device (SQUID) for the magnetic field measurement (*17*). Feasibility with *in vivo* applications is also demonstrated.

THEORY

In free space or regions without susceptibility heterogeneity and no macroscopic currents, all the components of the static magnetic field **H** satisfy Laplace's equation, i.e., $\nabla^2 \mathbf{H}_i = 0$, $i = x, y, z$, or $\nabla^2 \mathbf{H} = 0$, which can be easily derived by setting the temporal derivative of the magnetic field in the electromagnetic wave equation (*18*) to zero. Therefore, local magnetic induction (*4, 19*) experienced by a nucleus, $(1 + \chi/3)H$, also satisfies Laplace's equation. Since the spatial

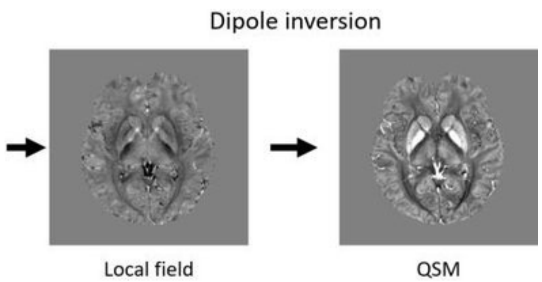
VSHARP

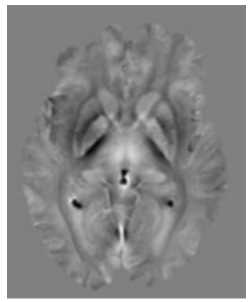
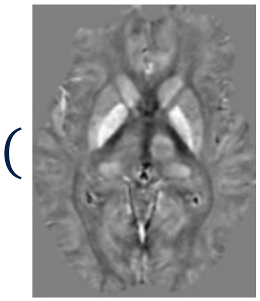
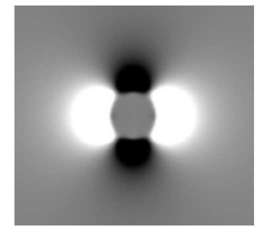
RESHARP

HARPERELLA

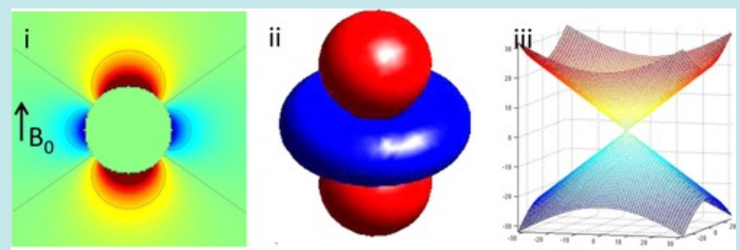
PDF

QSM - Dipole Inversion



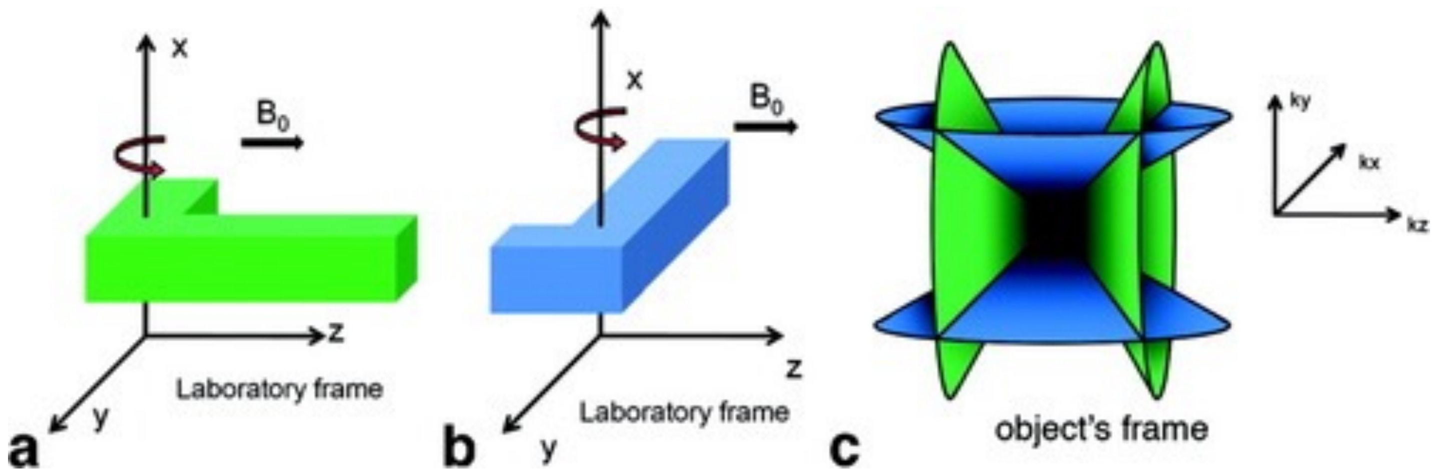
$FT($  $) = B_0[FT($  $) \cdot$  $]$

ill-posed inversion problem
Noise amplification near the zero cone surfaces



QSM - Dipole Inversion

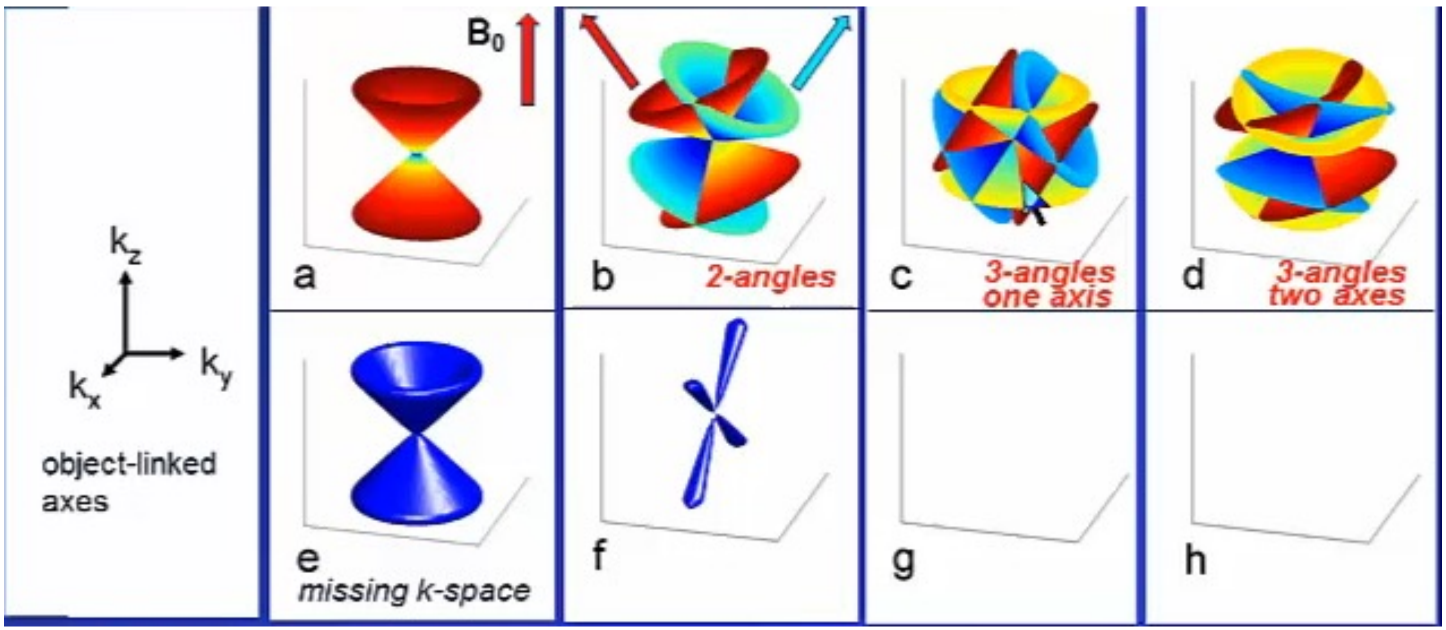
COSMOS: calculation of susceptibility using multiple orientation sampling



Liu T, MRM, 2008

QSM - Dipole Inversion

COSMOS: calculation of susceptibility using multiple orientation sampling

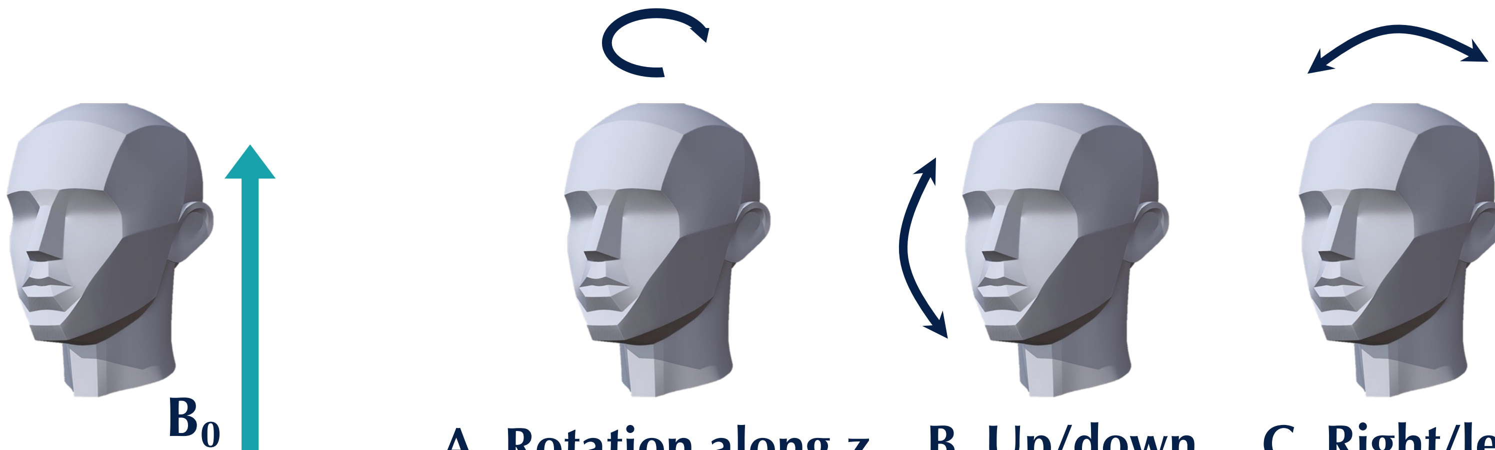


Liu T, MRM, 2008

QSM - Dipole Inversion

COSMOS: calculation of susceptibility using multiple orientation sampling

How would you reorient?



A. Rotation along z

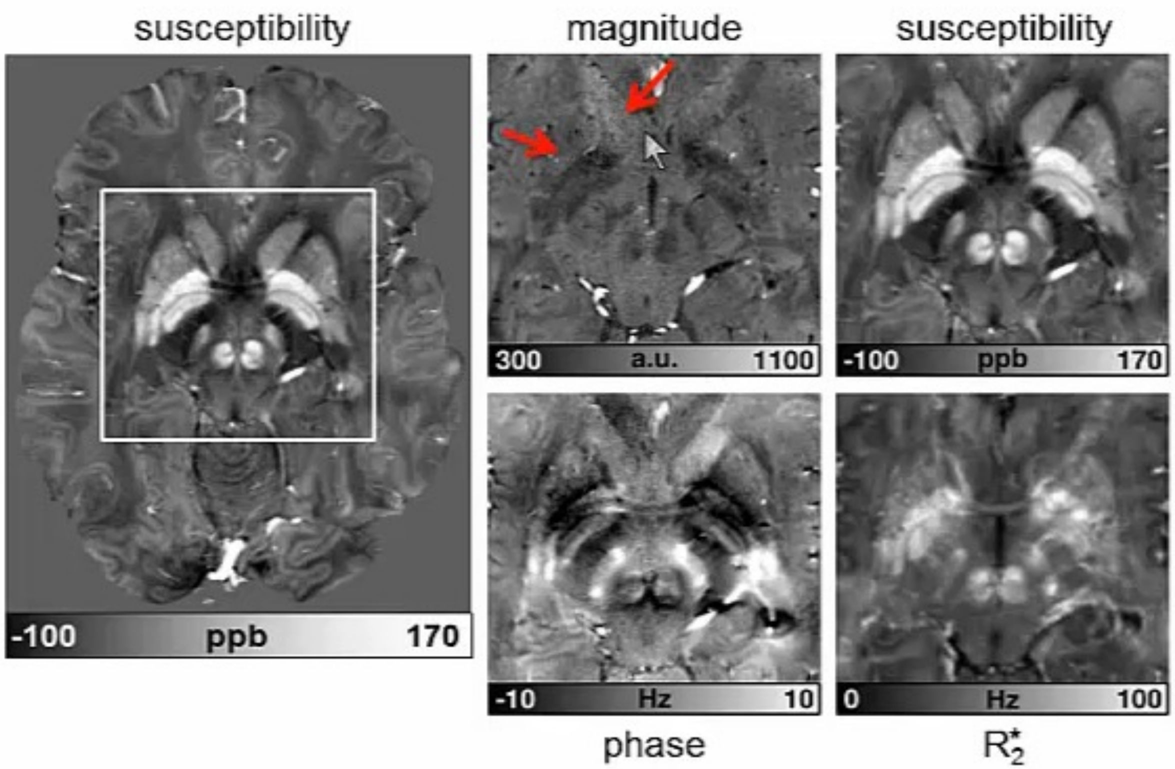
B. Up/down nodding

C. Right/left head tilt

PollEv.com/smoothdove577

QSM - Dipole Inversion

COSMOS:



Deistung, A et al. **NeuroImage** 2013, 65, 299–314.

QSM Dipole Inversion: TKD

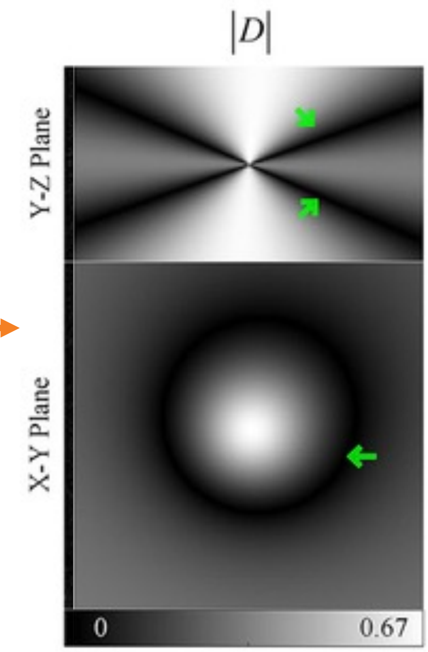
TKD: Truncated K-space Division

Dipole kernel

$$\psi(\mathbf{k}) = D_2(\mathbf{k}) \cdot \chi(\mathbf{k})$$

Field perturbation

Susceptibility distribution



$$D_2(k) = \begin{cases} D_2(k) & if\ |D_2(k)| \geq \delta \\ \delta \cdot sign(D_2(k)) & otherwise \end{cases}$$

Shmueli K, 2009

QSM Dipole Inversion: iLSQR

iLSQR: iterative method solving least square using the orthogonal and right triangular decomposition

Dipole kernel

$$\psi(\mathbf{k}) = D_2(\mathbf{k}) \cdot \chi(\mathbf{k})$$

Field perturbation

Susceptibility distribution

1st order derivative

$$\psi'(\mathbf{k}) + \left[2\left(k_x^2 + k_y^2\right)k_z / k^4\right] \cdot \chi(\mathbf{k}) - D_2(\mathbf{k}) \cdot \chi'(\mathbf{k}) = 0$$

$$D_3(\mathbf{k}) \cdot \chi(\mathbf{k}) + D_2(\mathbf{k}) \cdot FT[i \cdot r_z \chi(\mathbf{r})] = FT[i \cdot r_z \psi(\mathbf{r})]$$

$$\chi(\mathbf{k}) = D_2(\mathbf{k})^{-1} \cdot \psi(\mathbf{k}), \quad \text{when } D_2(\mathbf{k}) \geq \varepsilon$$

$$\chi(\mathbf{k}) \approx D_3(\mathbf{k})^{-1} \cdot FT[i r_z \psi(\mathbf{r})], \quad \text{when } D_2(\mathbf{k}) < \varepsilon$$

Where: $D_3(\mathbf{k}) = (k_x^2 + k_y^2)k_z / \pi k^4$

Li W, NeuroImage, 2011, 2015

QSM Dipole Inversion: iterative inversion methods with regularization

Recon problem

$$\operatorname{argmin}_{\chi} \frac{1}{2} \left\| W\left(F^{H} D F \chi - \Phi\right) \right\|_2^2 + \alpha \Omega(\chi)$$

Data consistency term

Regularization term

Nonlinear variant

$$\operatorname{argmin}_{\chi} \frac{1}{2} \left\| W\left(e^{iF^{H} D F \chi} - e^{i\Phi}\right) \right\|_2^2 + \alpha \Omega(\chi)$$

Method	Data consistency term	Regularization term
STAR-QSM (STreaking Artifact Reduction for QSM)	Linear L2-norm	Total variation
FANSI (FAst Nonlinear Susceptibility Inversion)	Nonlinear L2-norm	Total variation
HD-QSM (Hybrid Data fidelity)	Linear L1+L2-norm	Total variation
MEDI (Morphology Enabled Dipole Inversion)	Linear L1-norm	L1 norm of morphologically weighted gradients

Wei H, NMR in Biomed, 2015; Milovic C, MRM, 2018; https://github.com/mglambert/HD-QSM; Liu T, IEEE Trans Med Imaging, 2012

QSM Dipole Inversion: single step methods

QSIP

Quantifying Susceptibility by Inversion of a Perturbation model

$$\chi_1^* = arg \min_{\chi_1} \left[\lambda_1 |W \circ (\Delta B - \Delta (K_s * \chi_1))|_1 + \lambda_2 |M \circ (B - (K_s * \chi_1 + B_e))|_2^2 + \lambda_3 |M^C \circ (\chi_1 + \chi_0/\delta)|_2^2 \right]$$

Simultaneously estimating the external susceptibility outside the brain

SSTV, SSTGV

Single Step QSM with Total Variation / Total Generalized Variation penalties

$$\min_{\chi} \frac{1}{2} \sum_i \left\| M_i F^{-1} H_i D F \chi - M_i F^{-1} H_i F \Psi(\phi) \right\|_2^2 + R(\chi)$$

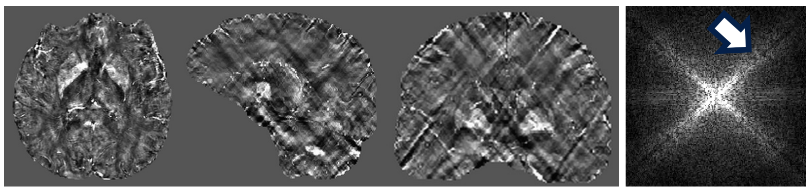
Perform VSHARP background field removal and dipole inversion in a single step

Poynton C, IEEE Trans Med Imaging, 2015
Chatnuntawech I, NMR in Biomed, 2017

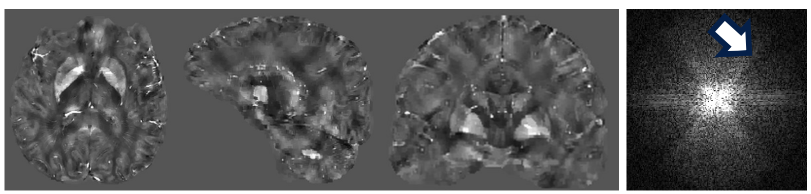
QSM Dipole Inversion: iterative inversion methods with regularization

Parameter optimization

(A) $\boldsymbol{\lambda = 10^{-6}}$

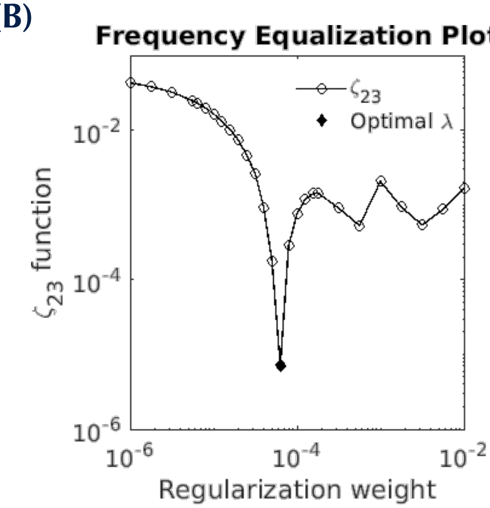


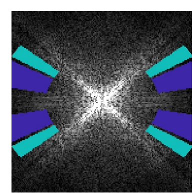
$\boldsymbol{\lambda = 10^{-4.2}}$



$\boldsymbol{\lambda = 10^{-3.5}}$

(B)





M2
M3

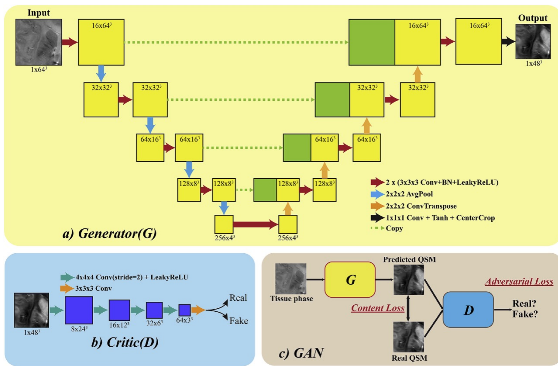
$$\xi_{ij} = \left(\frac{A_i - A_j}{A_i + A_j}\right)^2$$

A_i: Mean power in M_i

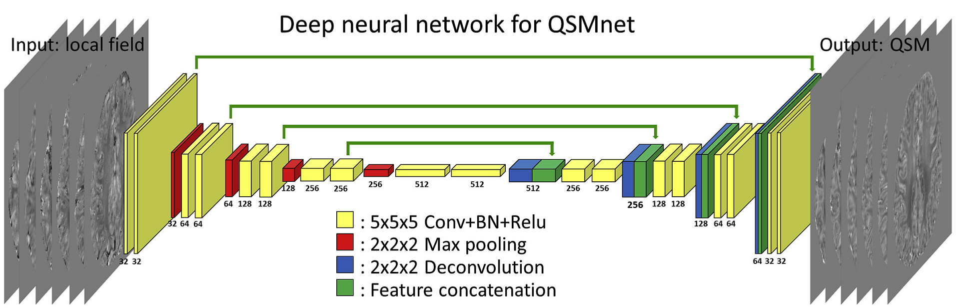
Yao J et al., NeuroImage, 2022

QSM Dipole Inversion: deep learning-based methods

QSMGAN



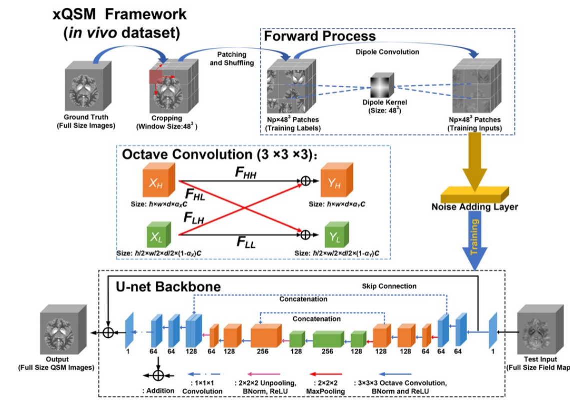
QSMnet+

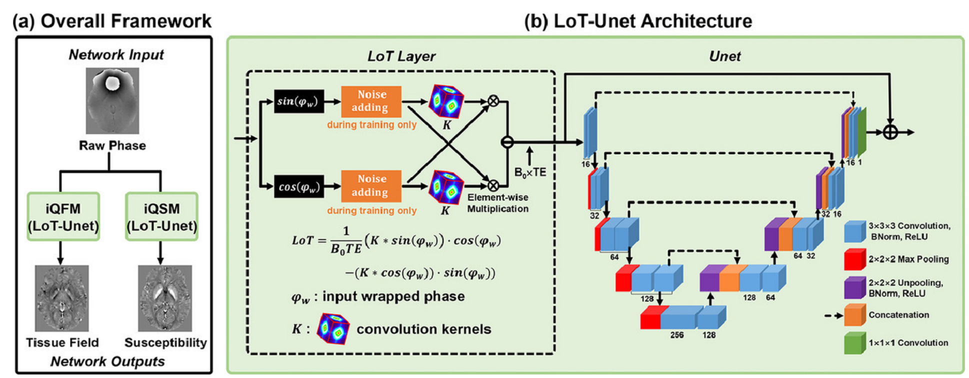


xQSM

Single step

iQSM





$$LoT = \frac{1}{B_0TE}(K * sin(\varphi_w)) \cdot cos(\varphi_w) - (K * cos(\varphi_w)) \cdot sin(\varphi_w))$$

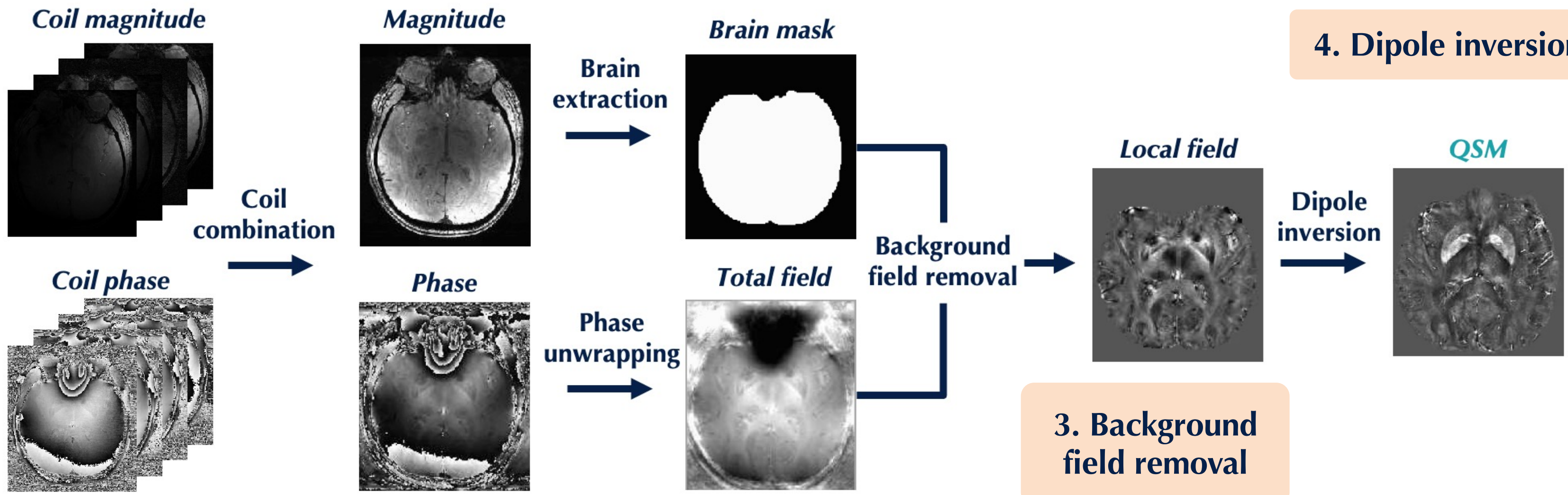
Chen Y, NeuroImage, 2020;
Yoon J, NeuroImage, 2018;
Jung W, NeuroImage, 2020

Gao Y, NMR in Biomed, 2020
Gao Y, NeuroImage, 2022

QSM - Processing

1. Coil combination

T_2*-weighted sequence



4. Dipole inversion

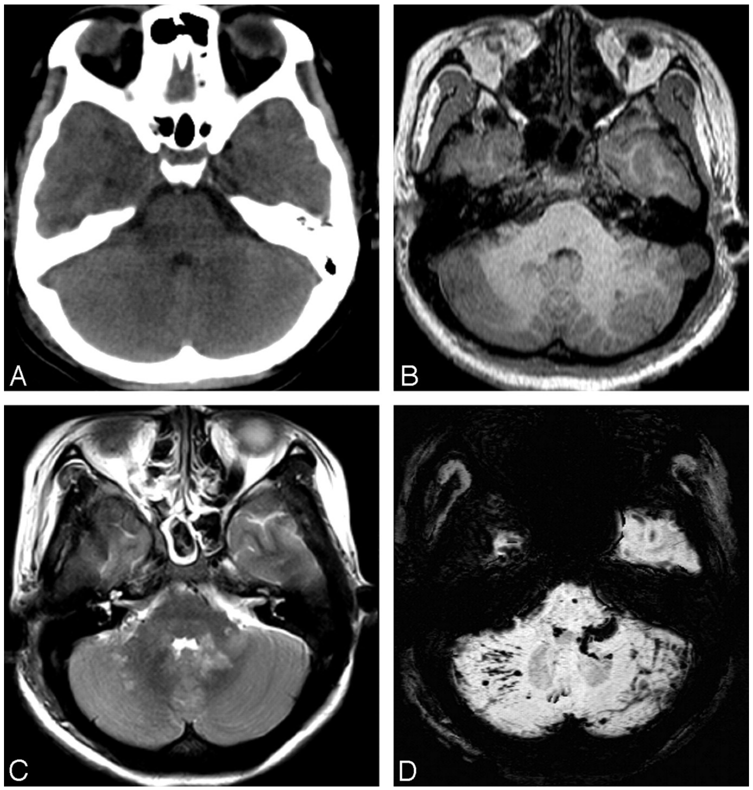
3. Background field removal

2. Phase unwrapping

SWI – Traumatic brain injury

Iron
- Blood product
- Vascular abnormalities
- Iron deposition

Calcium
- Infection
- Tumor



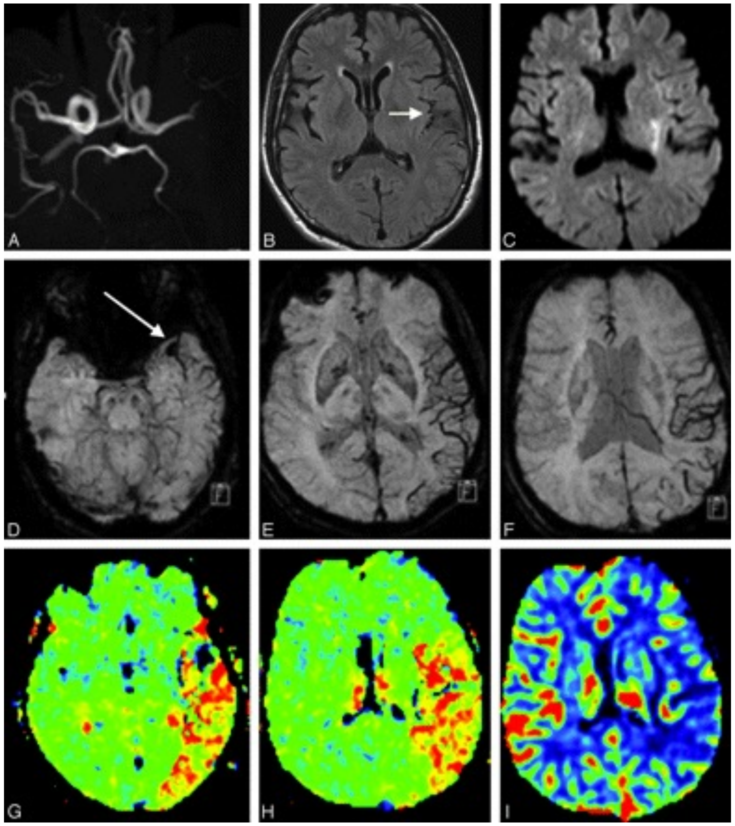
- SWI is **3–6 times more sensitive** than conventional T_2*-weighted gradient-echo sequences in detecting the size, number, volume, and distribution of hemorrhagic lesions in DAI.
- Number and volume of SWI hemorrhagic lesions appear to **correlate with specific neuropsychological deficits**.
- Especially helpful in **detecting traumatic lesions in the brainstem**, whereas other MR imaging sequences have failed to detect structural abnormalities.

Mittal S et al. AJNR 2009

SWI – Stroke

Iron
- Blood product
- Vascular abnormalities
- Iron deposition

Calcium
- Infection
- Tumor



- A 59-year-old man with a history of atrial fibrillation presented with sudden-onset right hemiparesis and speech deficits.
- MRI at 1.5T performed 2 hours after the onset of symptoms.
- SWI shows **prominent hypointense signals in the draining veins within areas of impaired perfusion**.

Mittal S et al. AJNR 2009

SWI – Multiple Sclerosis

Iron

Blood product

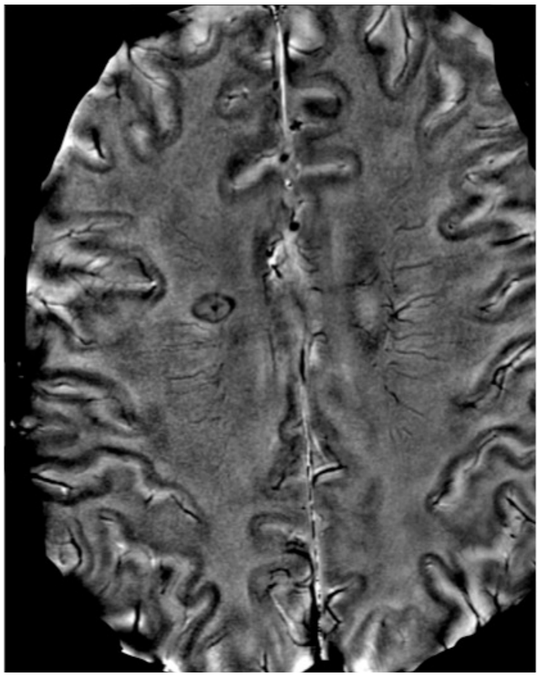
Vascular abnormalities

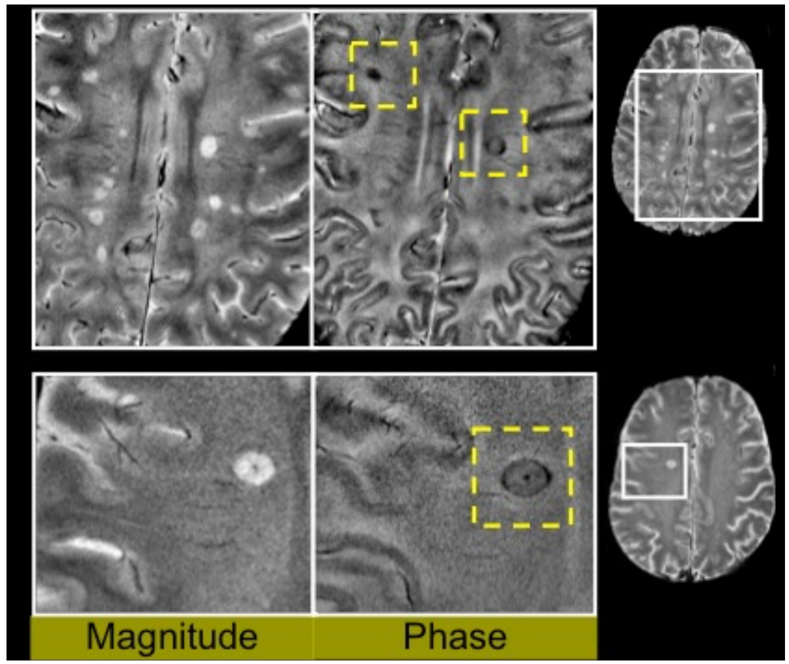
Iron deposition

Calcium

Infection

Tumor

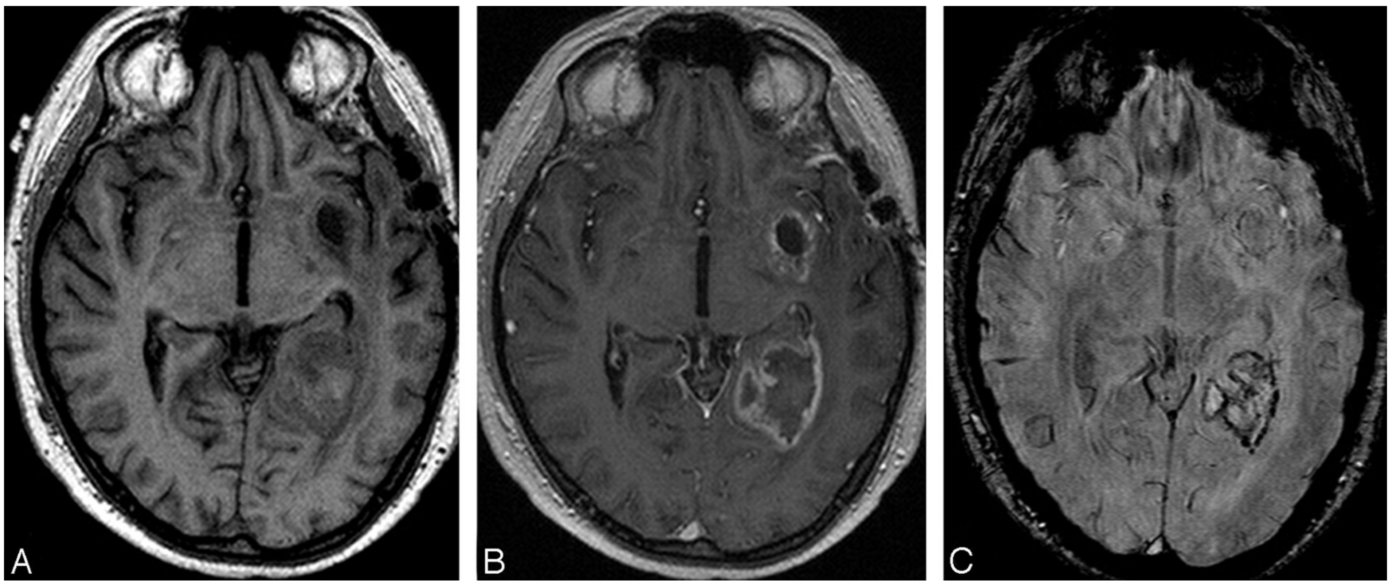




- Central vein sign
- Hypointense rim

Hammond et al, Annals of Neurology, 2008

SWI – Brain tumors



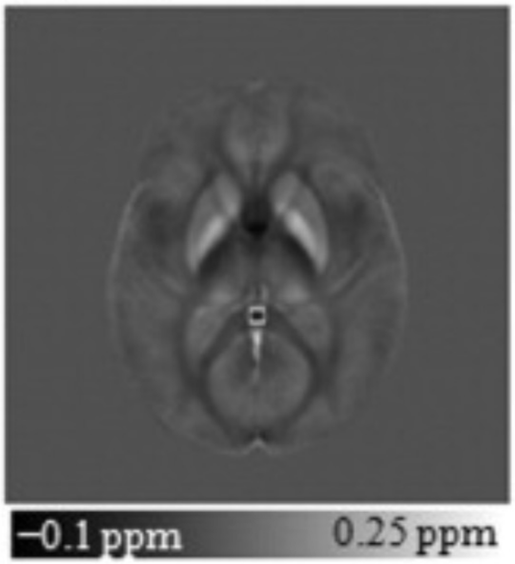
Iron
- Blood product
- Vascular abnormalities
- Iron deposition

Calcium
- Infection
- Tumor

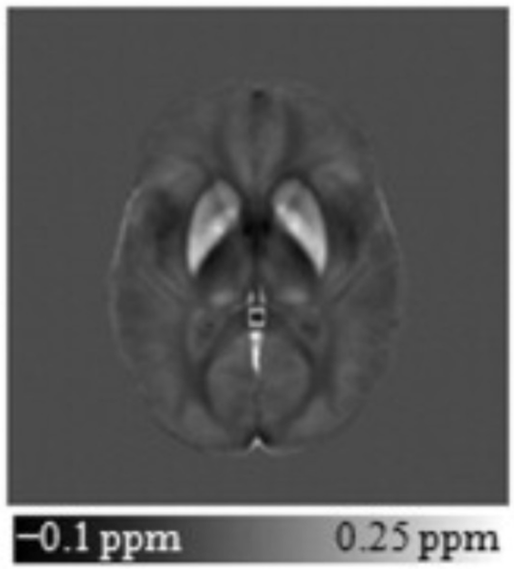
- Hemorrhage within tumors
- Pathologic vessels, angiogenesis

Mittal S et al. AJNR 2009

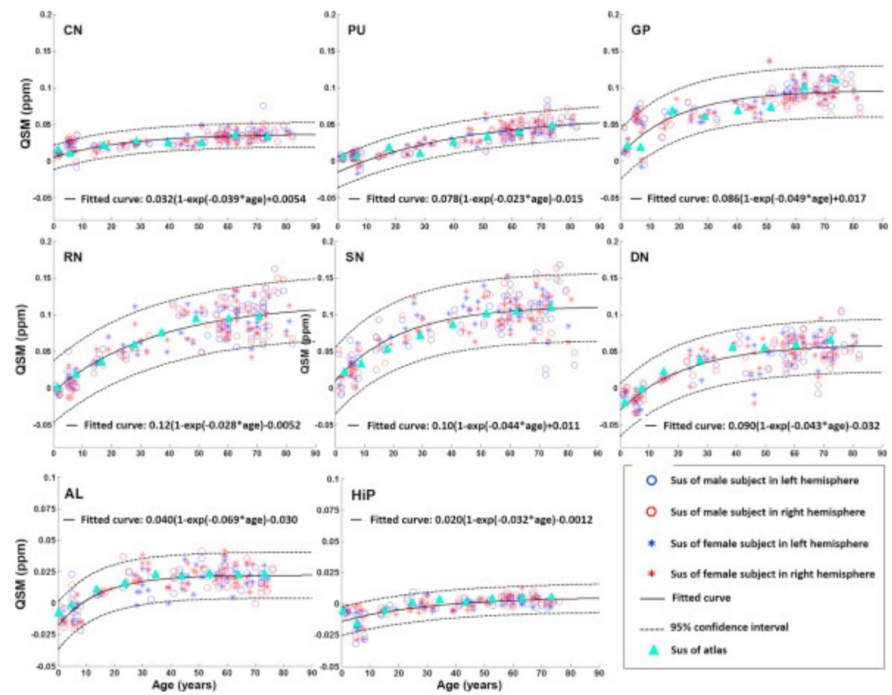
QSM applications - Aging



Young



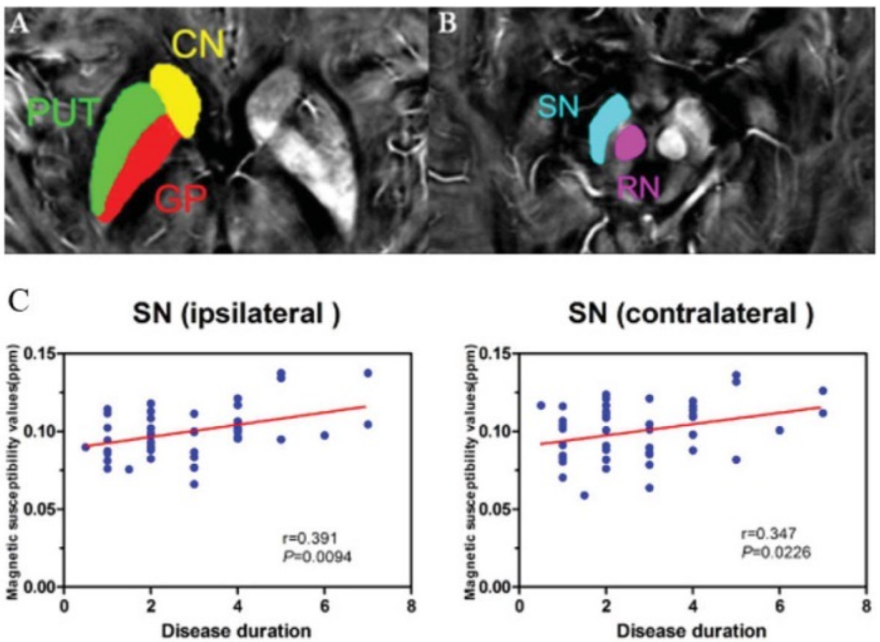
Elderly



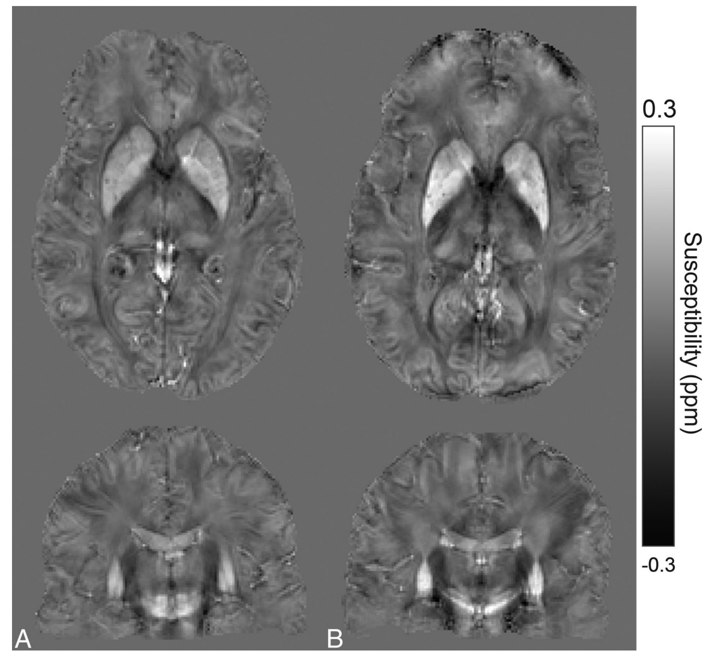
Bilgic B et al., NeuroImage, 2012
Zhang Y et al. Data Brief 2018

QSM applications – Neurodegenerative diseases

Parkinson's Disease

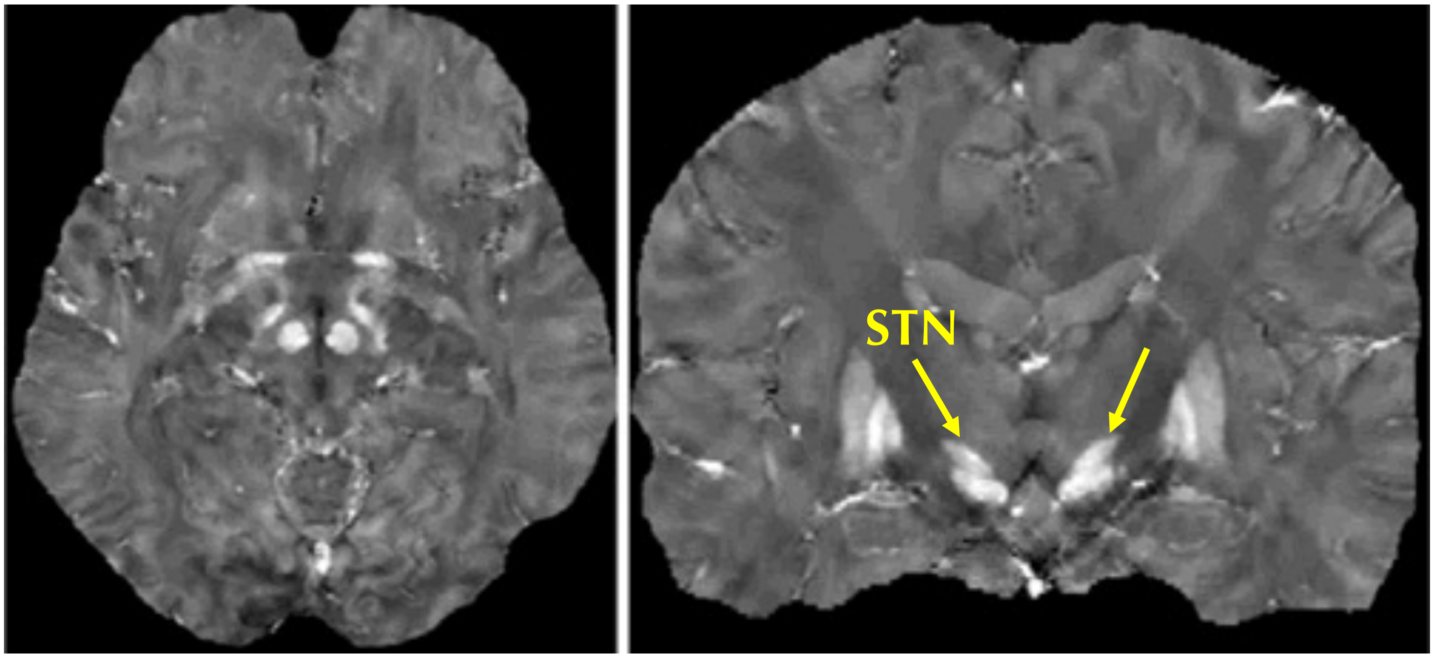


Huntington's Disease

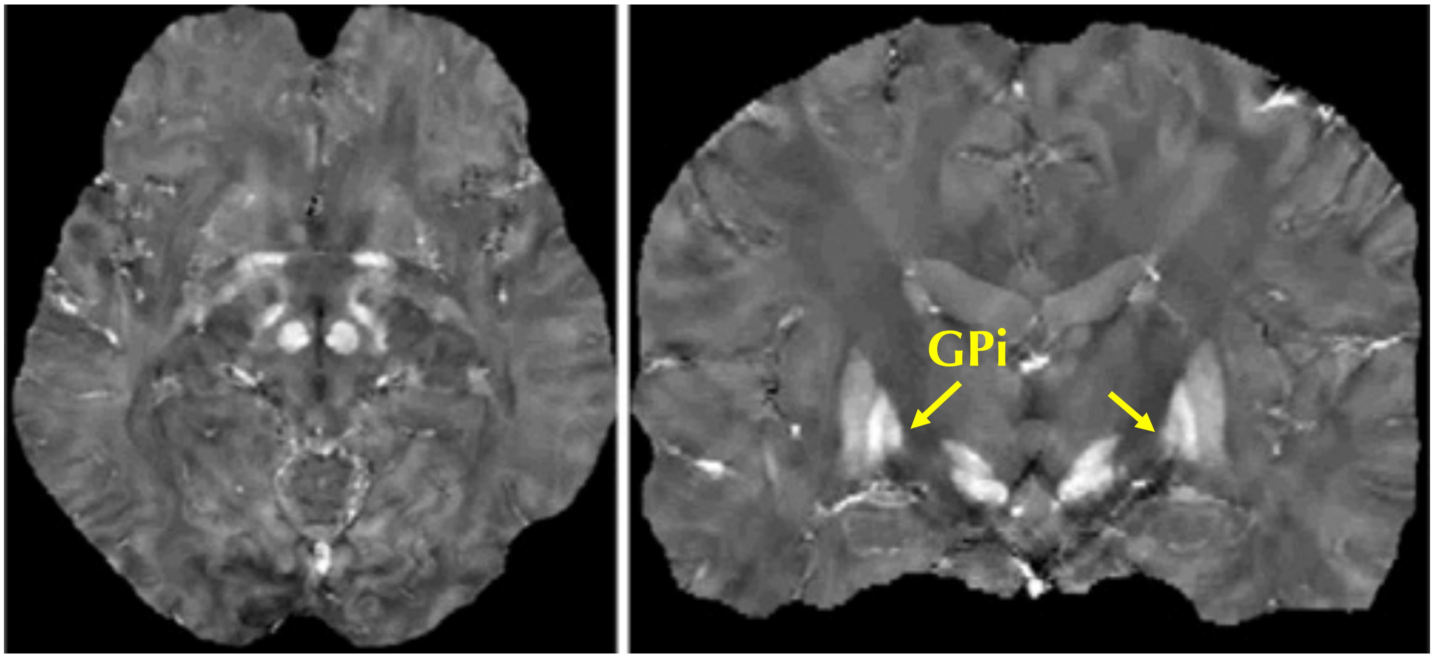


Azuma M et al., AJNR, 2016
van Bergen JM et al., AJNR, 2016

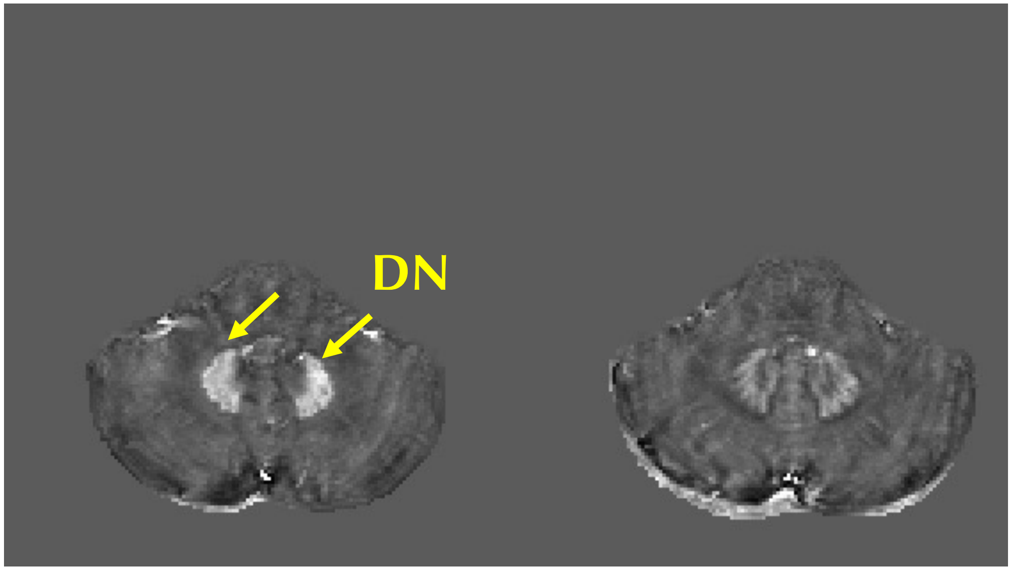
QSM applications – Functional neurosurgery

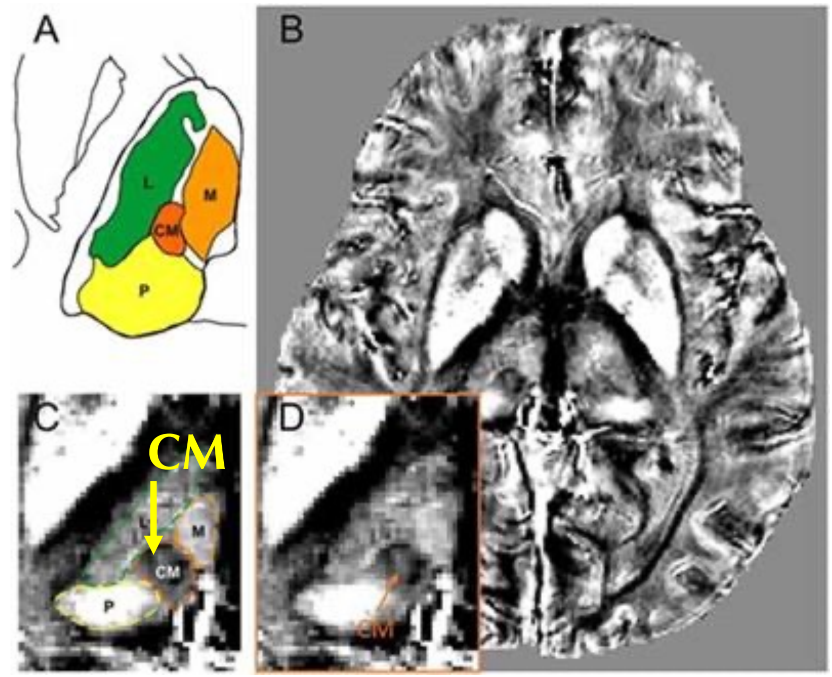


QSM applications – Functional neurosurgery



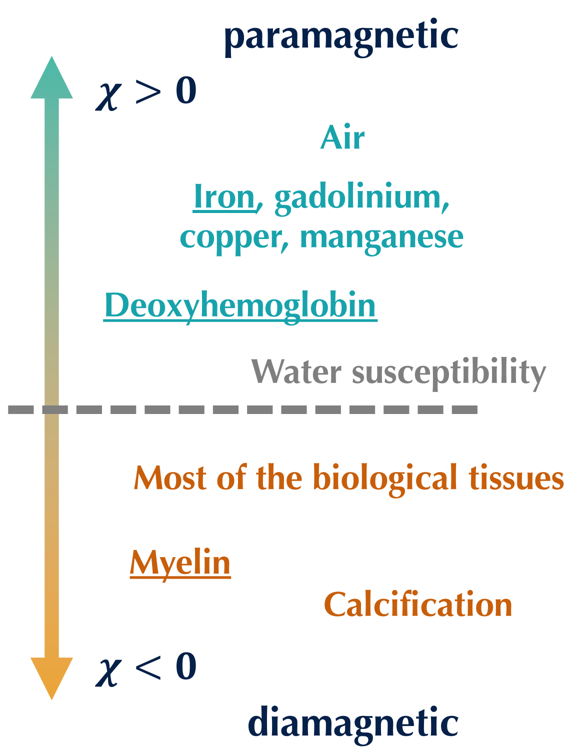
QSM applications – Functional neurosurgery

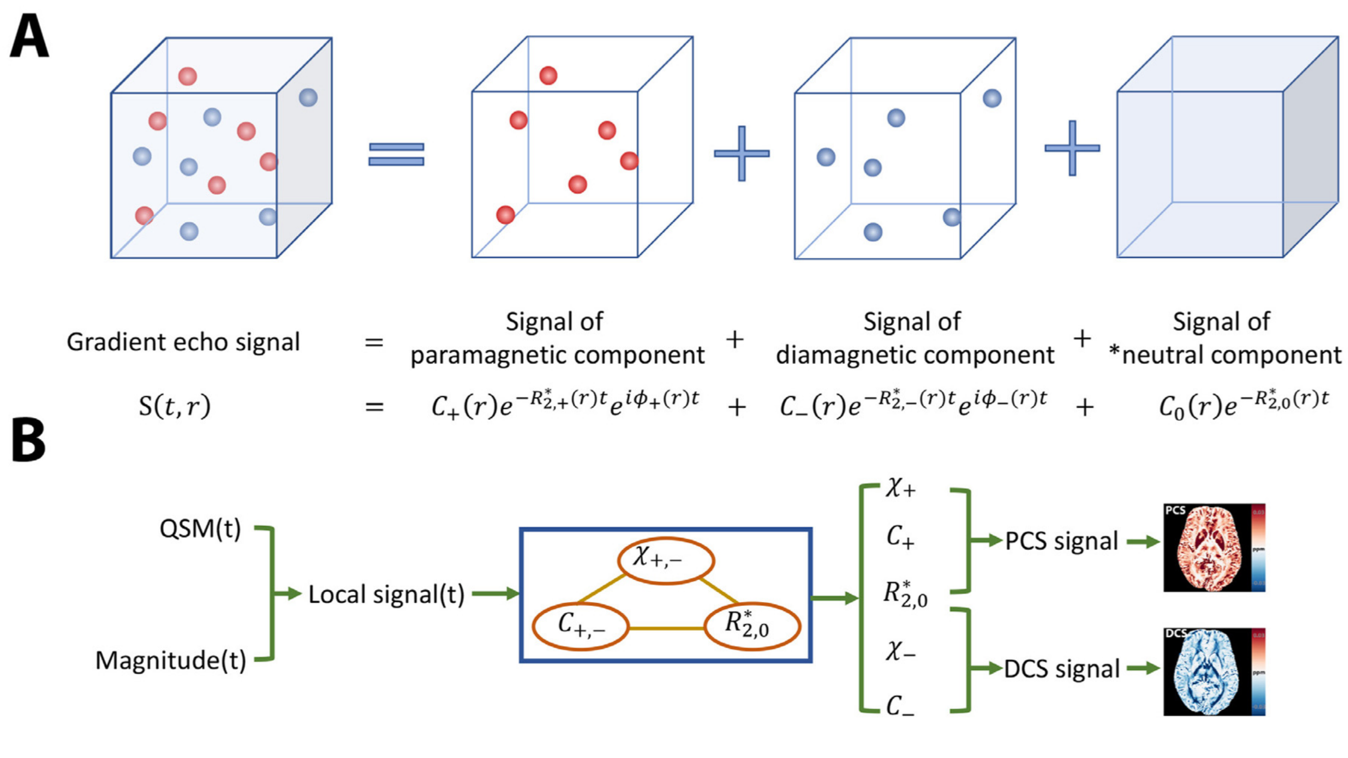




Li J et al. Front Hum Neurosci 2020

QSM: Susceptibility source decomposition

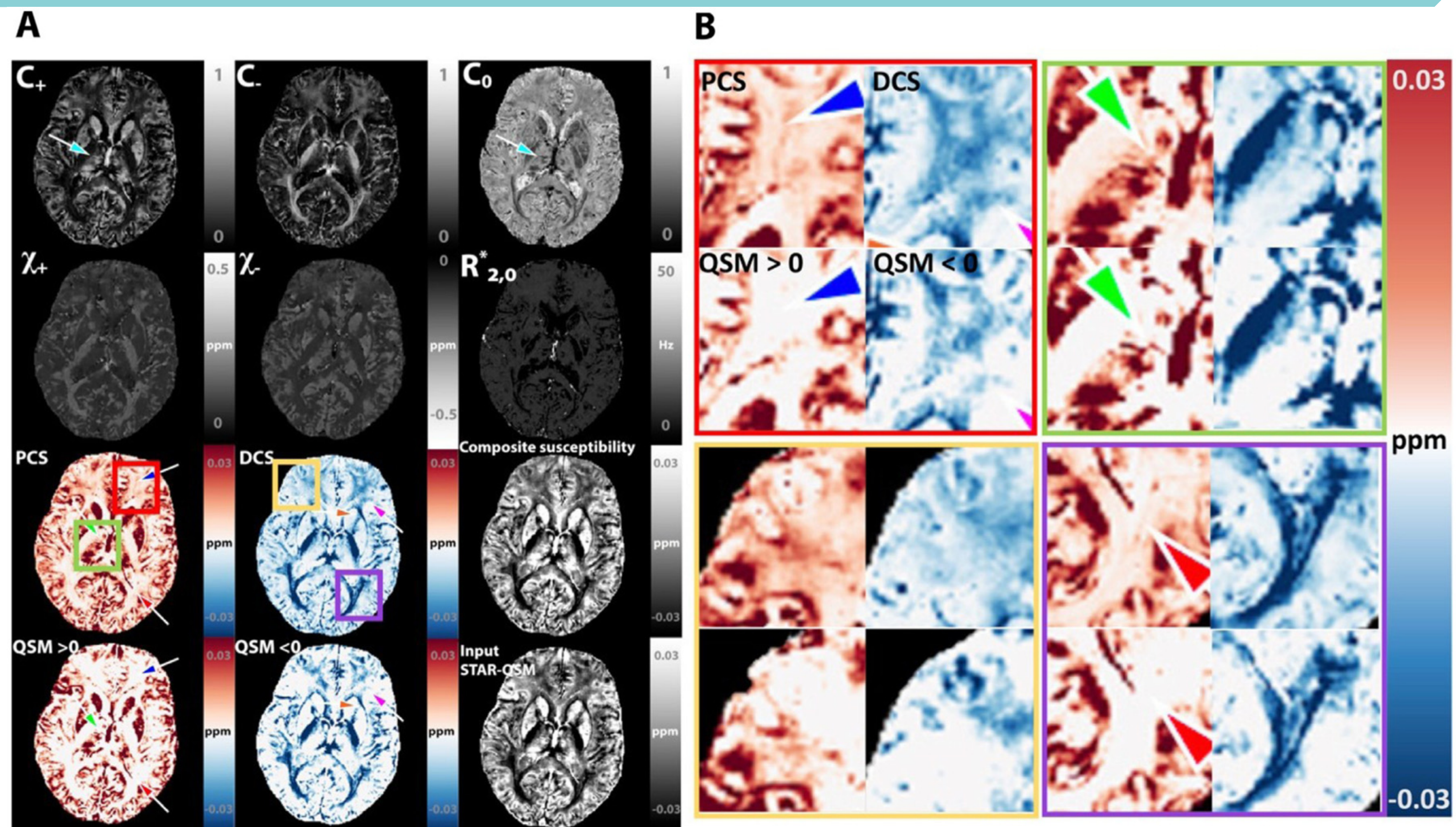




*neutral component means the component is susceptibility neutral relative to the susceptibility reference.

Chen J et al., NeuroImage, 2023

QSM: Susceptibility source decomposition



Chen J et al., NeuroImage, 2023

Summary

Learning Objectives

- Explain the biological and physical basis of susceptibility effects in MRI.
- Interpret and process phase images for SWI.
- Understand the procedure of processing QSM.
- Recognize common applications of susceptibility imaging in neuroimaging.
- Appreciate the challenges and assumptions in going from phase to susceptibility maps.