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# Fast Imaging Trajectories: Non-Cartesian Sampling (1)

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M229 Advanced Topics in MRI

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**UCLA**

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David Geffen School of Medicine at UCLA*

# Class Business

- Homework 2 - due 5/3
- Project proposal - due 5/10
- Office hours
- Course evaluations on Thu 5/2

# Outline

- Review of k-space sampling (2DFT)
- Radial
- Concentric rings

# MR Signal Equation

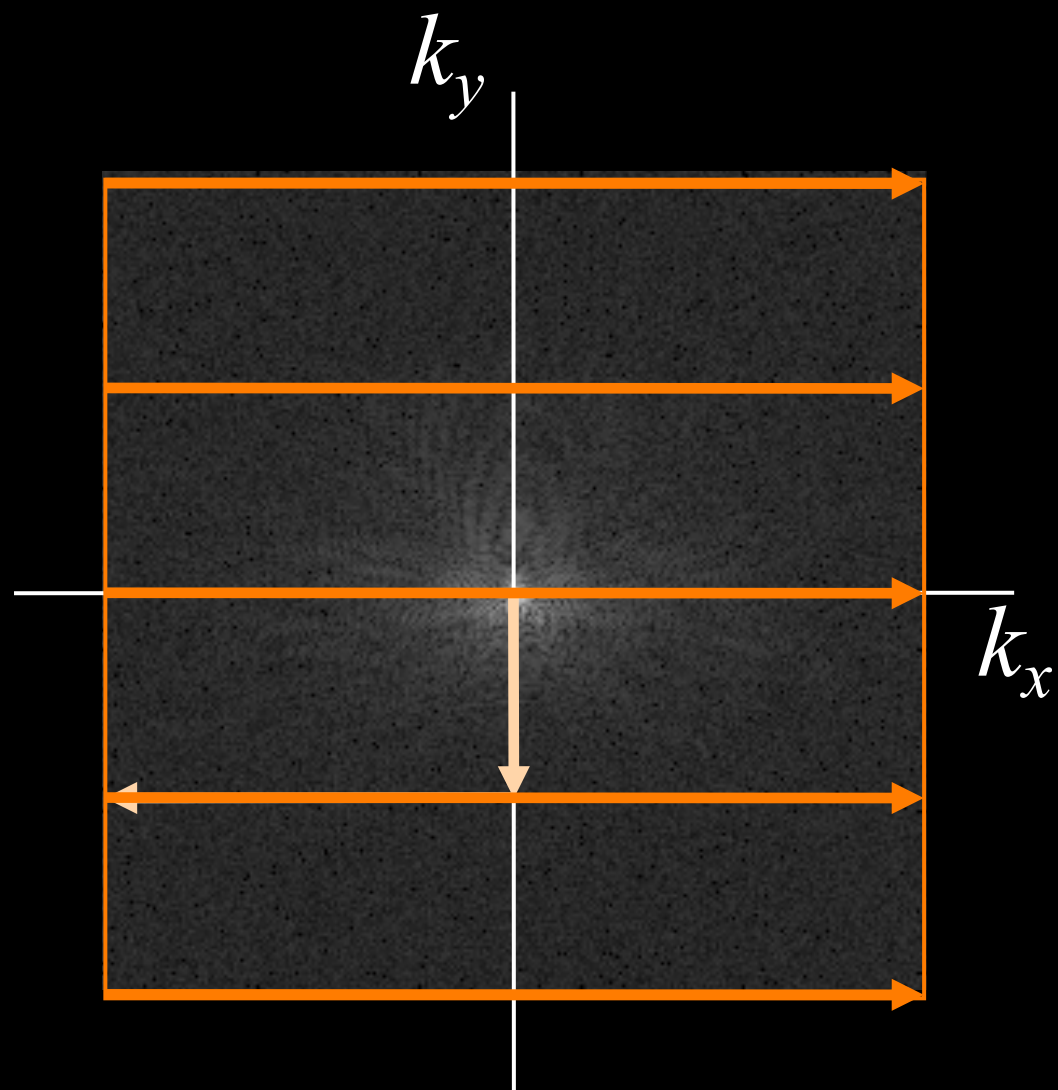
$$s(t) = \iint_{X,Y} M(x, y) \cdot \exp(-i2\pi \cdot [k_x(t)x + k_y(t)y]) dx dy$$

$$= m(k_x(t), k_y(t))$$

$$k_x(t) = \frac{\gamma}{2\pi} G_x t, \quad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

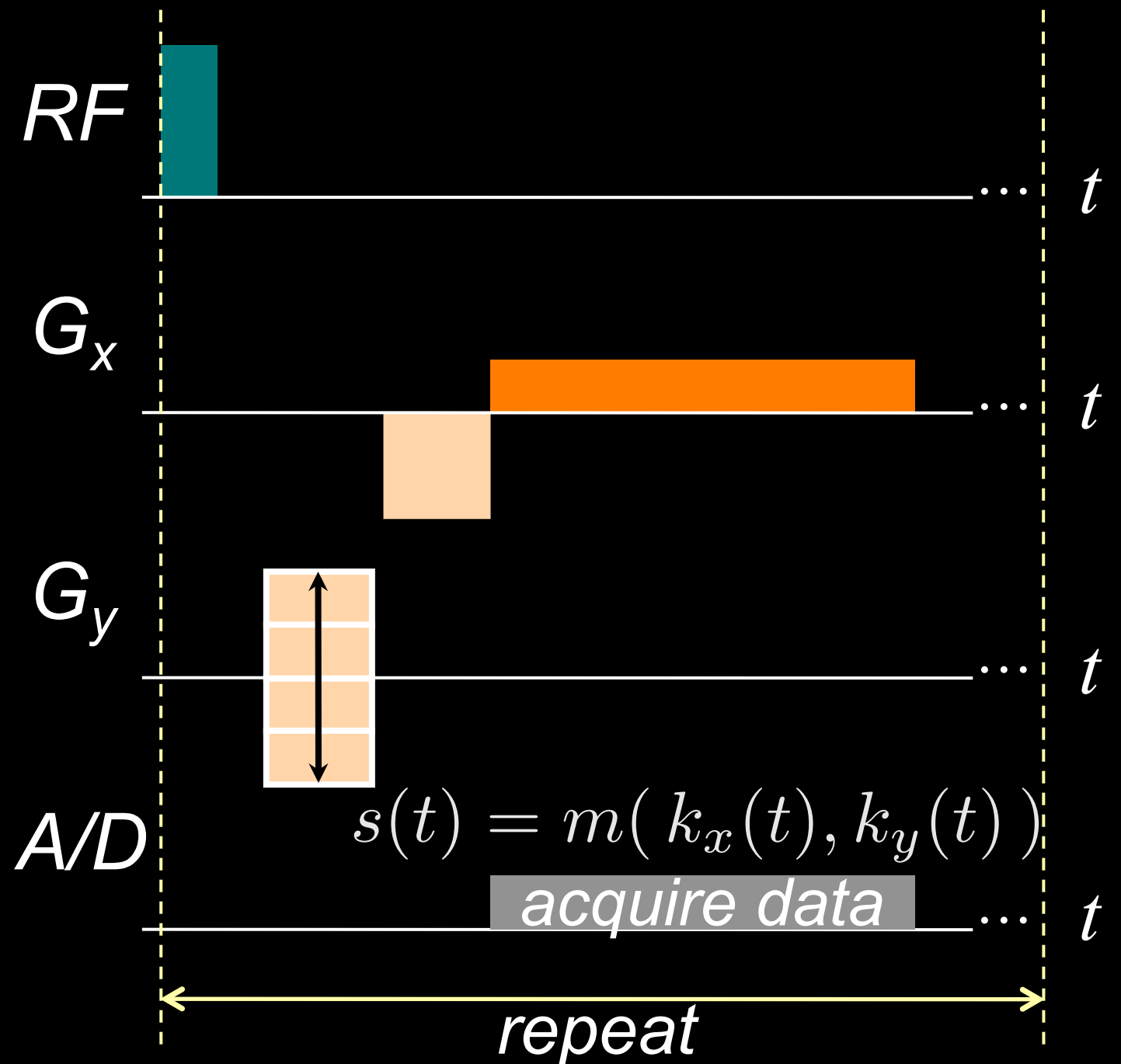
$$m = \mathcal{FT}(M(x, y))$$

# k-Space Sampling

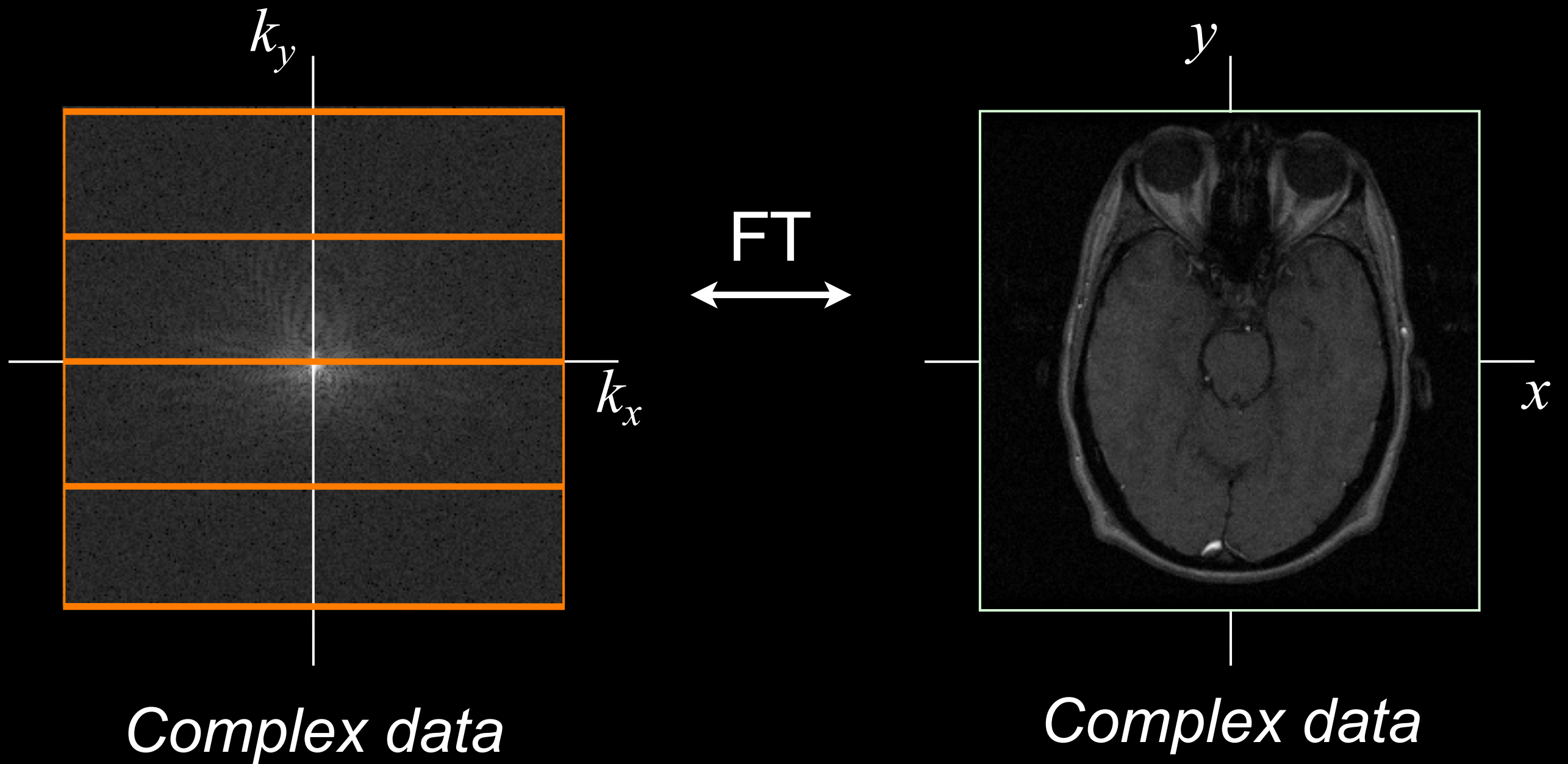


set of  $s(t)$  covers  $m(k_x, k_y)$

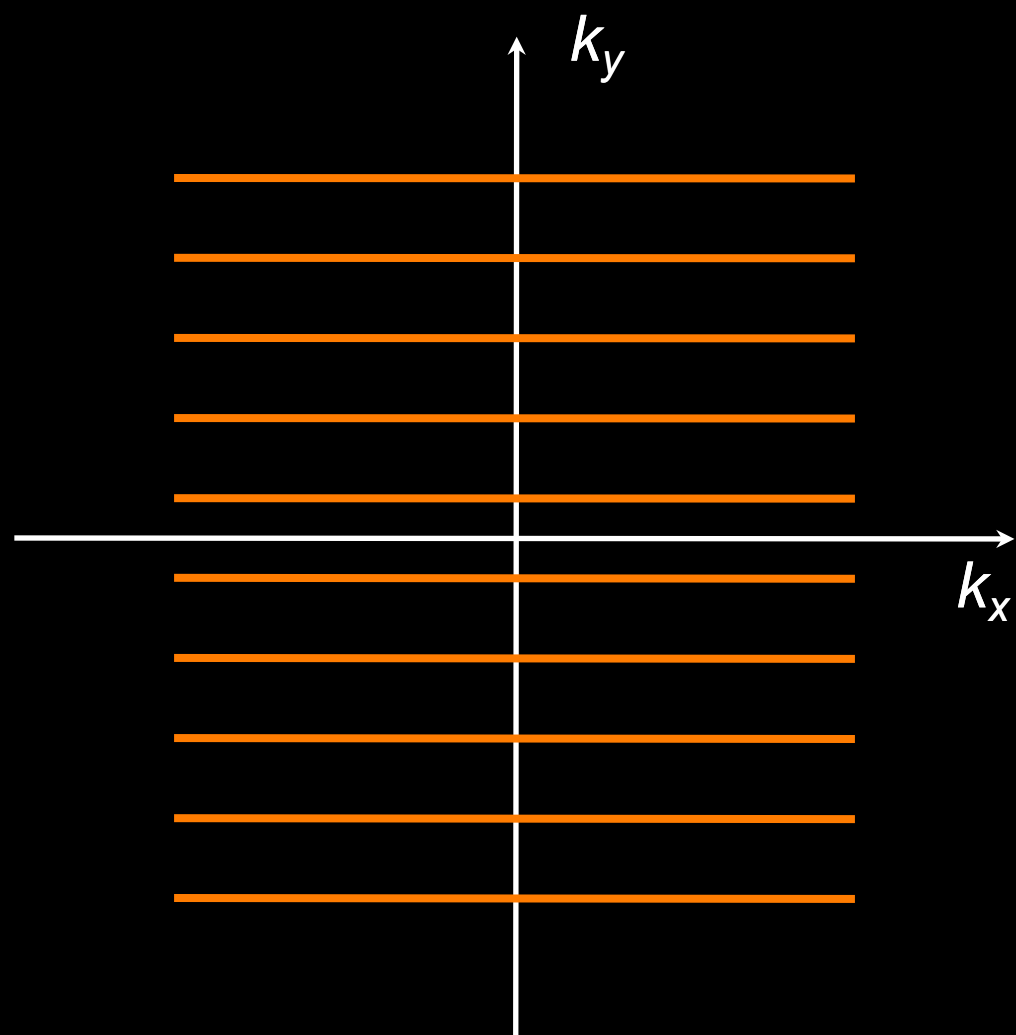
## Pulse Sequence Diagram



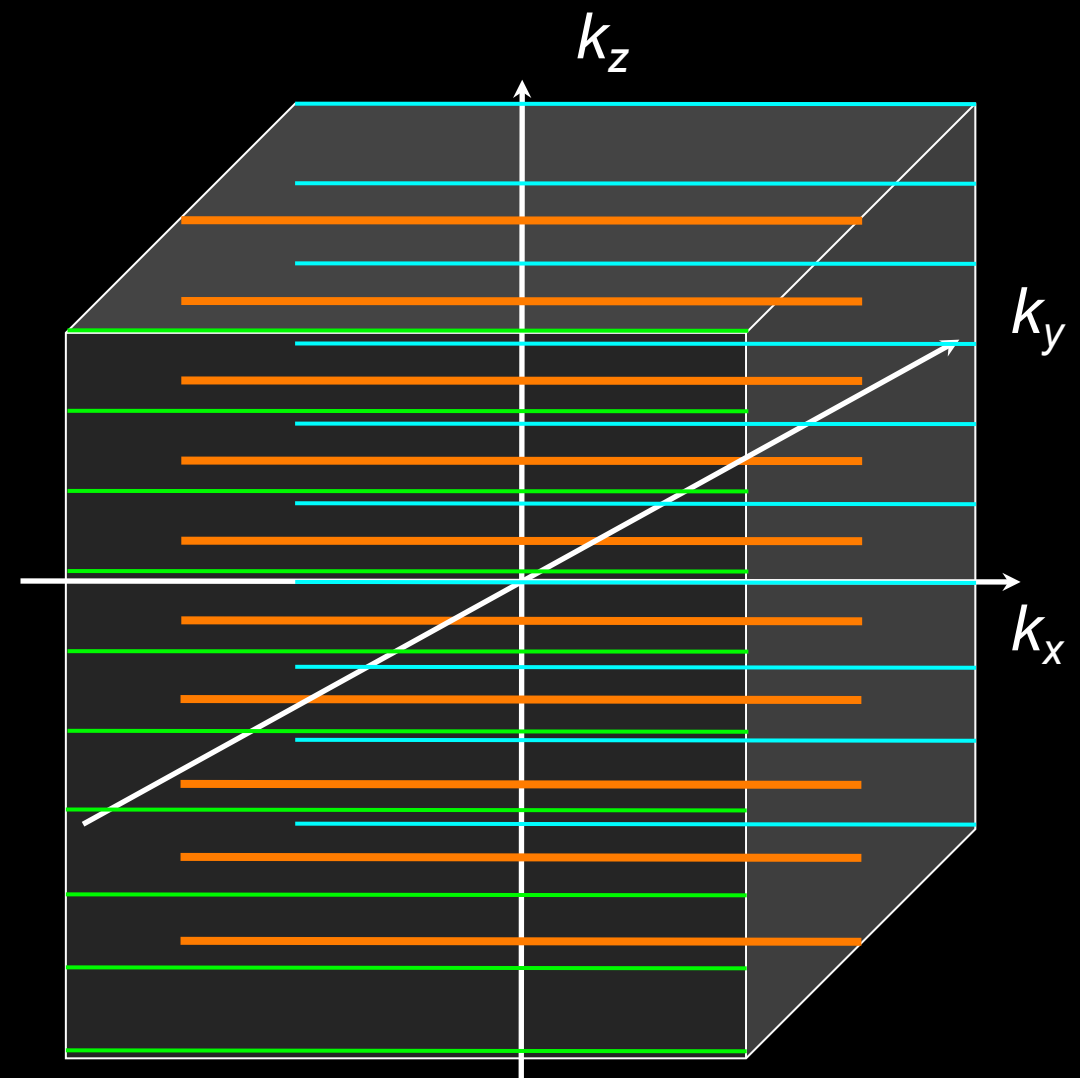
# Image Reconstruction



# Cartesian Sampling



Cartesian 2DFT



Cartesian 3DFT

# MR Signal Equation

$$s(t) = \iint_{X,Y} M(x, y) \cdot \exp(-i2\pi \cdot [k_x(t)x + k_y(t)y]) dx dy$$

$$= m(k_x(t), k_y(t))$$

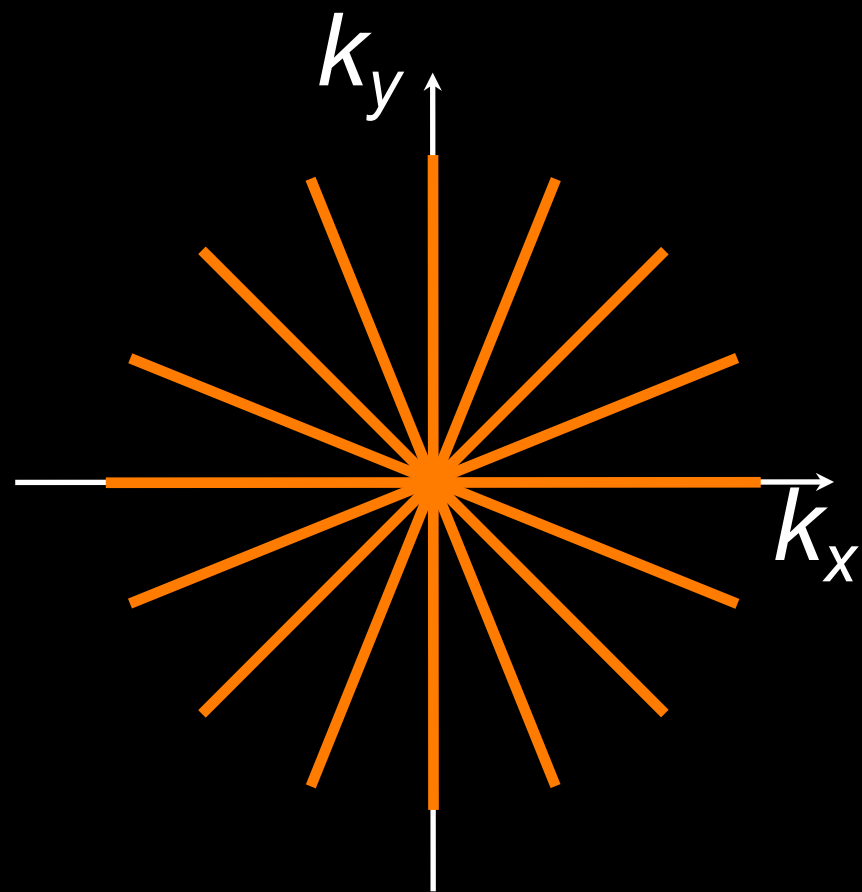
$$k_x(t) = \frac{\gamma}{2\pi} G_x t, \quad k_y(t) = \frac{\gamma}{2\pi} G_y t$$

$$m = \mathcal{FT}(M(x, y))$$

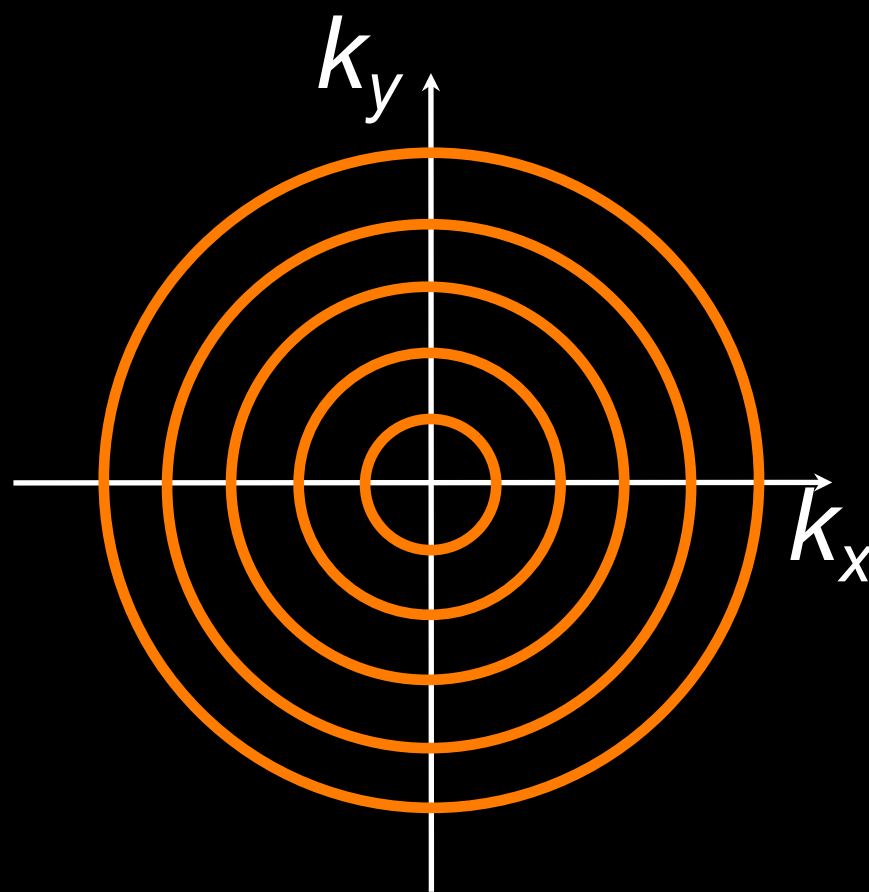
$$k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau) d\tau, \quad k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$$



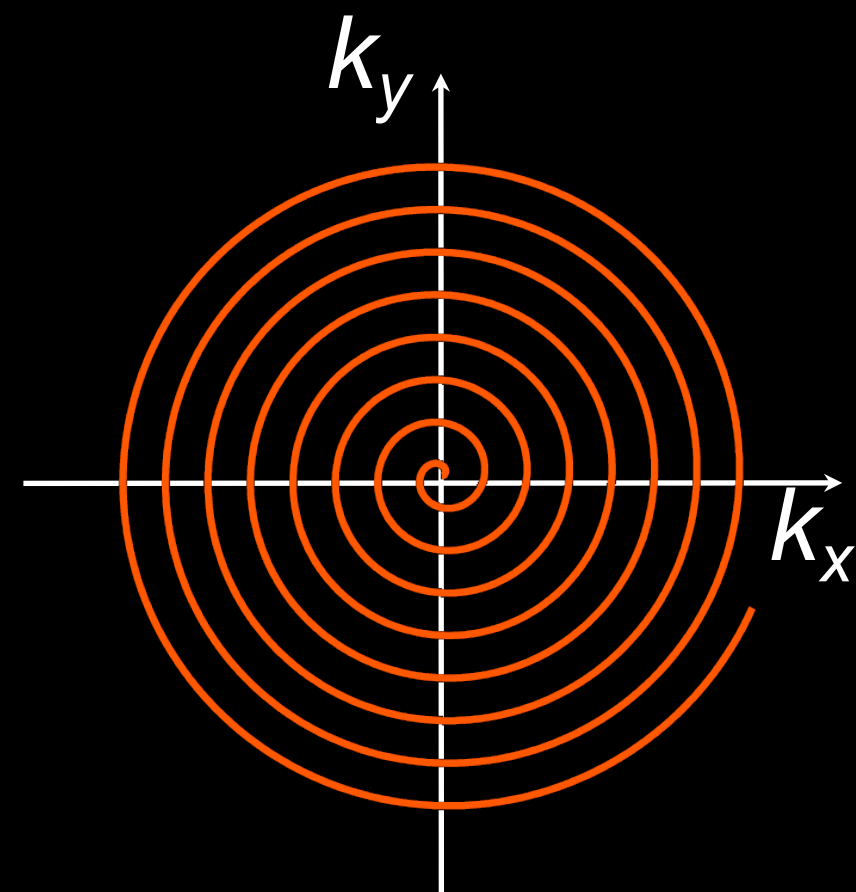
# Non-Cartesian Sampling



2D Radial



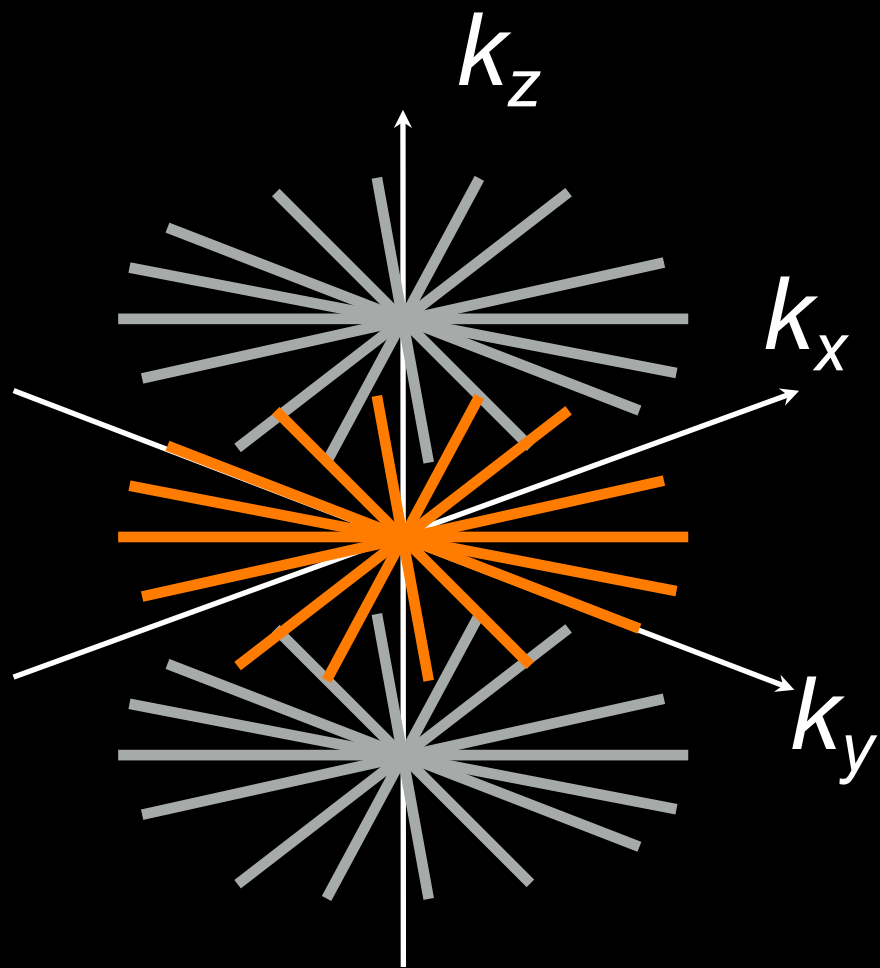
2D Concentric Rings



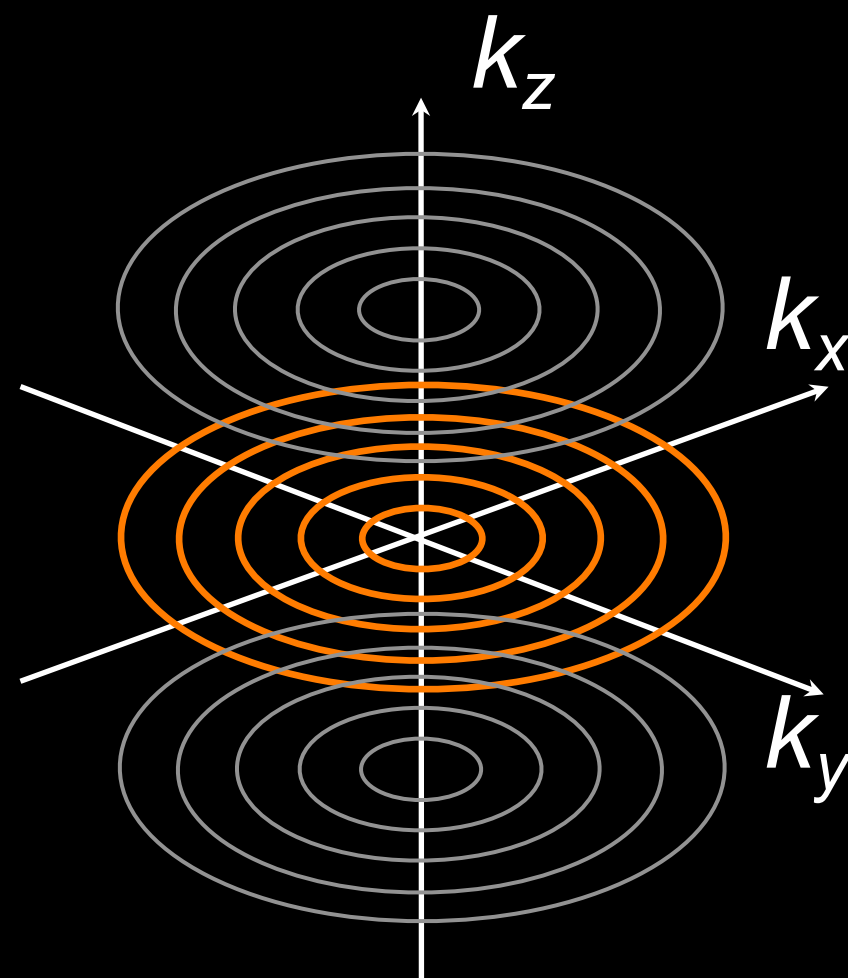
2D Spiral

*and much more ...*

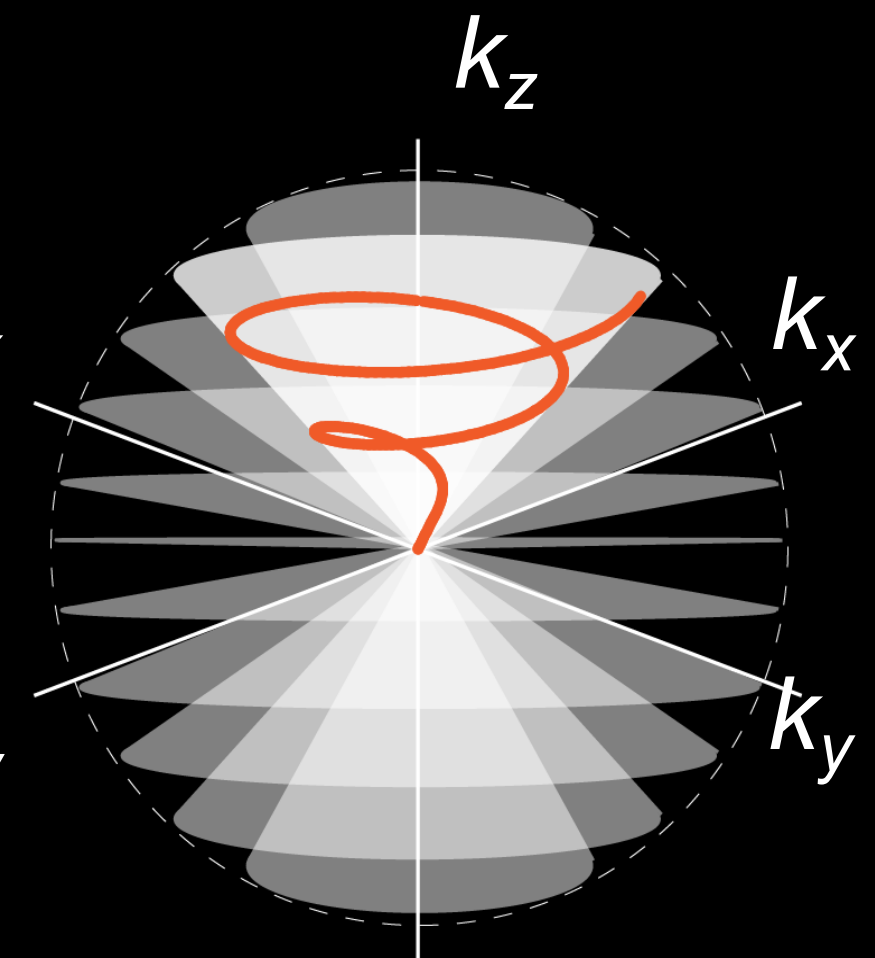
# Non-Cartesian Sampling



3D Stack of Stars



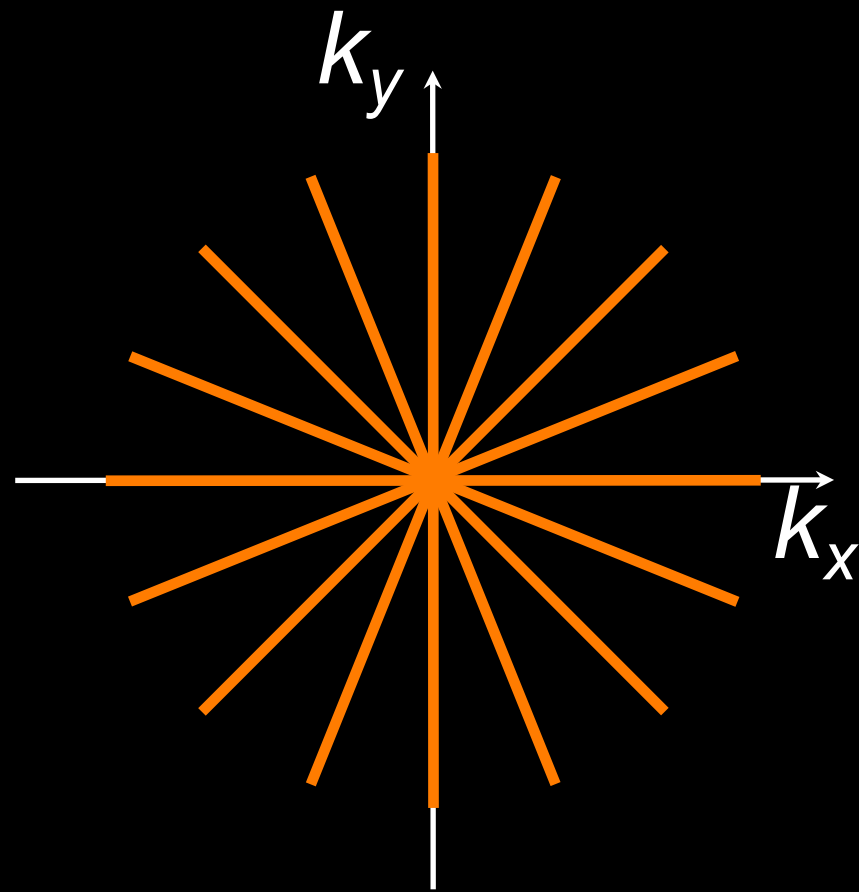
3D Stack of Rings



3D Cones

*and much more ...*

# Radial



The original MRI trajectory!

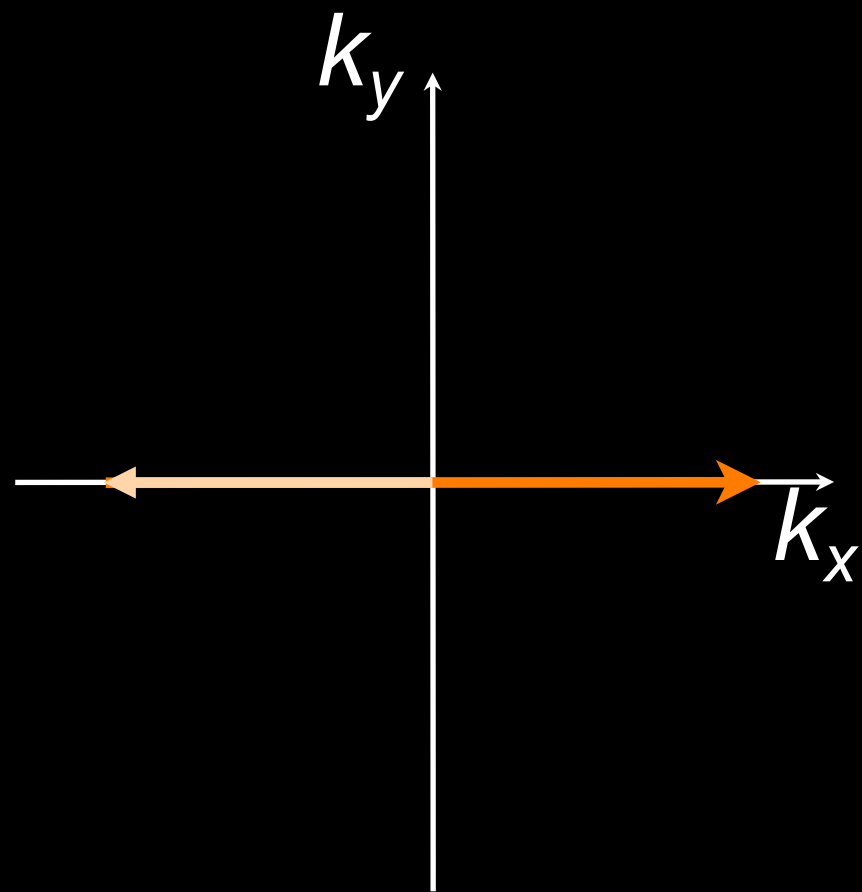
- *Lauterbur, Nature 1973*

Samples k-space on a polar grid

- “Spokes” correspond to projections

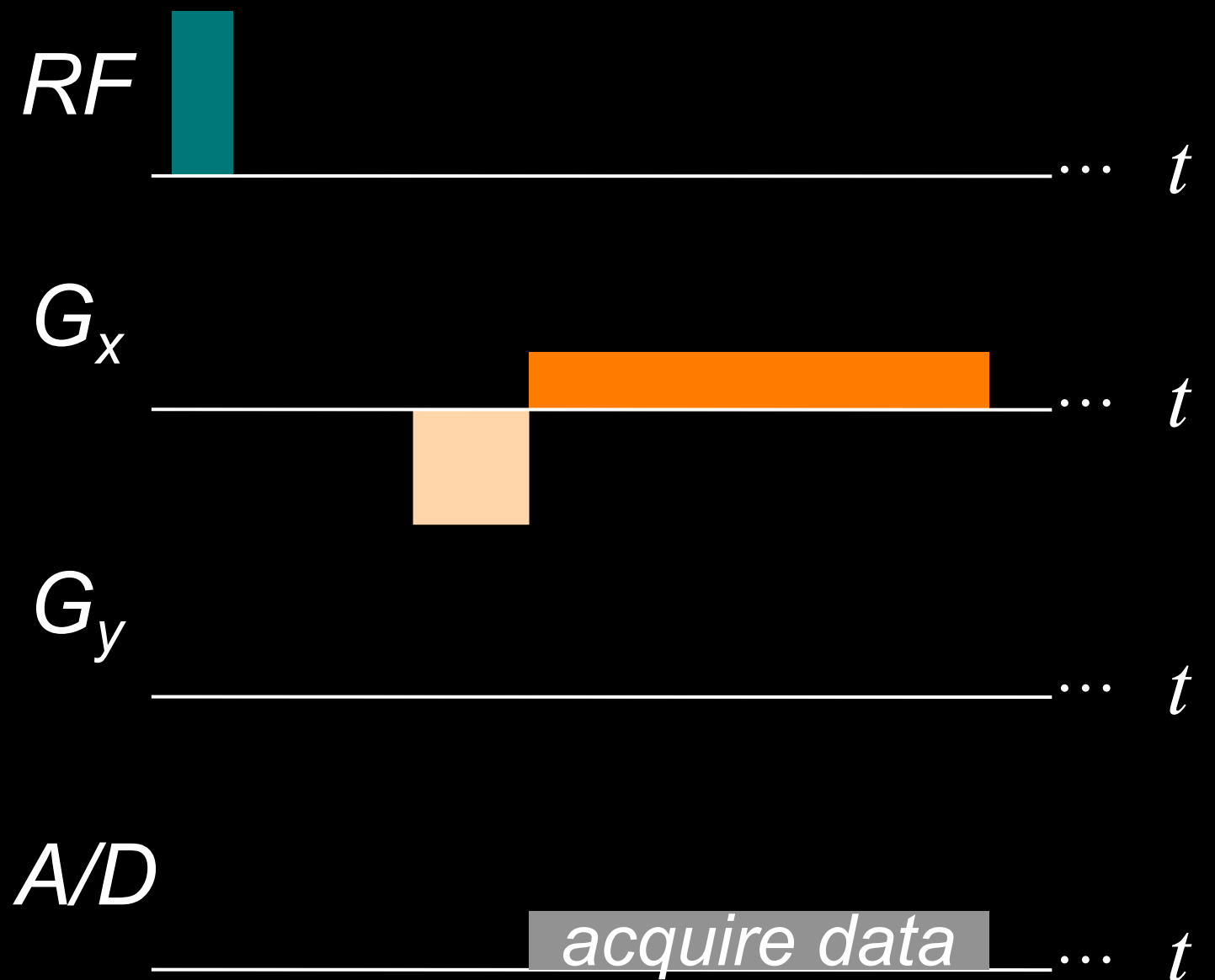
- Projection reconstruction (2DPR)

# Radial: Gradient Design

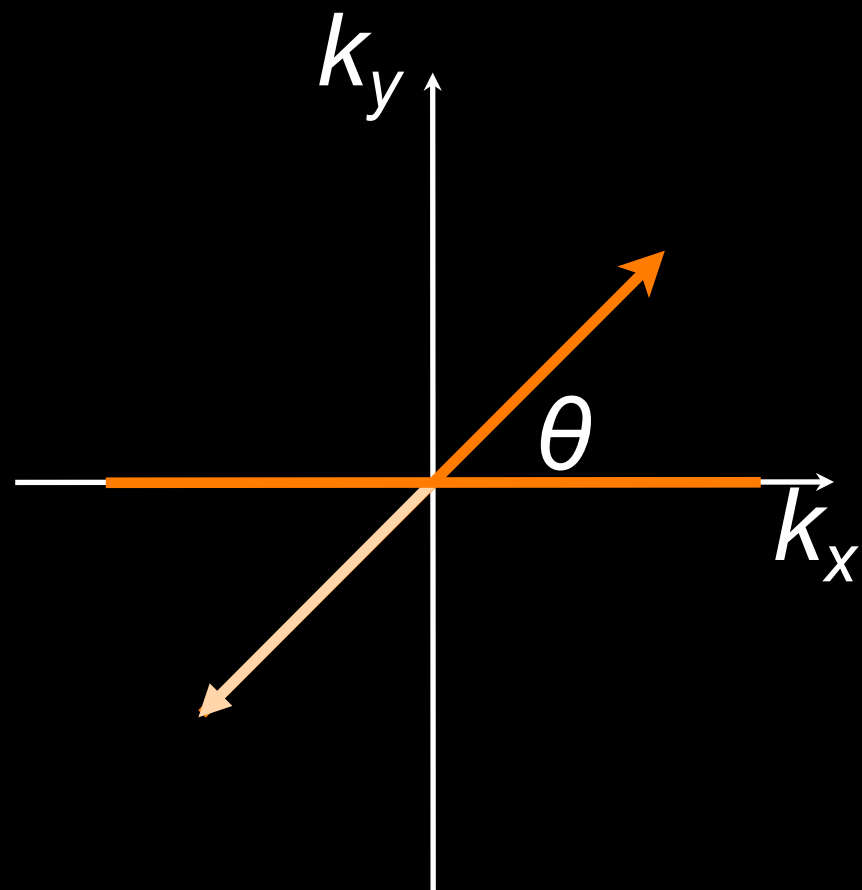


one "spoke"

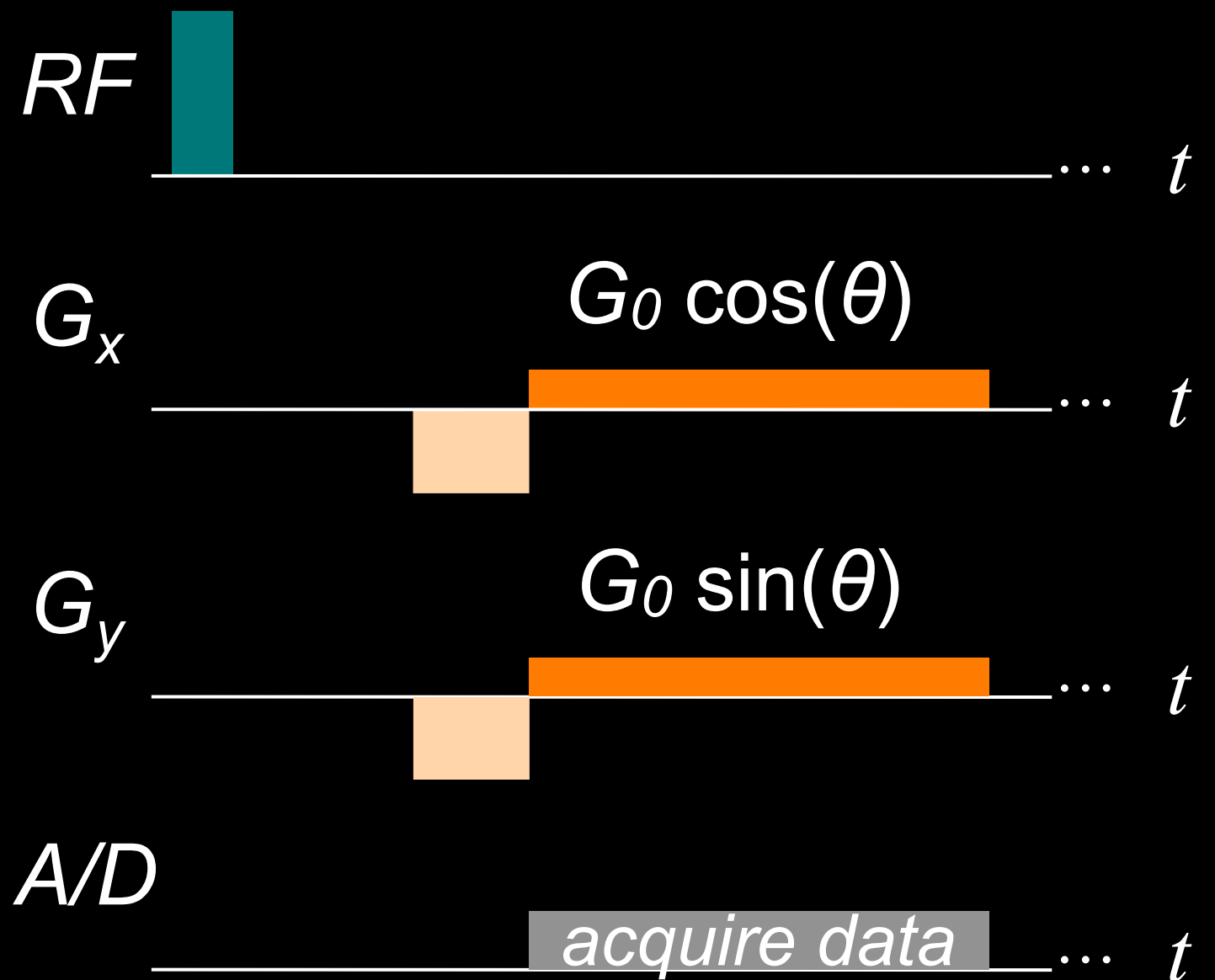
## Pulse Sequence Diagram



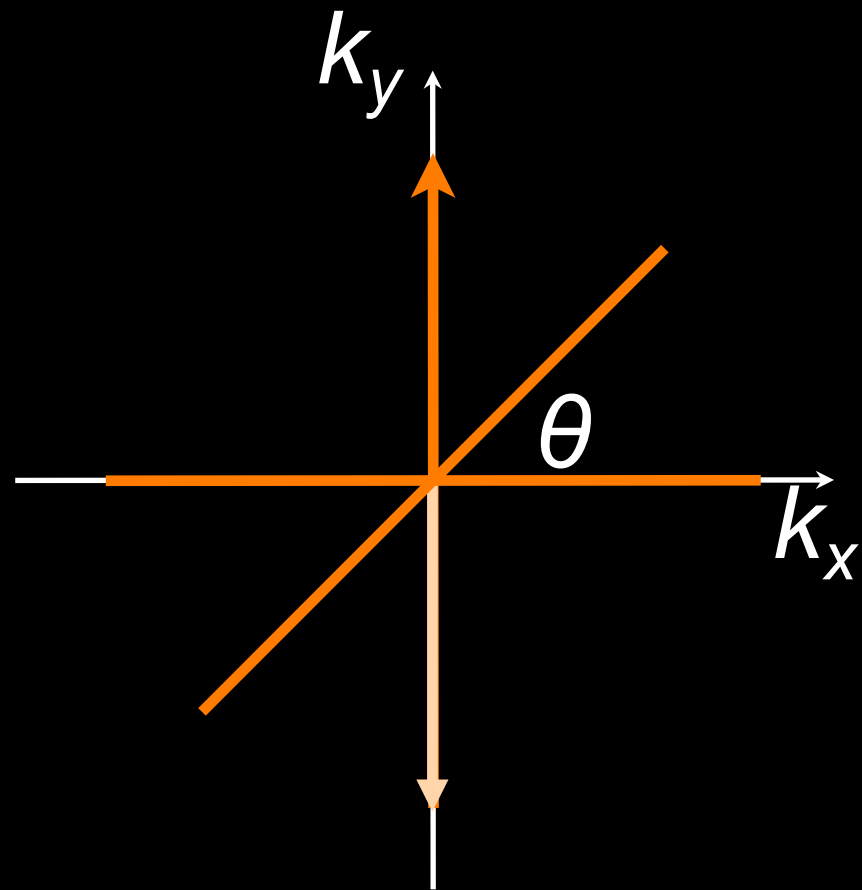
# Radial: Gradient Design



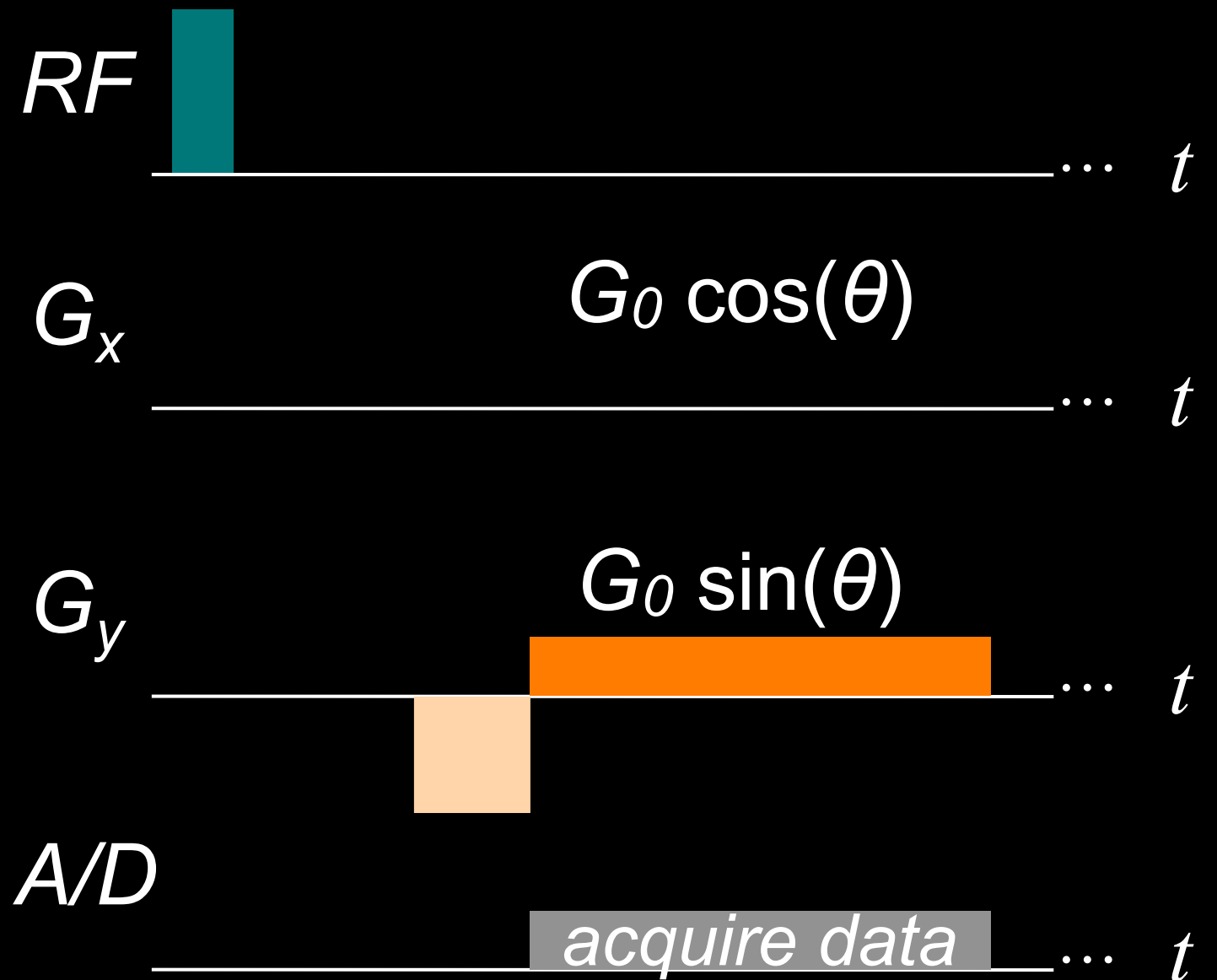
## Pulse Sequence Diagram



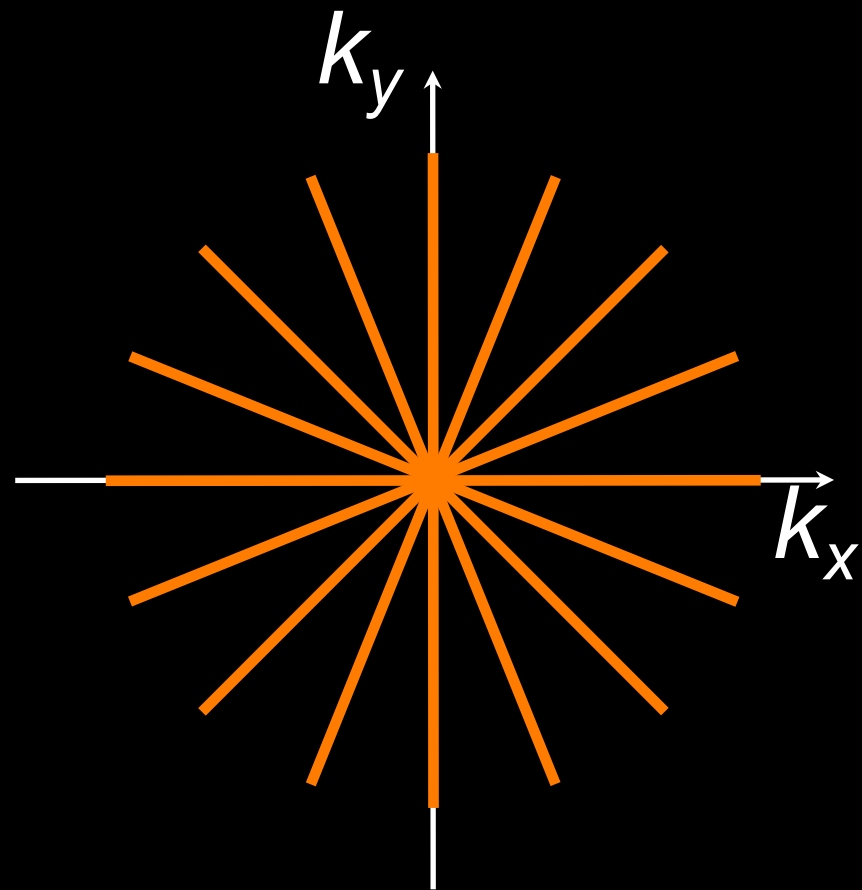
# Radial: Gradient Design



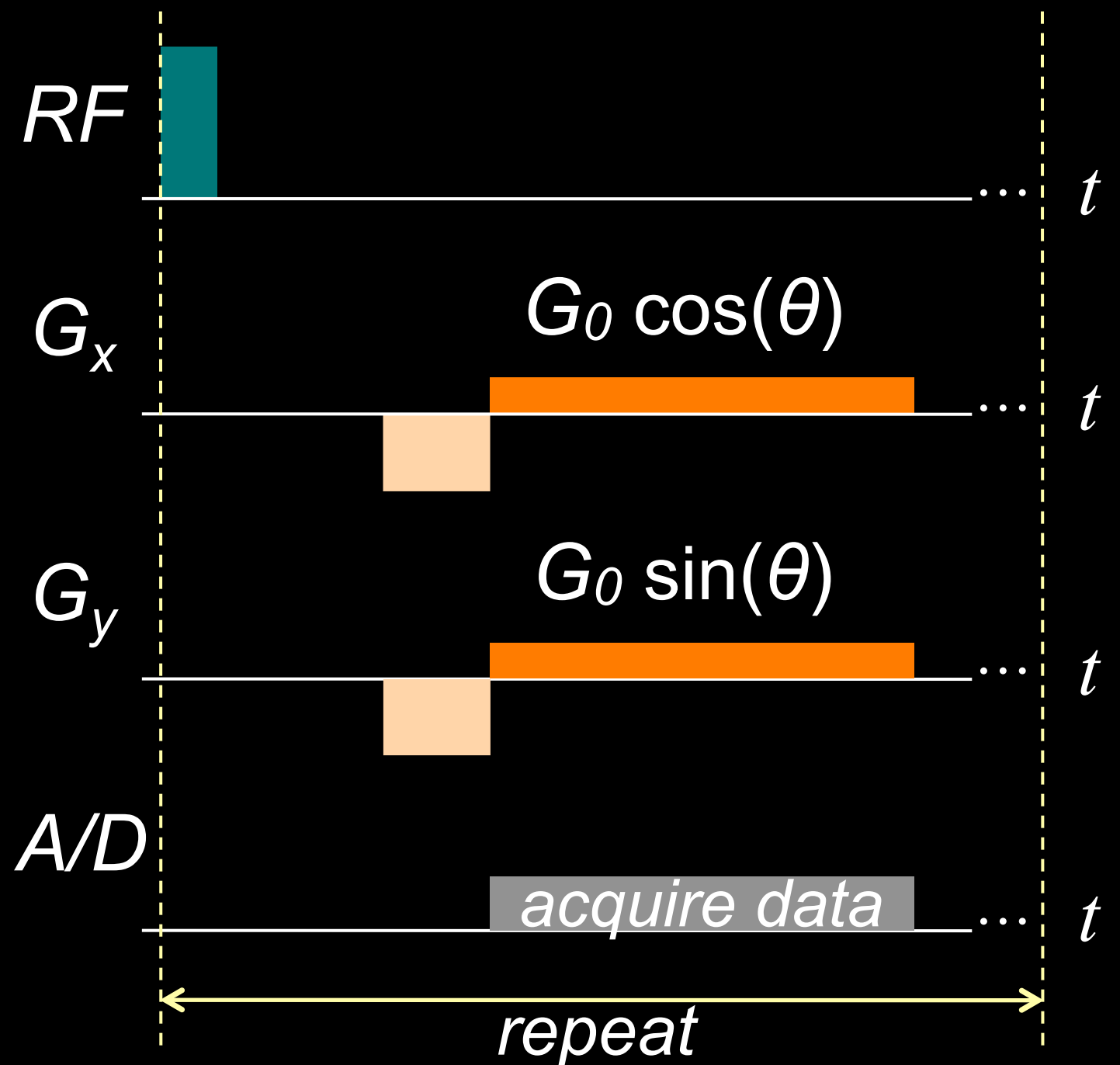
## Pulse Sequence Diagram



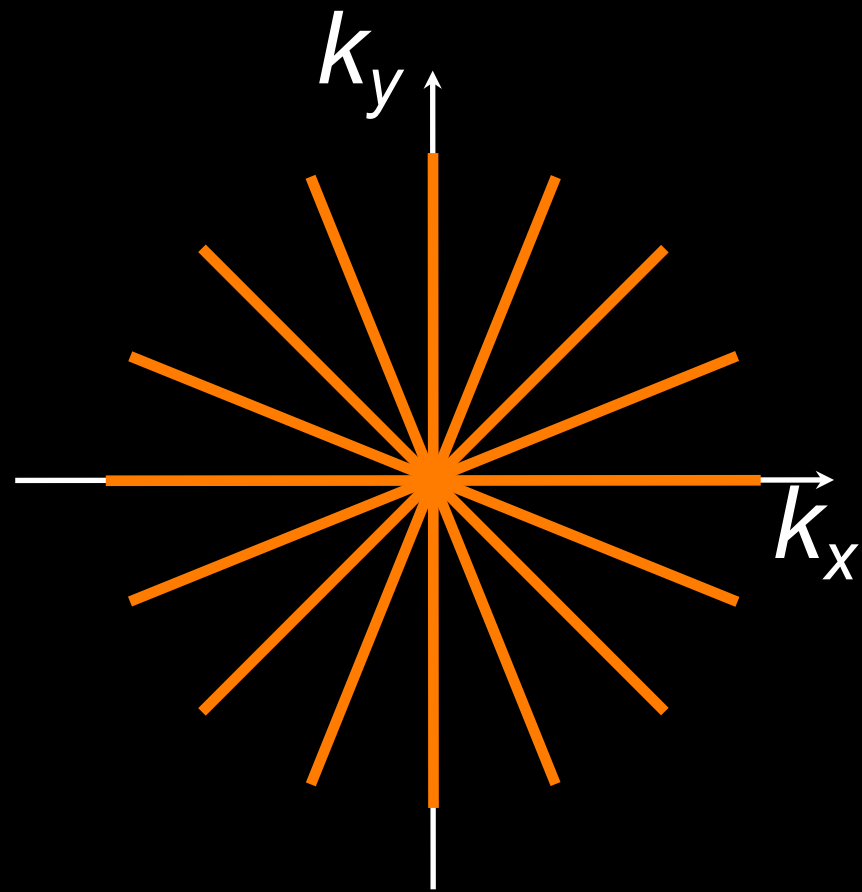
# Radial: Gradient Design



## Pulse Sequence Diagram



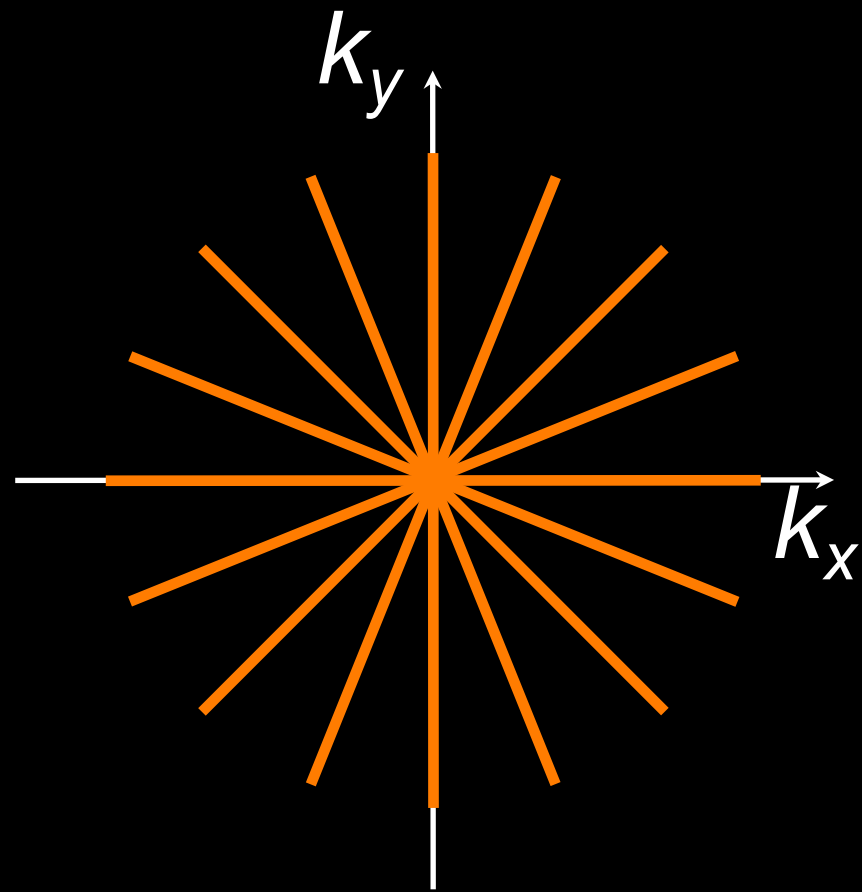
# Radial: Sampling Requirements



To the board ...



# Radial: Sampling Requirements



$N_r$  points/spoke

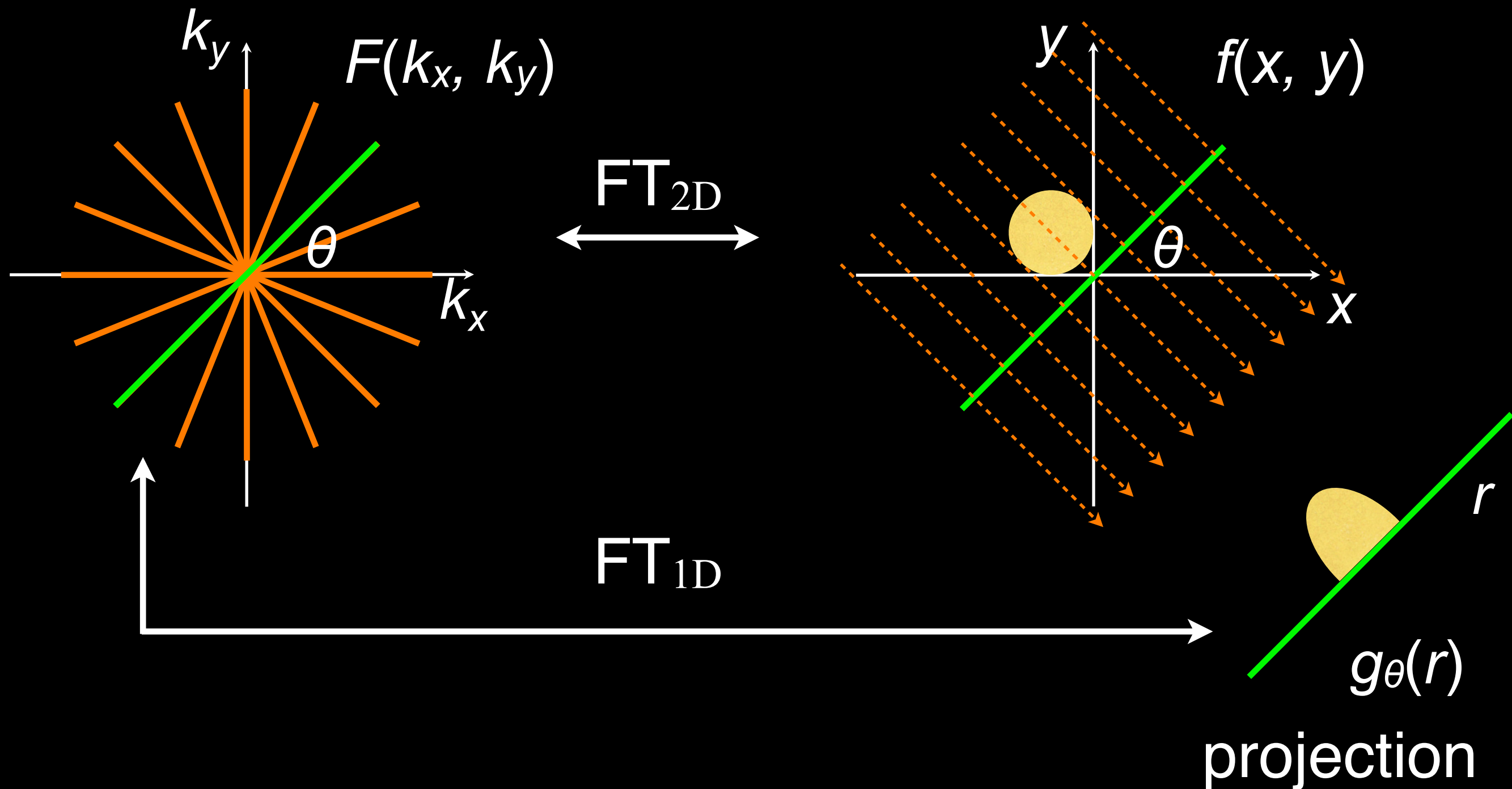
$$dk_r \leq \frac{1}{\text{FOV}}$$

To satisfy Nyquist at edges of k-space:

$$N_{sp} \geq \frac{\pi}{2} N_r$$

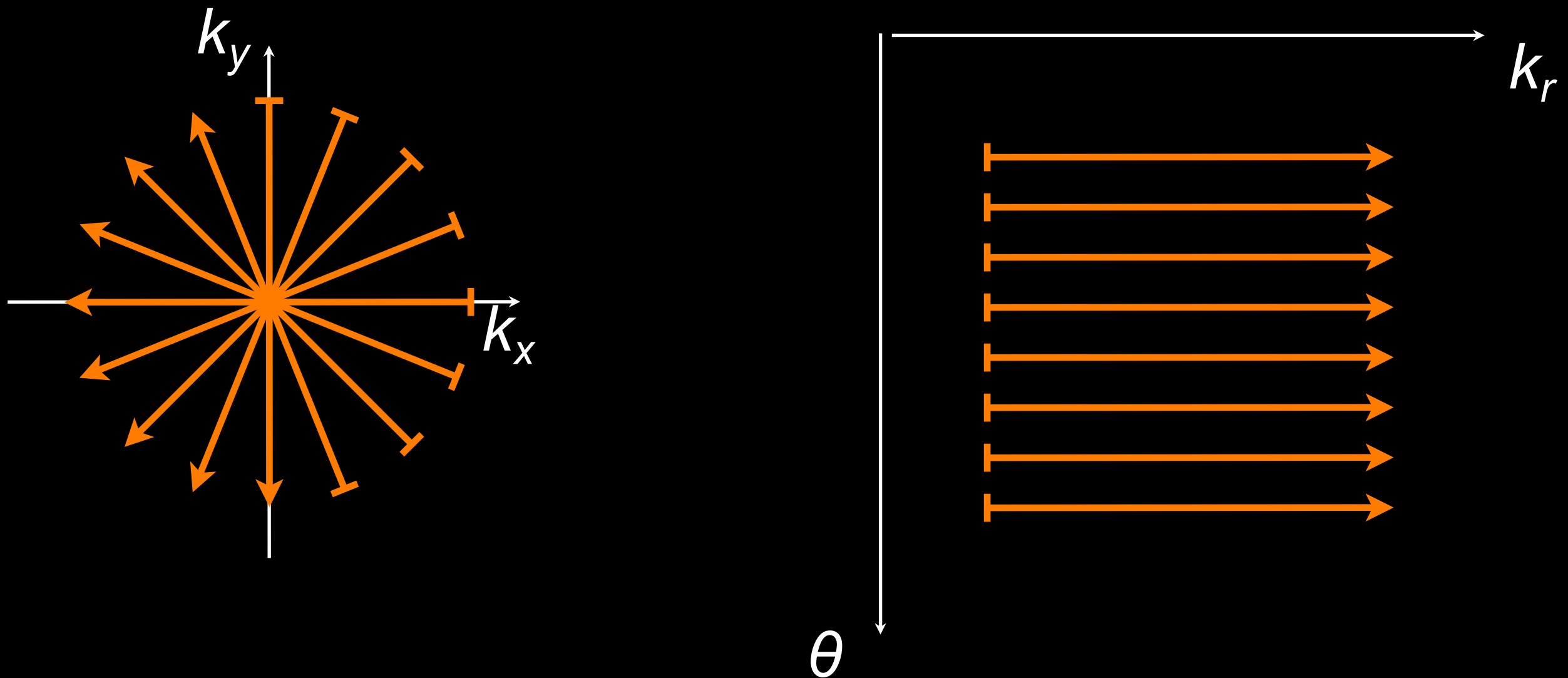
# Radial: Image Reconstruction

## Central Section Theorem



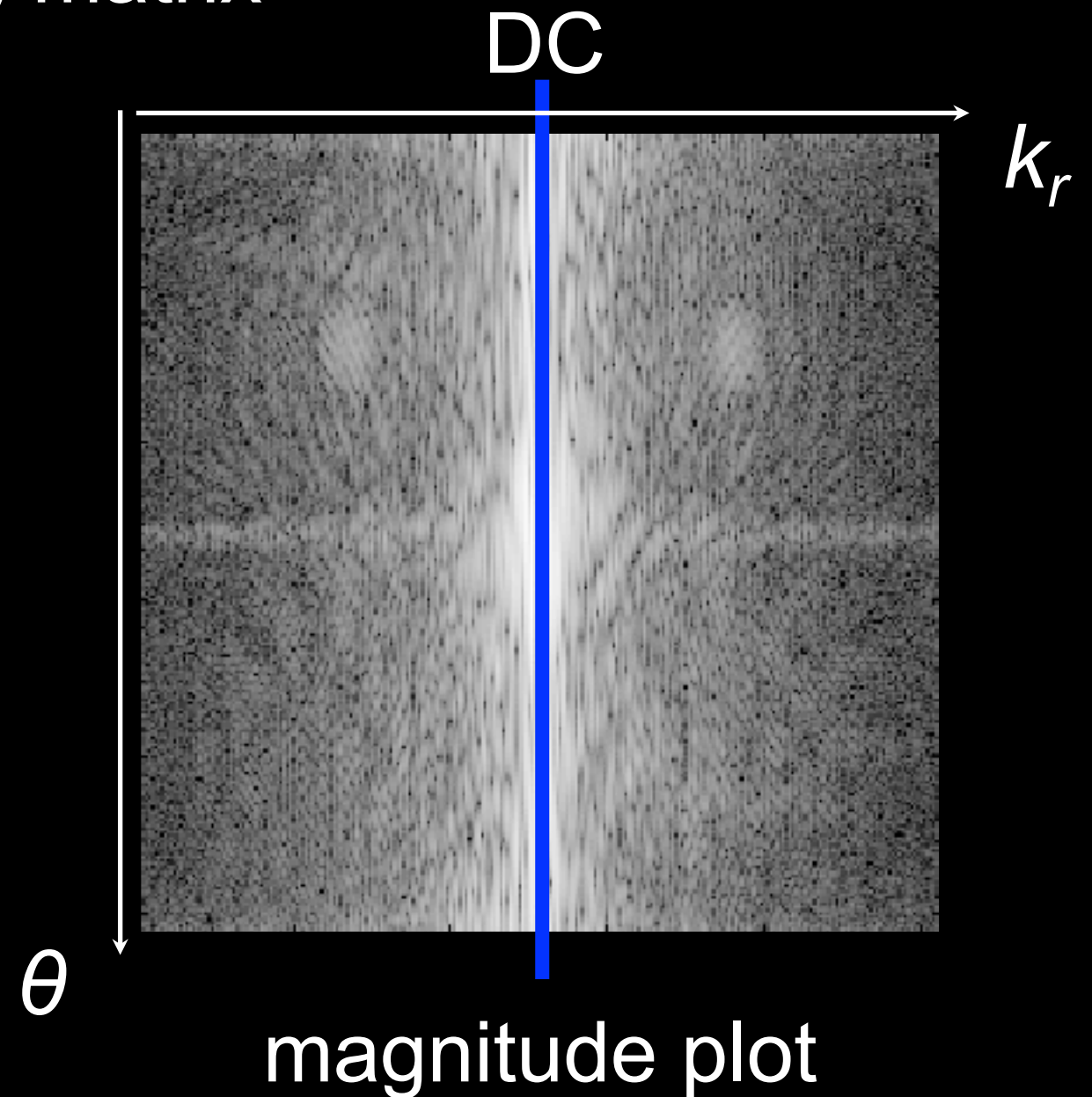
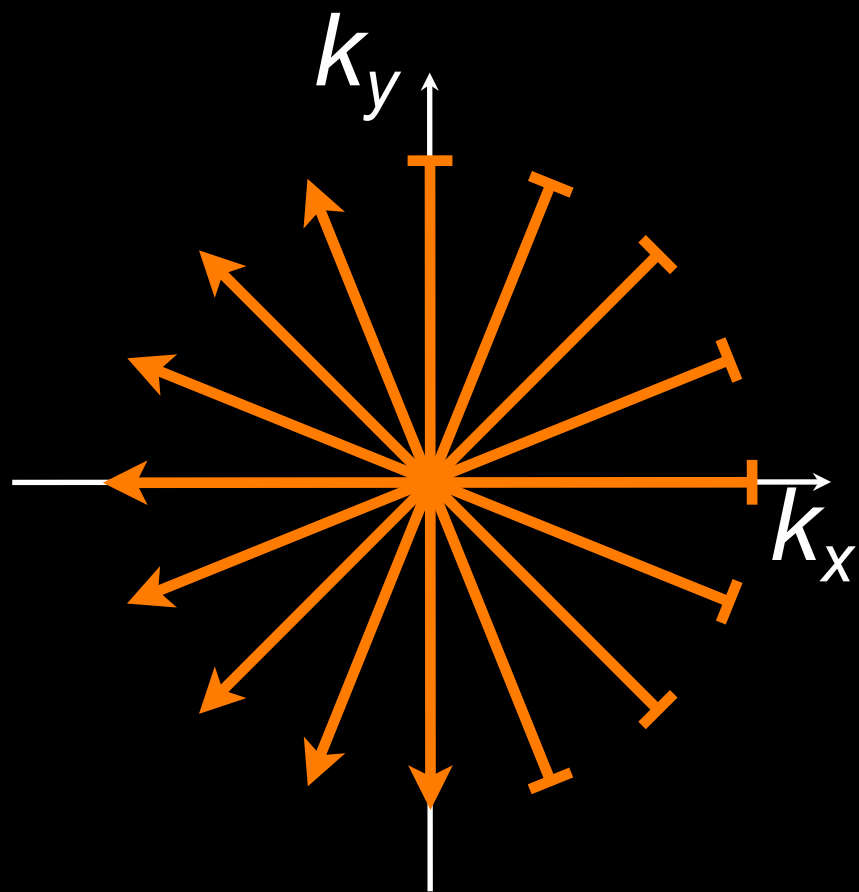
# Radial: Image Reconstruction

Collect spokes into  $(k_r, \theta)$  matrix  $\rightarrow$



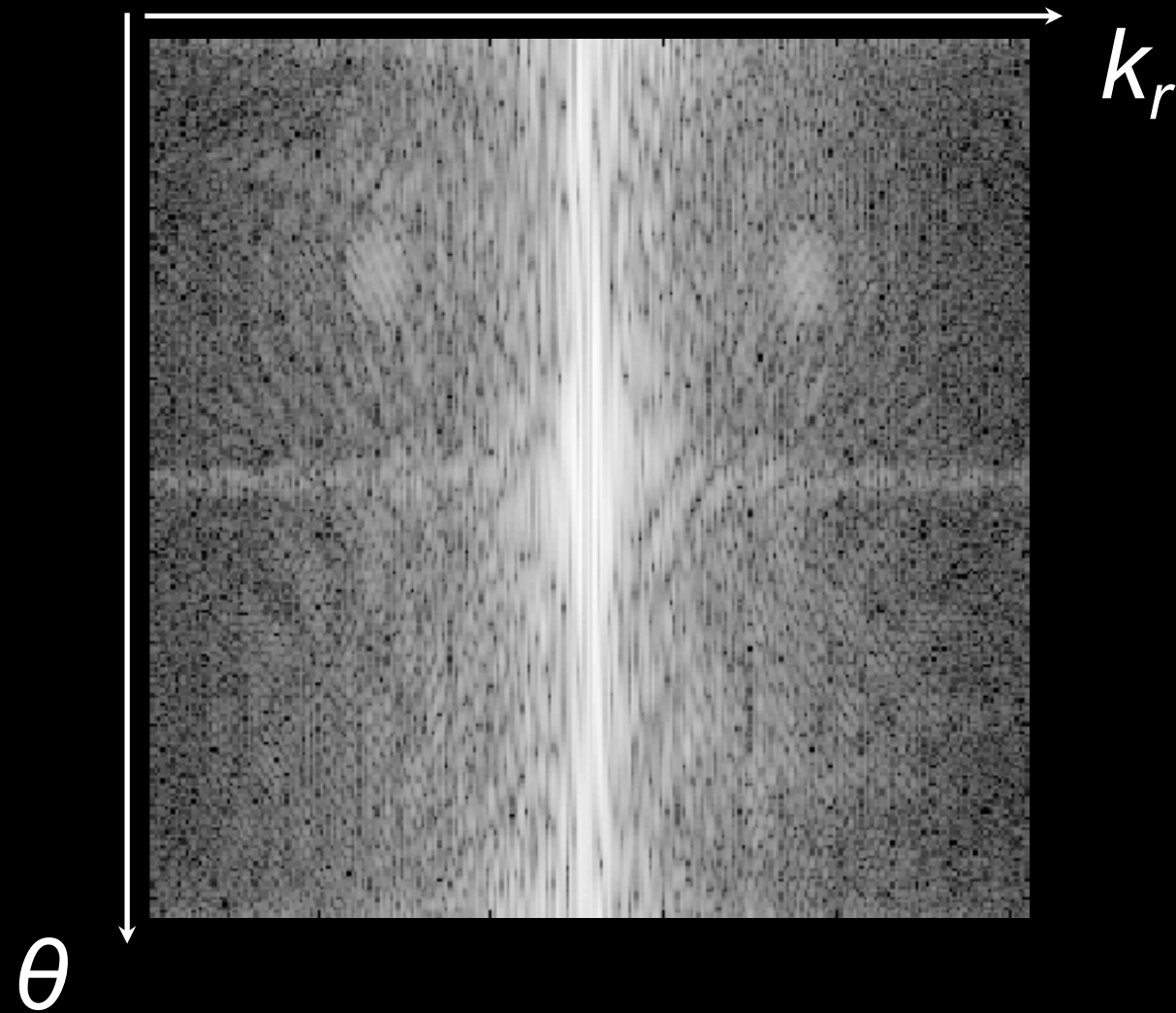
# Radial: Image Reconstruction

Collect spokes into  $(k_r, \theta)$  matrix  $\rightarrow$

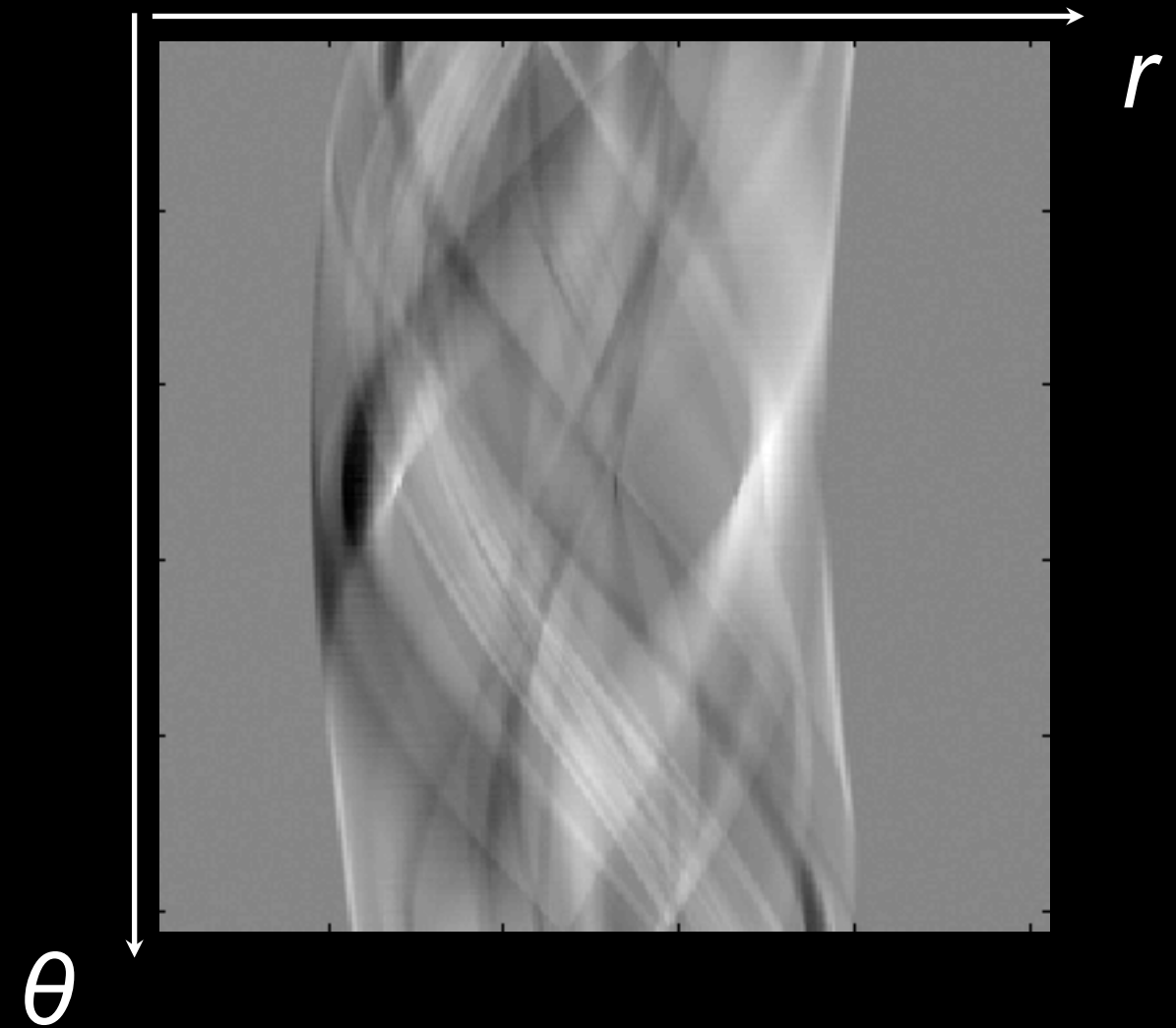


# Radial: Image Reconstruction

1DFT of each spoke along  $k_r \rightarrow$  "Sinogram"



magnitude plot

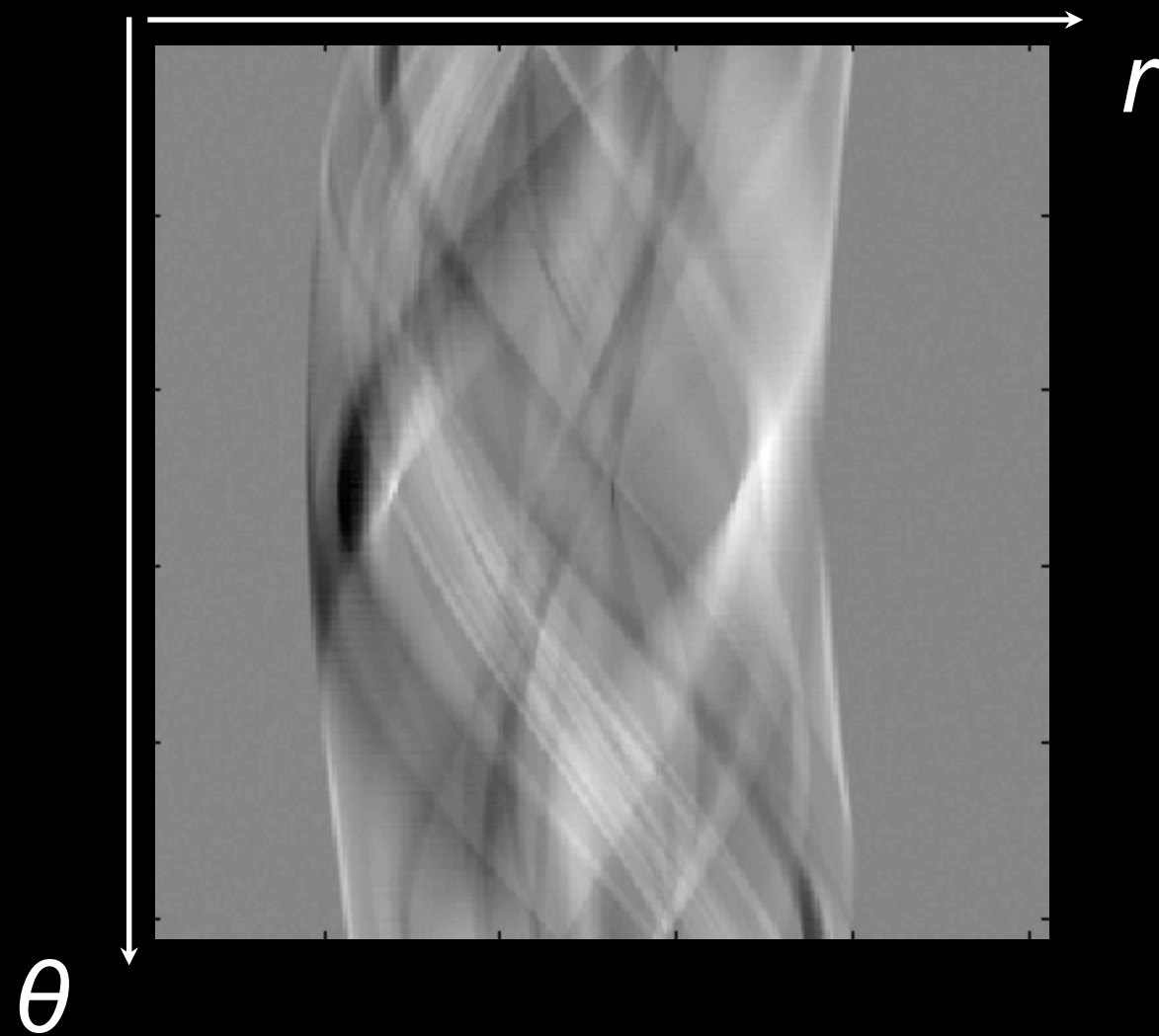


real channel

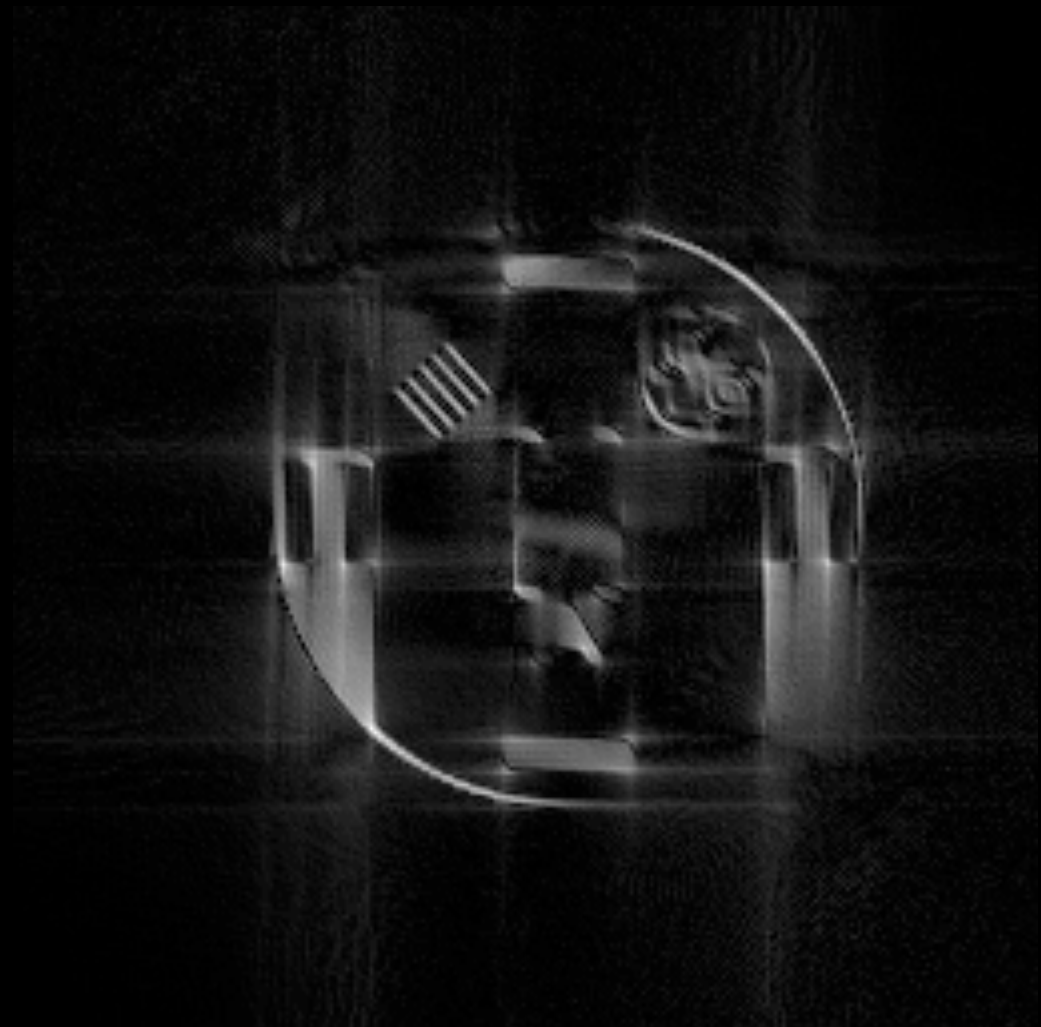
# Radial: Image Reconstruction

Filtered back projection  $\rightarrow$

Image



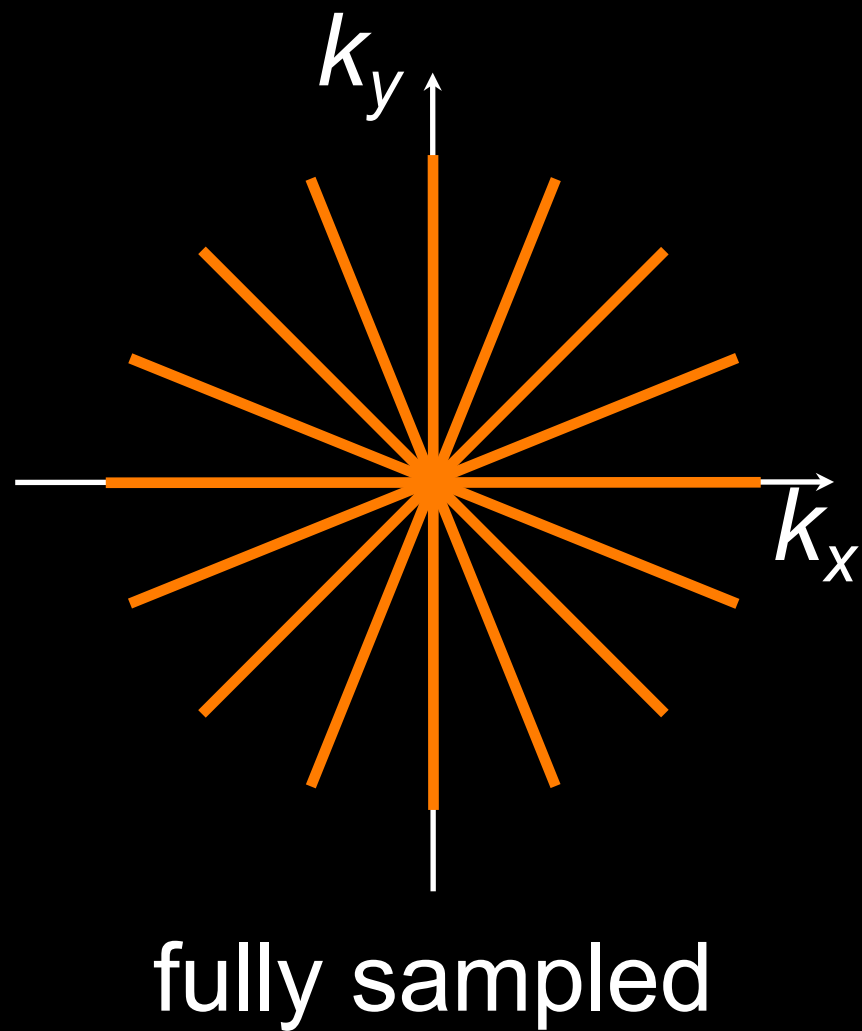
real channel



magnitude

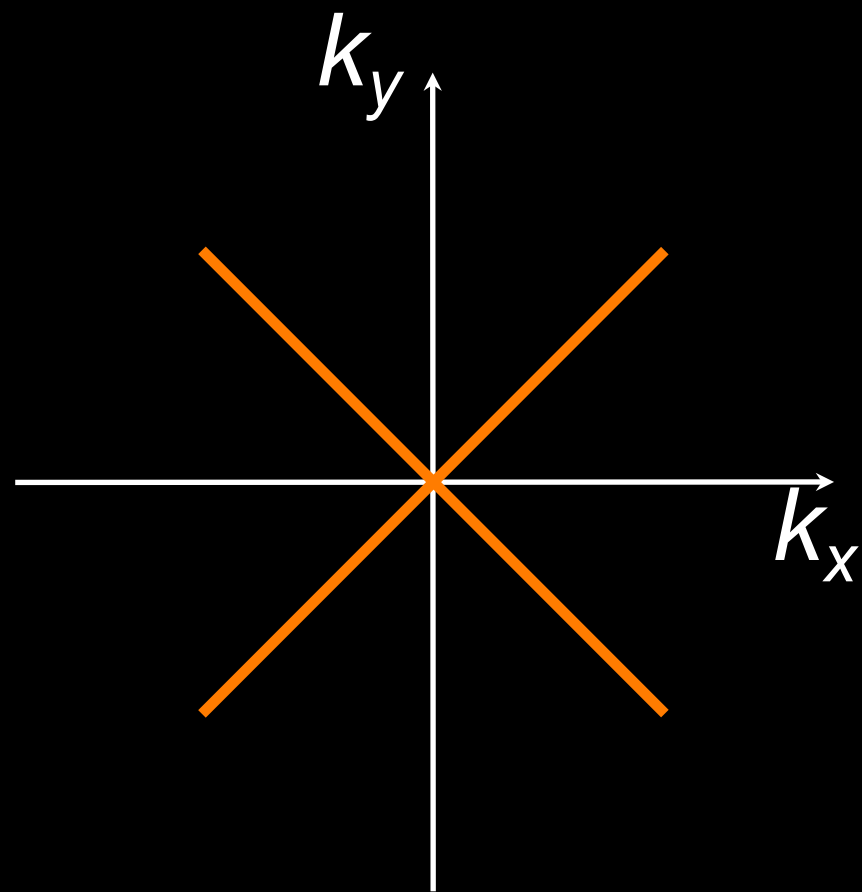
*alternatively, can use "gridding" reconstruction*

# Radial: Undersampling





# Radial: Undersampling



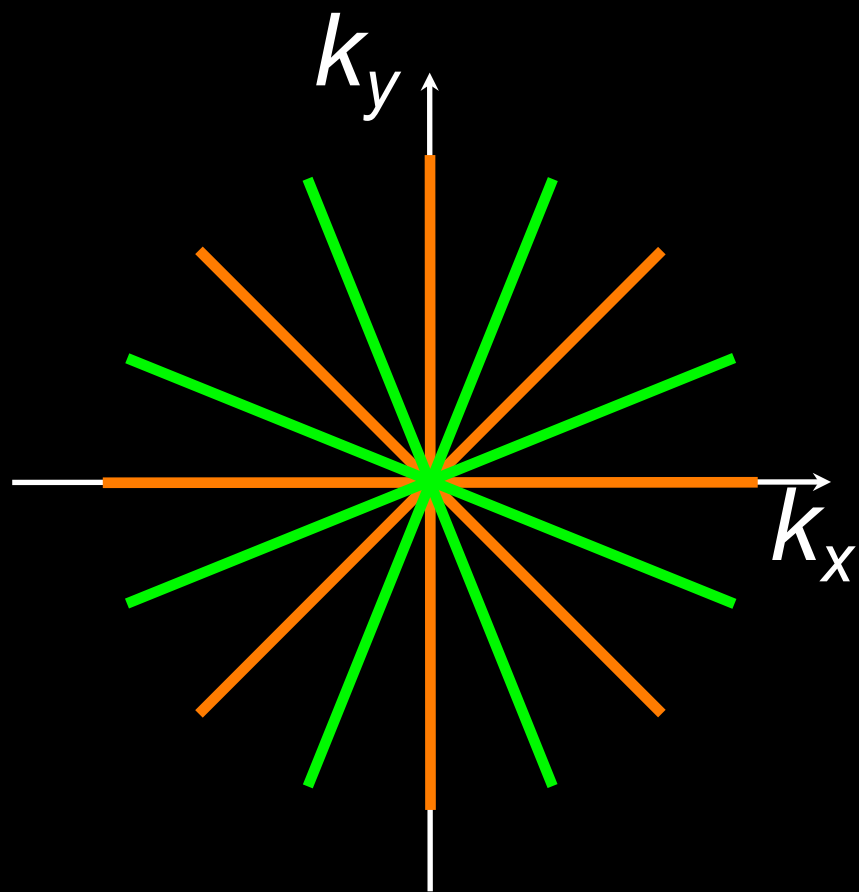
undersampled



streaking artifacts



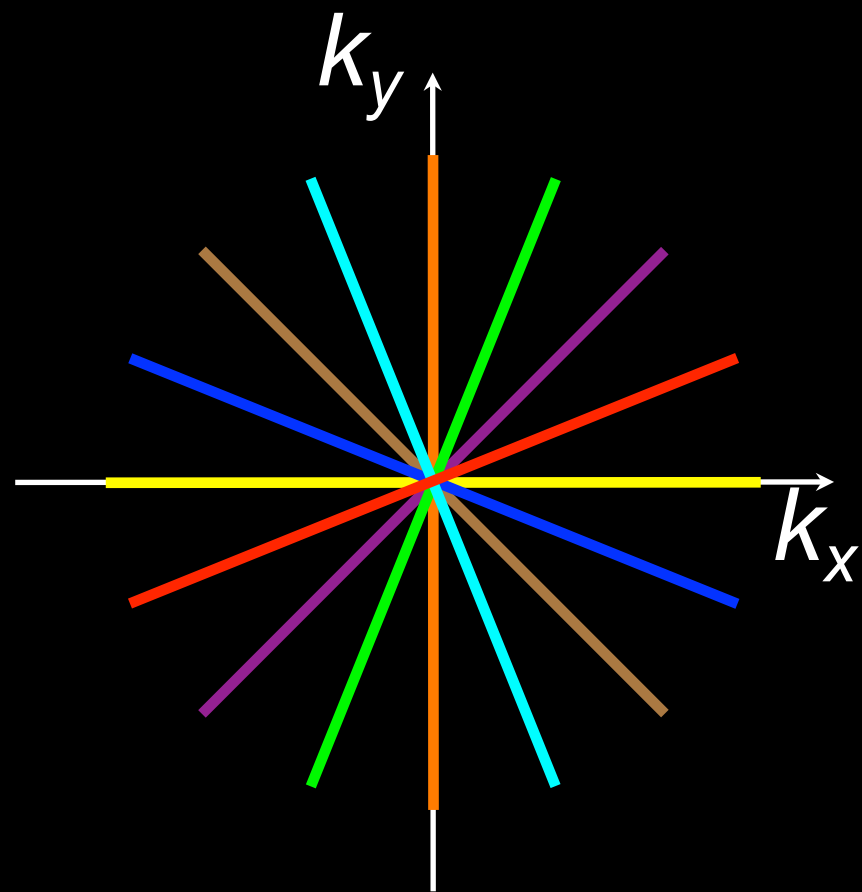
# Radial: Acq Ordering



interleaved acq



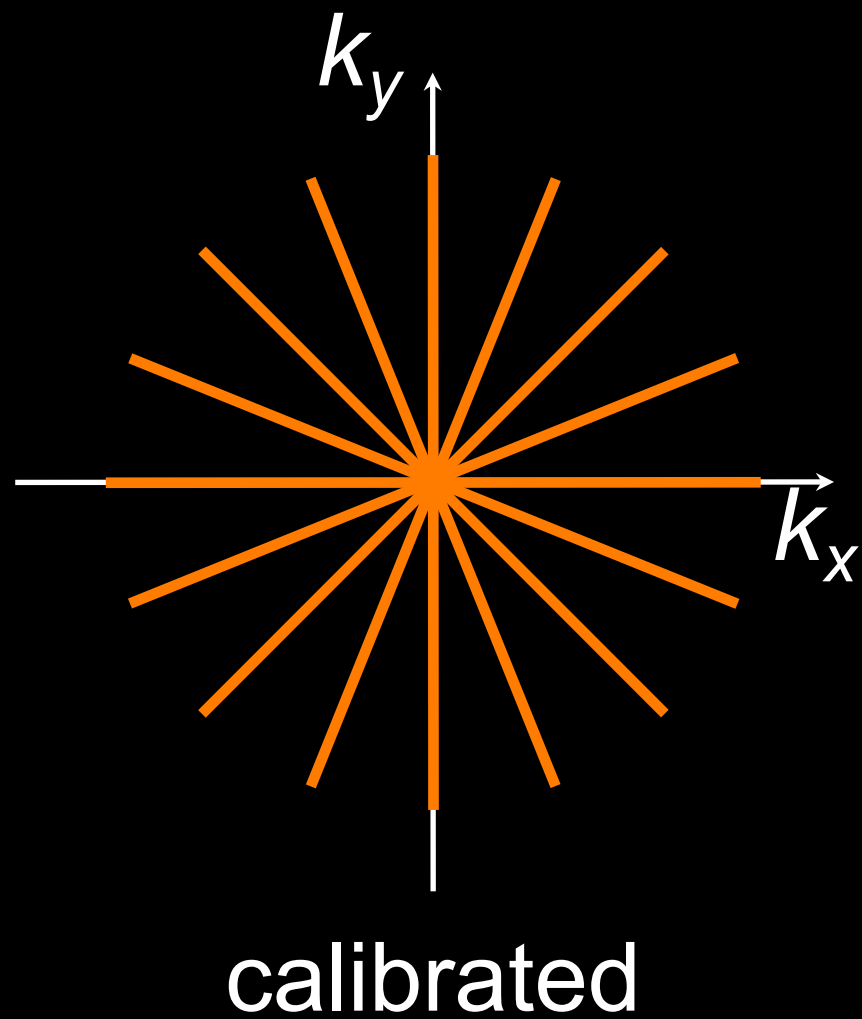
# Radial: Acq Ordering



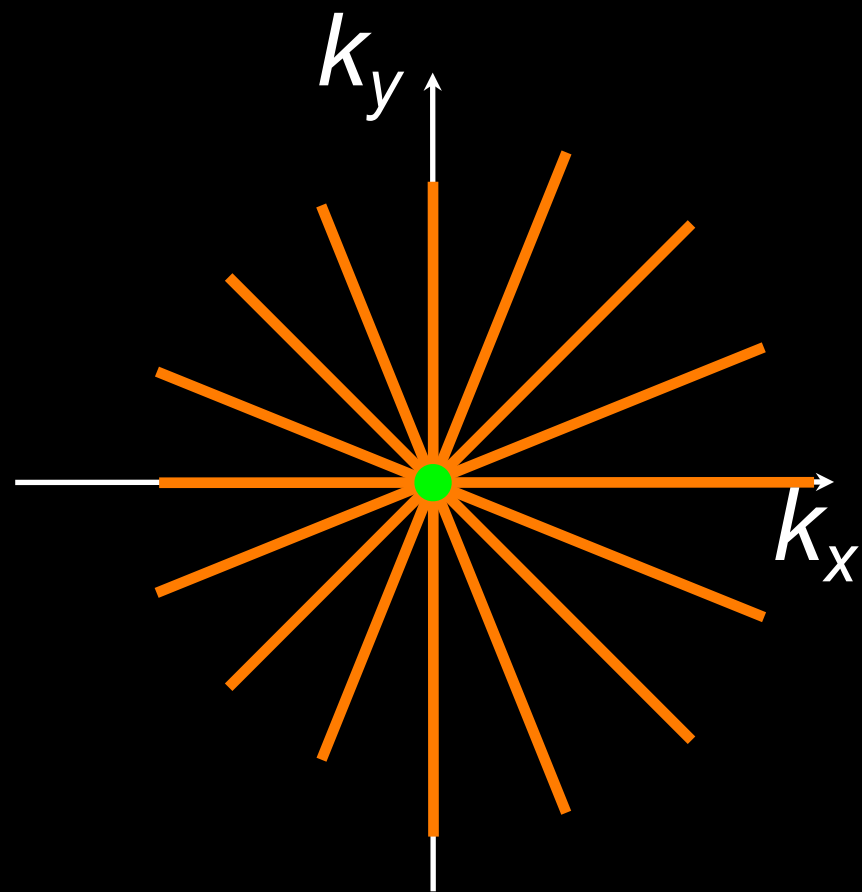
random acq



# Radial: Gradient Delays

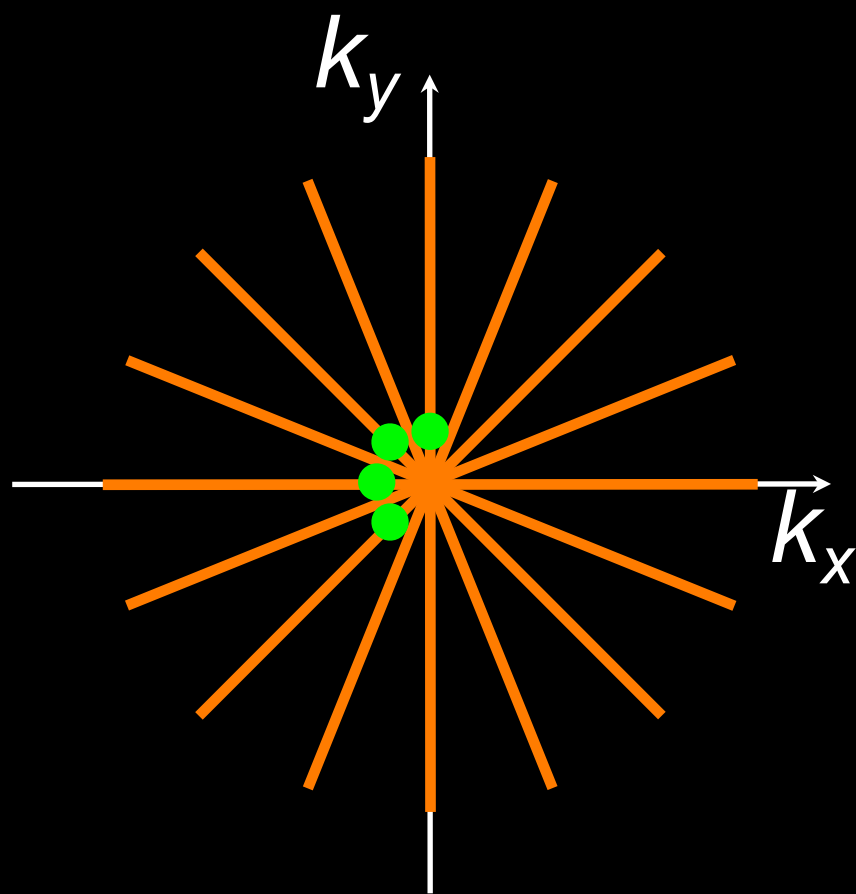


# Radial: Gradient Delays

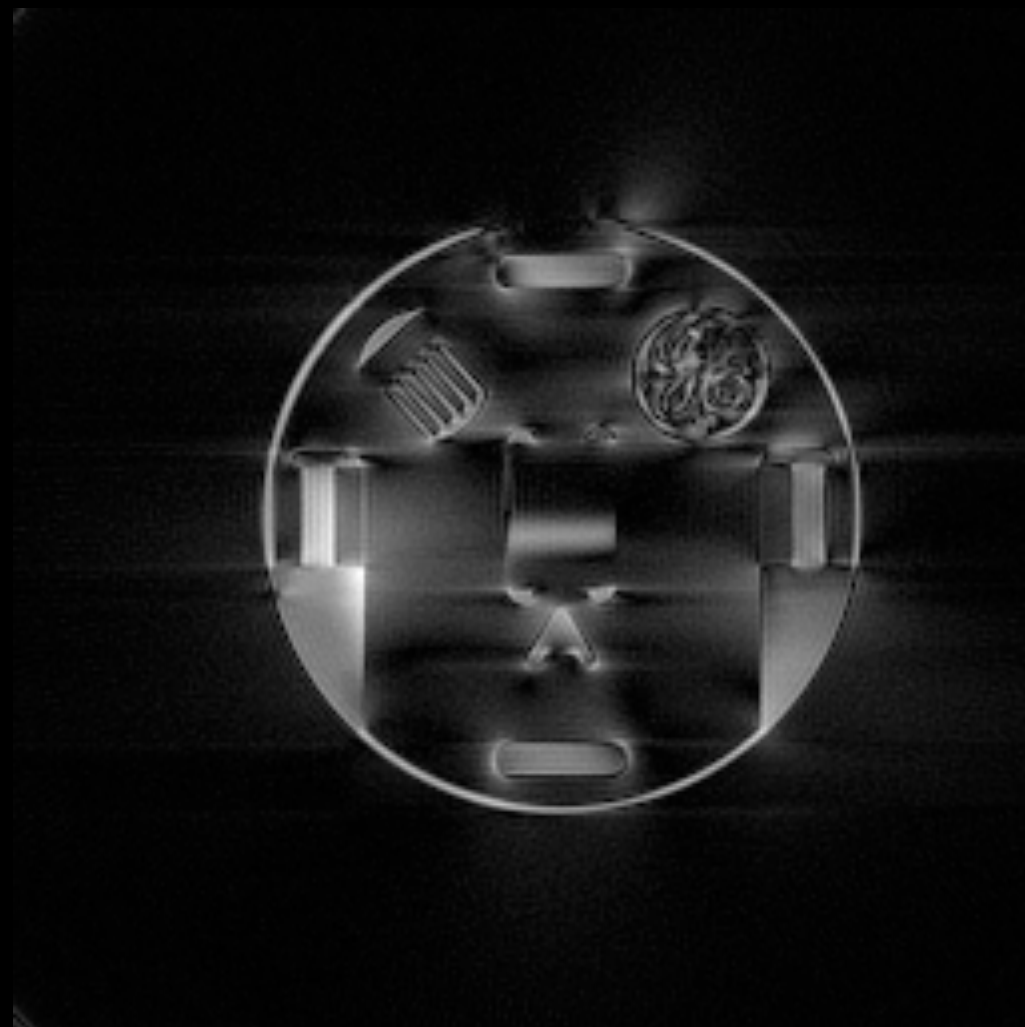


grad-acq delay

# Radial: Gradient Delays

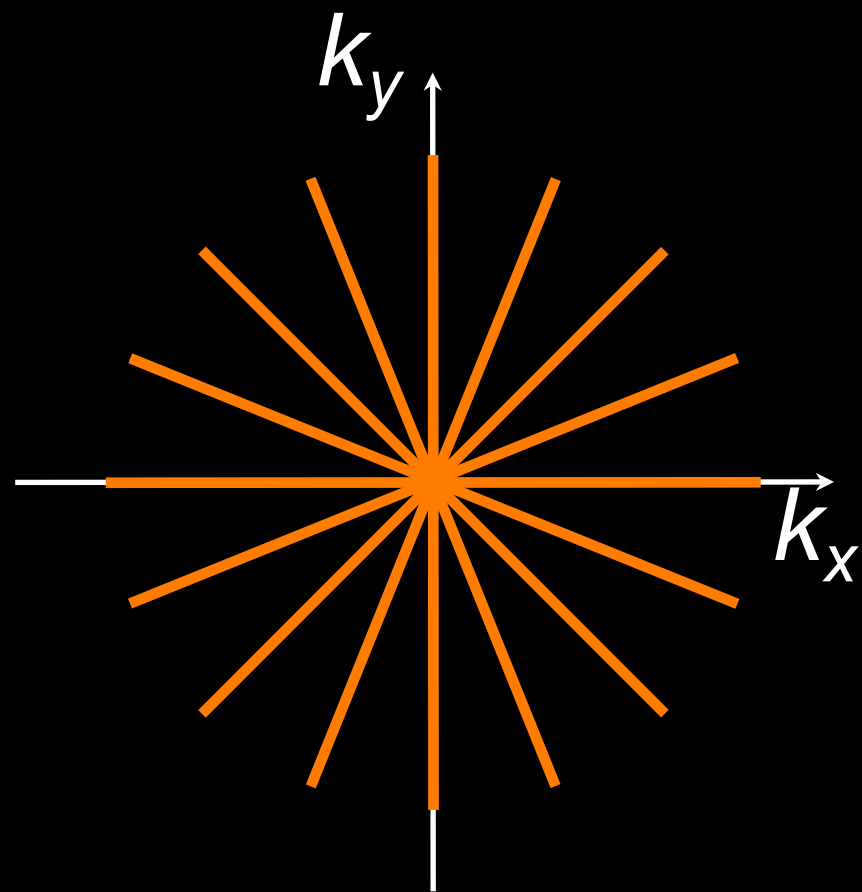


recon unaware of delays  
mis-aligned DC



misalignment artifacts

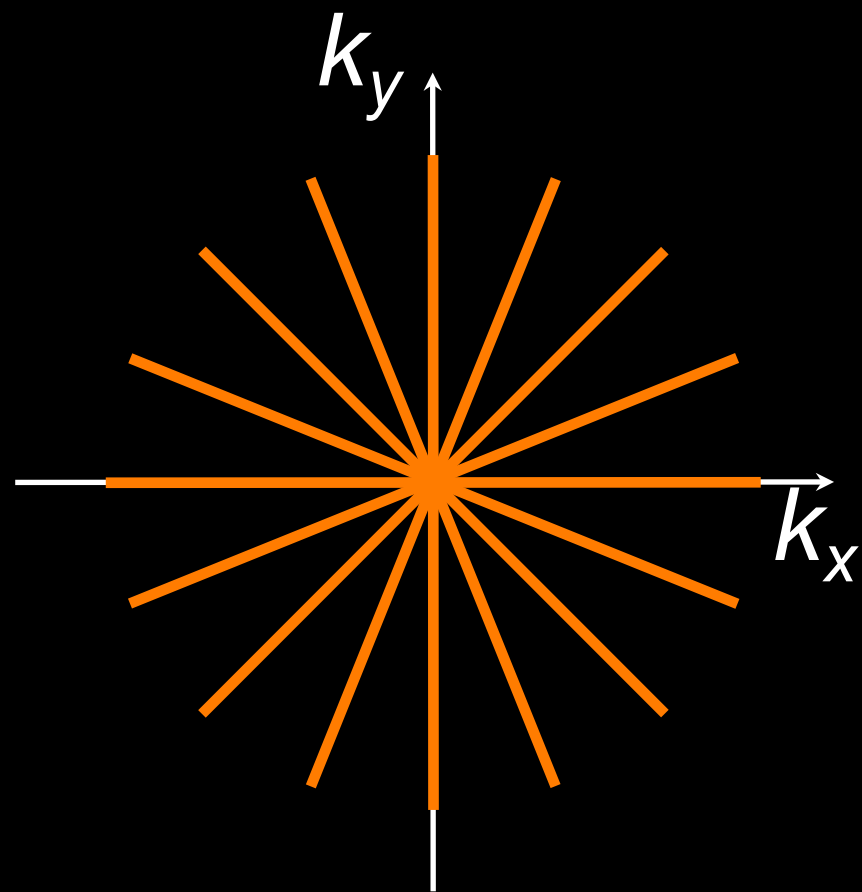
# Radial: Off-resonance Effects



on resonance



# Radial: Off-resonance Effects



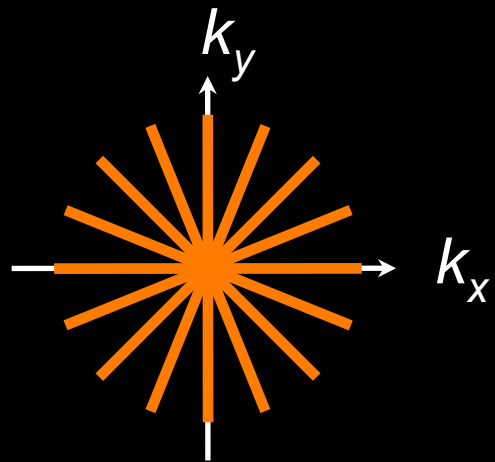
+200 Hz globally



off-res blurring

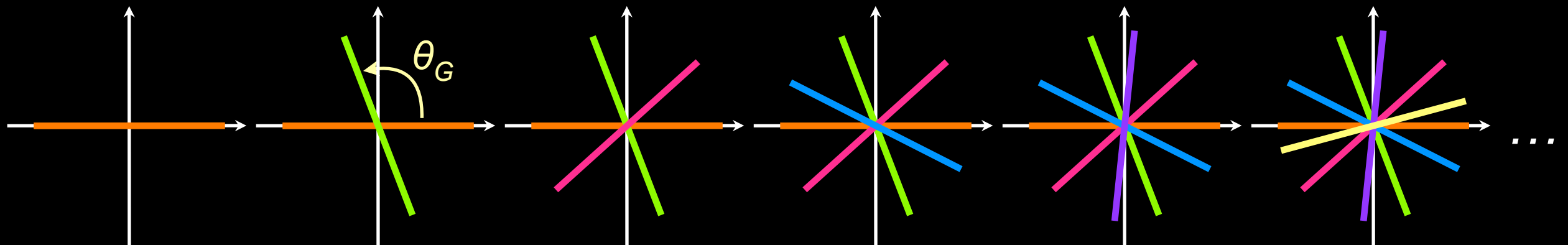
# Radial: Real-time MRI

## 2D Radial MRI



- Robust to motion (oversample center of k-space)
- Can tolerate a lot of undersampling

## Golden Angle Ordering



- Almost uniform sampling of  $k-t$  space
- Flexible choice of temporal frame location and width



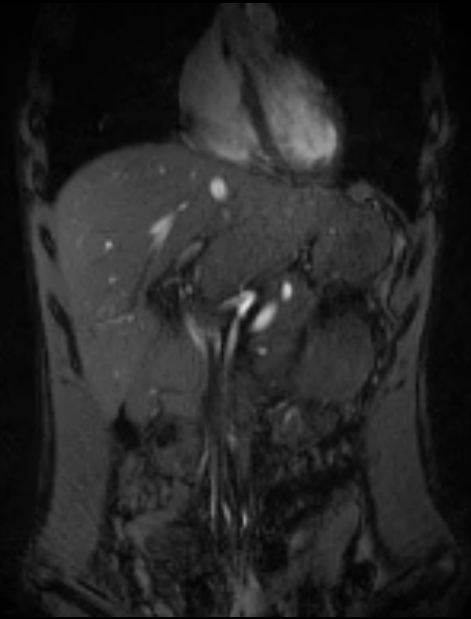
# Radial: Real-time MRI

## Radial FLASH

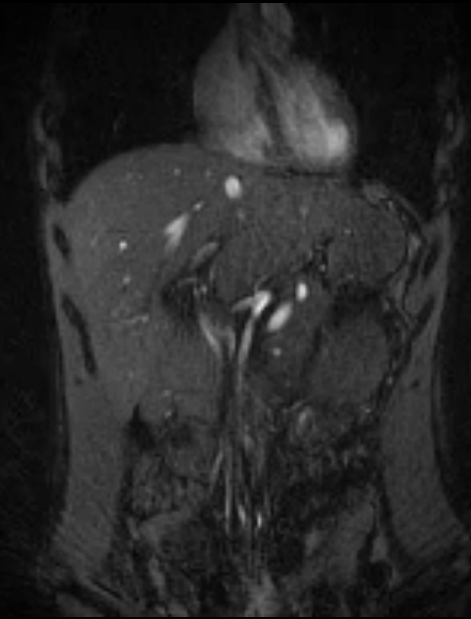
- golden-angle ordering
- 192 x 192 matrix
- TR = 3.1 ms  
(1 spoke per TR)
- 3.0 T

## Reconstruction

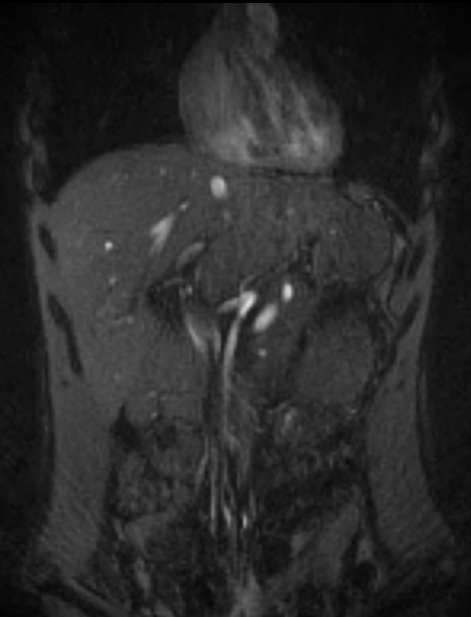
- sliding window of 20 TRs  
(display at 16 frames/sec)
- gridding only  
(300 spokes for Nyquist)



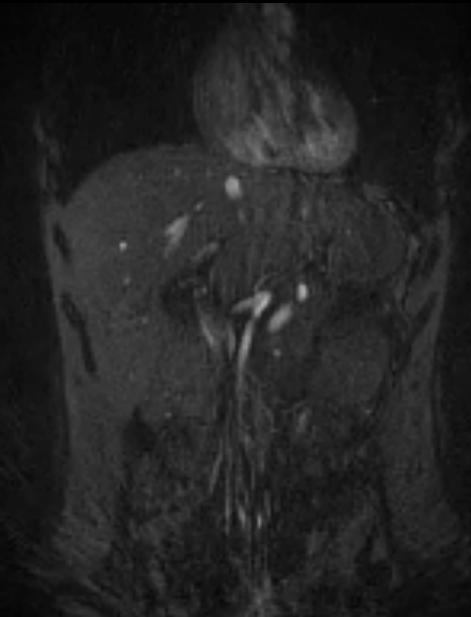
255 spokes/frame  
(791 ms/frame)



144 spokes/frame  
(446 ms/frame)



89 spokes/frame  
(276 ms/frame)



55 spokes/frame  
(171 ms/frame)

*courtesy of Samantha Mikael*

# Radial: Real-time MRI

## Radial FLASH

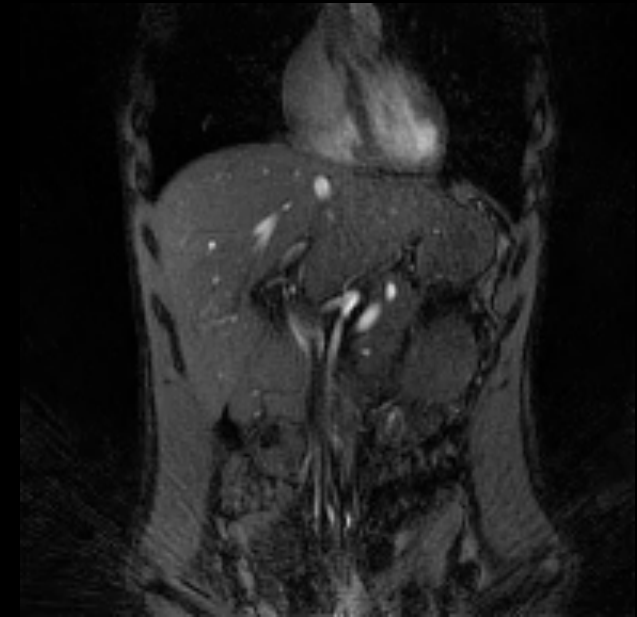
- golden-angle ordering
- 192 x 192 matrix
- TR = 3.1 ms  
(1 spoke per TR)
- 3.0 T

## Reconstruction

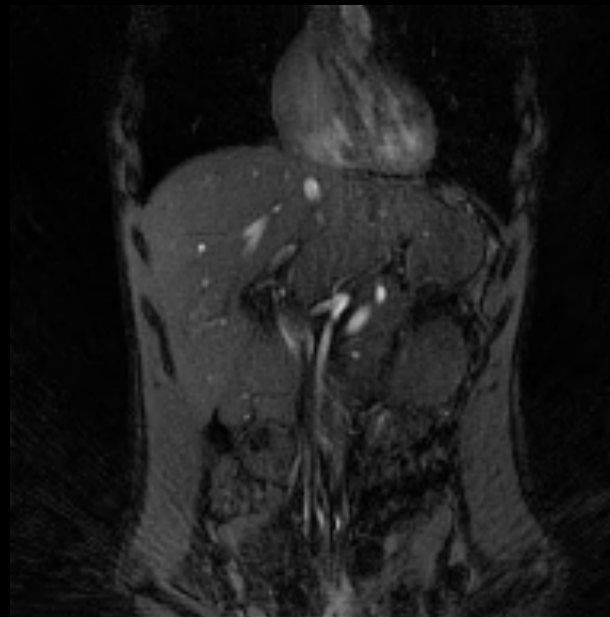
- sliding window of 20 TRs  
(display at 16 frames/sec)
- **parallel imaging (SPIRiT)**  
(300 spokes for Nyquist)



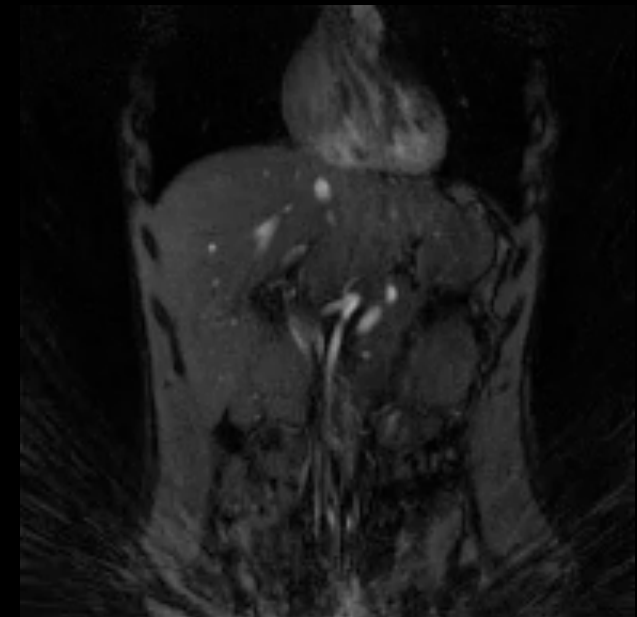
255 spokes/frame  
(791 ms/frame)



144 spokes/frame  
(446 ms/frame)



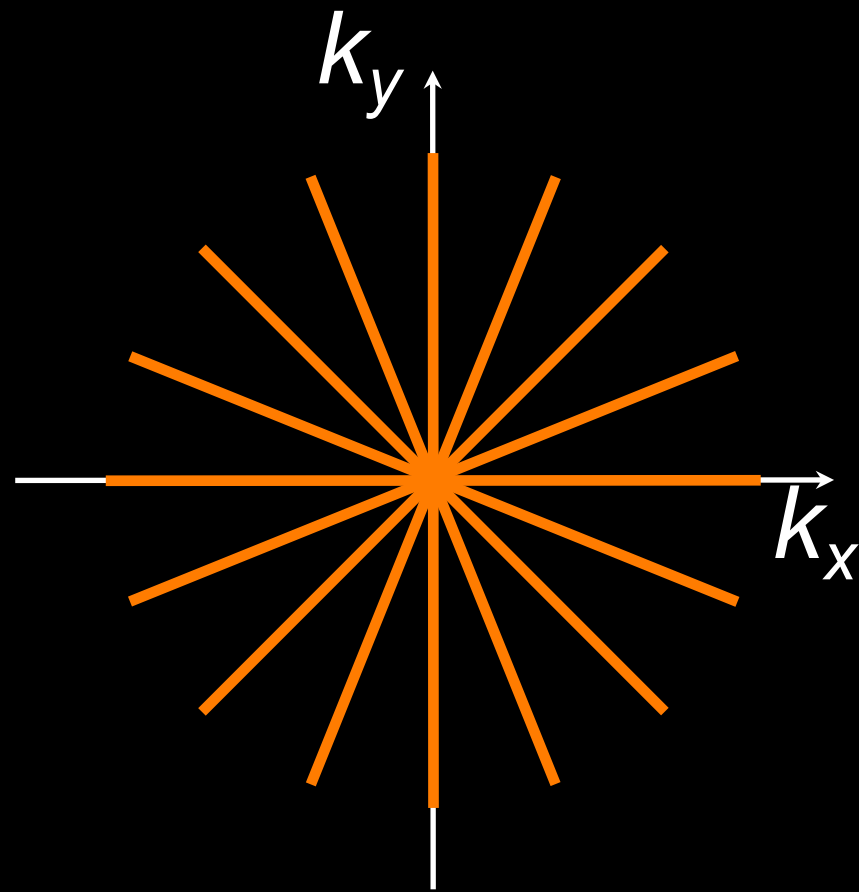
89 spokes/frame  
(276 ms/frame)



55 spokes/frame  
(171 ms/frame)

*courtesy of Samantha Mikael*

# Radial: Pros and Cons



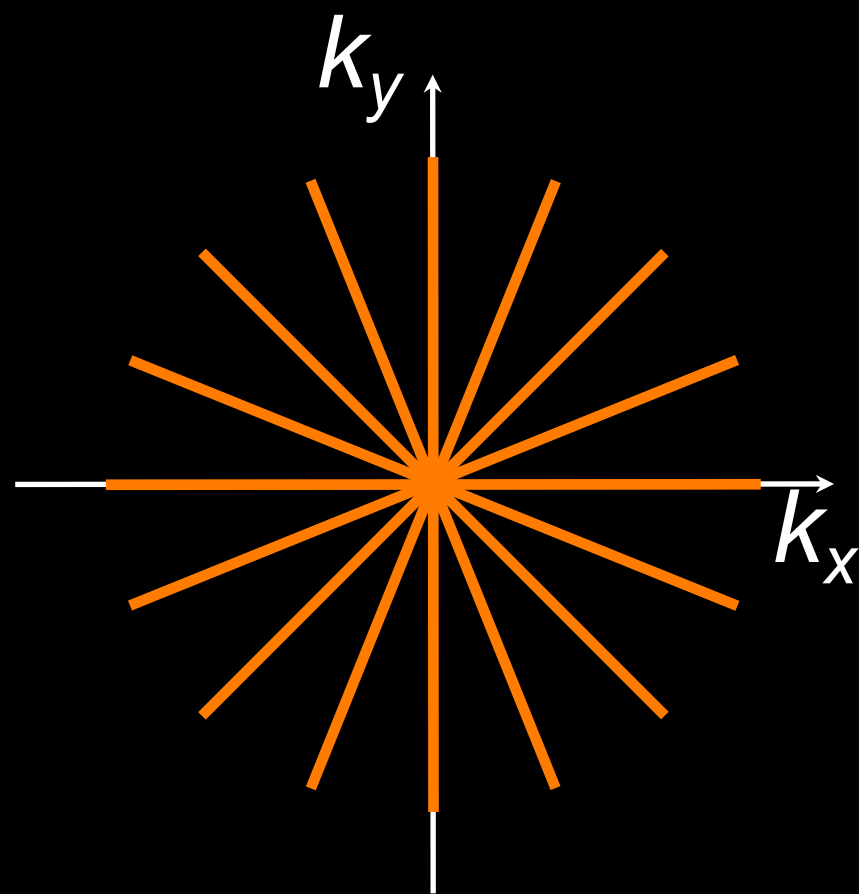
## Pros

- Robust to motion (get DC every TR)
- Can tolerate a lot of undersampling
- Half-spoke PR has very short TE

## Cons

- SNR penalty (non-uniform density)
- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

# Radial: Extensions



3D stack of stars

3D koosh ball

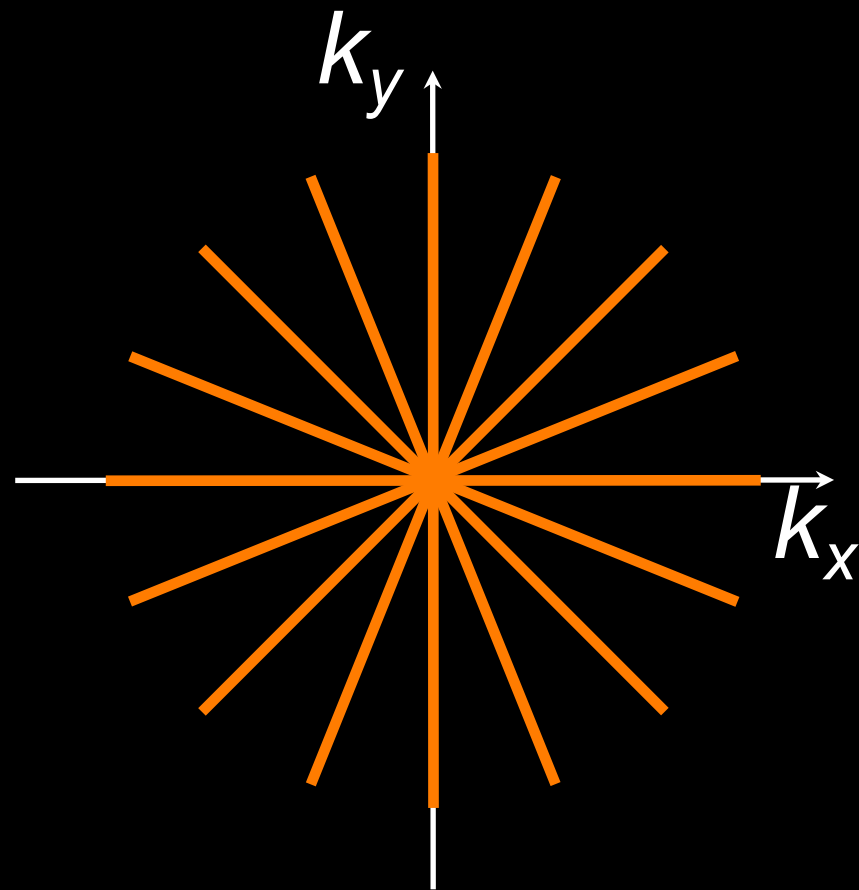
Multiple spokes per TR

Golden angle ordering

Parallel imaging

Partial Fourier

# Radial: Applications



## Fast imaging

- Cardiac MRI

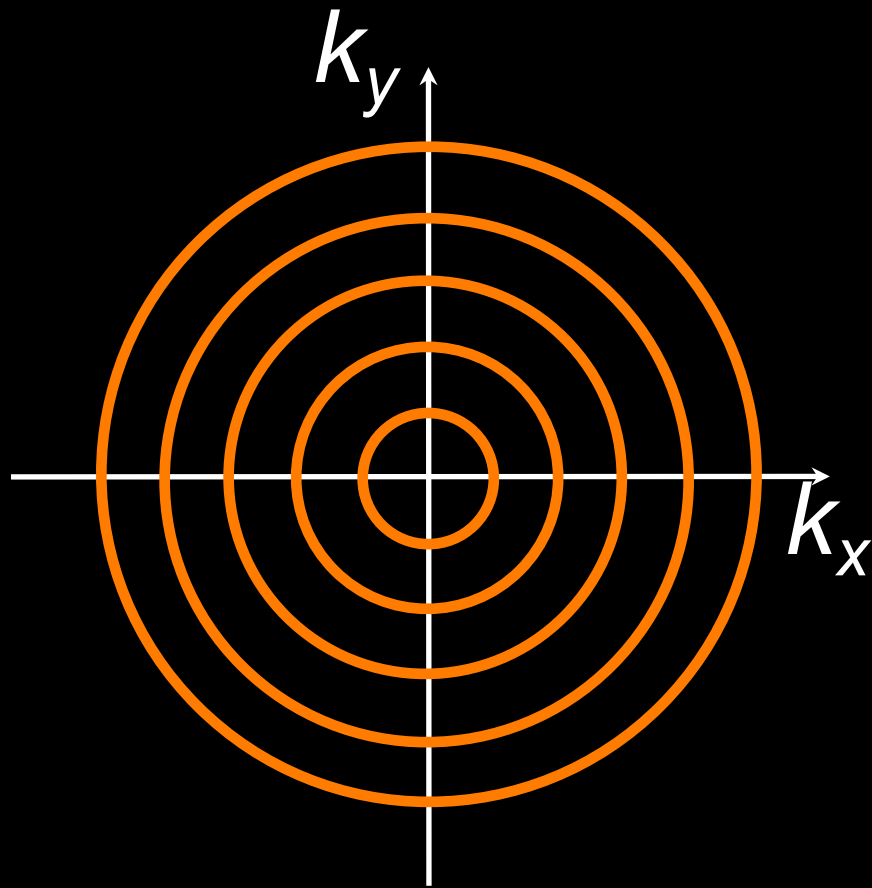
## Improve motion robustness

- Cardiac MRI
- Abdominal MRI

## Ultra-short TE (UTE) imaging

- Musculoskeletal MRI
- Lung MRI

# Concentric Rings

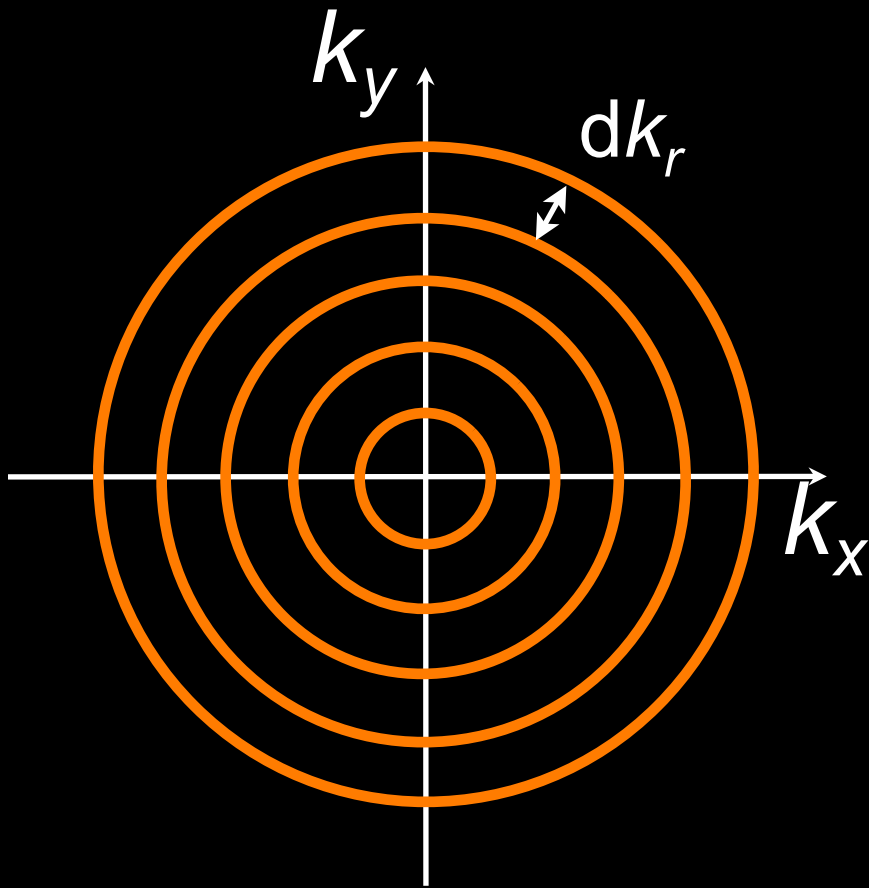


Non-rectilinear sampling!

Samples k-space on a polar grid

- “dual” of radial sampling
- shares some properties of 2DPR
- exhibits distinct characteristics

# Rings: Sampling Requirements



$N$  concentric rings

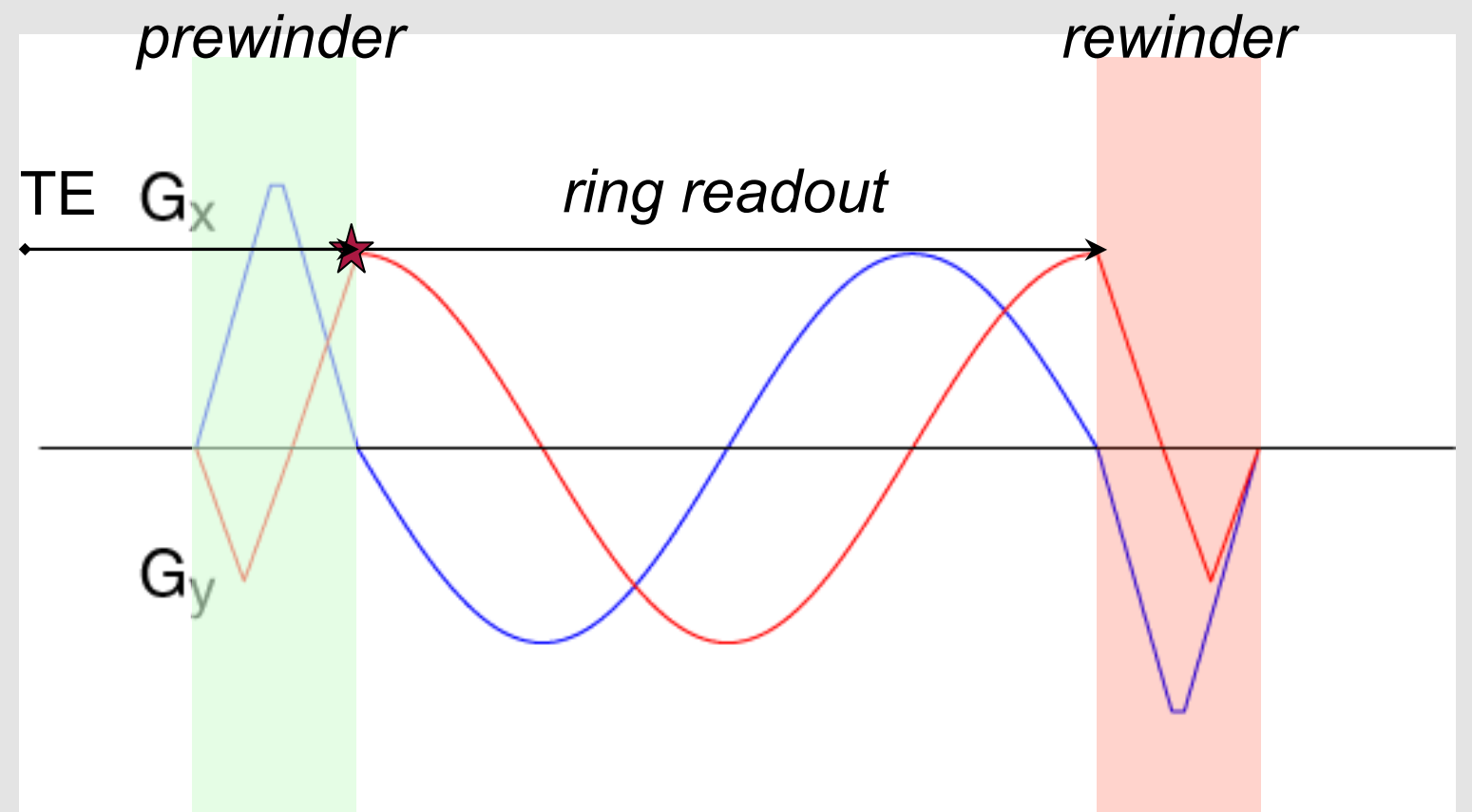
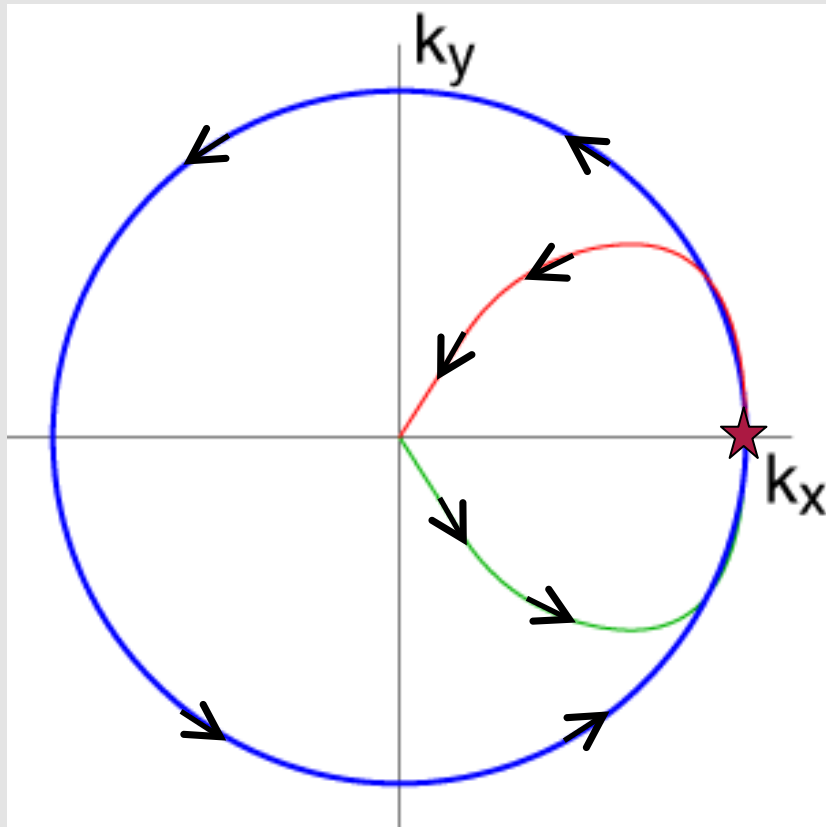
uniform spacing of  $dk_r$

$$\frac{1}{\text{FOV}} = dk_r$$

$$k_{r,max} = (N - 1) \cdot dk_r$$

Subject to hardware limits

# Rings: Gradient Design

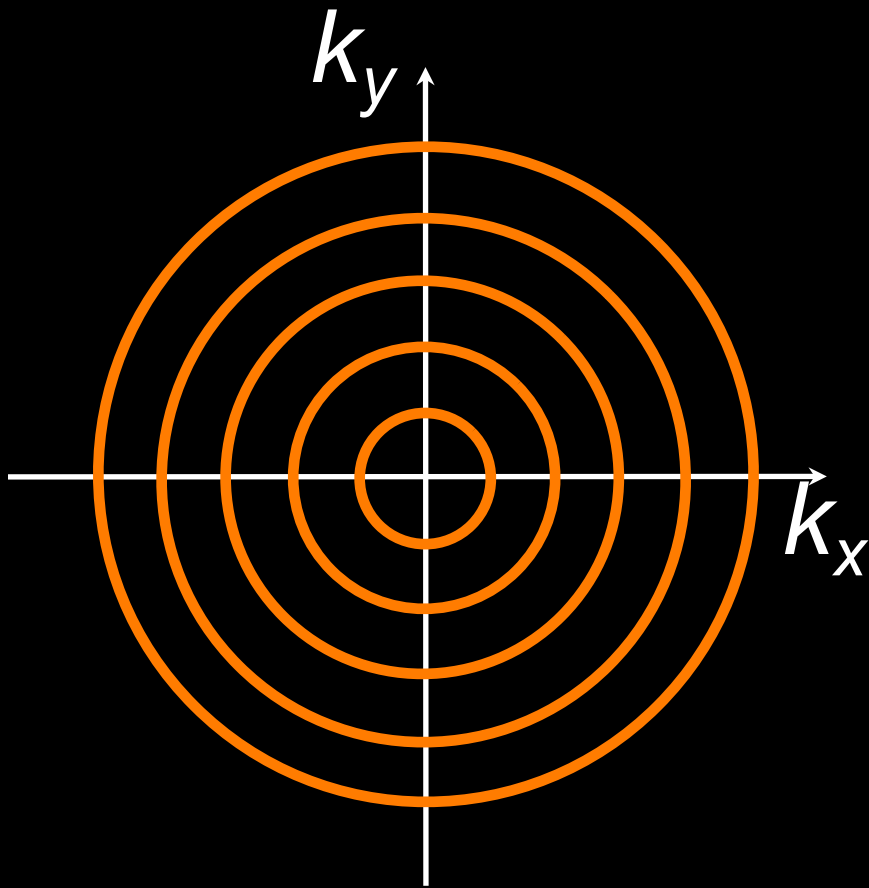


Scale down gradients for outermost ring

- Sampling density identical to 2DPR
- Robust to gradient delays & timing errors



# Rings: Scan Time



For an  $M \times M$  image,  
need  $N = M/2$  rings

Scan time =  $(M/2) \times TR_{\text{ring}}$

Compare with 2DFT:

Scan time =  $M \times TR_{\text{line}}$

Rings offer  $\sim 2x$  acceleration

# Rings: Motion and Flow

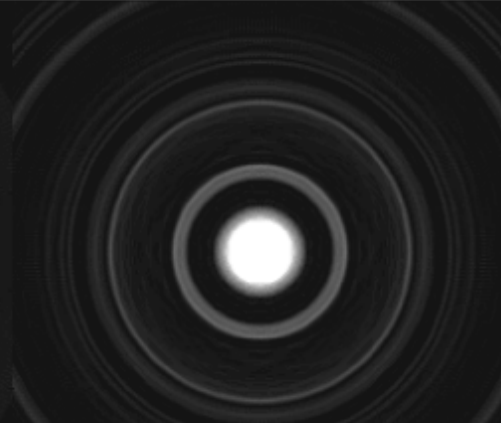
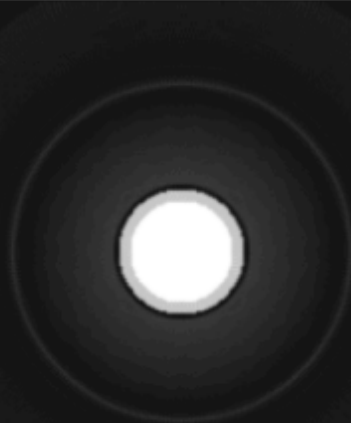
a. Pulsatile flow    b. Contraction/Dilation    c. Linear drift



2DFT, PE in x

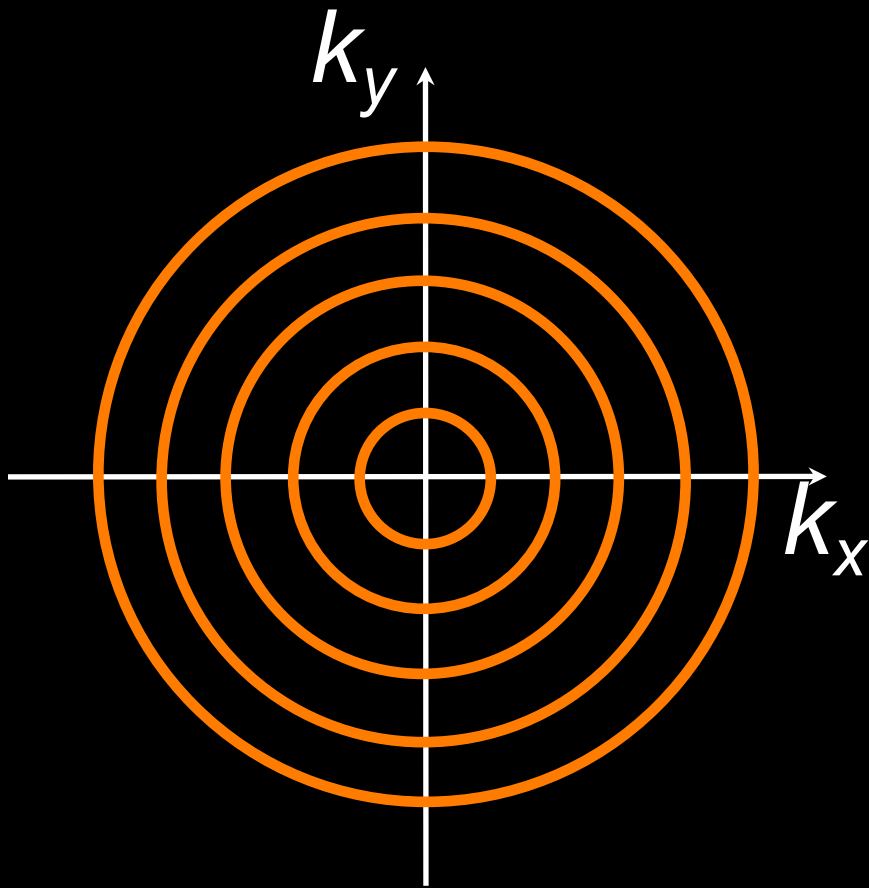


2DPR



Rings

# Rings: Image Reconstruction



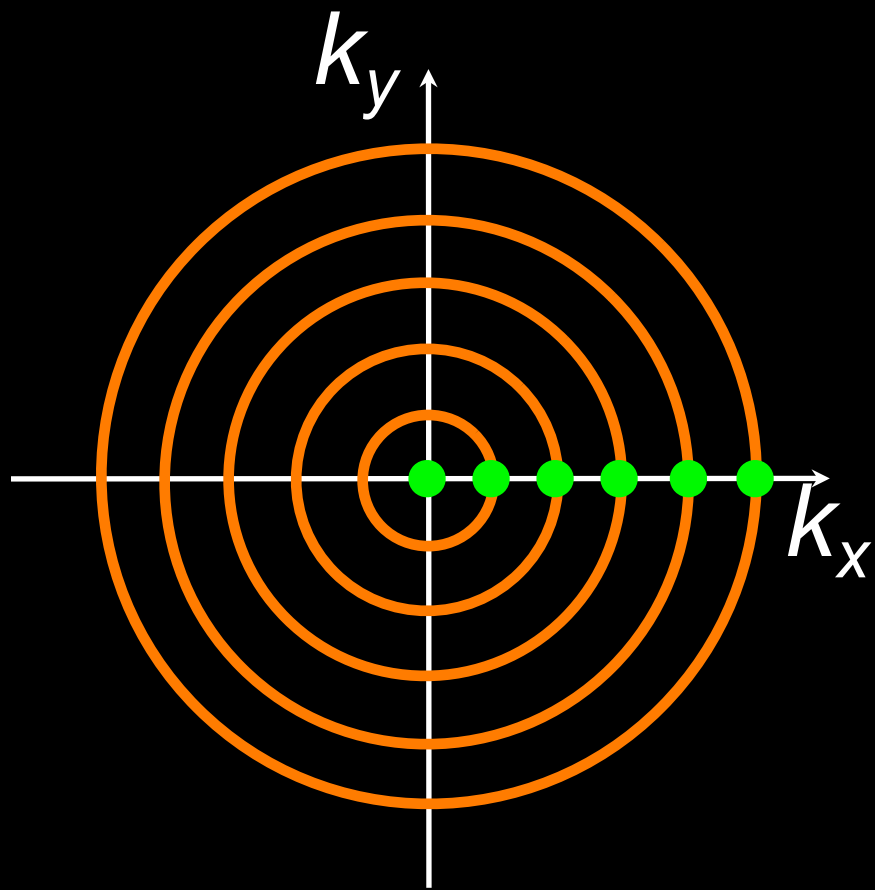
Reformat into spokes

- filtered back projection

Resample onto Cartesian grid

- “gridding” reconstruction

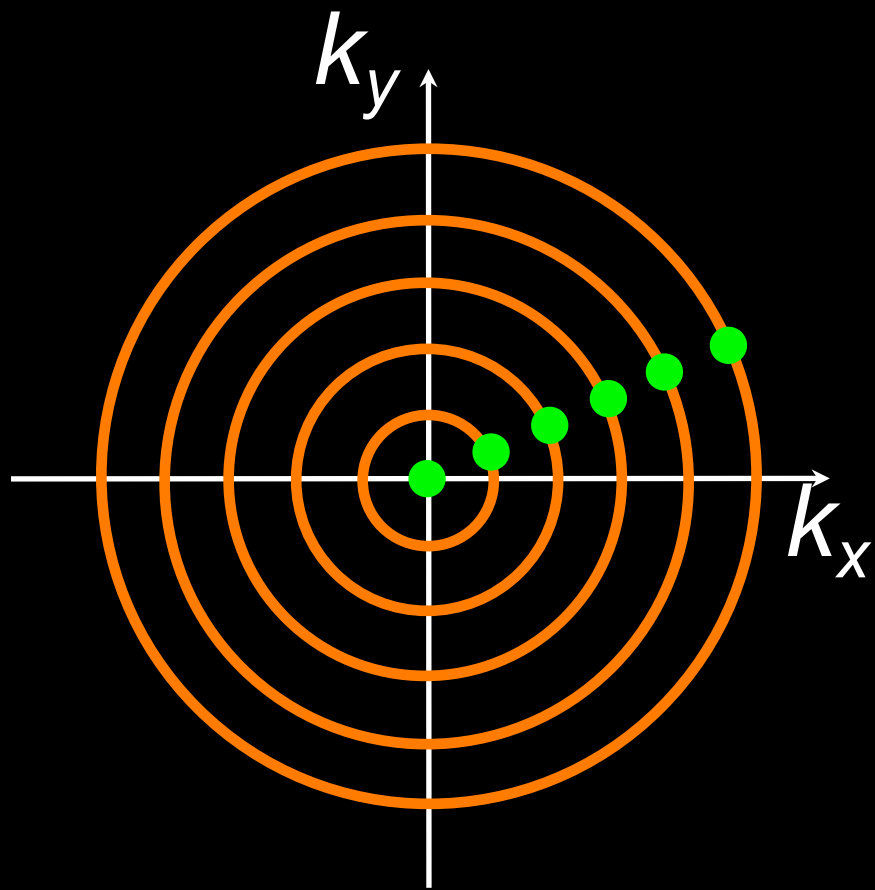
# Rings: Gradient Delays



calibrated



# Rings: Gradient Delays

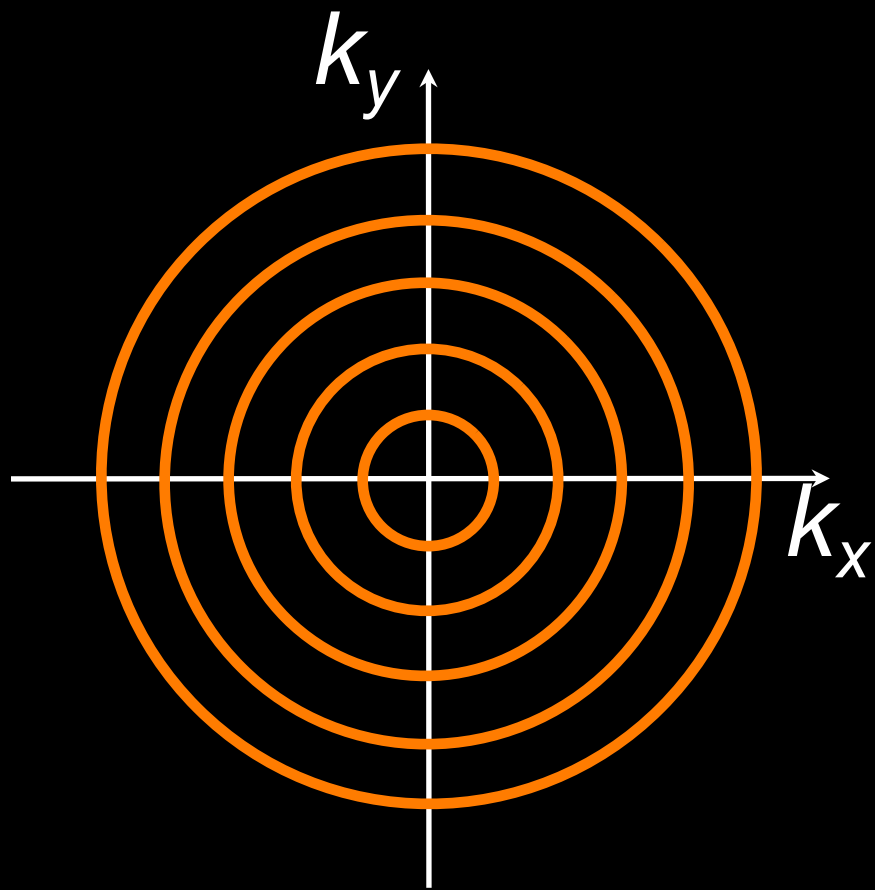


grad-acq delay

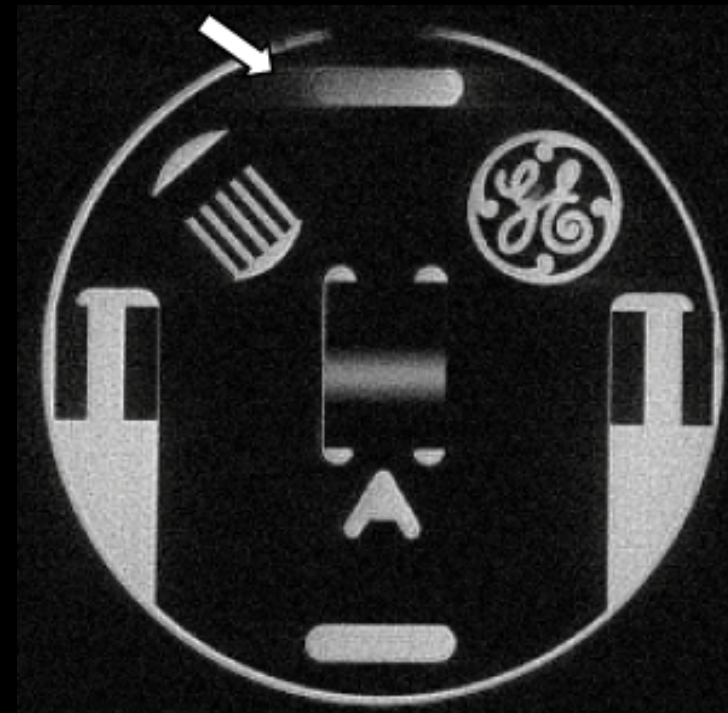


in-plane rotation

# Rings: Off-resonance Effects

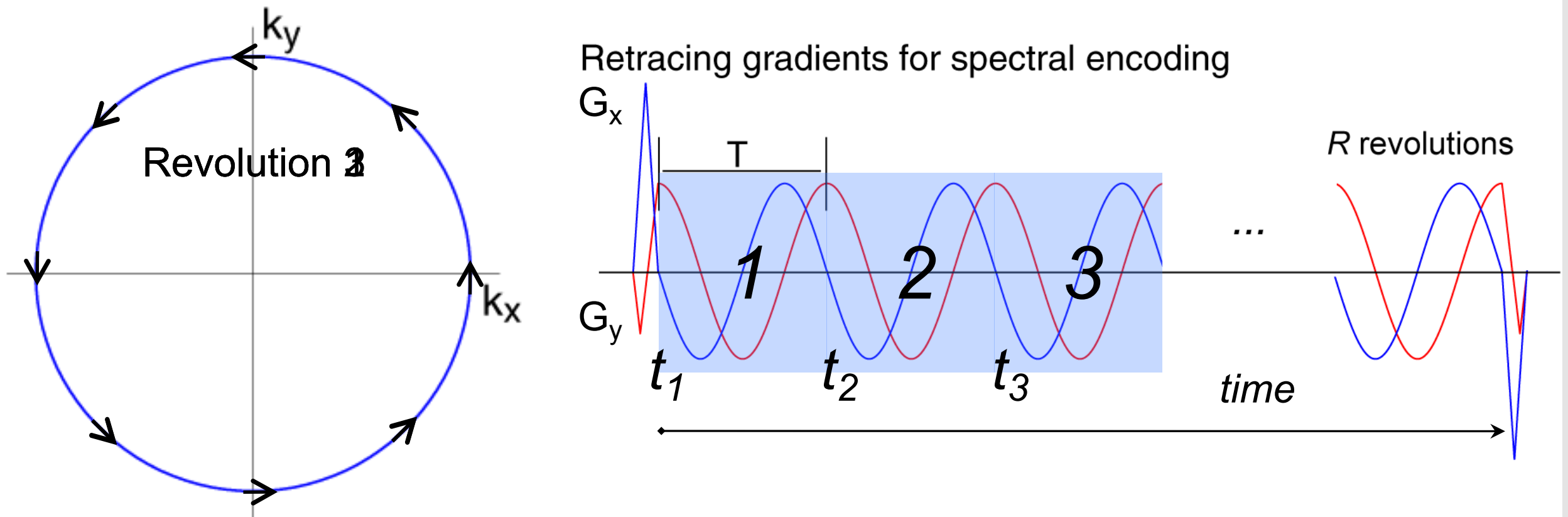


w/spatially  
varying off-res



off-res blurring

# Rings: Resolving Off-Res Effects



Encodes  $(k_x, k_y, time)$  simultaneously

- Resolve off-resonance effects
- "Spectral" encoding



# Rings: Resolving Off-Res Effects

Concentric Rings with 2 Revolutions / TR



Regular recon

Field map

ORC image



# Rings: Resolving Off-Res Effects

Concentric Rings with 3 Revolutions / TR



Field map



Water image

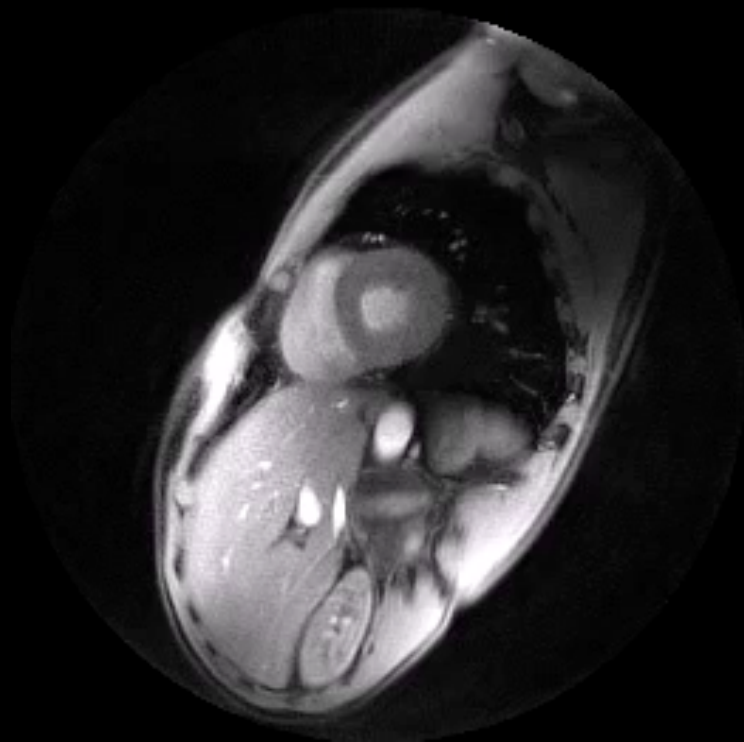


Fat image

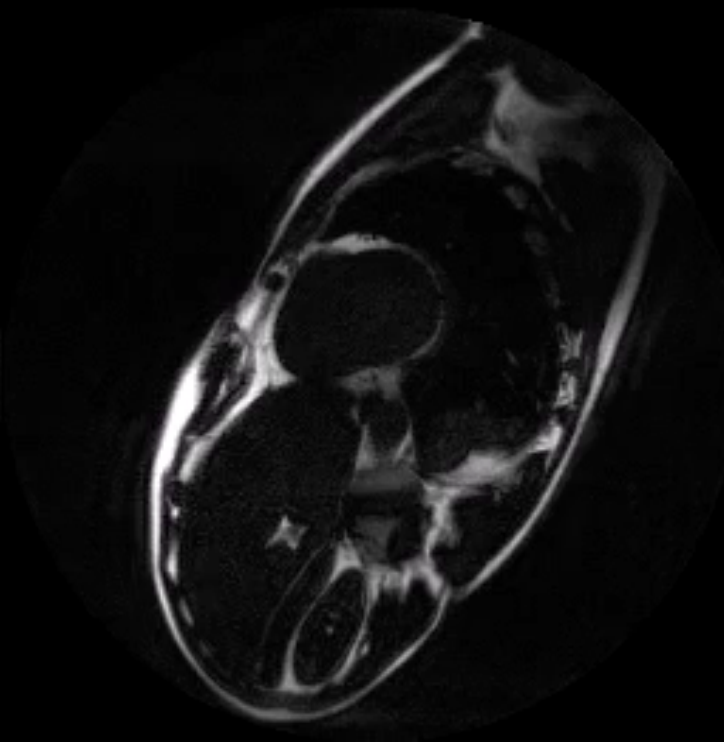
# Rings: Resolving Off-Res Effects

1.5 T, 2D GRE, Cardiac F/W Cine

13-HB BH scan (with add'l 3-fold *k-t* BLAST acceleration)



*Water*



*Fat*

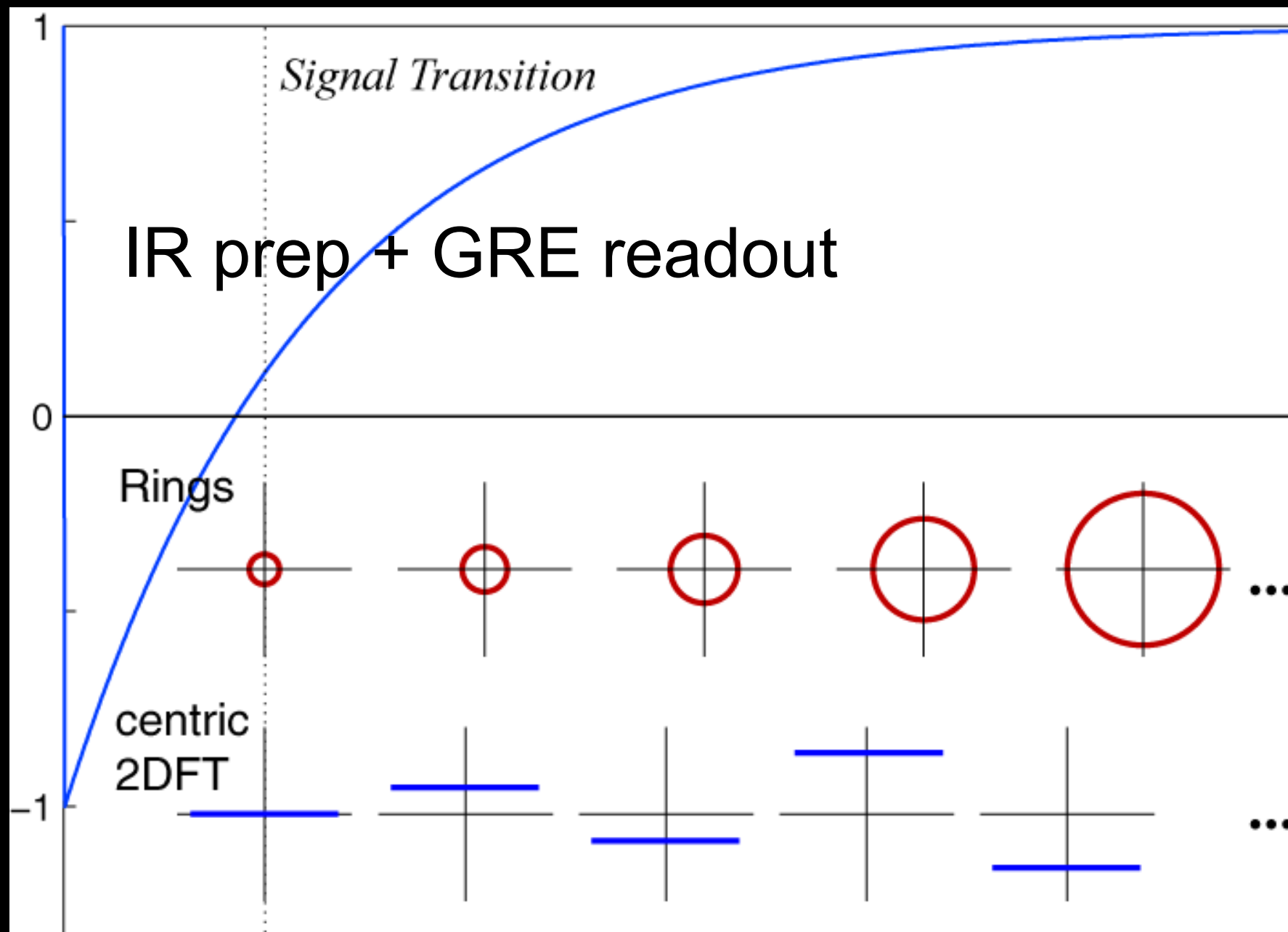


*Combined*

# Rings: Magnetization-Prepared MRI

## Inherent 2D centric ordering

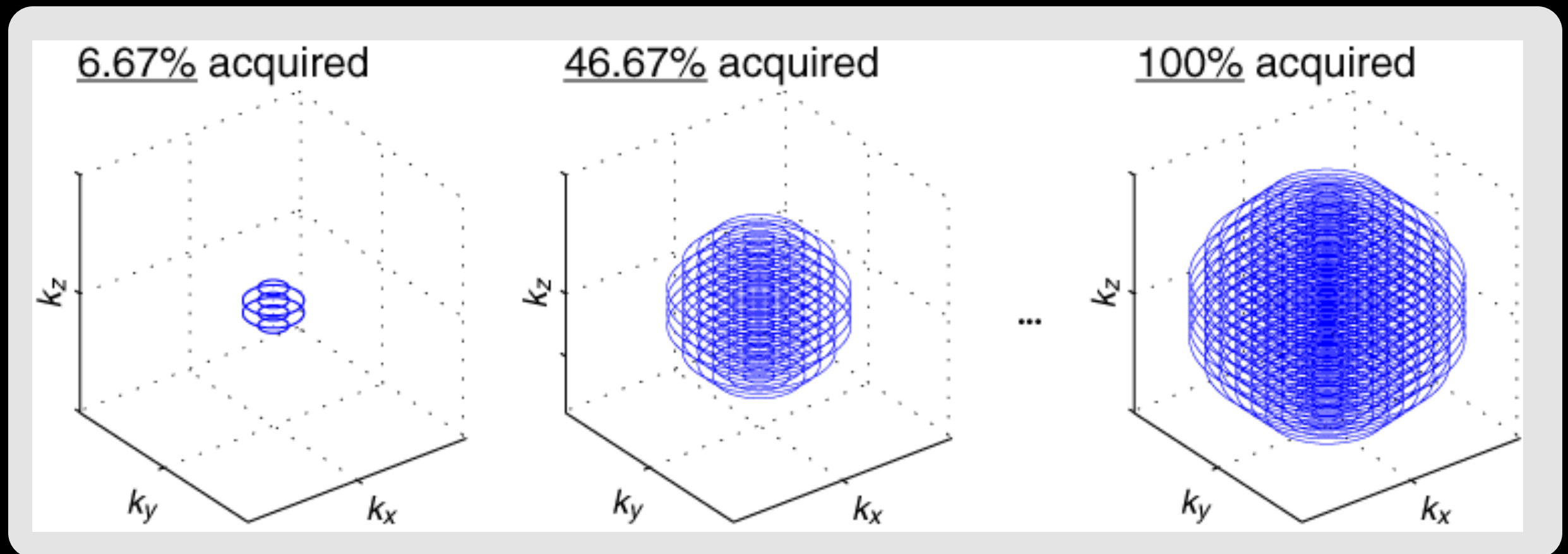
- improved mag-prep contrast and k-space weighting



# Rings: 3D Mag-Prep MRI

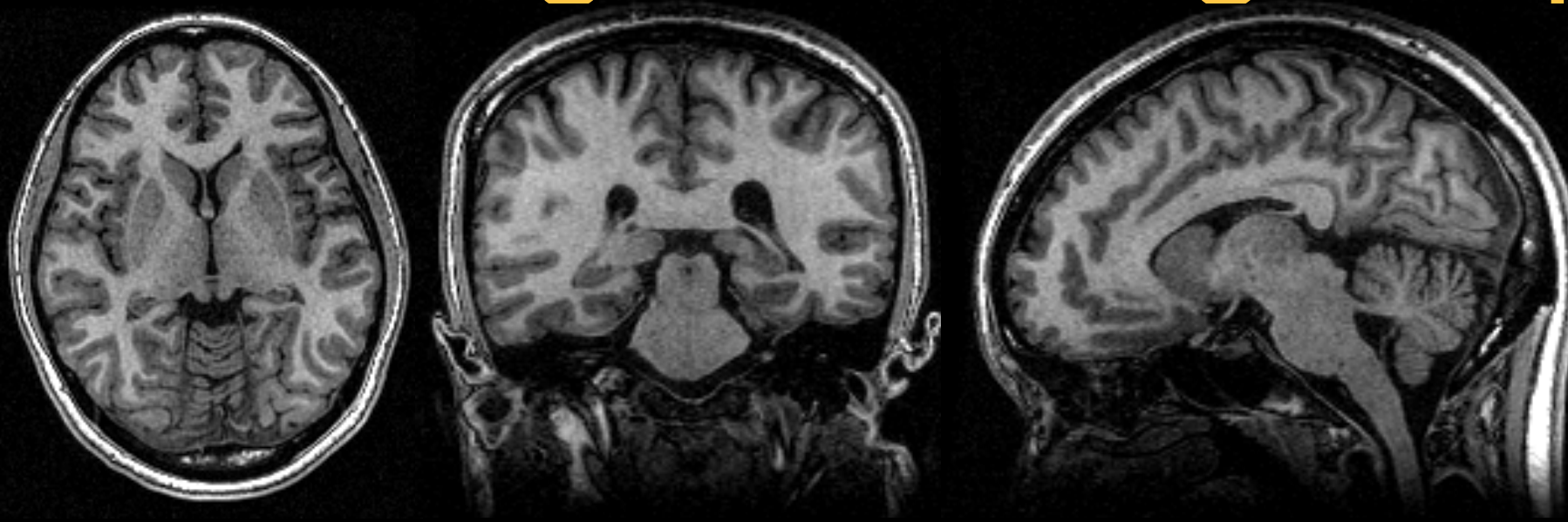
## Fully 3D centric ordering

- improved mag-prep contrast and k-space weighting
- spherical k-space coverage saves time





# Rings: 3D Mag-Prep MRI



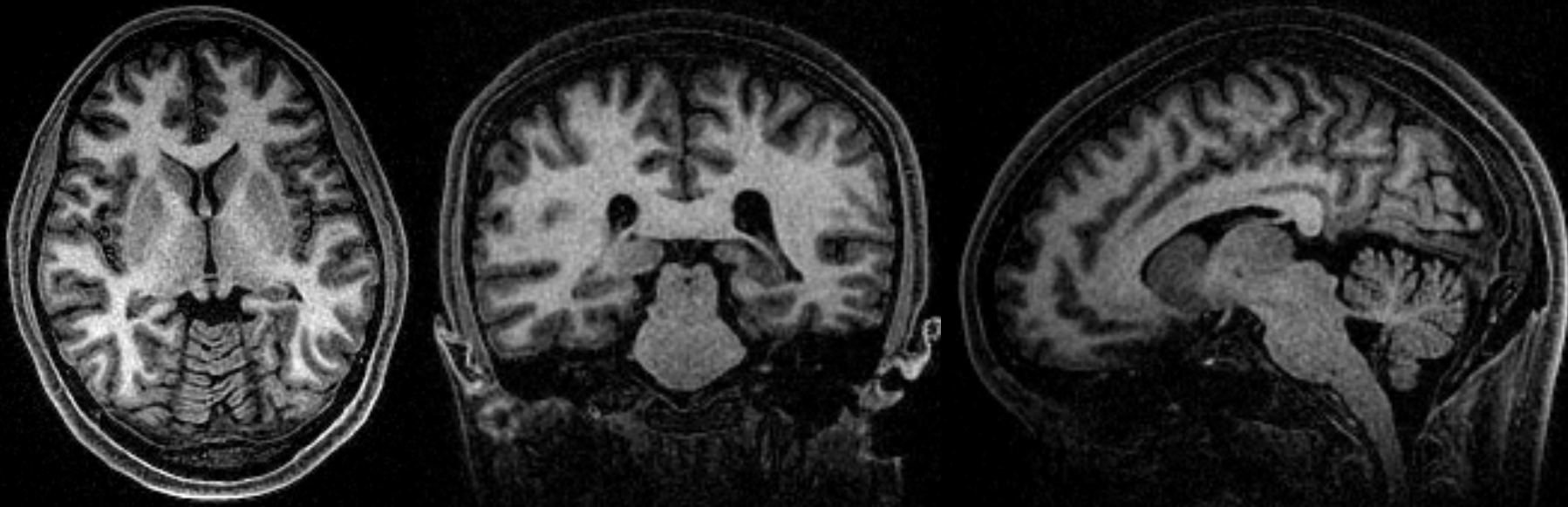
Product 3DFT

TI/TD = 600/----- ms

9 min 34 s

SNR<sub>WM</sub> 24.07

CNR<sub>GW</sub> 8.86



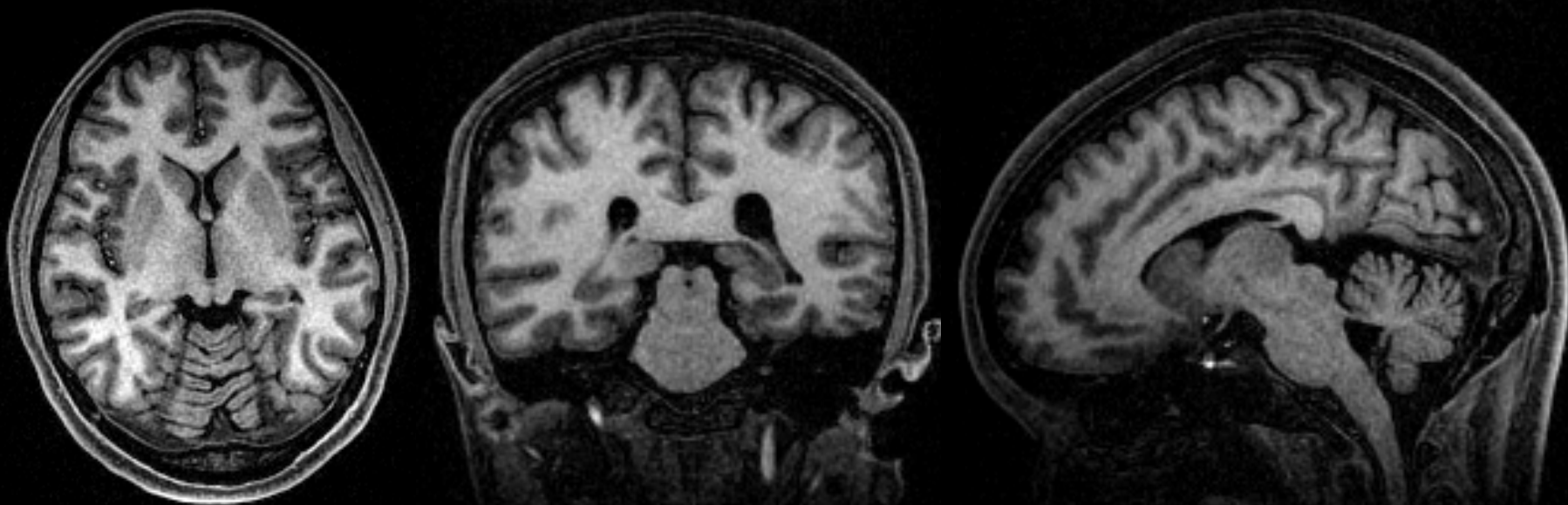
3D Rings, Protocol A

TI/TD = 600/----- ms

4 min 52 s

SNR<sub>WM</sub> 25.78

CNR<sub>GW</sub> 12.05



3D Rings, Protocol B

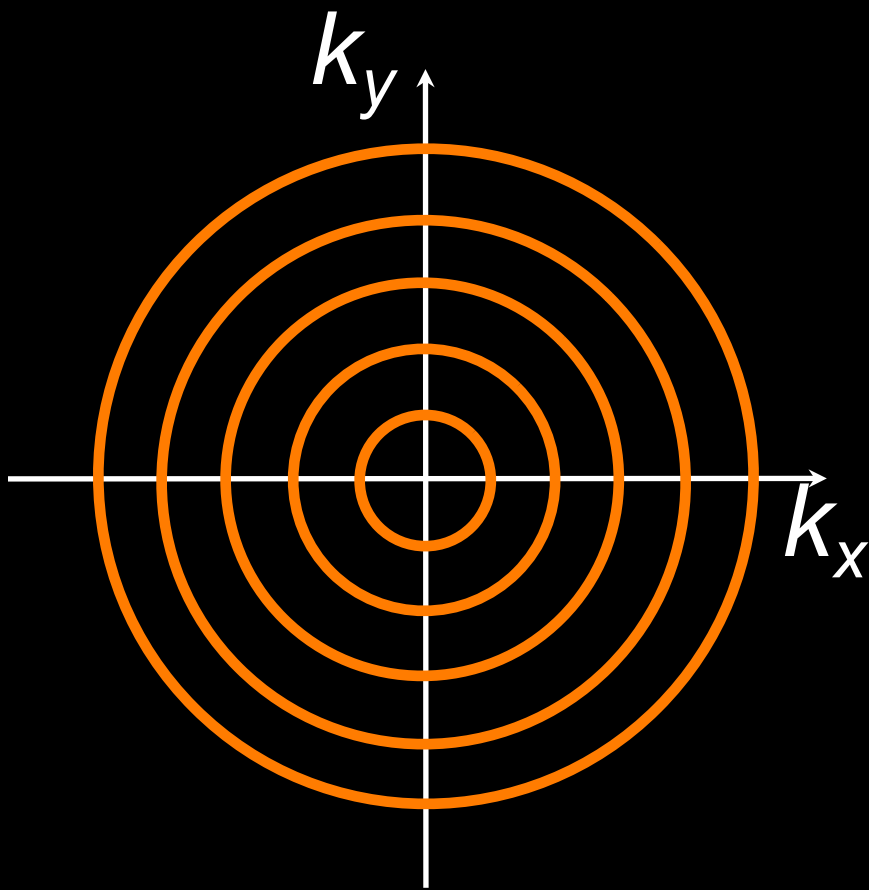
TI/TD = 900/600 ms

7 min 00 s

SNR<sub>WM</sub> 33.46

CNR<sub>GW</sub> 16.19

# Rings: Pros and Cons



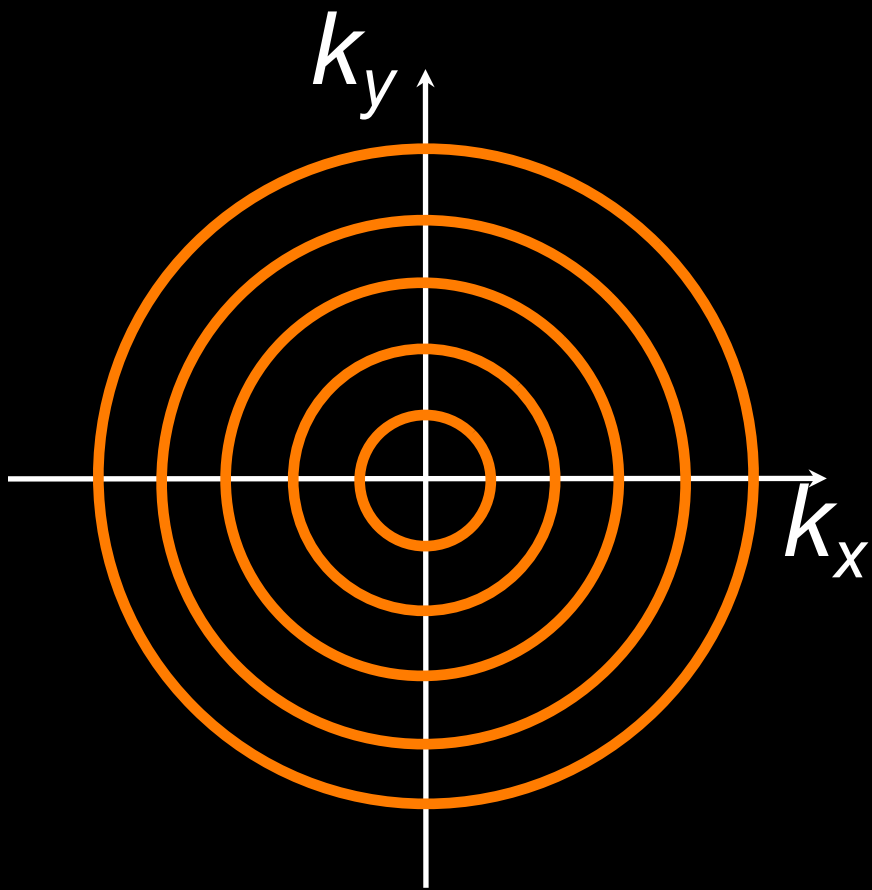
## Pros

- 2x reduction in #TRs (vs. Cartesian)
- Favorable motion/flow properties
- Robust to gradient delays
- Efficient spatial/spectral encoding
- Effective for mag-prep MRI

## Cons

- SNR penalty (non-uniform density)
- Scale-down design not optimal

# Rings: Extensions



Variable density sampling

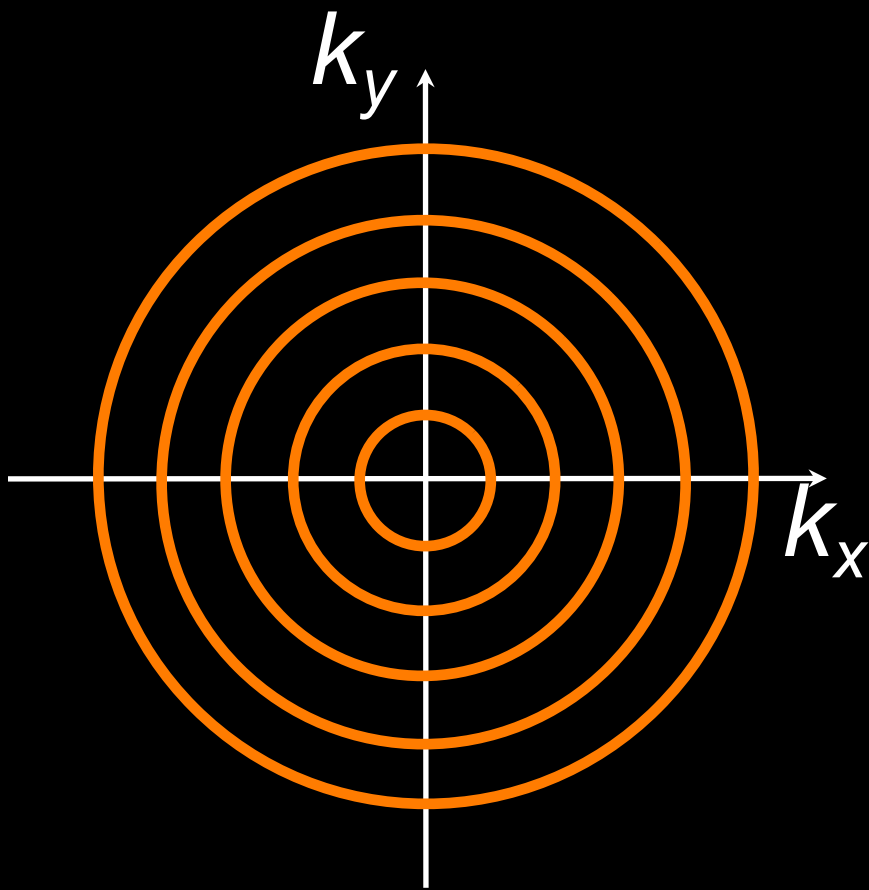
Multiple rings per TR

3D concentric cylinders

Parallel imaging

Partial Fourier

# Rings: Applications



## Fast imaging

- Cardiac MRI

## Chemical shift imaging

- Fat/water separation
- MR spectroscopic imaging

## Mag-prep imaging

- Neuro MRI
- Non-con MR angiography (MRA)
- Contrast-enhanced MRA



# Non-Cartesian Sampling

- **Benefits**

- Reduced scan time
- Robustness to motion and flow
- Short echo time

- **Applications**

- Dynamic MRI
- Real-time MRI
- Cardiovascular MRI
- Short-TE MRI

- **Challenges**

- Hardware performance
- Gradient fidelity
- Off-resonance effects
- Implementation

- **Challenges addressed**

- **On-going research**

- **Use judiciously!**

# Thanks!

- Further reading
  - Bernstein et al., Handbook of MRI Sequences
- Next week
  - Spiral, 3D Non-Cartesian trajectories
  - Gridding reconstruction
  - Trajectory measurement
  - Off-resonance correction

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