
Pulse Sequences: Rapid Gradient Echo

M229 Advanced Topics in MRI

Holden H. Wu, Ph.D.

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UCLA

*Department of Radiological Sciences
David Geffen School of Medicine at UCLA*

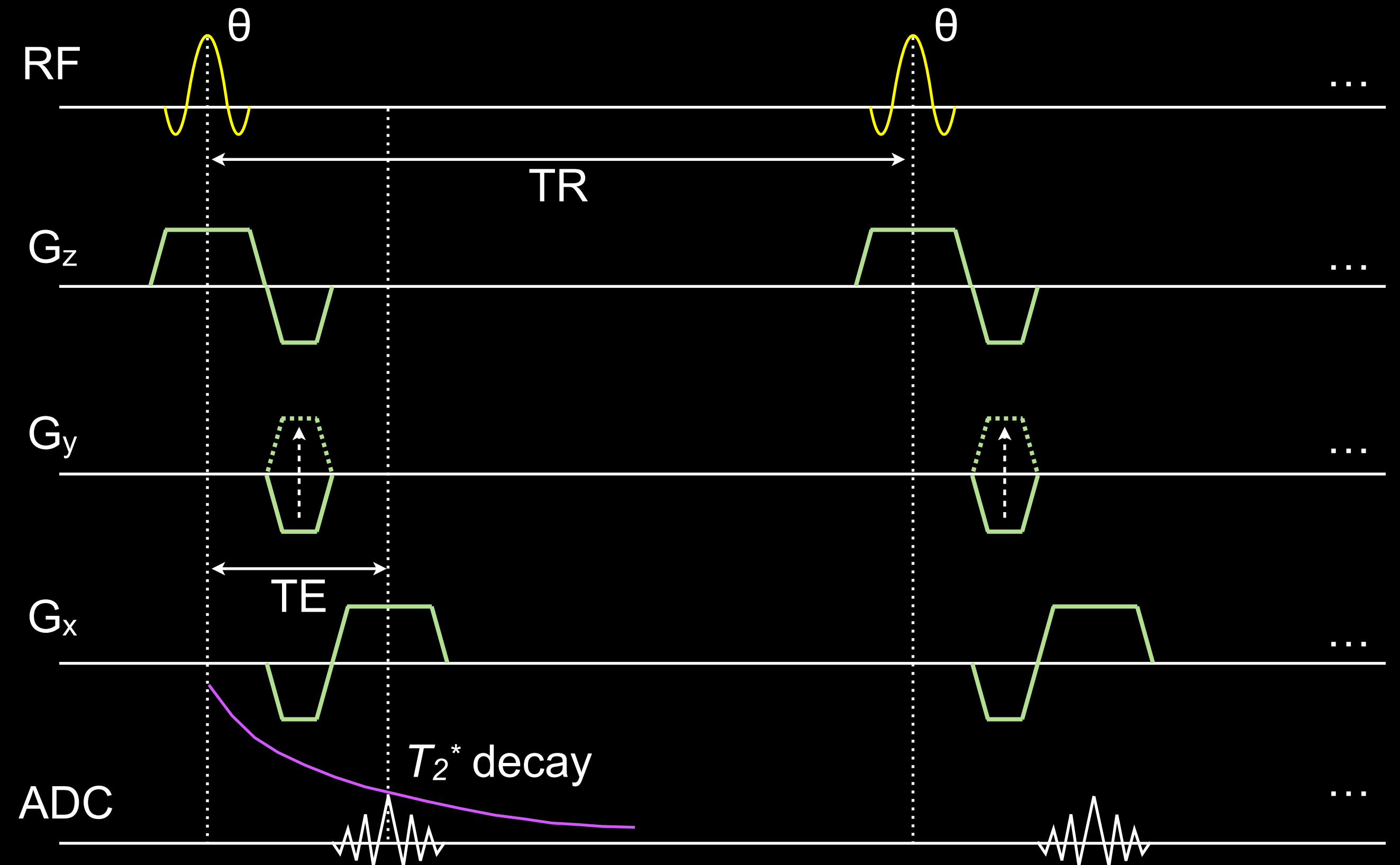
Class Business

- Textbook, email list, and class website
- Office hours
 - Instructors: Fri 10-11 am
just this time: reschedule to 4/5 Mon 11-12
 - TA for HW1: Sevgi Kafali
4/8, 4/15, 4/22 Thu 8-10 am
 - email beforehand would be helpful
- Follow Brian's Bloch sim tutorial
 - <http://www-mrsrl.stanford.edu/~brian/bloch/>

Outline

- Gradient Echo (GRE)
- Rapid Gradient Echo
 - Balanced SSFP
 - Gradient-spoiled GRE
 - RF-spoiled GRE
- Comparison
- Extensions and Variations
- Applications

Gradient Echo



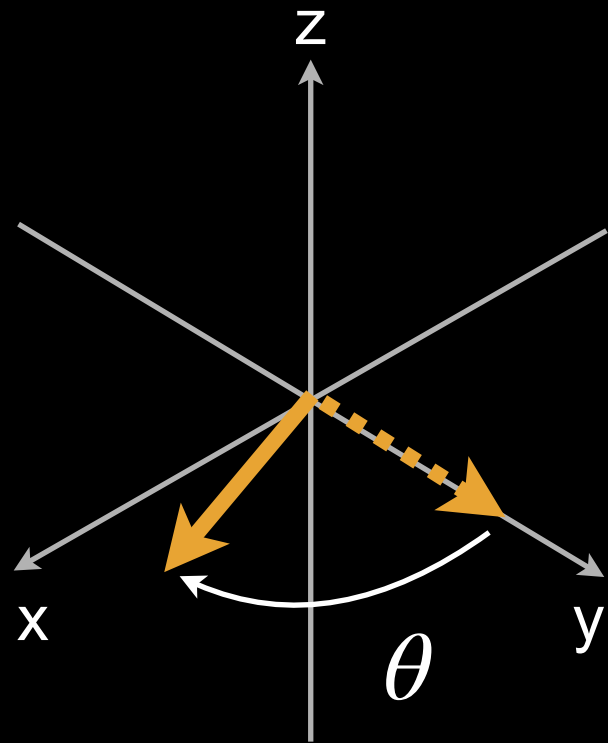
Gradient Echo

- Gradient reversal on the readout axis forms the echo (vs. RF spin echo)
- A.k.a. gradient-recalled echo, gradient-refocused echo, field echo
- Flip angle θ typically $< 90^\circ$
- M_{xy} has T_2^* instead of T_2 decay
- Advantageous for fast 3D imaging

Gradient Echo

- Basic steps
 - RF excitation (flip angle θ and phase ϕ)
 - Free precession (from G and ΔB)
 - T_1 and T_2 (or T_2^*) relaxation
- Steady state
 - “Dynamic equilibrium”
 - Established after initial transient state
 - M_z and M_{xy} remain the same, TR to TR
 - Need to meet certain conditions

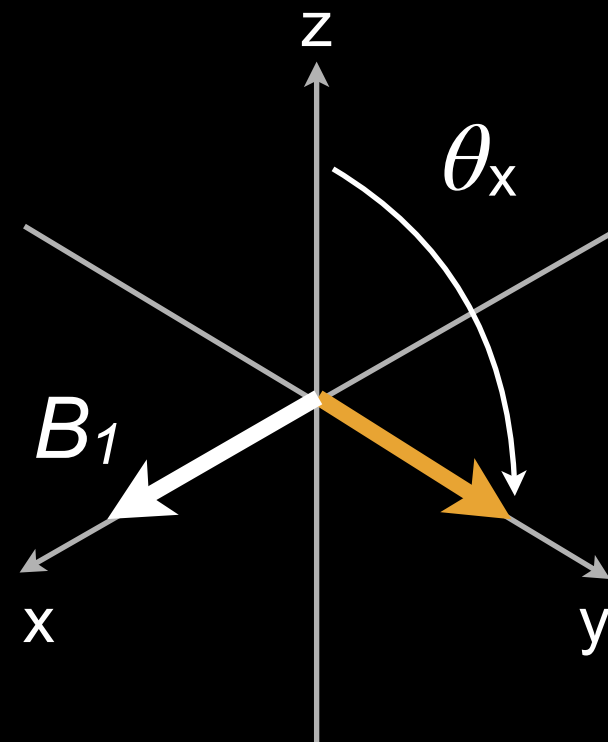
Notations



Free Precession (FP)

$$R_z(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Excitation



$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

rotating frame at ω_0

Notations

Relaxation + Free Precession:

$$M(t) = \begin{bmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{bmatrix} R_z(\Delta\omega t) M(0) + \begin{bmatrix} 0 \\ 0 \\ M_0(1 - e^{-t/T_1}) \end{bmatrix}$$

$AM(0) + B$

Gradient Echo

- When $TR > 5 \cdot T_2^*$, M_{xy} naturally “spoiled”
 - *i.e.*, consider $M_{xy} = 0$ at the end of each TR

Gradient Echo

Steady-state signal equation:

$$M_{xy,ss}(\text{TE}) = \frac{M_0 \sin \theta (1 - E_1)}{1 - \cos \theta E_1} e^{-\text{TE}/T_2^*}$$

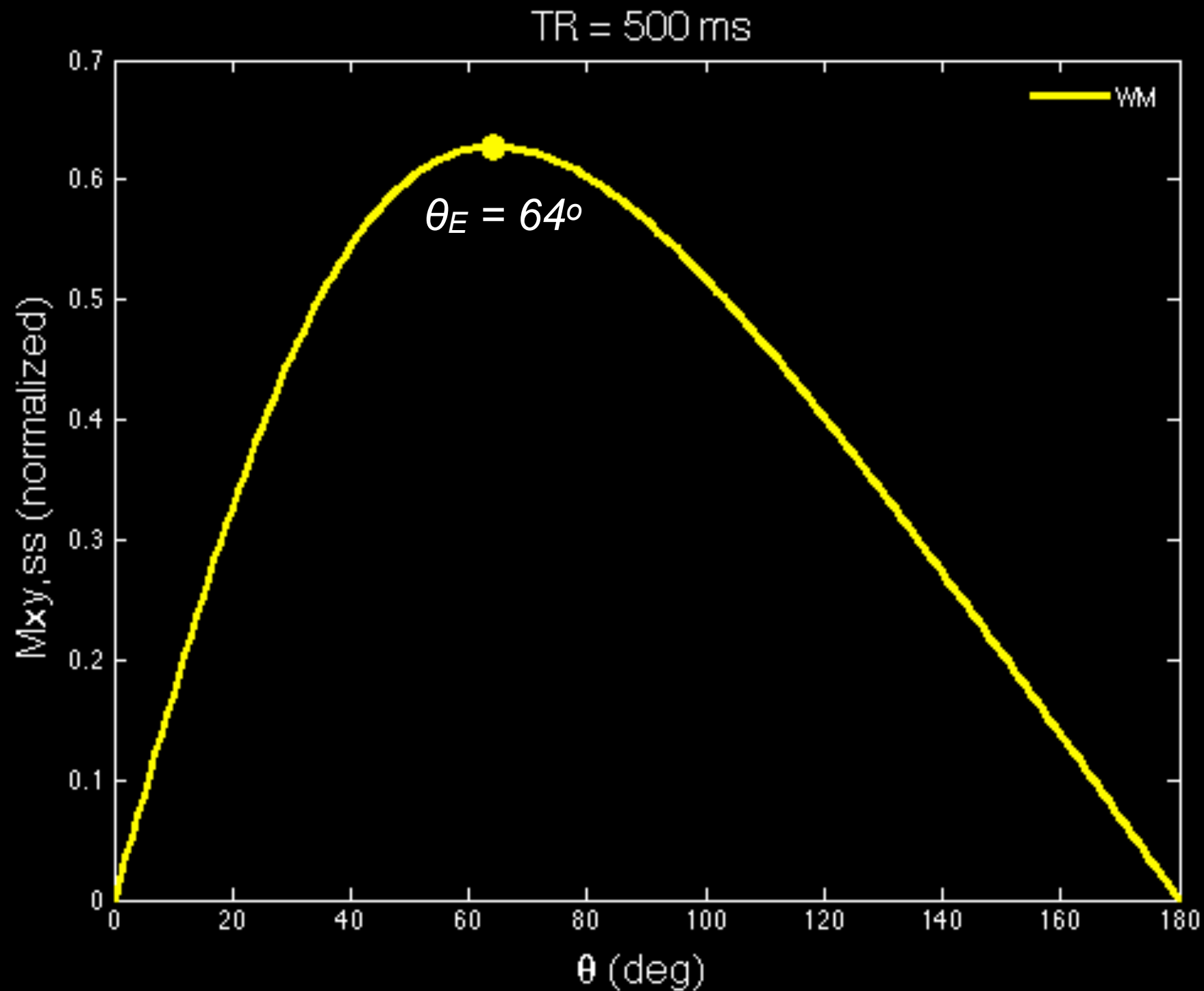
Ernst angle:

$$\theta_E = \cos^{-1}(E_1)$$

$$E_1 = e^{-\text{TR}/T_1}$$

Gradient Echo

Ernst angle:



WM $T_1 = 600$ ms, $T_2 = 80$ ms

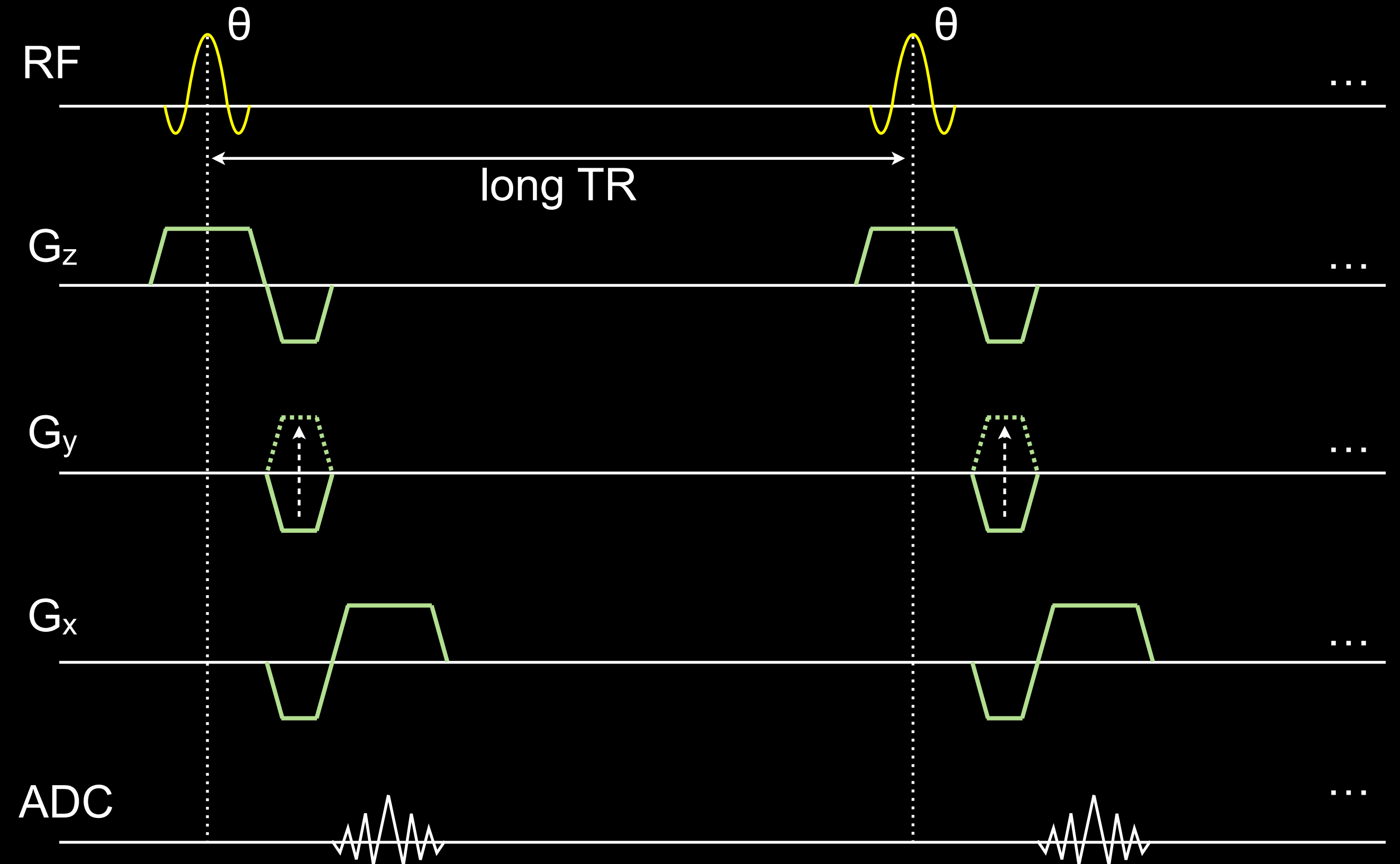
Gradient Echo

- T_1 -weighted image contrast
 - M_{xy} gone at end of each TR
 - TE controls T_2^* weighting
- Typical $T_2^* \sim 50$ ms
 - need TR ~ 250 ms for “natural” spoiling
- Reduce TR and maintain T1w contrast?
 - rapid GRE with appropriate spoiling

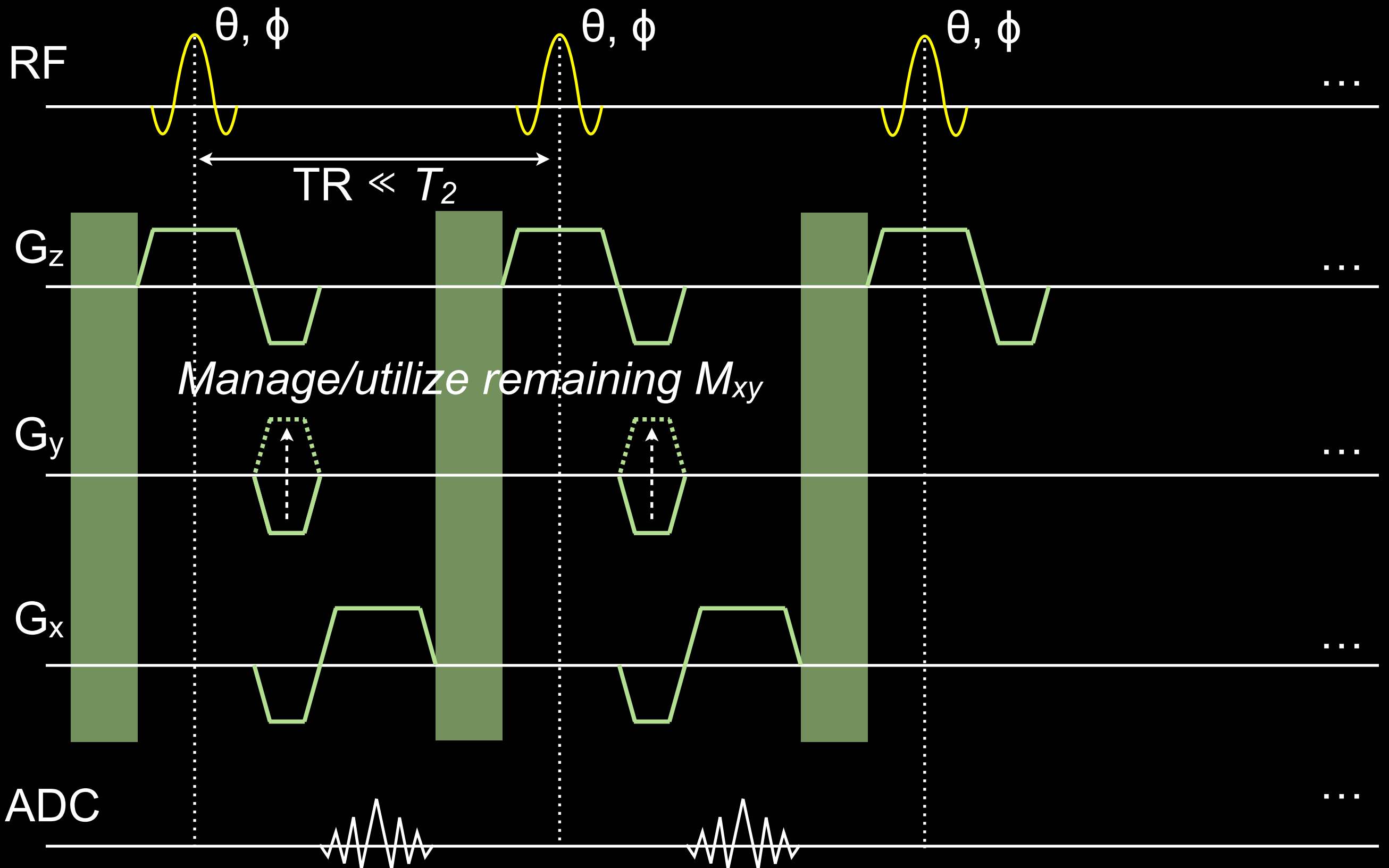
Rapid Gradient Echo

- Rapid imaging with $TR \ll T_2 < T_1$
- Steady state
 - Involves a mixture of M_z and M_{xy}
 - Necessary and sufficient conditions:
 1. Constant RF flip angle θ
 2. Constant TR
 3. Constant dephasing β between RF pulses
 4. RF phase $\phi_n = a + bn + cn^2$

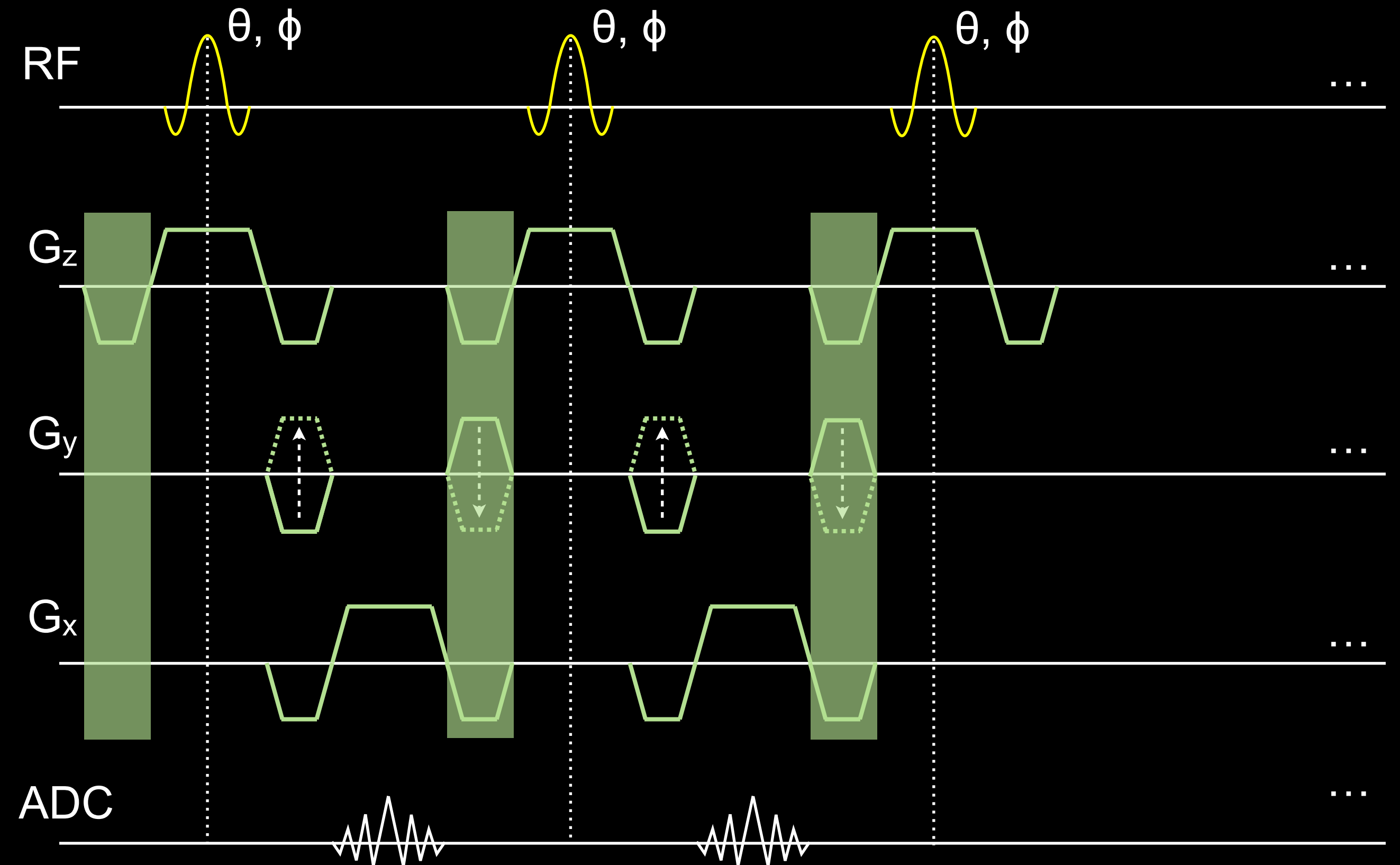
Gradient Echo



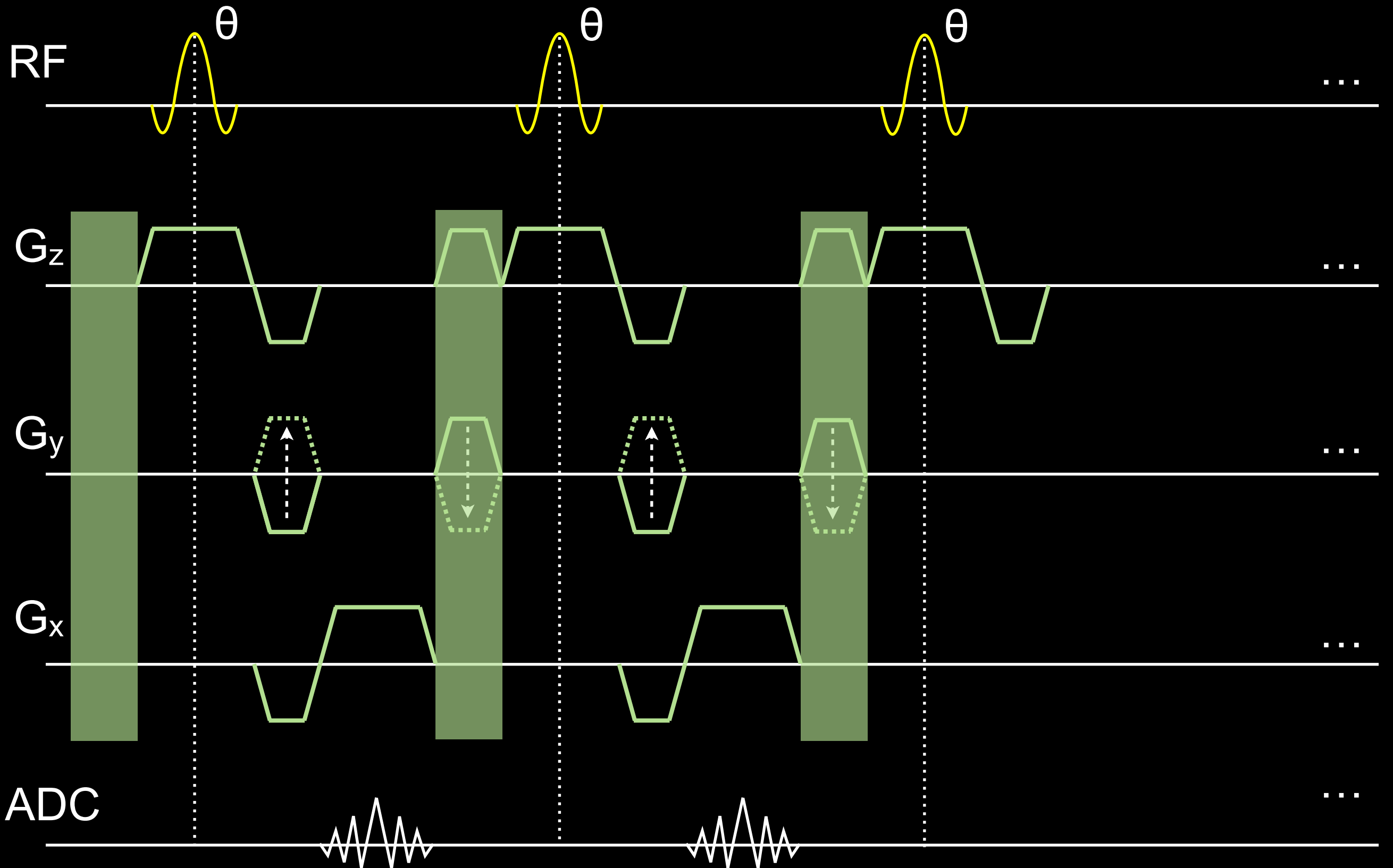
Rapid Gradient Echo



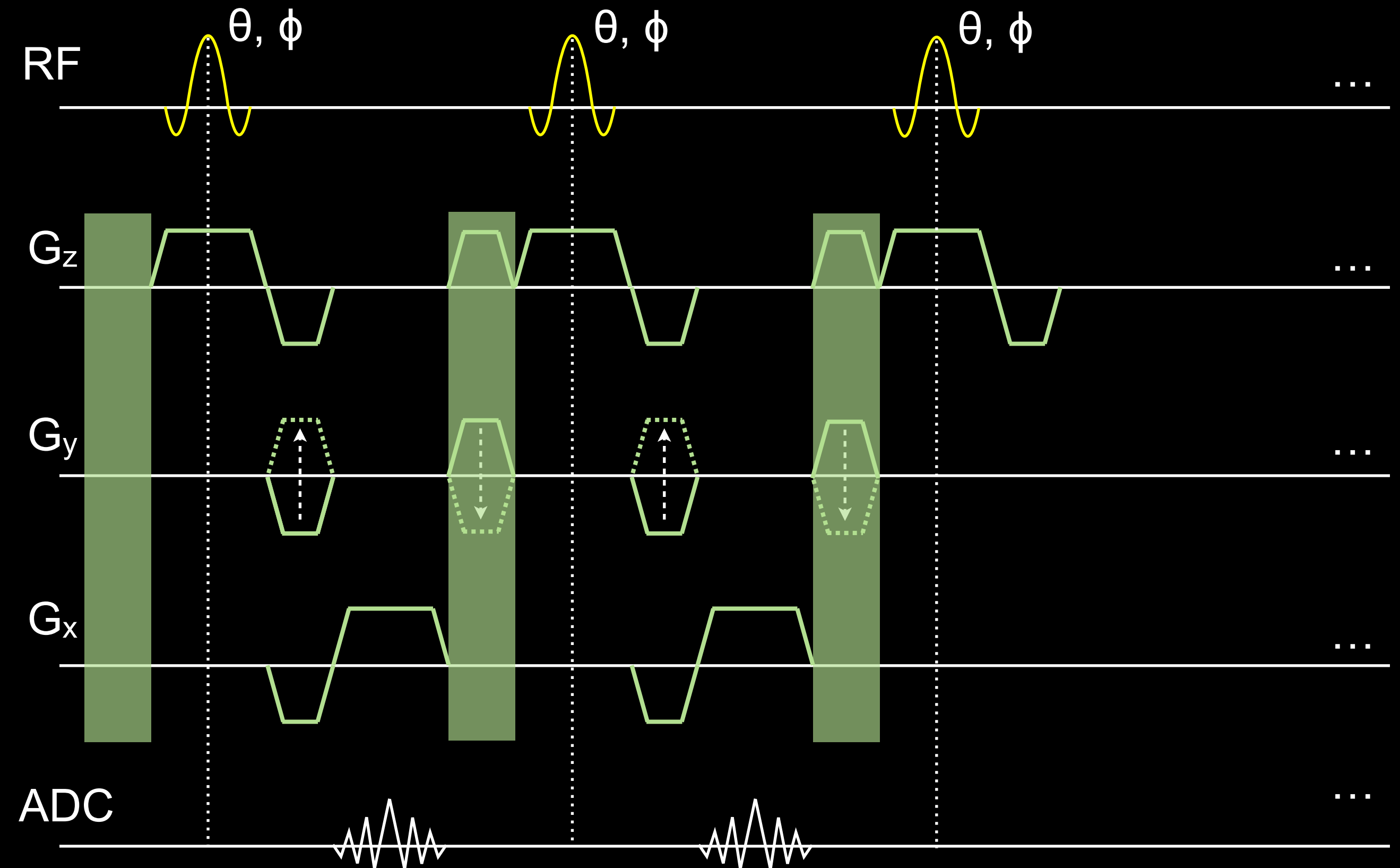
Balanced SSFP



Gradient-spoiled GRE



Gradient & RF-spoiled GRE

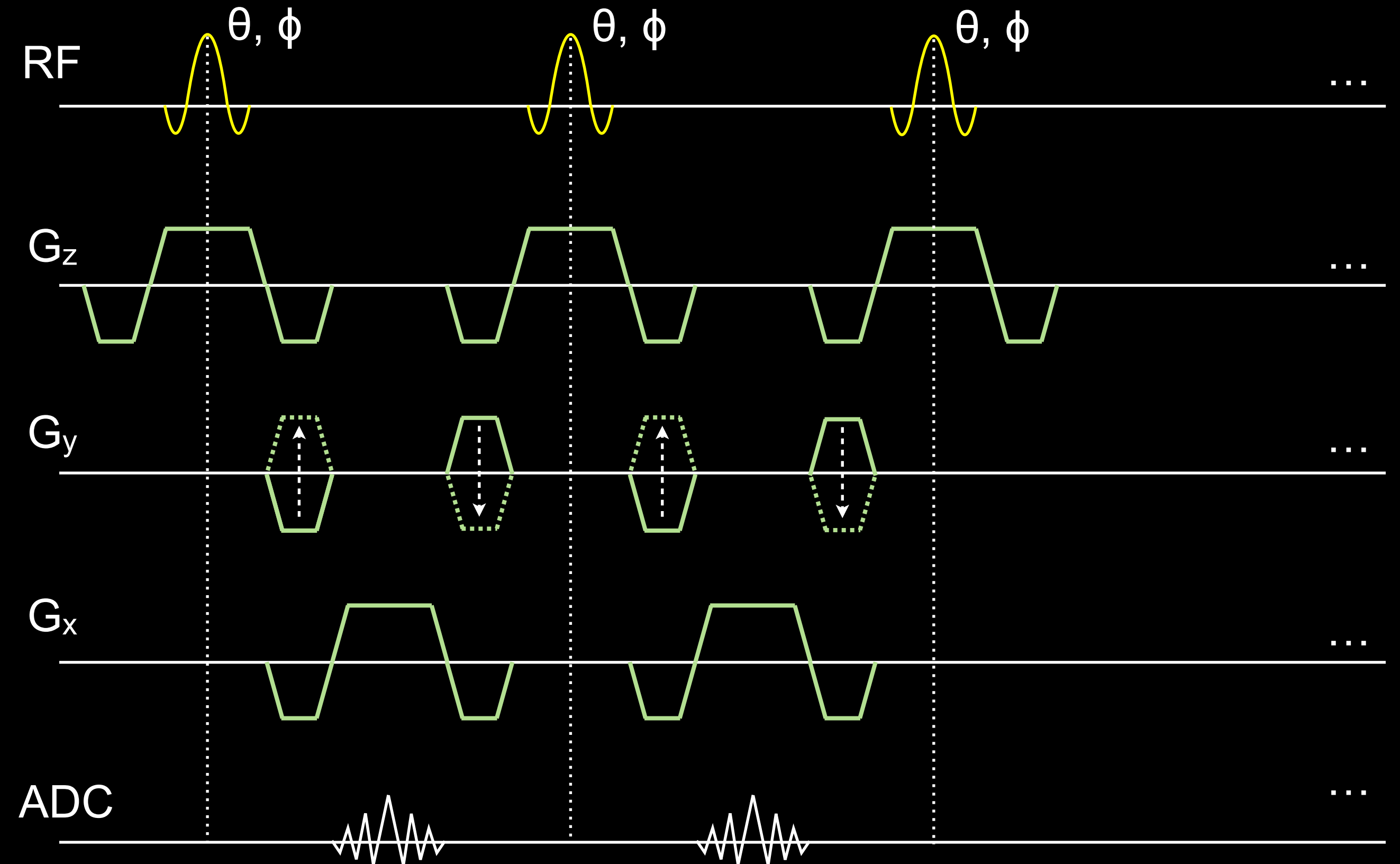


Rapid Gradient Echo

General terminology		Siemens	GE	Philips
Balanced SSFP	bSSFP	TrueFISP	FIESTA	Balanced FFE
Gradient-spoiled GRE	SSFP-FID	FISP	GRASS	FFE
	SSFP-Echo	PSIF	SSFP	T2-FFE
Gradient and RF-spoiled GRE	Spoiled GRE	FLASH	SPGR	T1-FFE

*cf. Table 14.1, Handbook of MRI Pulse Sequences
cf. "MRI Acronyms", Siemens Healthcare*

Balanced SSFP



Balanced SSFP

- All gradients are balanced
 - β from $G_x, G_y, G_z = 0$
 - β only comes from ΔB
- Typically use $\phi_n = n \cdot \pi$ ($\Delta\phi = \pi$)
- Typically use $TE = TR/2$
 - M_{xy} actually has T_2 (not T_2^*) decay¹
- Contrast depends on T_1 and T_2

¹Ganter C, MRM 2006; 56:687-691

Balanced SSFP

Steady-state signal equation ($\beta = 0$):

$$M_{xy,ss}(\text{TE}) = M_0 \sin \theta \frac{1 - E_1}{1 - (E_1 - E_2) \cos \theta - E_1 E_2} \sqrt{E_2}$$

$$E_1 = e^{-\text{TR}/T_1}$$

$$E_2 = e^{-\text{TR}/T_2}$$

$$\sqrt{E_2} = e^{-\text{TE}/T_2}$$

Balanced SSFP

Steady-state signal equation ($\beta = 0$):

If TR (3-5 ms) $\ll T_2$, $E_1 \sim 1 - TR/T_1$ and $E_2 \sim 1 - TR/T_2$:

$$M_{xy,ss}(\text{TE}) = \frac{M_0 \sin \theta}{(T_1/T_2)(1 - \cos \theta) + (1 + \cos \theta)} \sqrt{E_2}$$

T_2/T_1 contrast weighting

$$\theta_{max} = \arccos\left(\frac{T_1 - T_2}{T_1 + T_2}\right) \quad M_{xy,ss}(\theta_{max}) \sim \frac{M_0}{2} \sqrt{\frac{T_2}{T_1}}$$

When $T_1 = T_2$, $\theta_{max} = 90^\circ$, $M_{xy,ss} \sim 0.5 M_0$!

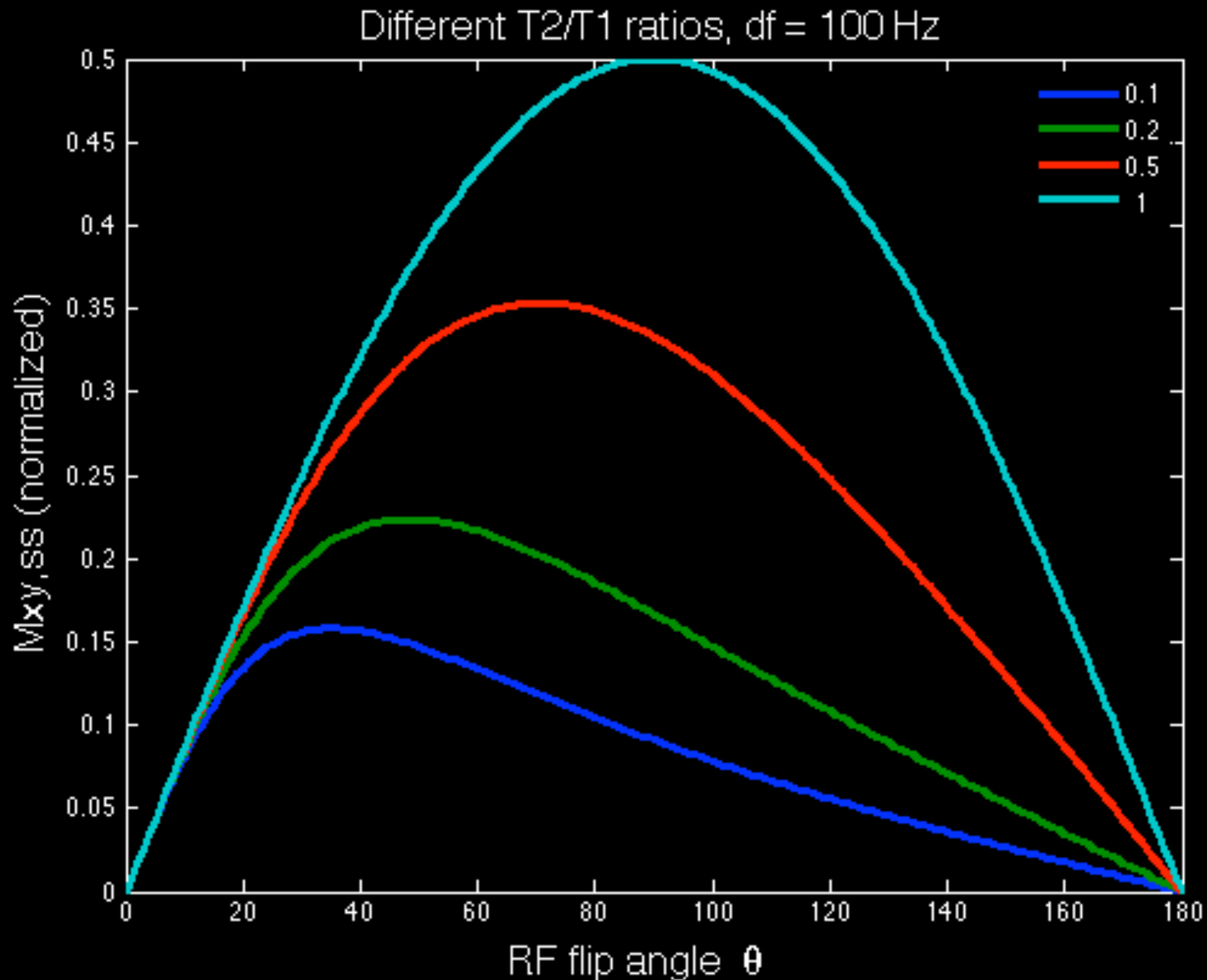
Balanced SSFP

SS signal as a function of flip angle:

$TR = 5 \text{ ms}$

$\Delta\phi = 0$

$\beta = \pi$



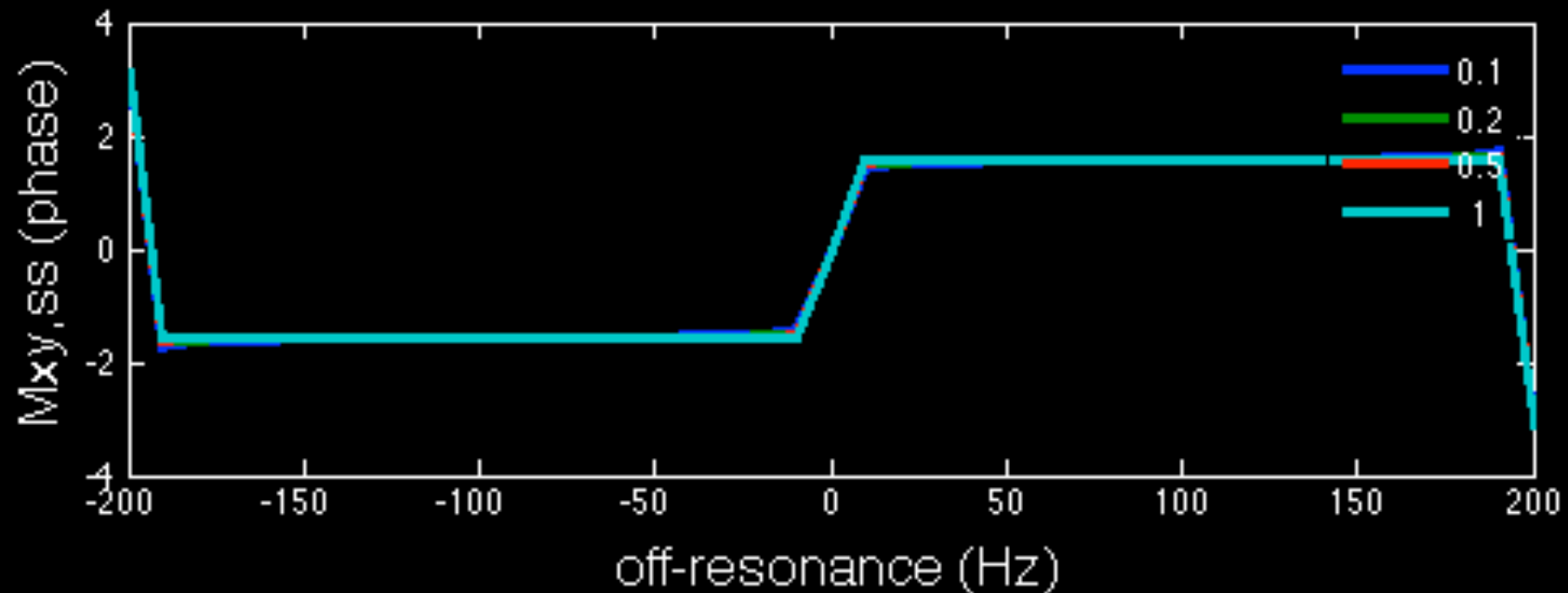
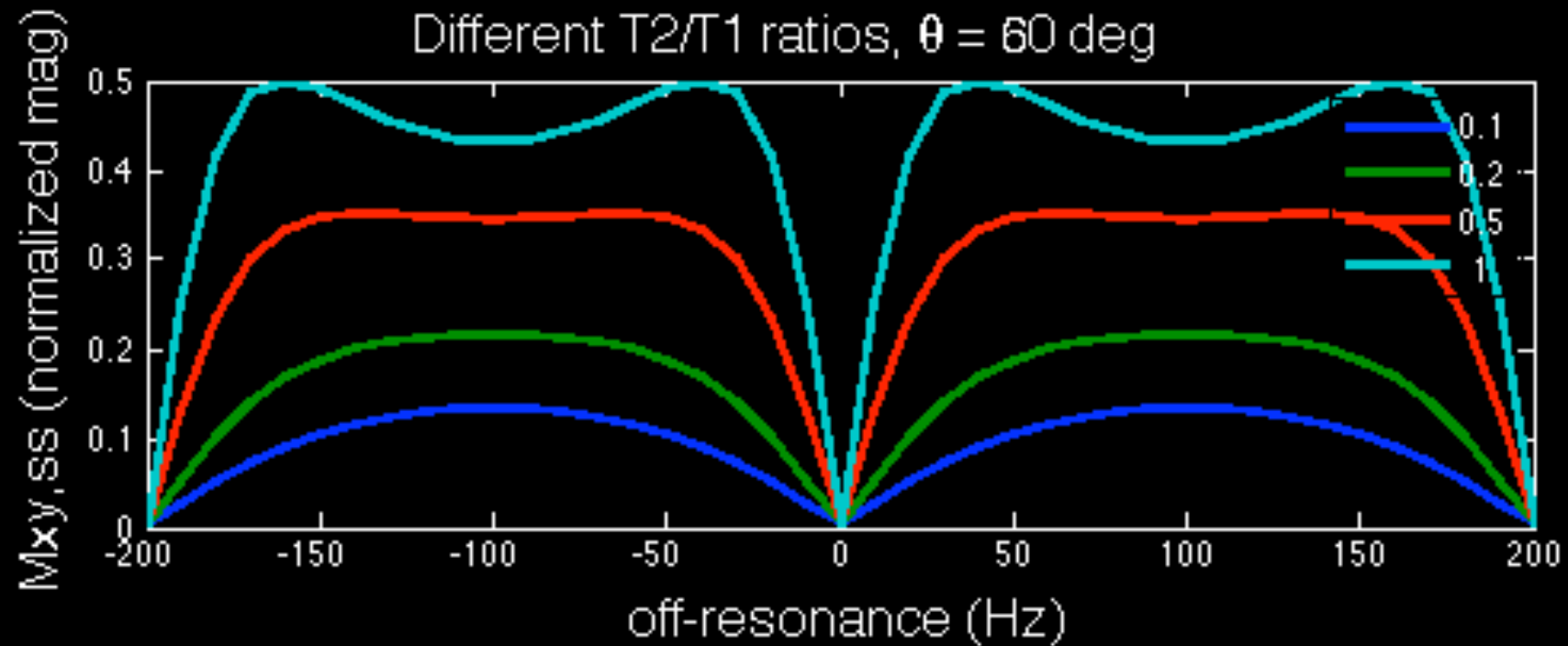
$T_1 = 1000 \text{ ms}, T_2 = 100, 200, 500, 1000 \text{ ms}$

Balanced SSFP

SS signal as a function of off-resonance:

TR = 5 ms

$\Delta\phi = 0$



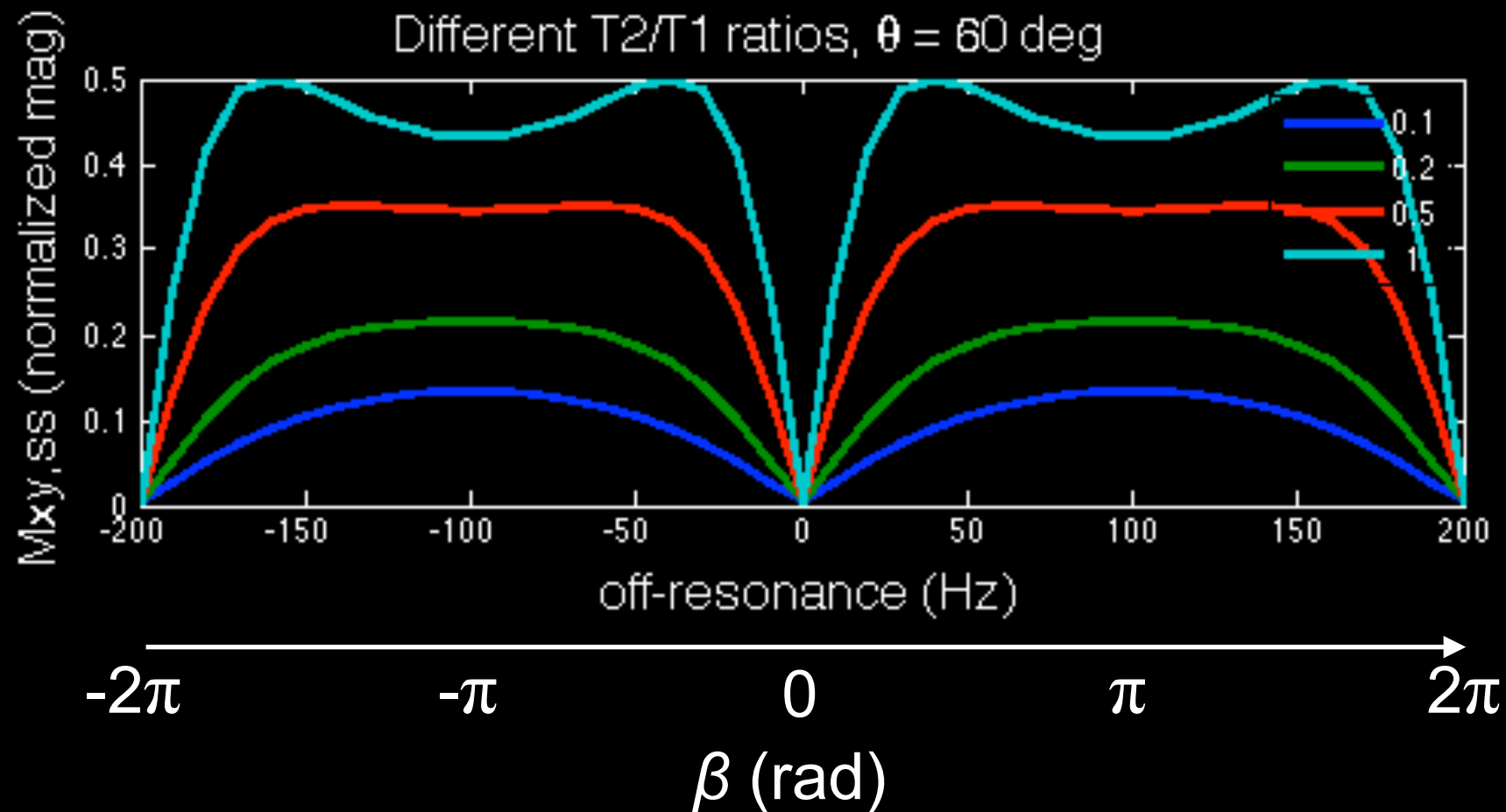
$T_1 = 1000$ ms, $T_2 = 100, 200, 500, 1000$ ms

Balanced SSFP

SS signal as a function of off-resonance:

TR = 5 ms

$\Delta\phi = 0$



Recall $\beta = 2\pi \Delta f \times \text{TR}$ and $\Delta f = \gamma B / 2\pi$
 $\beta = \pm\pi$ corresponds to $\Delta f = \pm 1/(2 \text{TR})$ Hz

TR = 5 ms: $\Delta f = \pm 100$ Hz

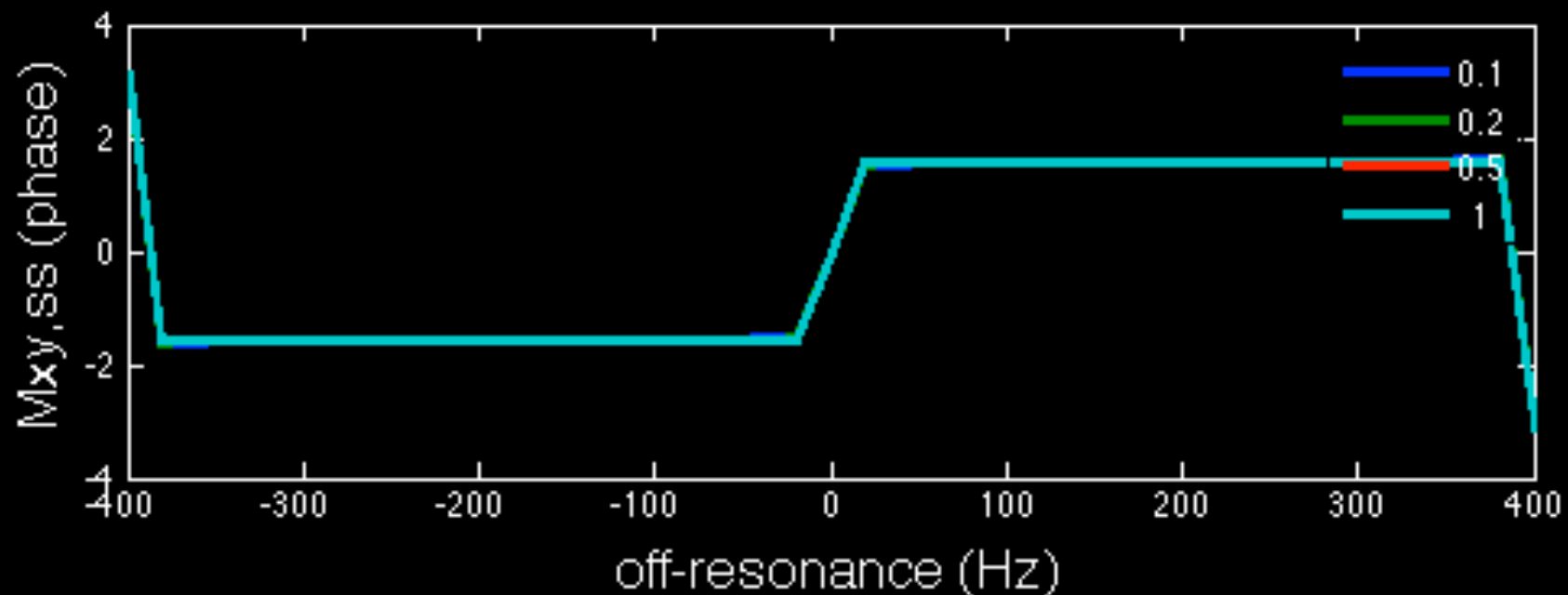
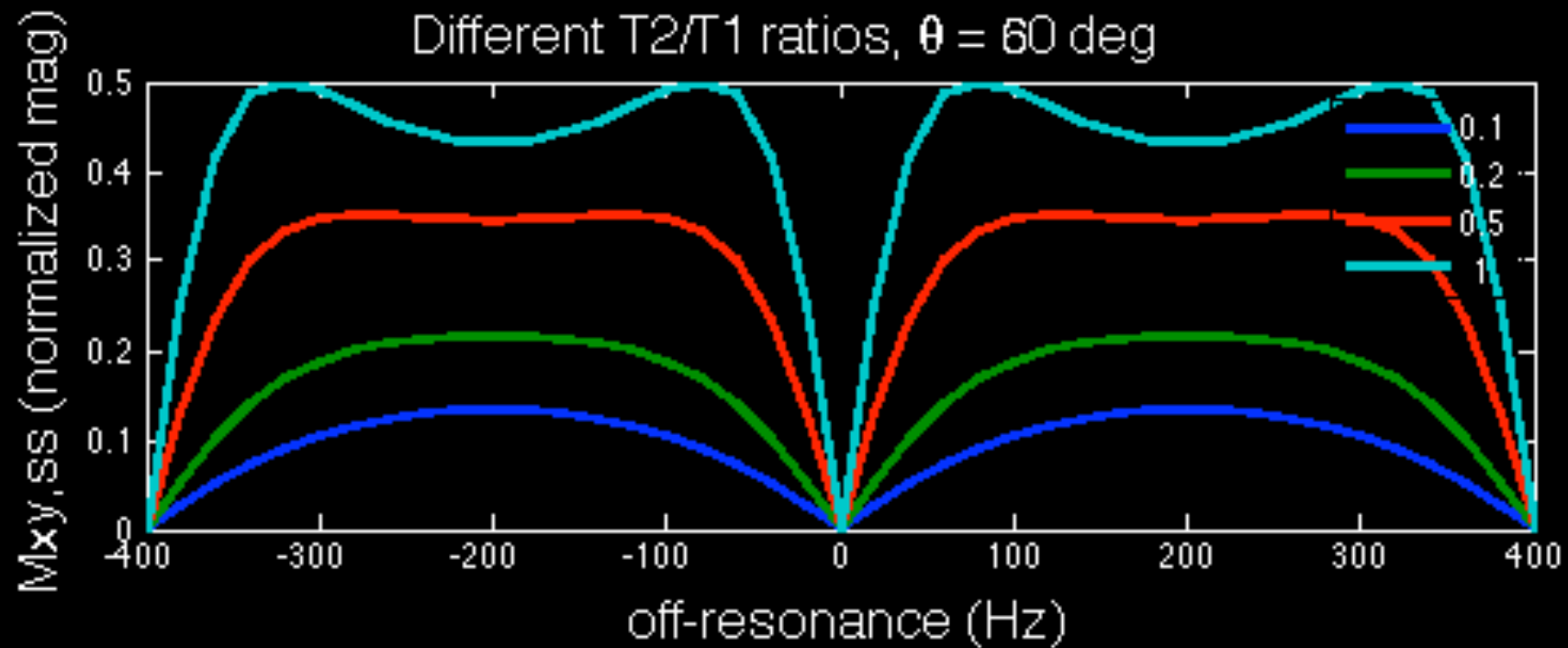
TR = 2.5 ms: $\Delta f = \pm 200$ Hz

Balanced SSFP

SS signal as a function of off-resonance:

TR = 2.5 ms

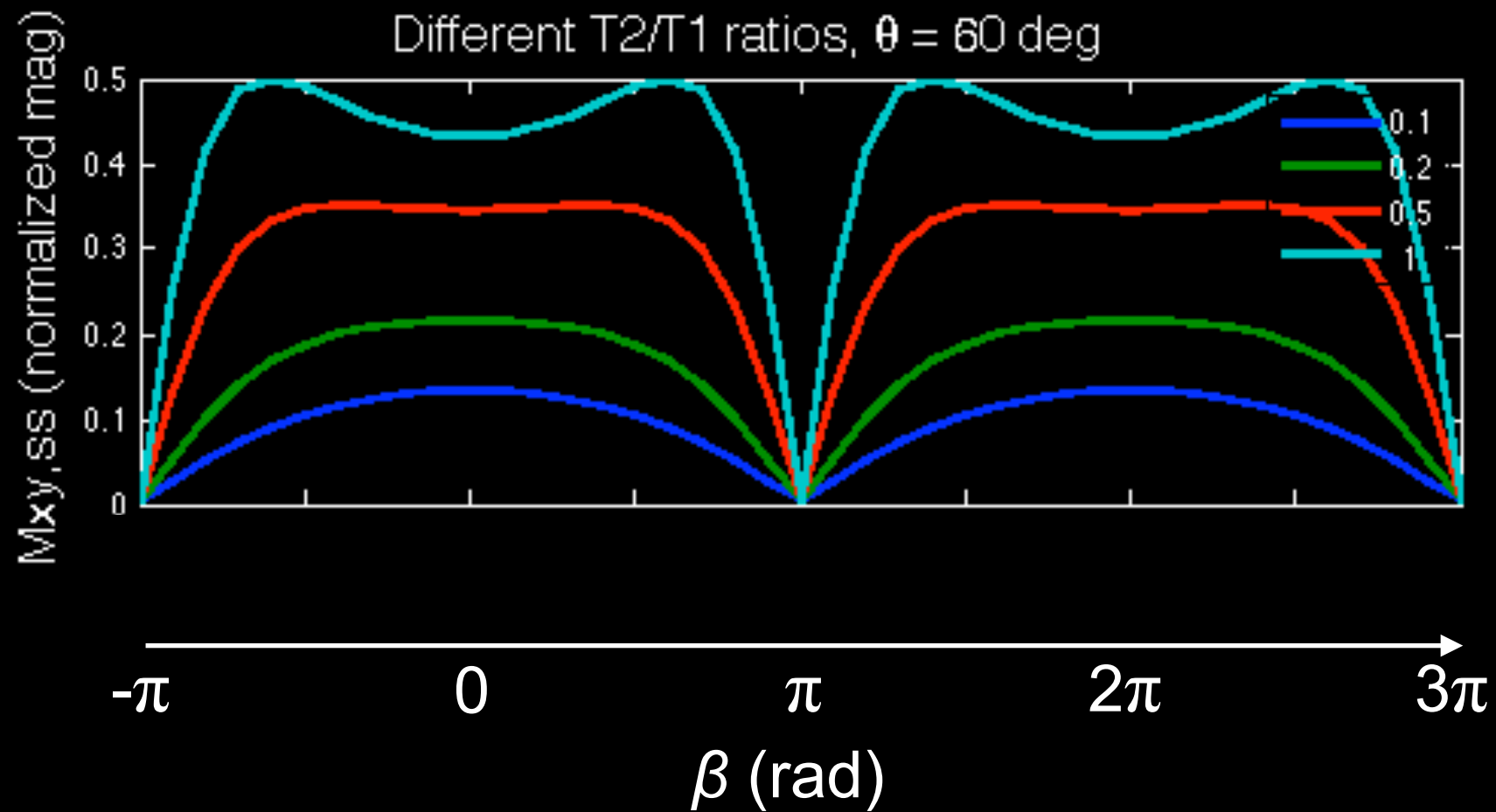
$\Delta\phi = 0$



$T_1 = 1000$ ms, $T_2 = 100, 200, 500, 1000$ ms

Balanced SSFP

SS signal as a function of off-resonance:



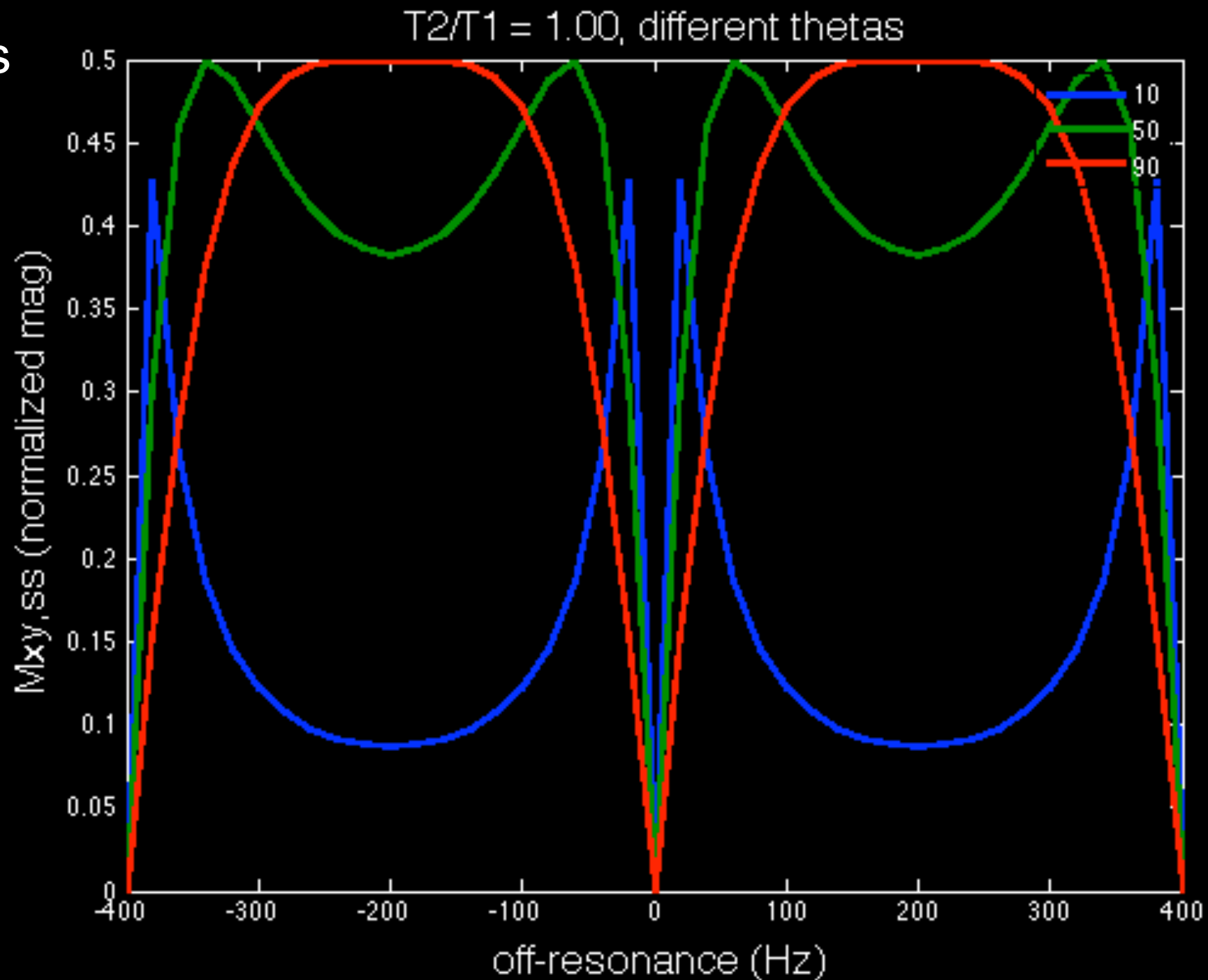
$\Delta\phi$ can shift the off-resonance response

Balanced SSFP

SS signal as a function of off-resonance:

TR = 2.5 ms

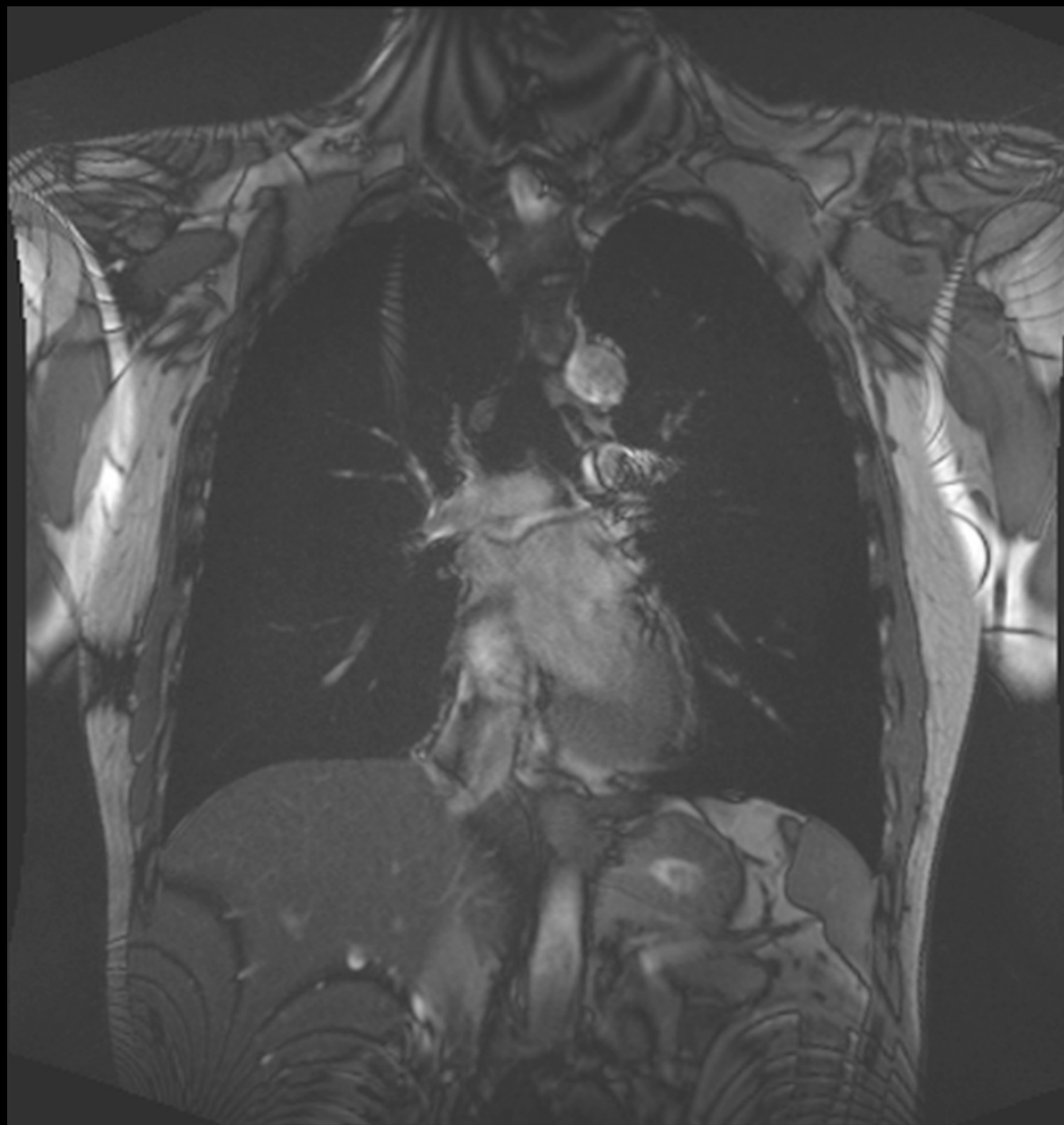
$\Delta\phi = 0$



$T_1 = 1000\text{ ms}$, $T_2 = 1000\text{ ms}$

Balanced SSFP

Banding artifacts at 3 T:

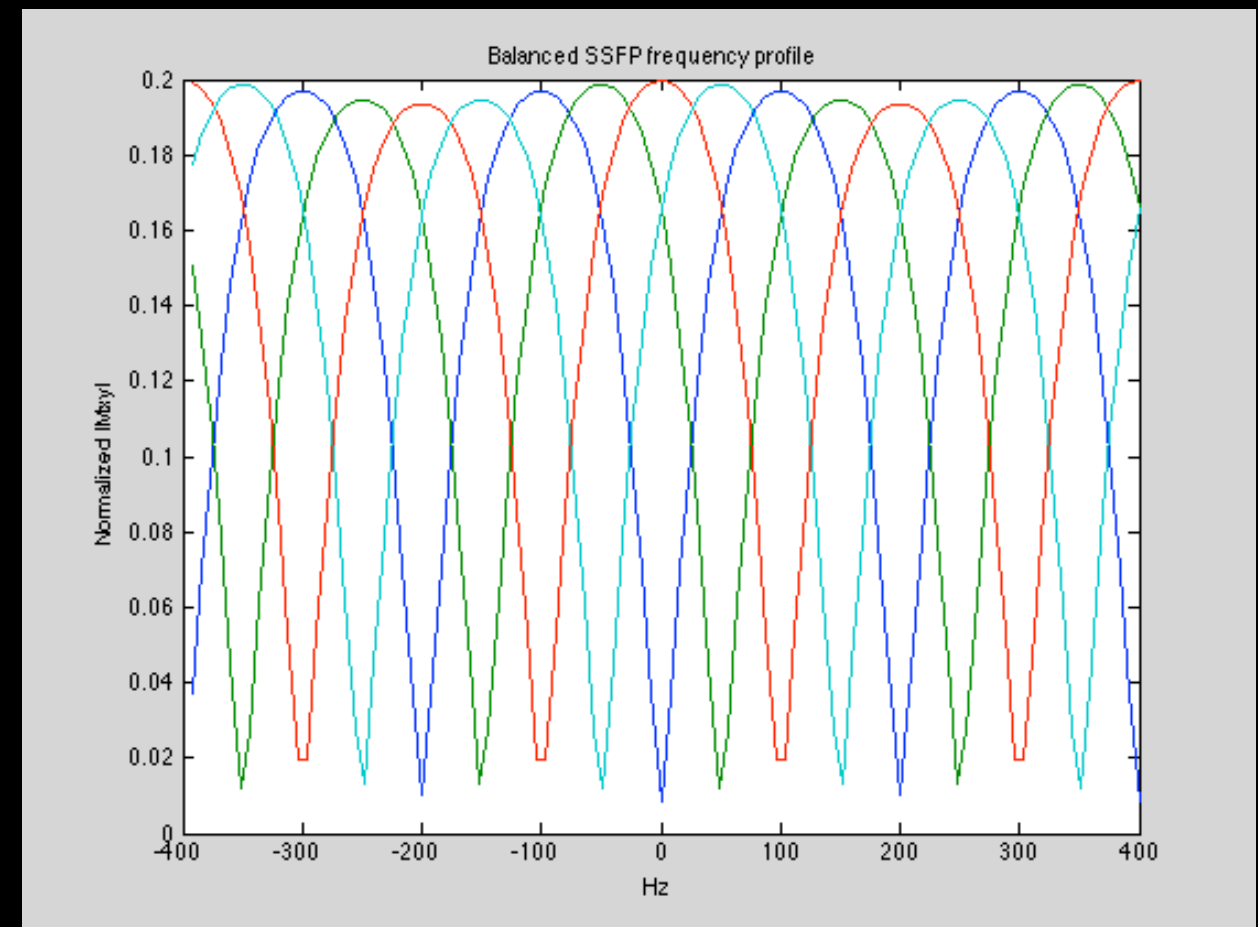
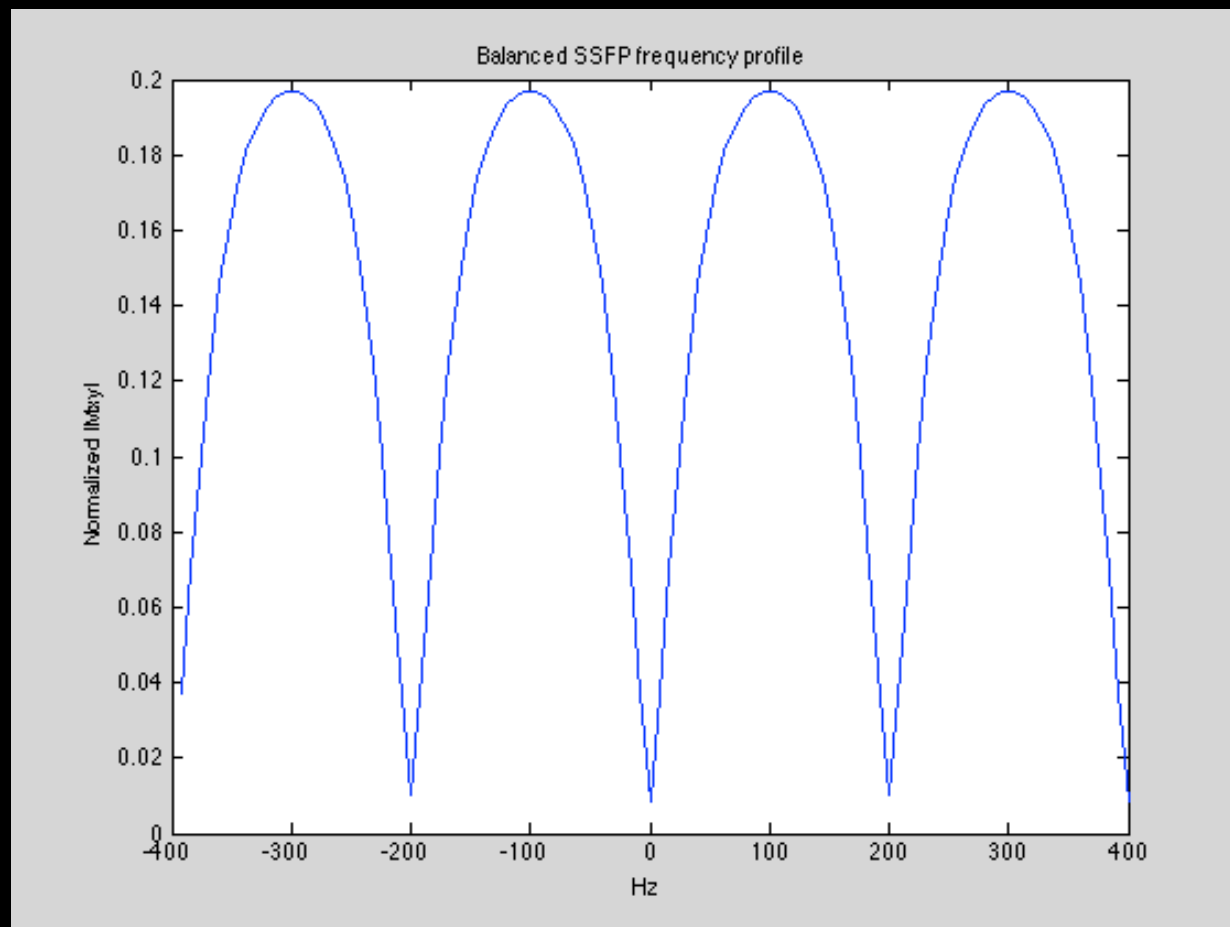


Balanced SSFP

- Banding artifacts
 - bSSFP has freq-dep null bands
 - spatially varying field inhomogeneity
 - shim not perfect
 - worse at high field (e.g., 3 T vs 1.5 T)
- Mitigating banding artifacts
 - reduce TR
 - custom shim; shift center freq
 - phase cycling

Balanced SSFP

- Removing banding artifacts
 - Multi-acquisition bSSFP (phase cycled)
 - Image reconstruction (rSoS, MIP, etc.)



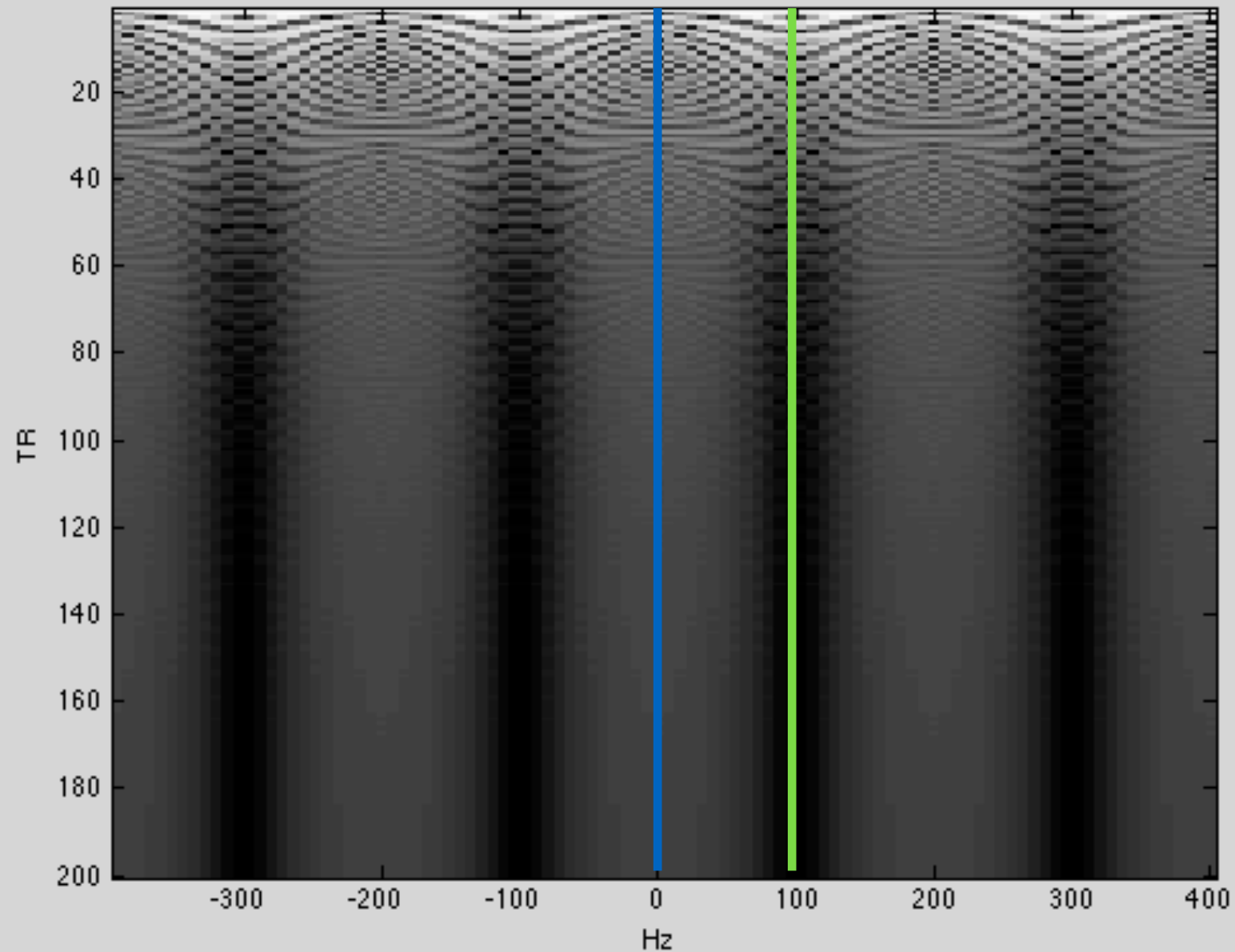
Balanced SSFP

Transition to steady state:

$TR = 5 \text{ ms}$

$\Delta\phi = \pi$

$\theta = 60^\circ$



$T_1 = 600 \text{ ms}, T_2 = 100 \text{ ms}$

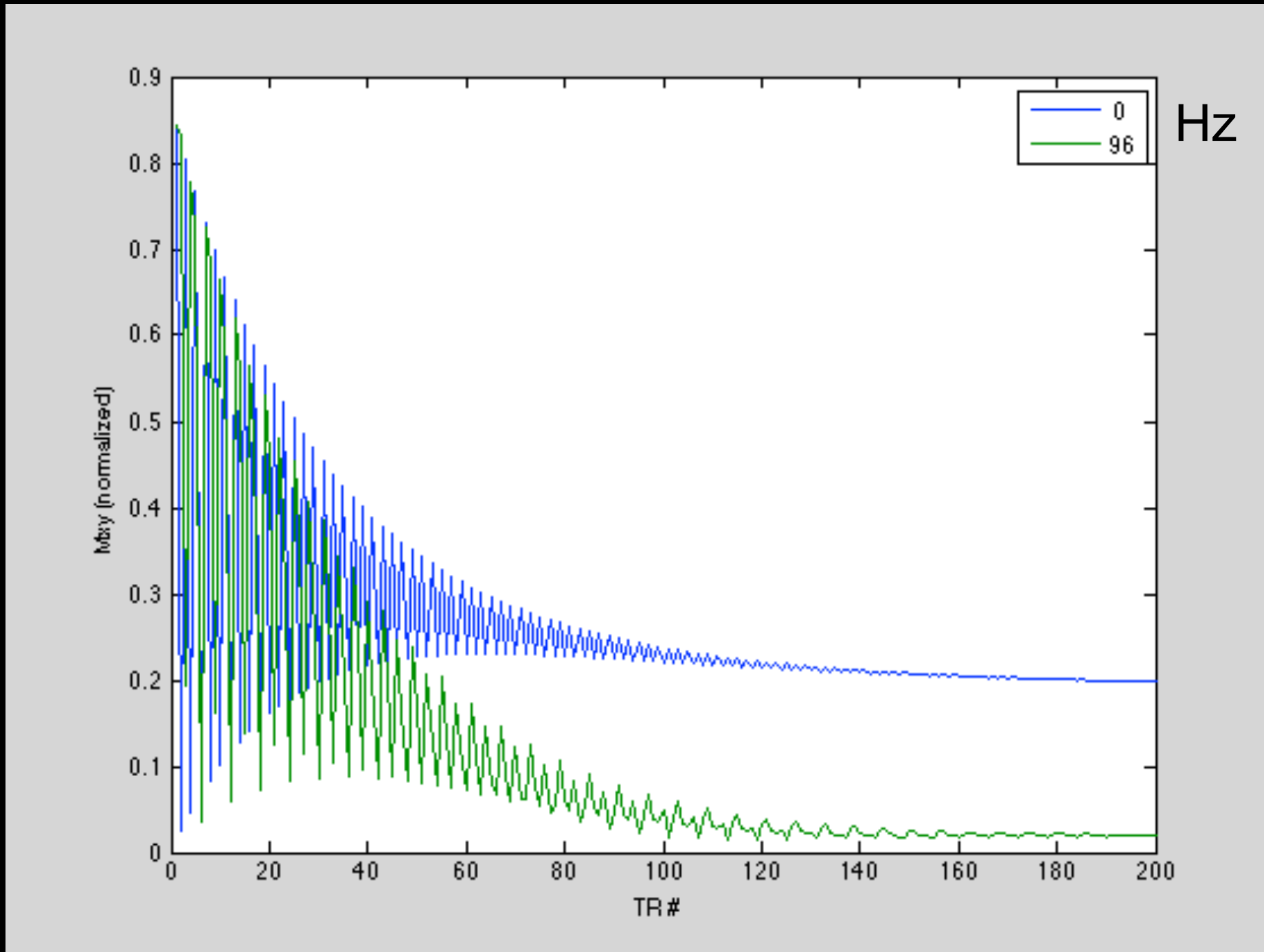
Balanced SSFP

Transition to steady state:

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^\circ$



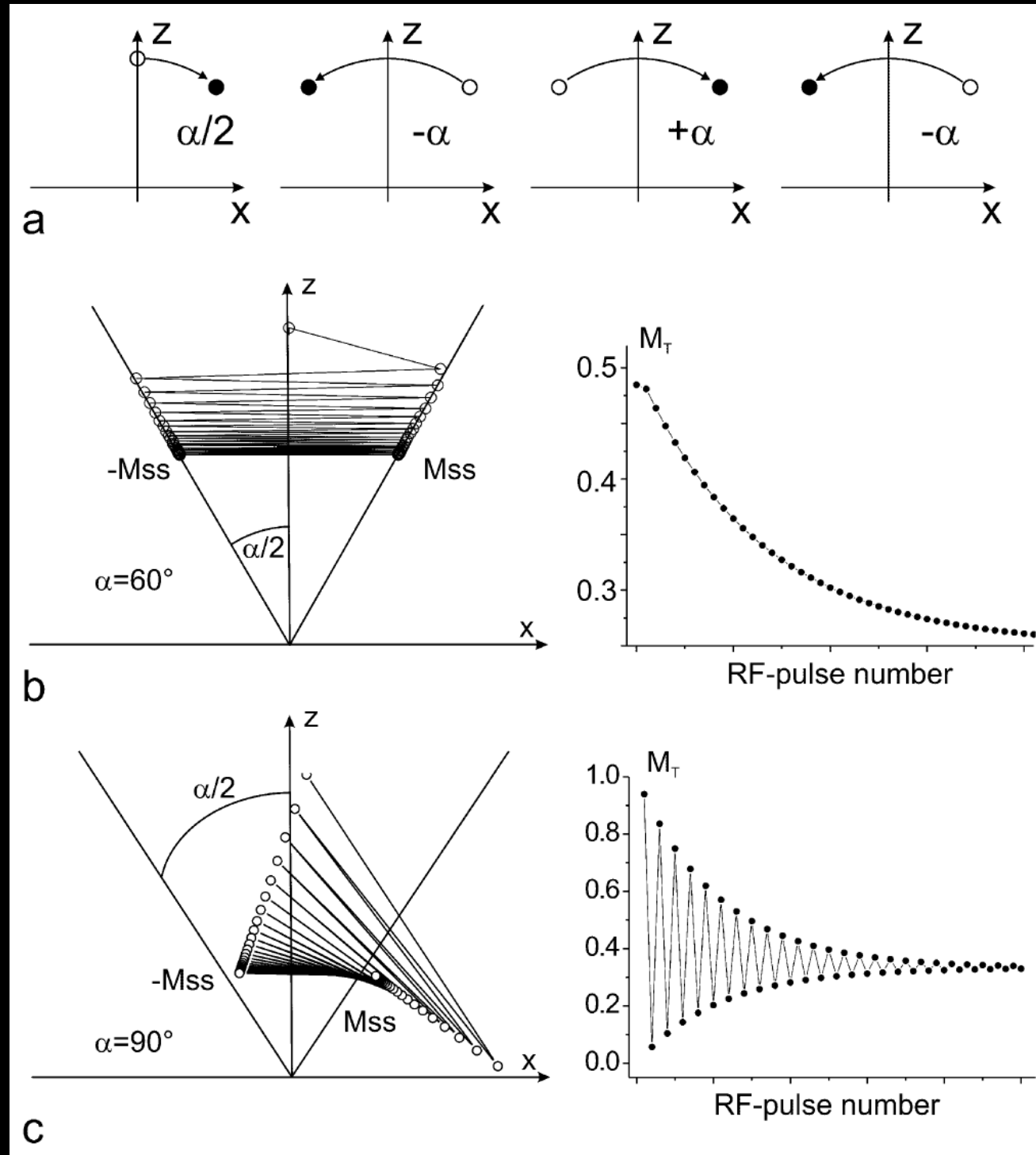
$T_1 = 600\text{ ms}, T_2 = 100\text{ ms}$

Balanced SSFP

- Transient state
 - approach to steady state can take $5 \cdot T_1$
 - depends on sequence and tissue params
 - longer transition for larger θ
 - artifacts and variable image contrast
- Catalyzation pulses
 - achieve smoother transition to steady state
 - simple approach: $\theta/2$ - $TR/2$ preparation
 - other sophisticated designs

Balanced SSFP

Transition to steady state ($\theta/2$ -TR/2 prep):



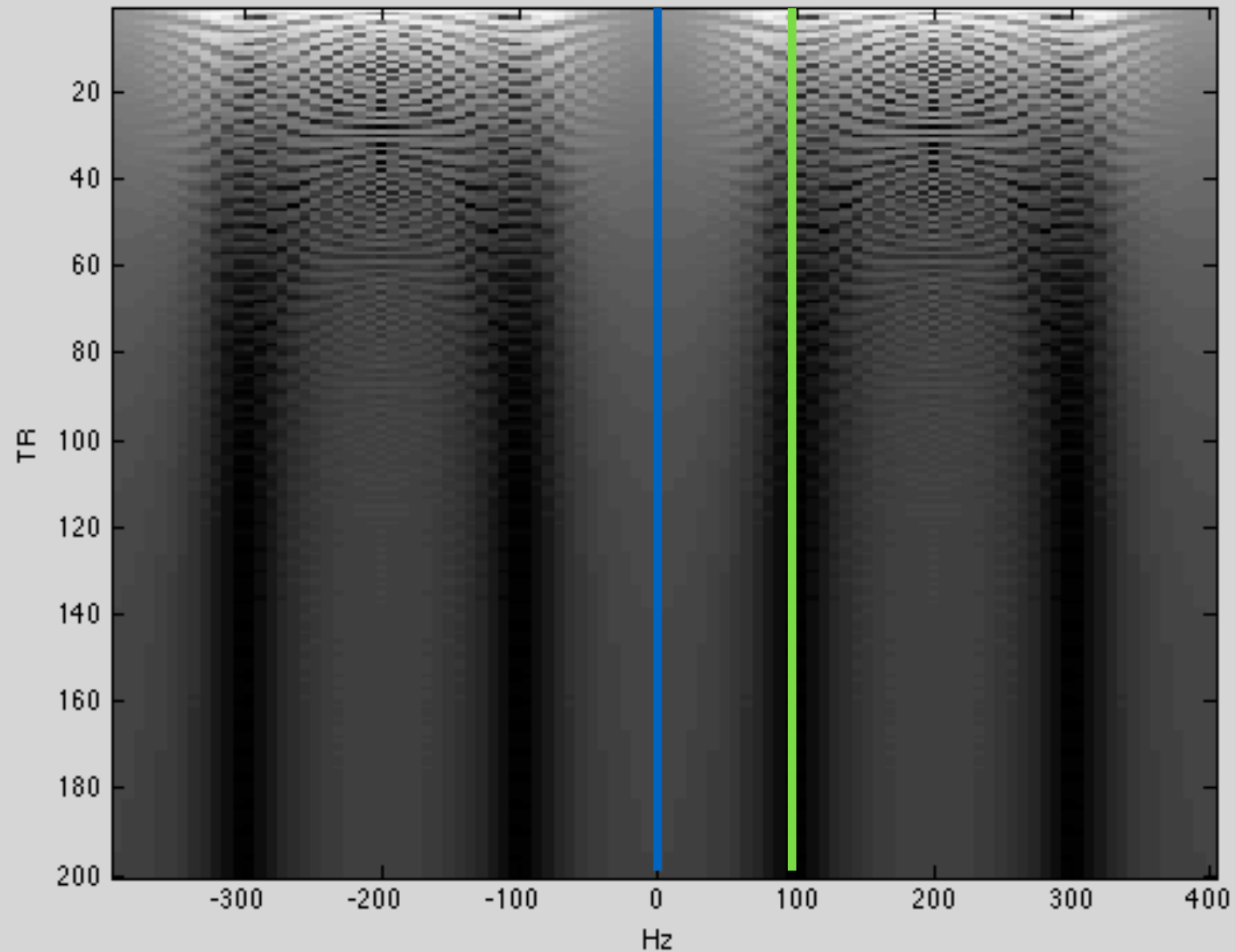
Balanced SSFP

Transition to steady state ($\theta/2$ -TR/2 prep):

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^\circ$



$T_1 = 600$ ms, $T_2 = 100$ ms

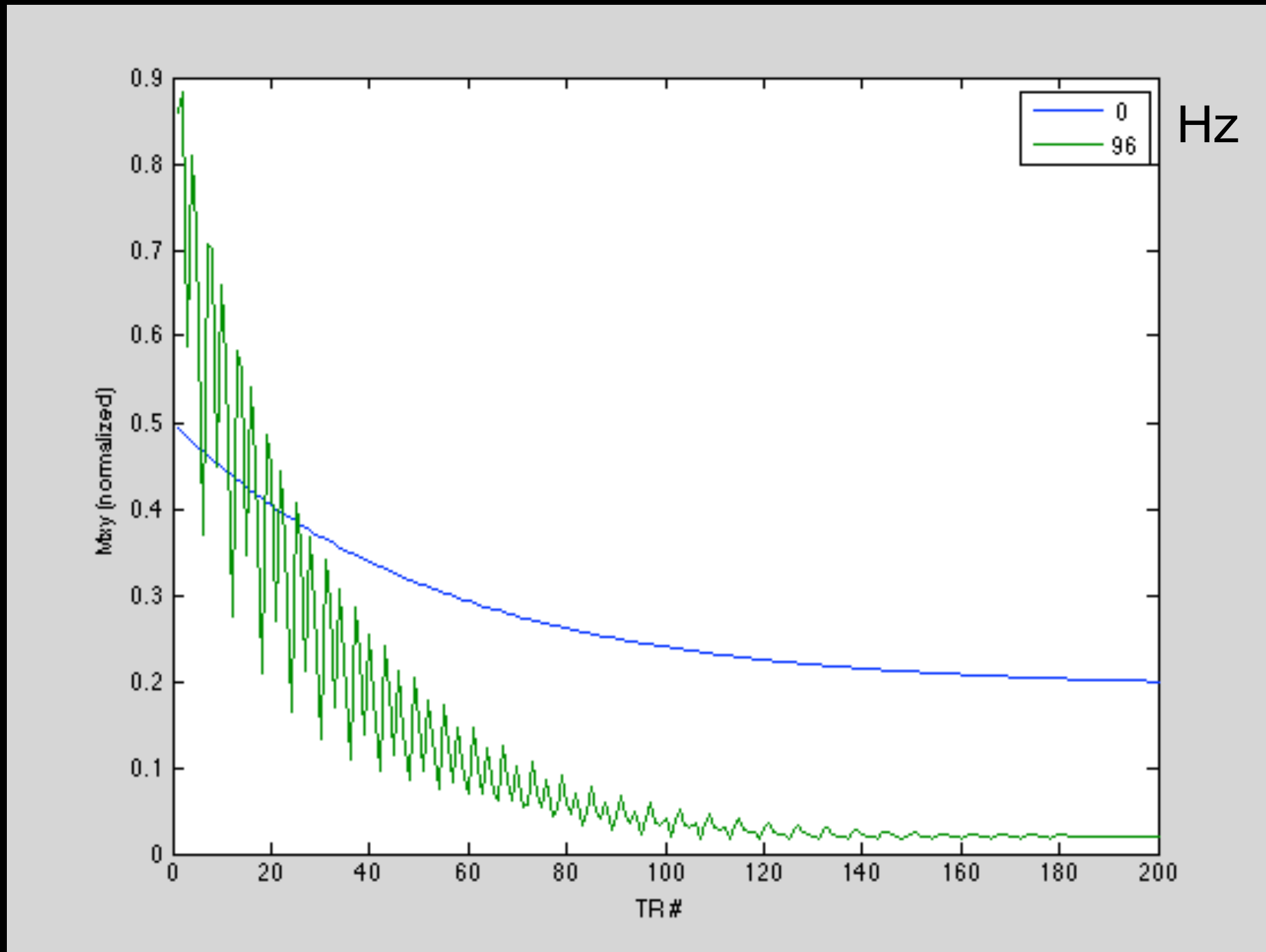
Balanced SSFP

Transition to steady state ($\theta/2$ -TR/2 prep):

TR = 5 ms

$\Delta\phi = \pi$

$\theta = 60^\circ$

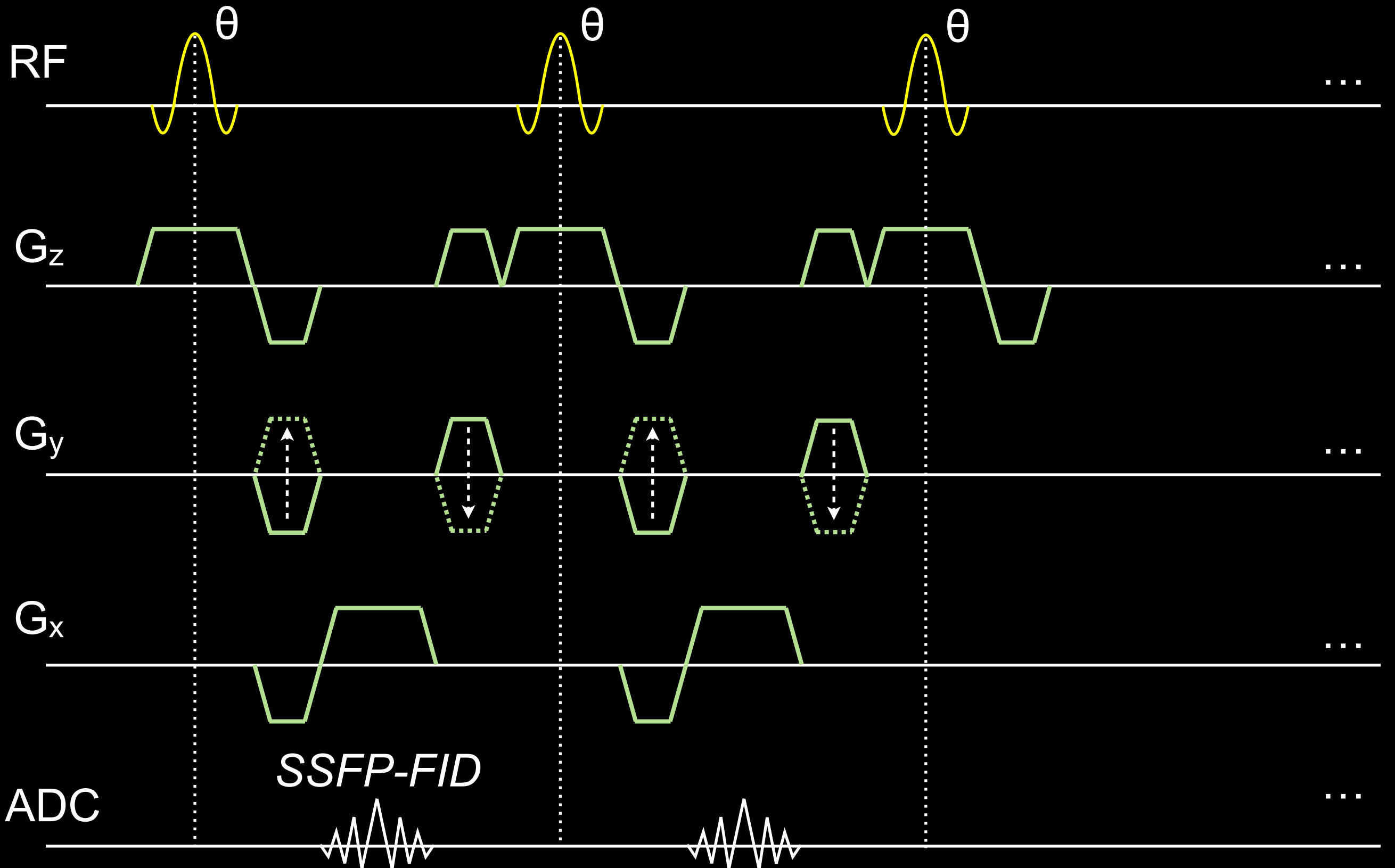


$T_1 = 600$ ms, $T_2 = 100$ ms

Balanced SSFP

- Advantages
 - High SNR efficiency
 - G_x and G_z first moments nulled
- Challenges
 - Field homogeneity
 - TR
 - SAR
 - Catalyzation
 - Bright fat

Gradient-spoiled GRE



Gradient-spoiled GRE

- End-of-TR gradient spoiler
 - typically on G_x and/or G_z
 - Range of β within each voxel
 - M_{xy} is a complex sum of all spins
- Contrast depends on T_1 and T_2

Gradient-spoiled GRE

Steady-state signal equation:

$$\text{SSFP}_{\text{FID}} = M_0 \frac{\sin \theta}{1 + \cos \theta} (1 - (E_1 - \cos \theta) f(E_1, E_2, \theta))$$

$$f(E_1, E_2, \theta) = \sqrt{\frac{1 - E_2^2}{(1 - E_1 \cos \theta)^2 - E_2^2 (E_1 - \cos \theta)^2}}$$

When $\text{TR} \gg T_2$:

$$\text{SSFP}_{\text{FID}} \rightarrow M_0 \sin \theta \frac{1 - E_1}{1 - E_1 \cos \theta}$$

same as ideally spoiled GRE

Gradient-spoiled GRE

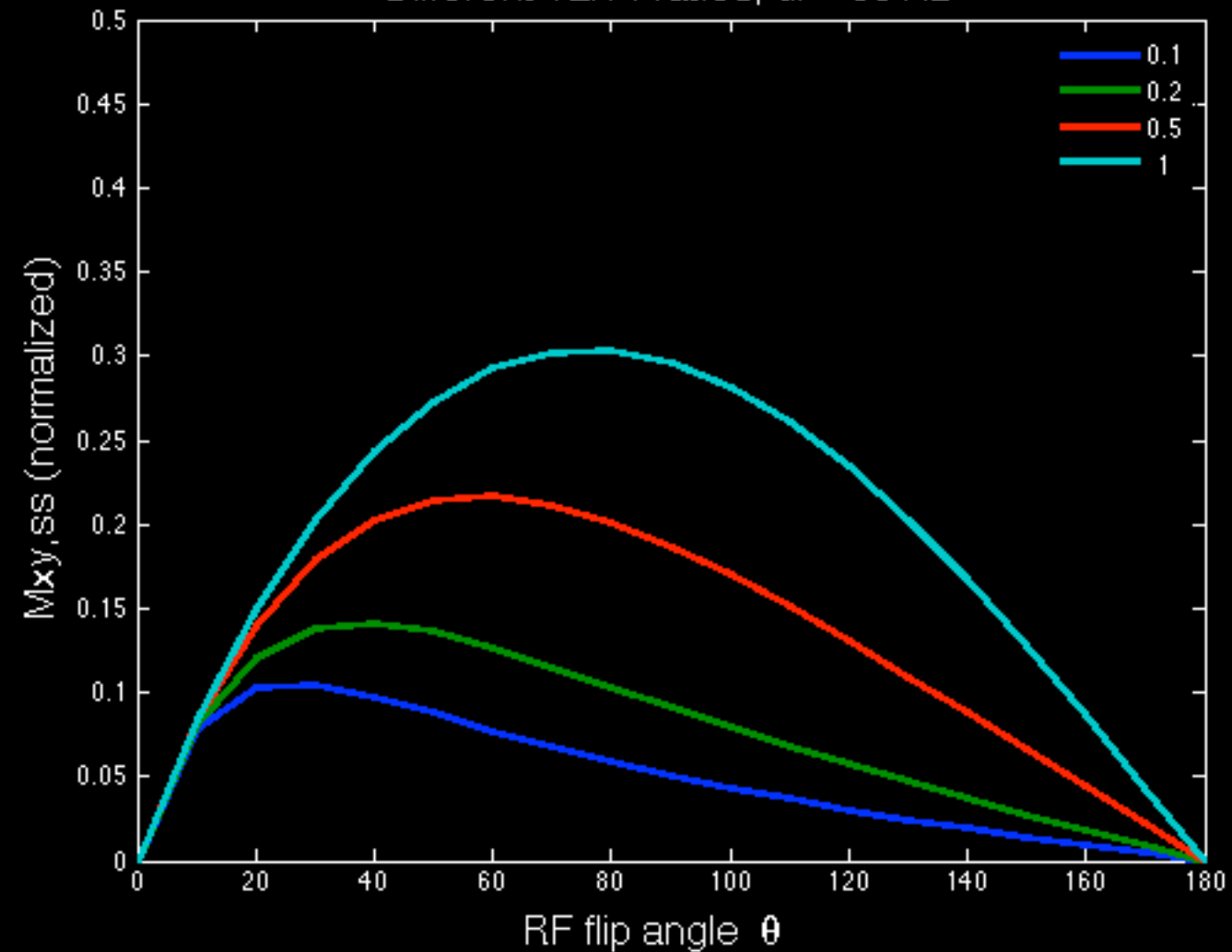
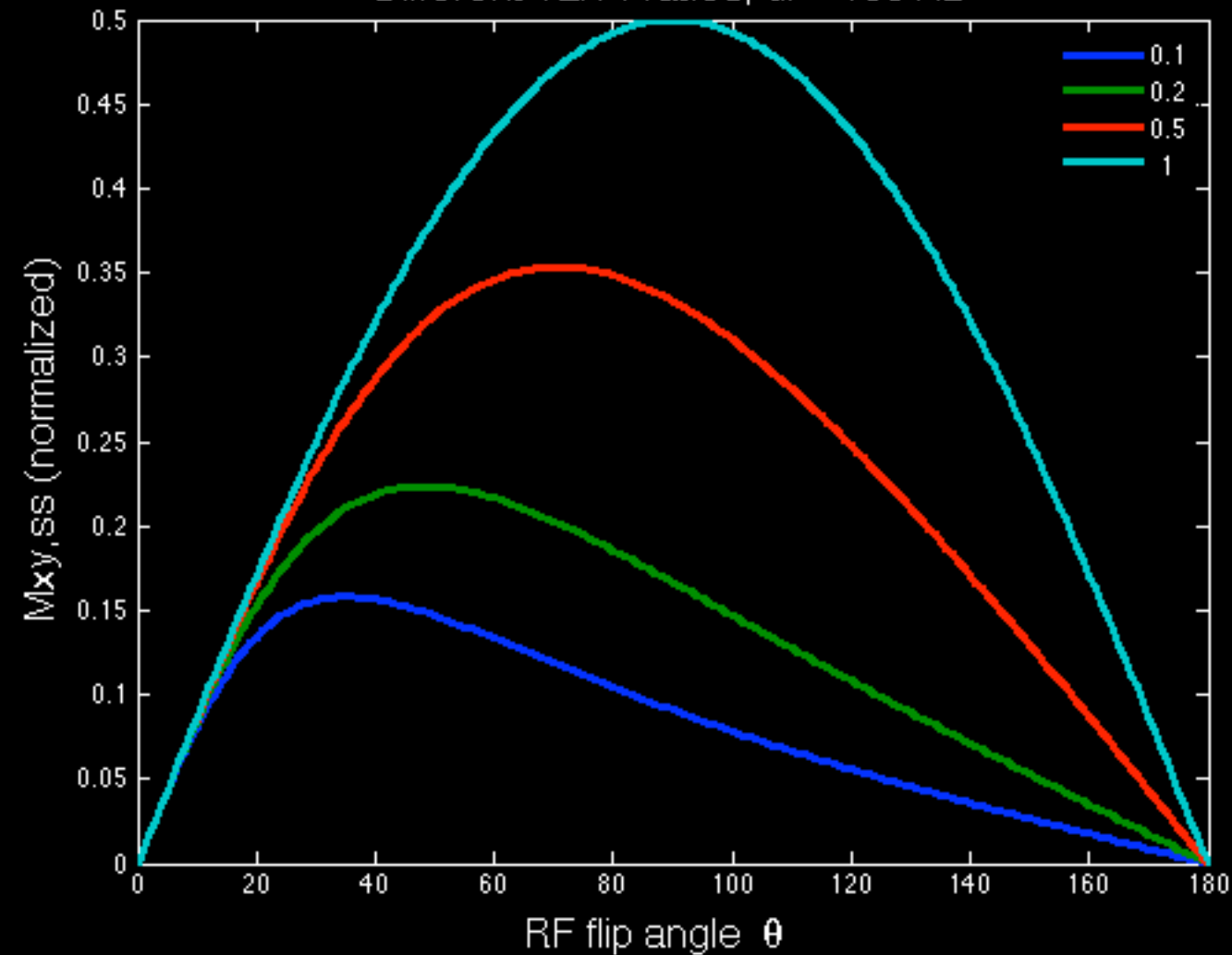
SS signal as a function of flip angle:

bSSFP

GRE (SSFP-FID)

Different T2/T1 ratios, df = 100 Hz

Different T2/T1 ratios, df = 50 Hz



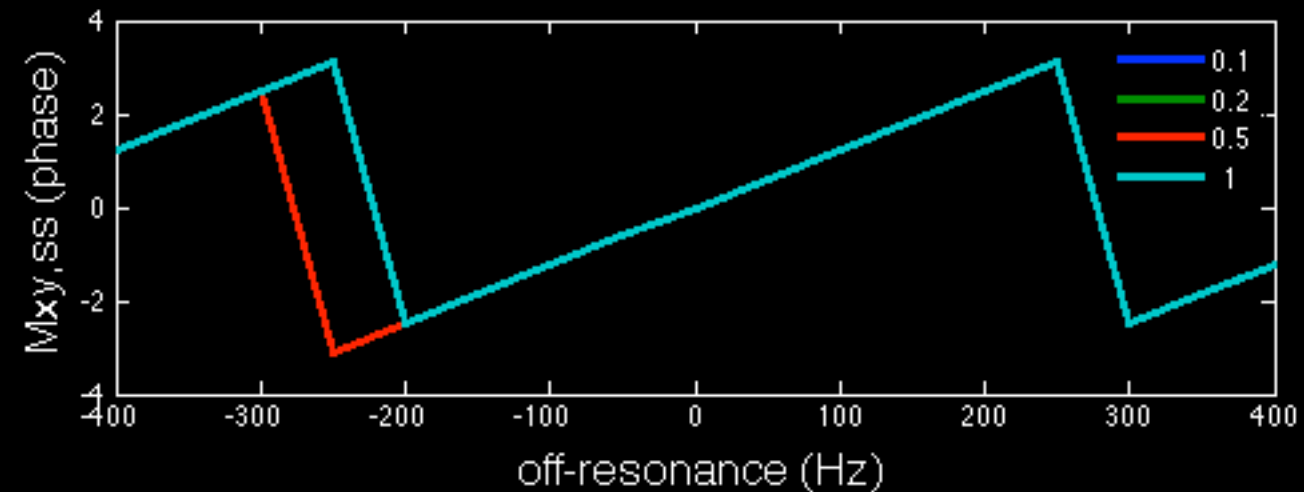
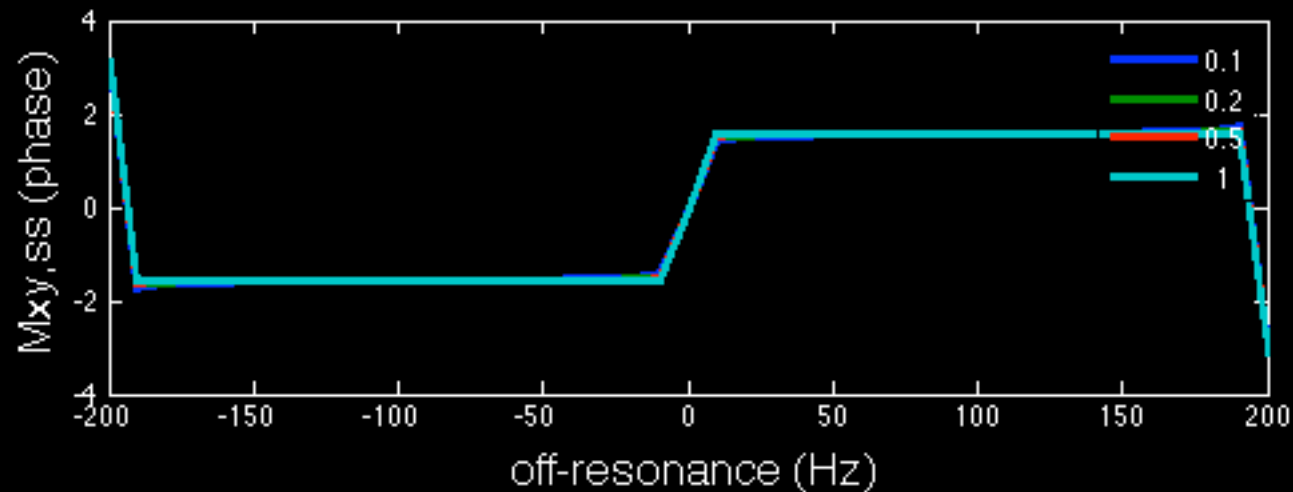
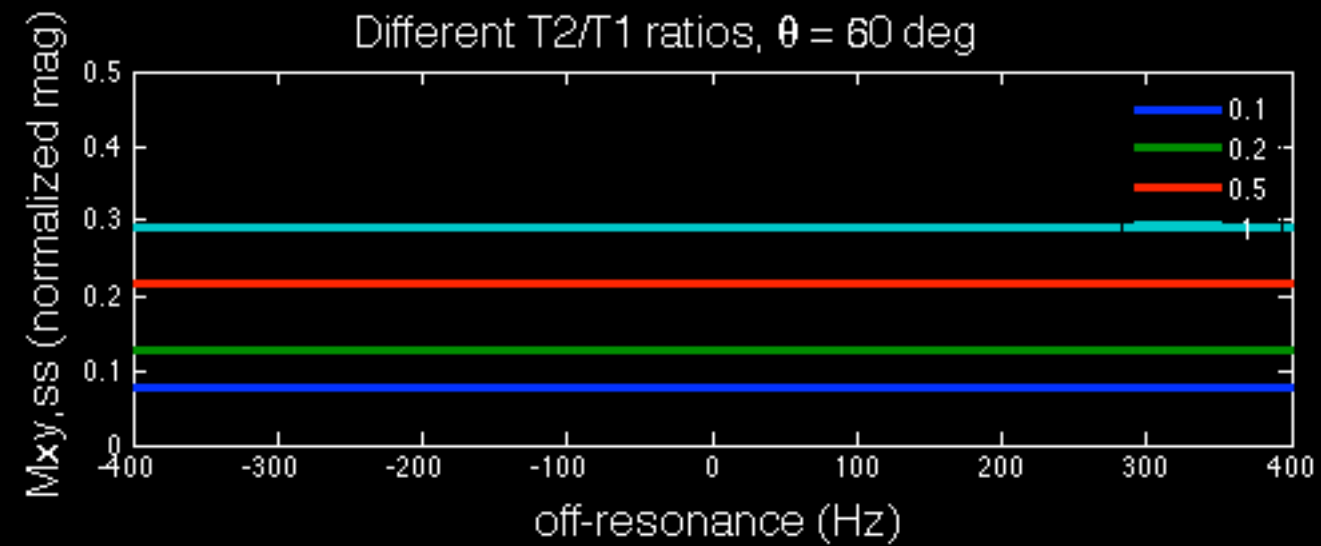
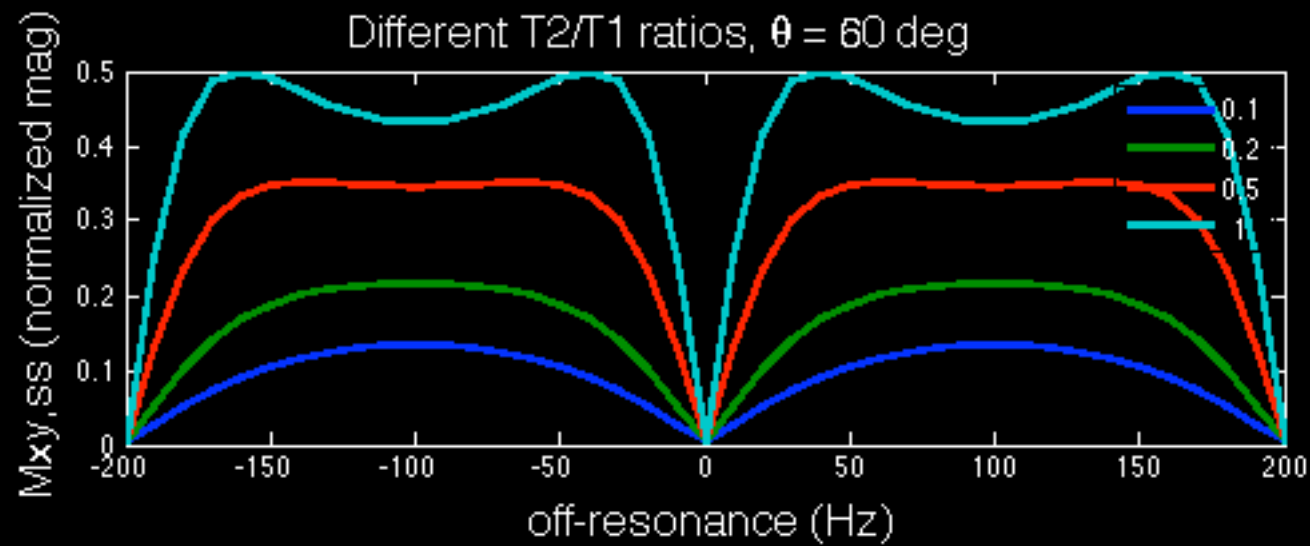
$T_1 = 1000 \text{ ms}$, $T_2 = 100, 200, 500, 1000 \text{ ms}$

Gradient-spoiled GRE

SS signal as a function of off-resonance:

bSSFP

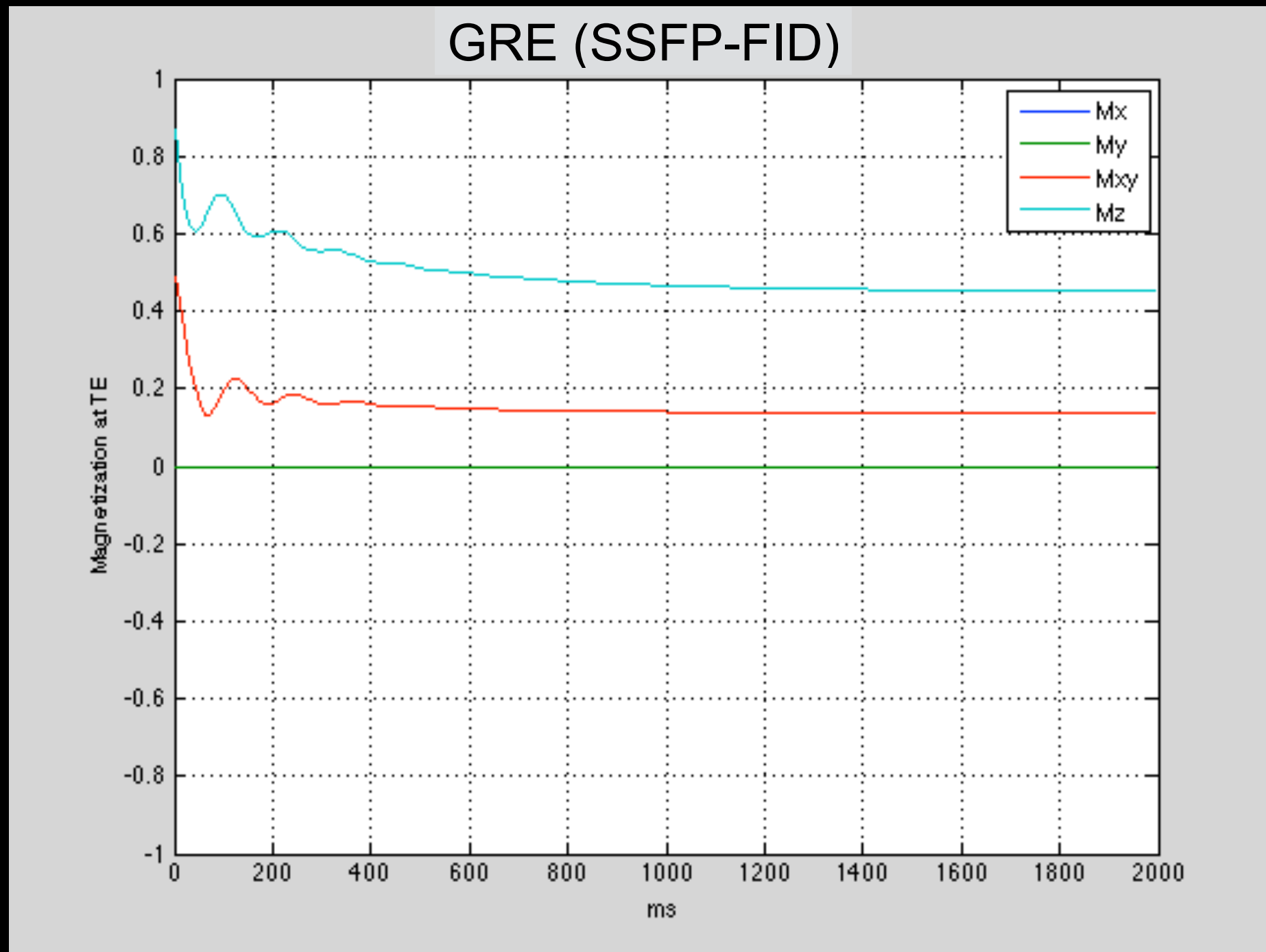
GRE (SSFP-FID)



$T_1 = 1000$ ms, $T_2 = 100, 200, 500, 1000$ ms

Gradient-spoiled GRE

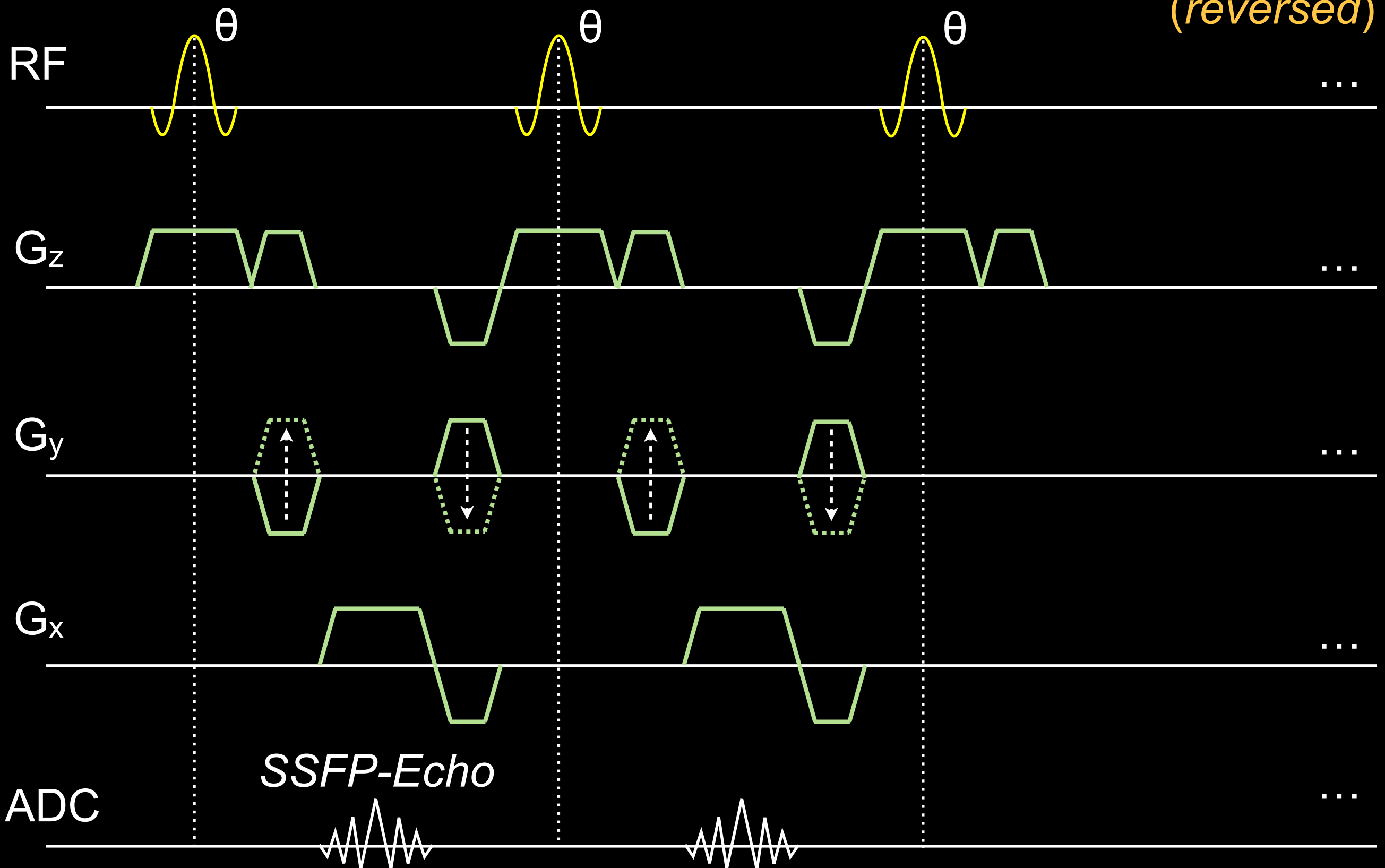
Transition to steady state:



$$T_1 = 600 \text{ ms}, T_2 = 100 \text{ ms}, TE/TR = 2/10 \text{ ms}, \theta = 30^\circ$$

Gradient-spoiled GRE

(reversed)



Gradient-spoiled GRE

(reversed)

Steady-state signal equation:

$$\text{SSFP}_{\text{Echo}} = M_0 \frac{\sin \theta}{1 + \cos \theta} (1 - (1 - E_1 \cos \theta) f(E_1, E_2, \theta))$$

$$f(E_1, E_2, \theta) = \sqrt{\frac{1 - E_2^2}{(1 - E_1 \cos \theta)^2 - E_2^2 (E_1 - \cos \theta)^2}}$$

When $\text{TR} \ll T_1$:

$$\frac{\text{SSFP}_{\text{Echo}}}{\text{SSFP}_{\text{FID}}} \sim E_2^2 = e^{-2\text{TR}/T_2}$$

higher T_2 contrast weighting than SSFP_{FID}

Gradient-spoiled GRE

(reversed)

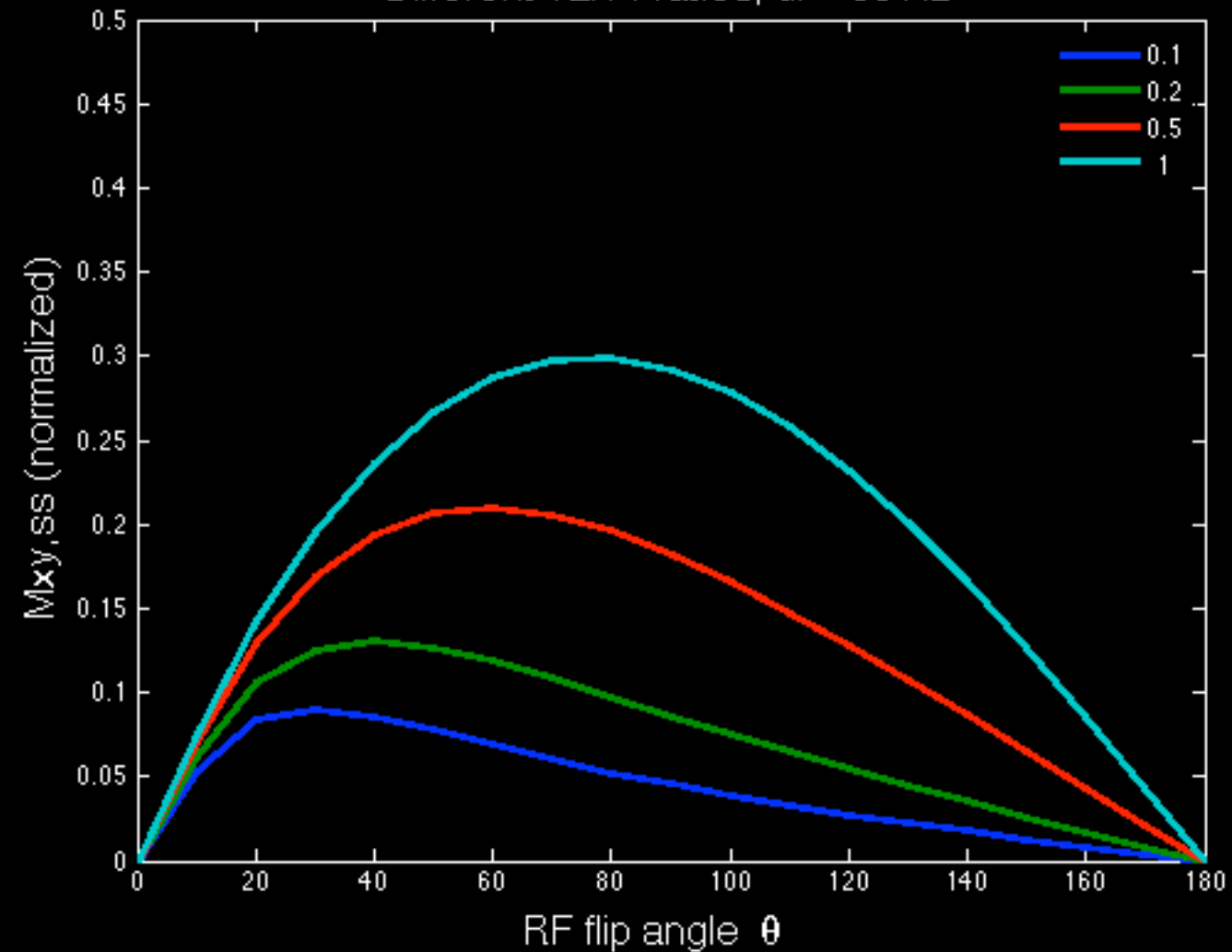
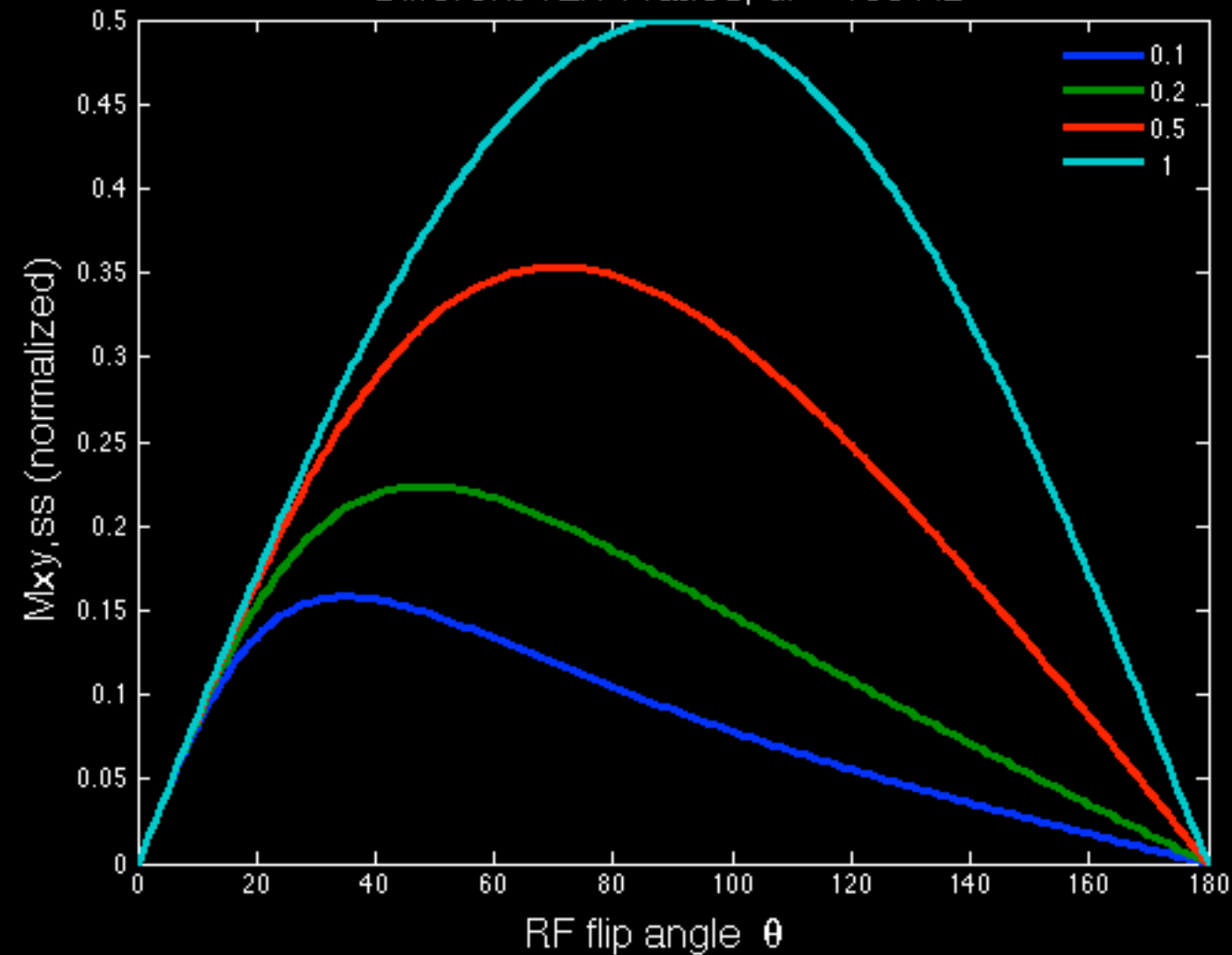
SS signal as a function of flip angle:

bSSFP

GRE (SSFP-Echo)

Different T2/T1 ratios, df = 100 Hz

Different T2/T1 ratios, df = 50 Hz

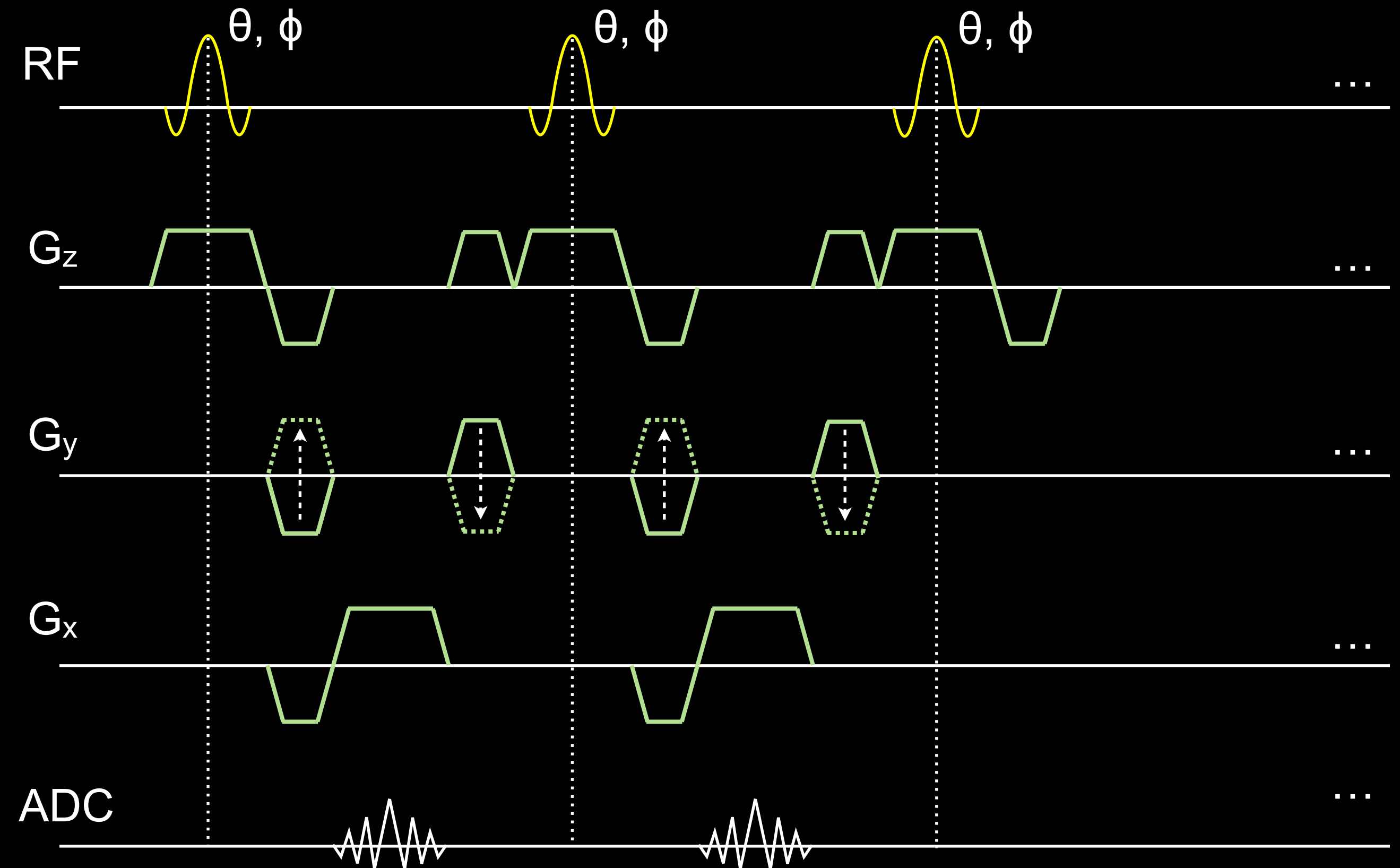


$T_1 = 1000$ ms, $T_2 = 100, 200, 500, 1000$ ms

Gradient-spoiled GRE

- Image characteristics
 - no banding (averaged in voxel)
 - SSFP-FID: T_2/T_1 contrast
 - SSFP-Echo: more T_2 contrast
 - sensitive to motion / flow / diffusion
- When all gradients are balanced
 - SSFP-FID and SSFP-Echo coalesce
 - T_2 instead of T_2^* weighting
 - Balanced SSFP!

Gradient & RF-spoiled GRE

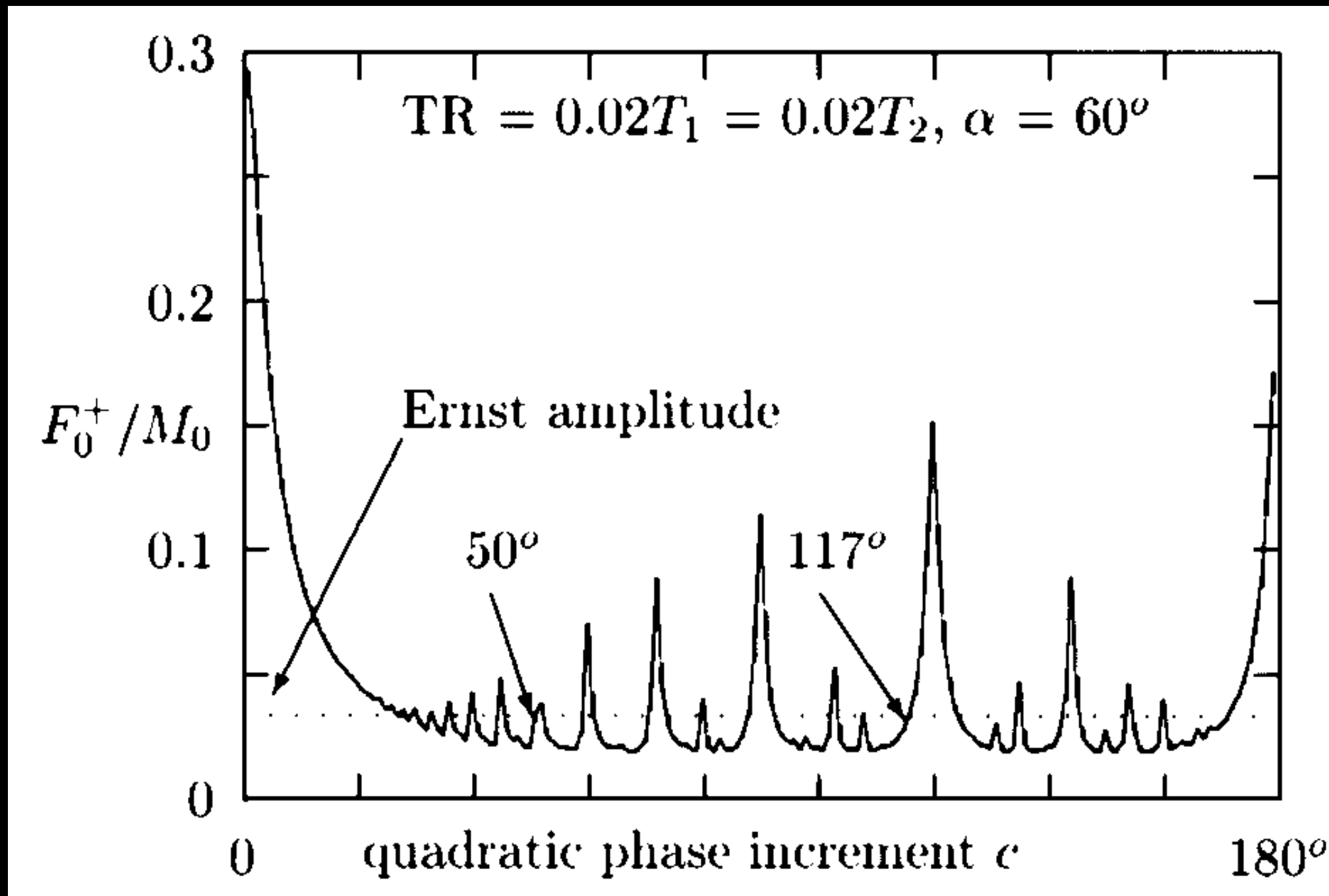


Gradient and RF-spoiled GRE

- RF spoiling (quadratic)
 - $\phi_n = \phi_{n-1} + n\phi_0 = (1/2)\phi_0(n^2 + n + 2)$
 - typically $\phi_0 = 50^\circ$ or 117°
 - ADC phase each TR also needs to match ϕ_n
- T_1 -weighted contrast
 - approaches contrast of ideally spoiled GRE
 - at expense of reduced SNR
(removes T2w contributions)

Gradient and RF-spoiled GRE

Choice of RF phase increment:



Gradient and RF-spoiled GRE

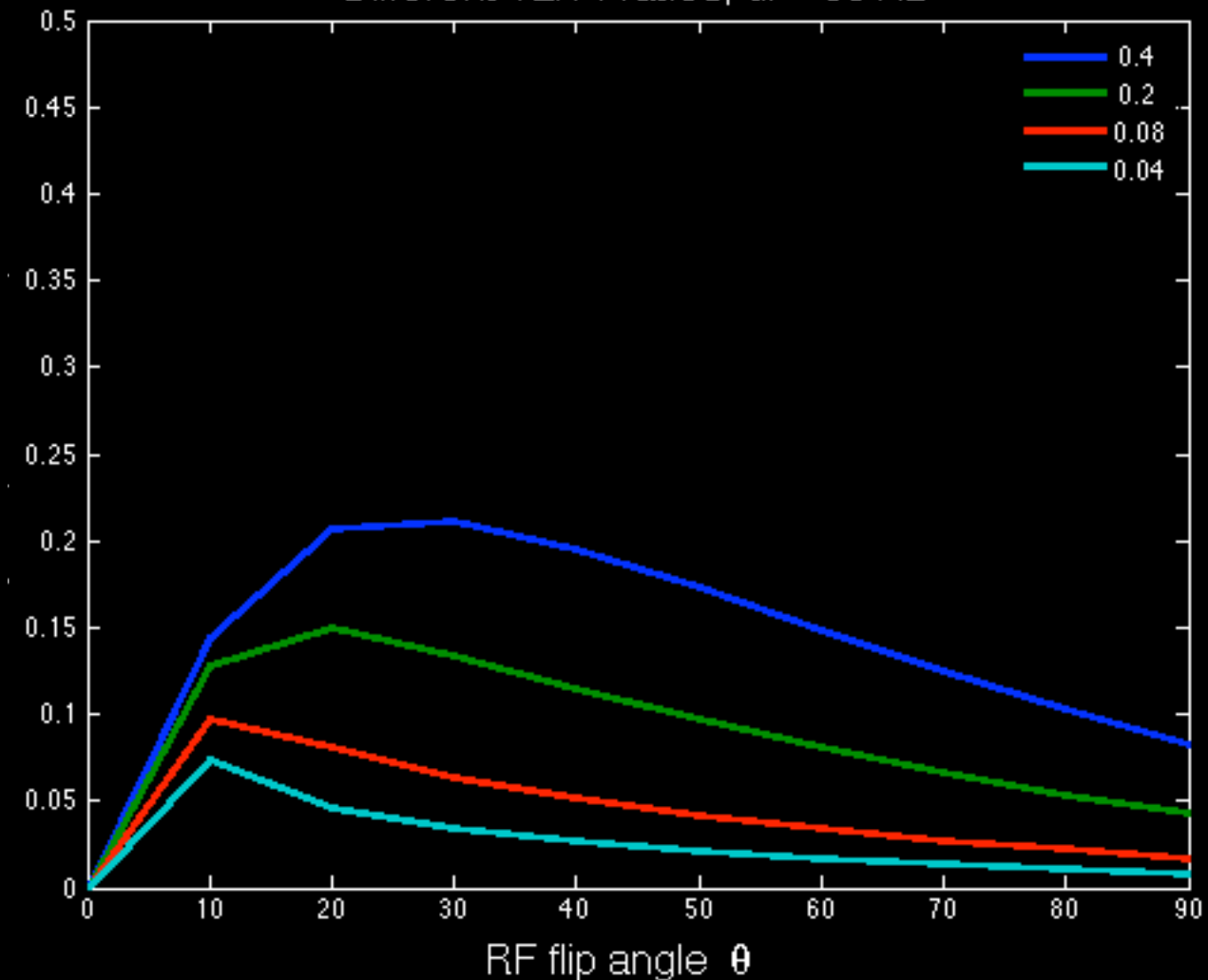
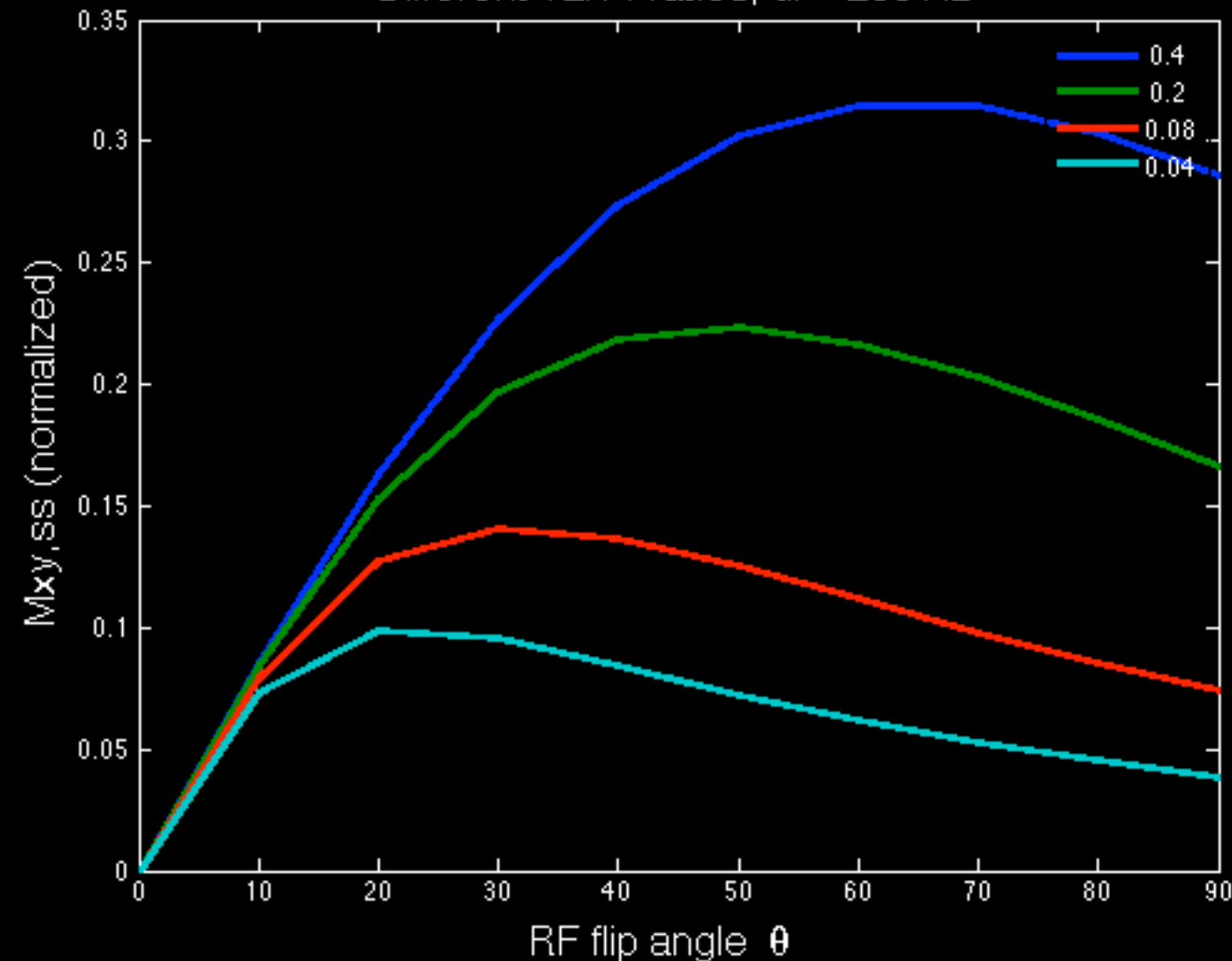
SS signal as a function of flip angle:

bSSFP

Spoiled GRE

Different T2/T1 ratios, df = 200 Hz

Different T2/T1 ratios, df = 50 Hz



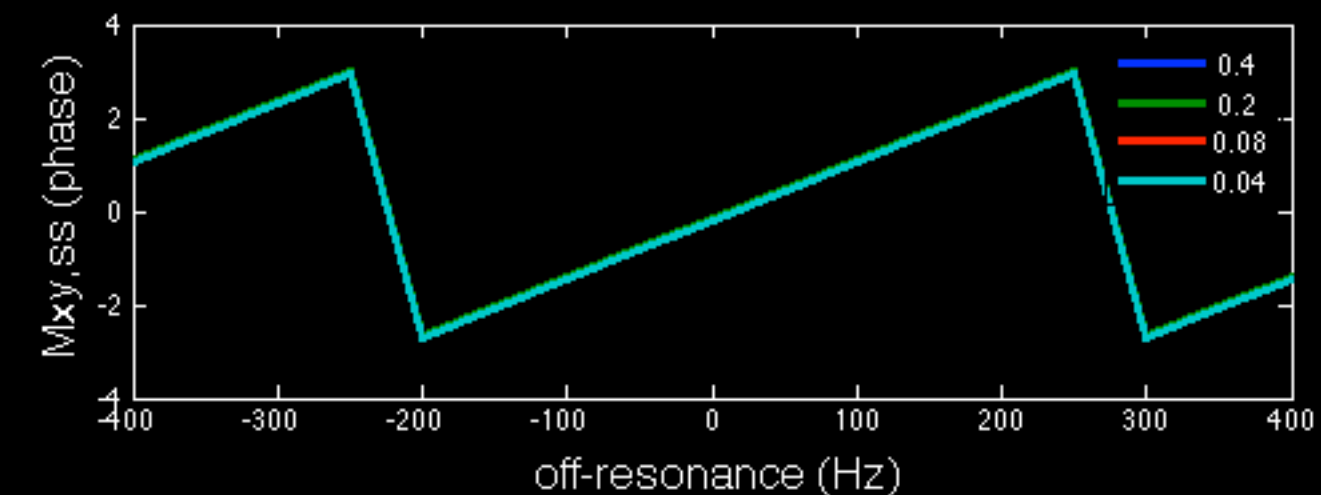
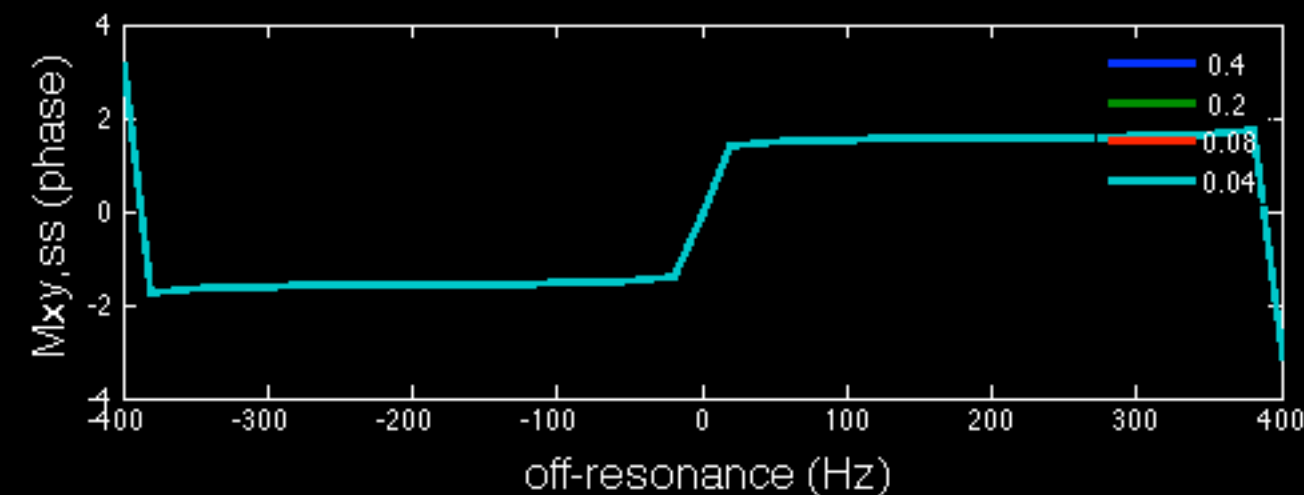
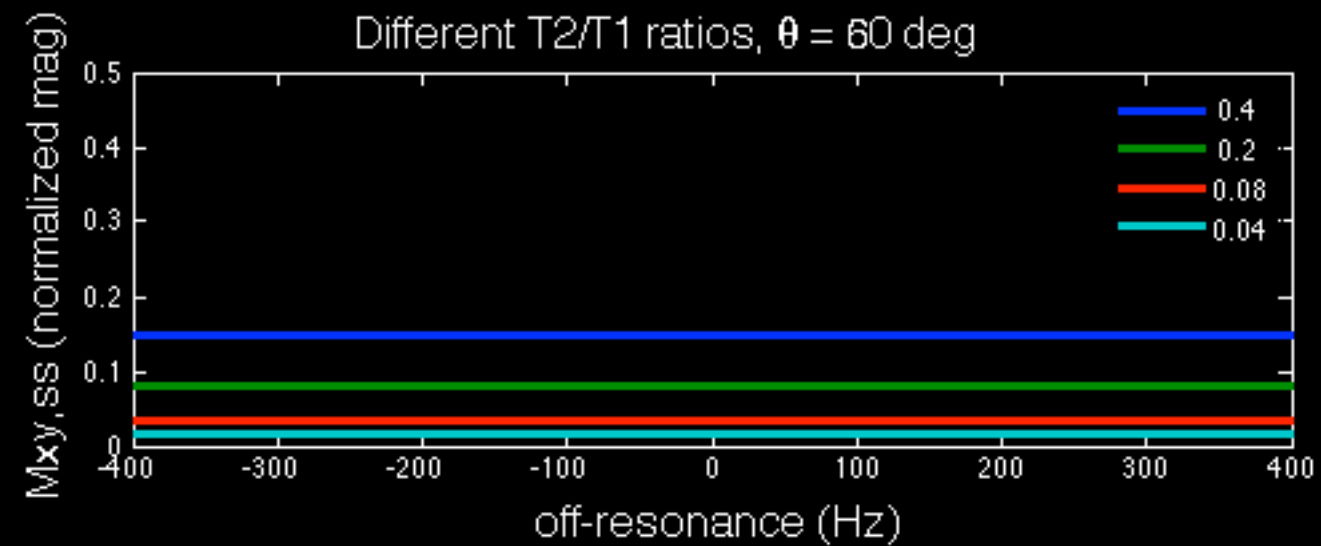
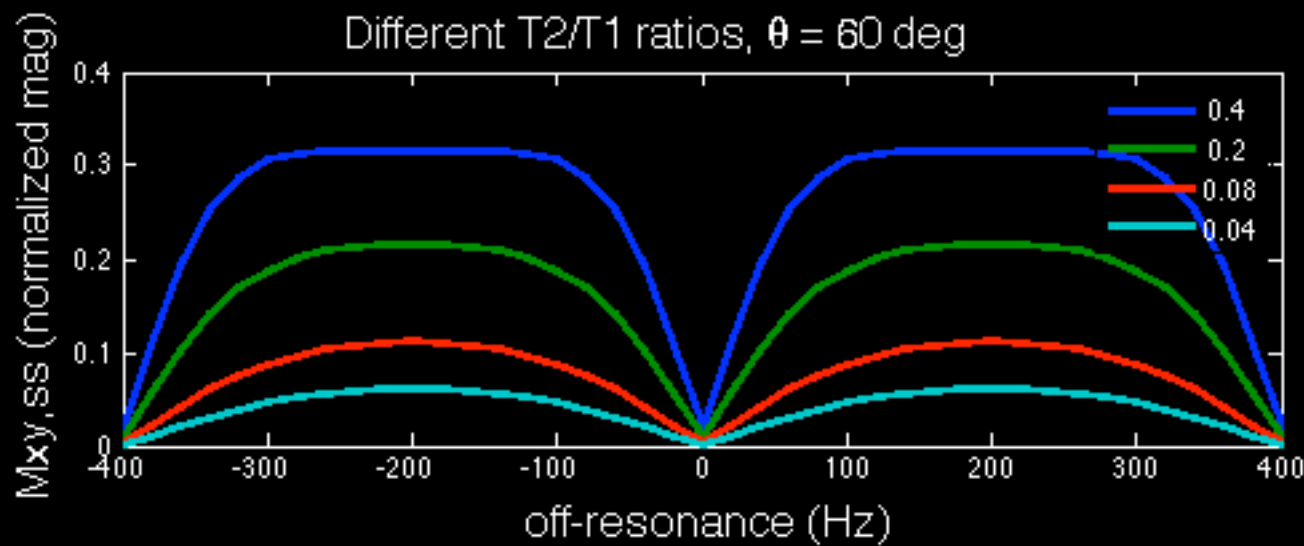
$T_1 = 100, 200, 500, 1000$ ms, $T_2 = 40$ ms

Gradient and RF-spoiled GRE

SS signal as a function of off-resonance:

bSSFP

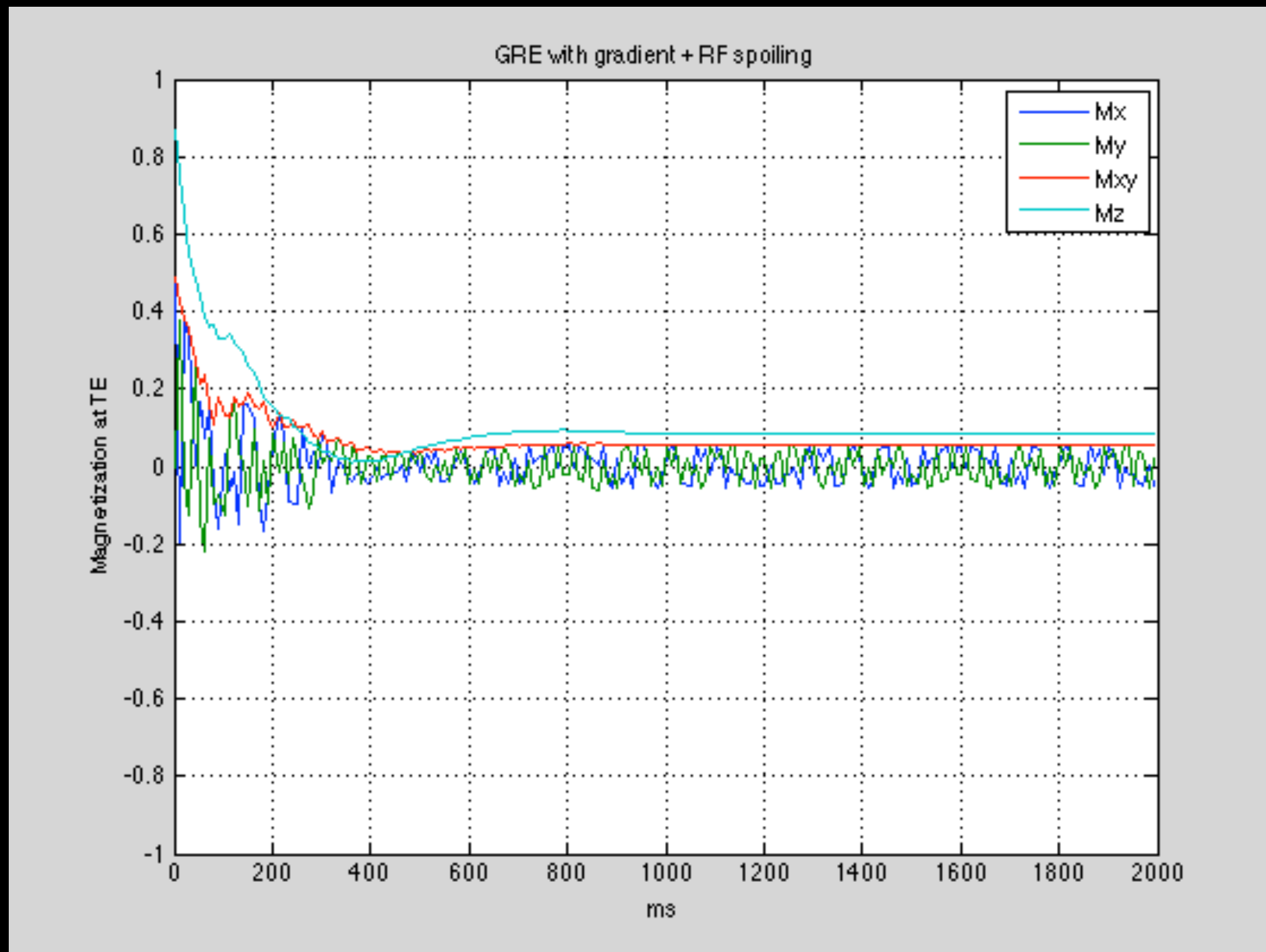
Spoiled GRE



$T_1 = 100, 200, 500, 1000$ ms, $T_2 = 40$ ms

Gradient and RF-spoiled GRE

Transition to steady state:



$$T_1 = 600 \text{ ms}, T_2 = 100 \text{ ms}, TE/TR = 2/10 \text{ ms}, \theta = 30^\circ$$

Gradient and RF-spoiled GRE

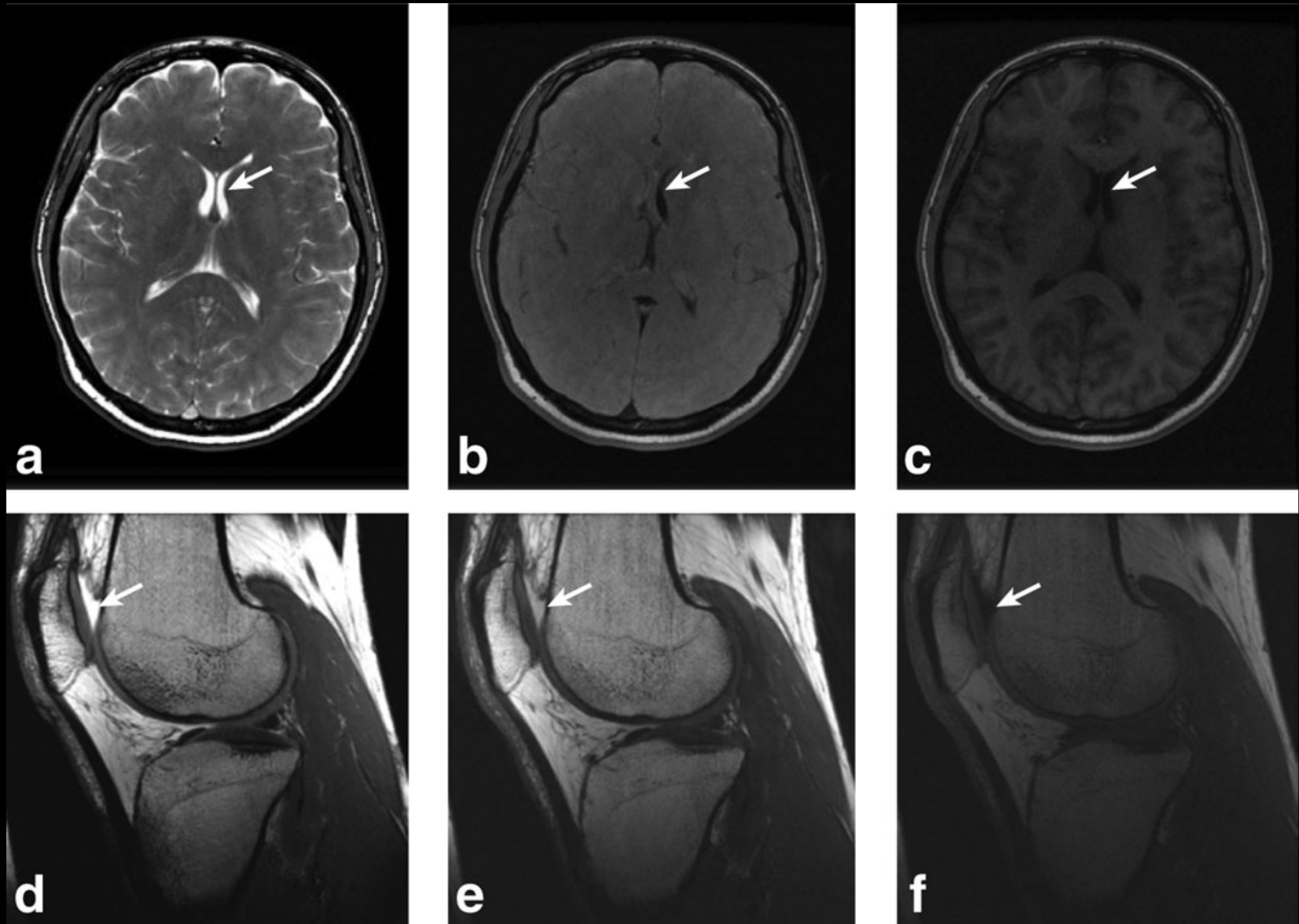
- Image characteristics
 - no banding
 - M_{xy} spoiled before next TR
 - T1w contrast with short TR
 - θ controls degree of T_1 contrast
 - TE controls degree of T_2^* contrast
 - robust to motion

Rapid GRE - Comparison

bSSFP

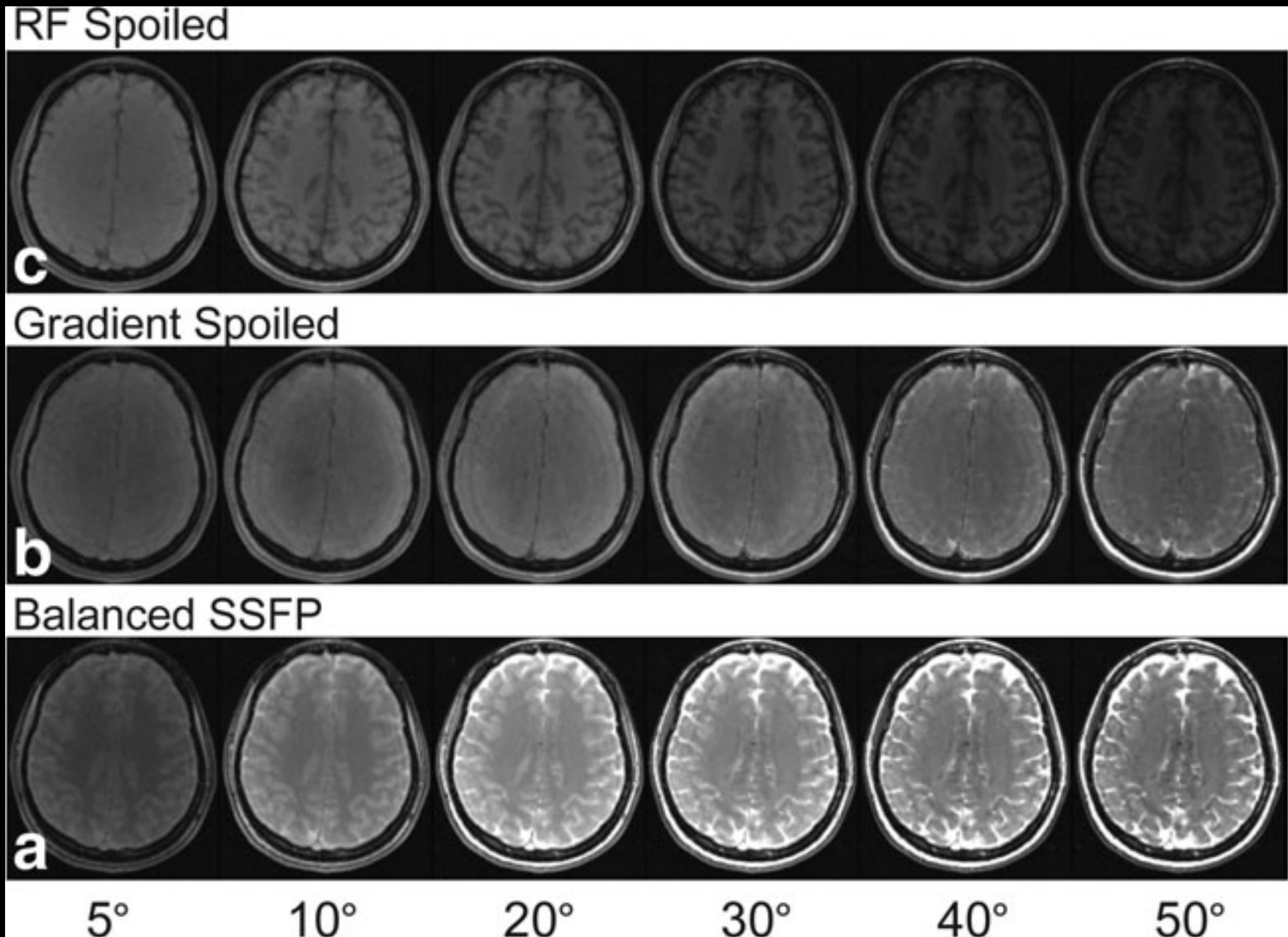
Grad spoiled

RF spoiled



Rapid GRE - Comparison

Flip angle



Rapid GRE - Comparison

Pulse Sequence		Mxy	Contrast	SNR	Artifacts
Balanced SSFP	bSSFP	retained	T_2/T_1	high	banding
Gradient-spoiled GRE	SSFP-FID	averaged	T_2/T_1	mid	motion
	SSFP-Echo	averaged	T_2+T_2/T_1	mid	motion
Gradient and RF-spoiled GRE	Spoiled GRE	cancelled	$T_1; T_2^*$	low	minimal

SS transition

Considerations

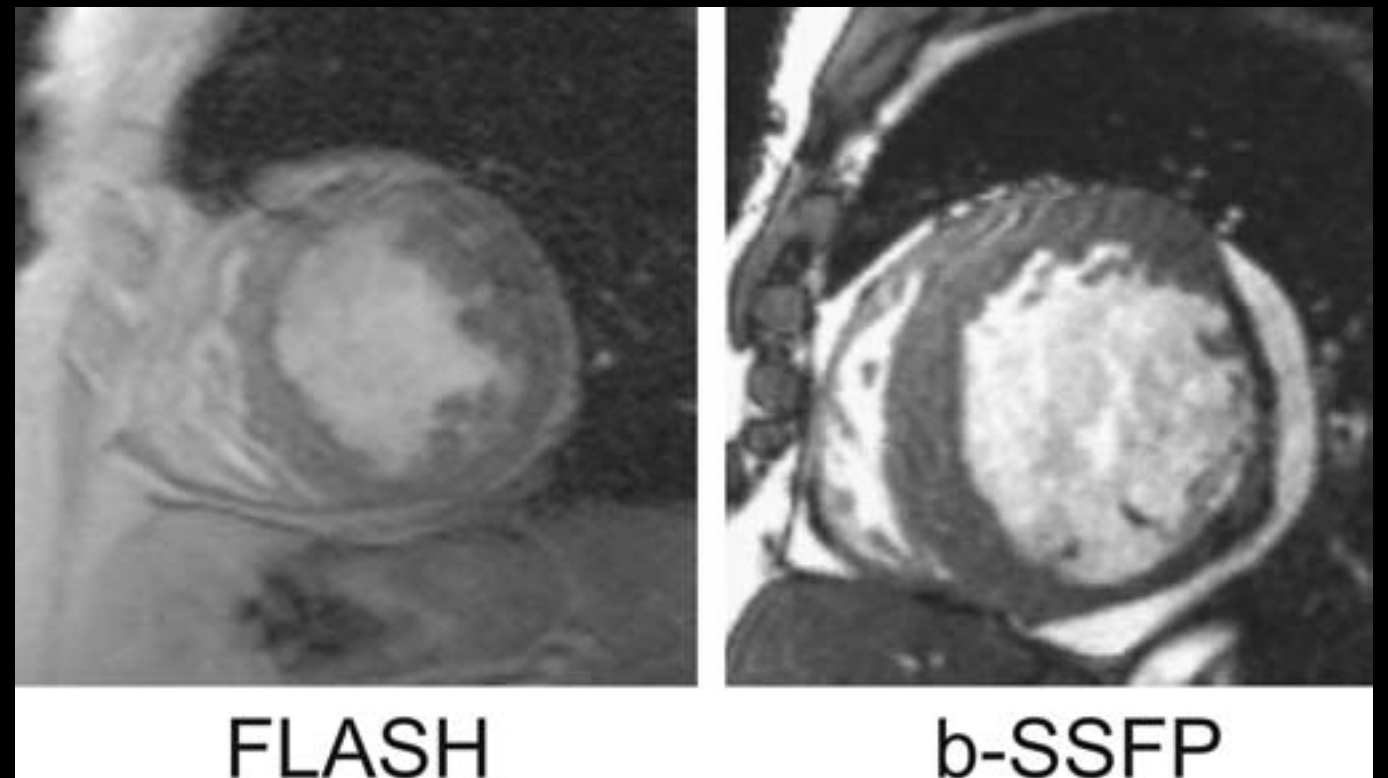
- Chemical shift
- Flow
- Diffusion

Extensions and Variations

- Partial echo
- Multi-echo
- Ultra-short TE
- Magnetization preparation
- Multiple steady states

Applications

- bSSFP
 - Cardiac
 - MRA
 - T_2 -like imaging
 - fMRI
 - phase contrast
 - Mag-prep



Applications

- SSFP-FID / Echo
 - T_2 -like imaging (e.g., cartilage)
 - Bright fluid (bSSFP-like without banding)
 - Diffusion-weighted imaging (SSFP-Echo)

Applications

- Spoiled GRE
 - T1w imaging
 - T_2^* BOLD fMRI
 - Susceptibility-weighted imaging (SWI)
 - Phase contrast
 - Thermometry
 - Time-of-flight MRA
 - Contrast-enhanced imaging
 - Mag-prep imaging

Thanks!

- Web resources
 - ISMRM 2010 Edu: Weigel, Bieri, Miller
 - ISMRM 2011 Edu: Weigel, Miller
 - ISMRM 2012 Edu: Miller, Bieri
- Further reading
 - Bernstein et al., Handbook of MRI Sequences
 - Haacke et al., Magnetic Resonance Imaging
 - Nishimura, Principles of MRI
 - pubmed.org

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Holden H. Wu, Ph.D.

HoldenWu@mednet.ucla.edu

<http://mrrl.ucla.edu/wulab>