Susceptibility and Conductivity MRI

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M229 Advanced Topics in MRI

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Outline

Phase MRI

Susceptibility MRI Contrast

Susceptibility MRI Processing

Susceptibility MRI Applications

Conductivity MRI

Phase MRI

Quadrature (Q) = "imaginary"

In phase (I) = "real"

$$Mag = \sqrt{Re^2 + Im^2}$$

$$\Phi = \tan^{-1}[Im/Re]$$

Imaginary Channel

$$Signal = Re + i\,Im$$

Mag

Im

Φ

Re

Real Channel

Real

Imaginary
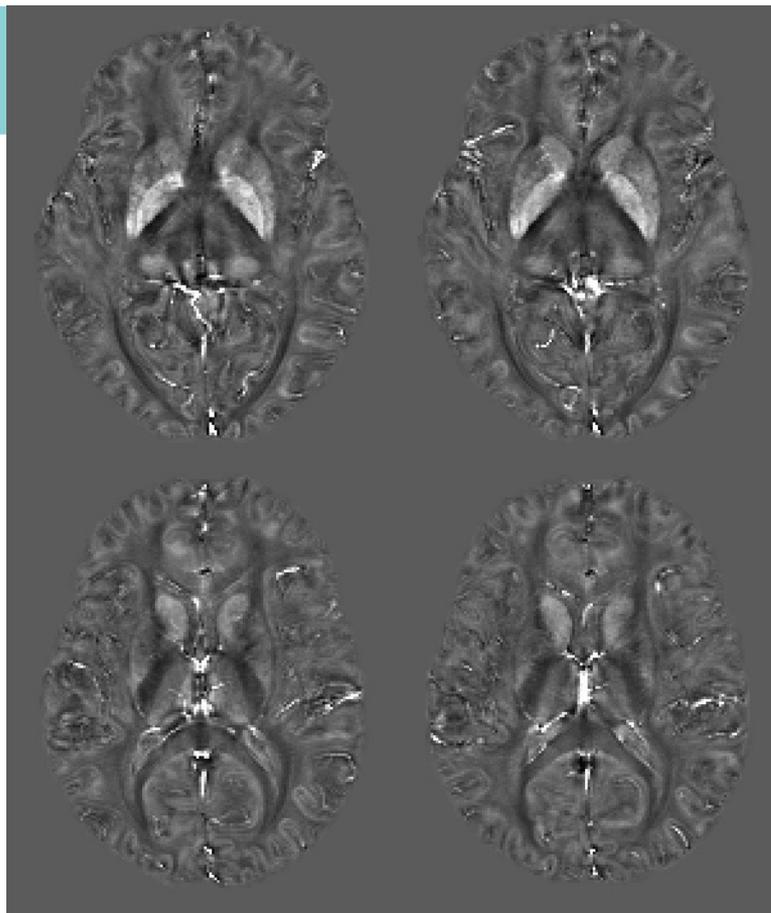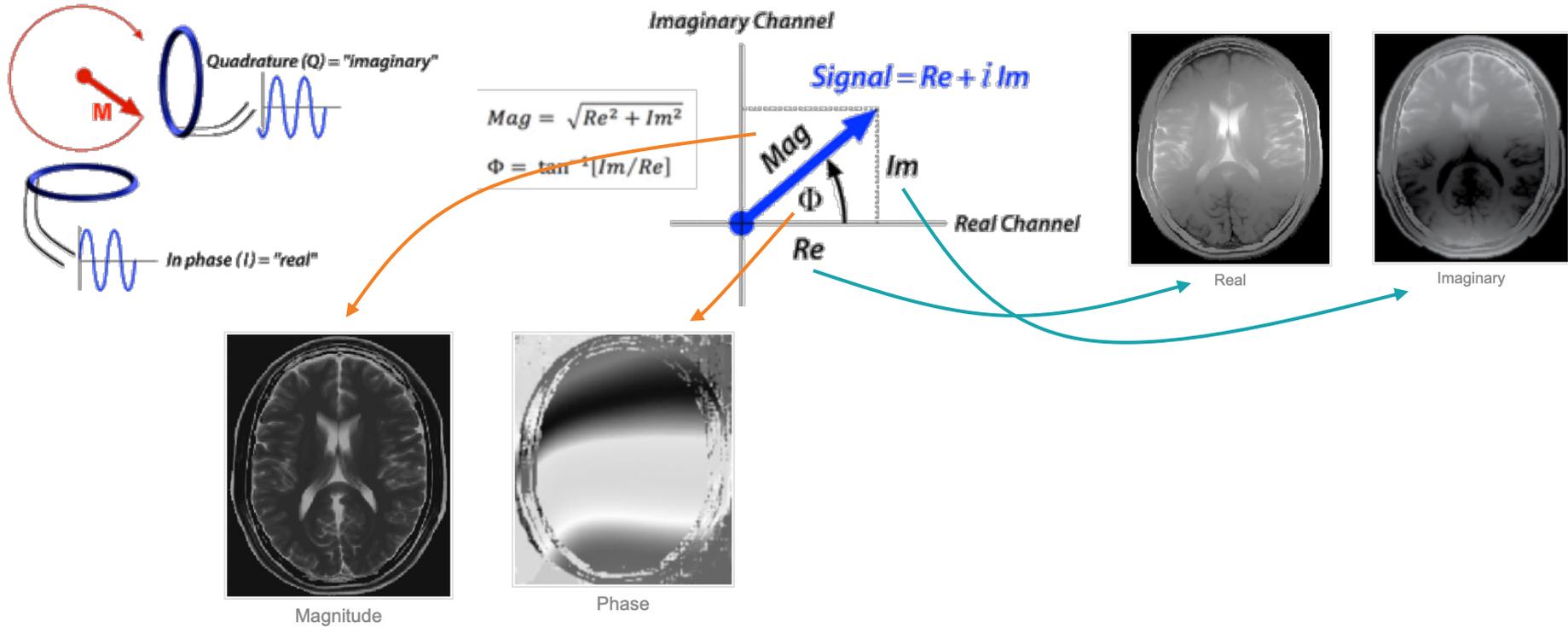
Magnitude

Phase

Phase MRI
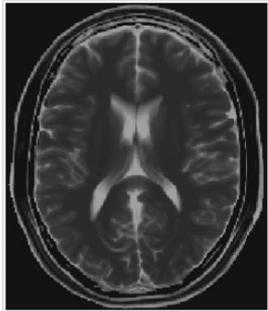
Magnitude
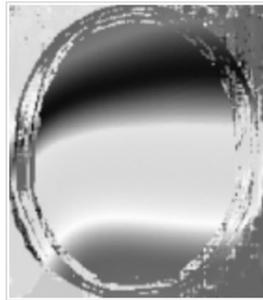
Phase

Magnitude

Phase

Brain imaging

Structural imaging
• T1-weighted, T2-weighted, FLAIR

Function imaging

Diffusion and perfusion imaging

Susceptibility imaging

Cardiac imaging

Structural imaging
• Dark-blood and bright-blood sequences

Cardiac motion imaging
• Cine

Perfusion imaging

Flow imaging

Phase MRI

	Encoding	Data	Applications
Susceptibility imaging	None	Raw phase	Iron, calcium, myelin imaging
Conductivity imaging	None	Raw phase	Tumors, ischemic lesions
MR thermometry	None	Phase shift	MR-guided procedures
Flow imaging	Velocity-encoding bipolar gradient	Subtracted phase data from opposite encodings	Cardiac flow, CSF flow
Phase contrast angiography	Bipolar gradients applied along the x, y, and z axes sequentially	Subtracted phase data from opposite encodings and combined across three directions	Angiogram, venogram, aneurysm
Elastography	Motion-encoding gradients	Phase differences	Liver fibrosis, brain

Phase MRI

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Phase MRI

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Phase MRI

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Elastography	Motion-encoding gradients

(a)

(b)

G_{SS} G_{FE}

G_{PE} Speed

(c)

(d)

MIP

MRI from Picture to Proton

	Encoding	Data	Applications
Susceptibility imaging			
Conductivity imaging			
MR thermometry			
Flow imaging			
Phase contrast angiography			
Elastography	Motion-encoding gradients	Phase differences	Liver fibrosis, brain

a — Mechanical Driver — Conventional MR Image
b — Wave Image — -70 0 +70 Displacement (µm)
c — Elastogram — 0 4 8 Shear Stiffness (kPa)

Phase MRI

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Elastography	Motion-encoding gradients	Phase differences	Liver fibrosis, brain

Susceptibility MRI – source of contrast

$$J = \chi B_0$$

Magnetic field (B_o)

Diamagnetic   **Para/Ferromagnetic**

$\chi < 0$ $\chi > 0$

paramagnetic

$\chi > 0$

Air

Iron, gadolinium, copper, manganese

Deoxyhemoglobin

Water susceptibility

Most of the biological tissues

Myelin

Calcification

$\chi < 0$

diamagnetic
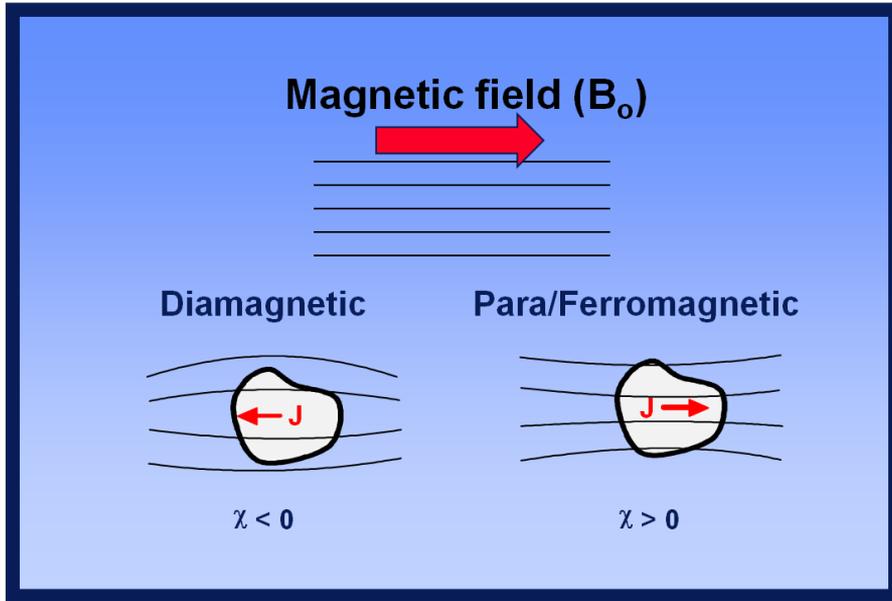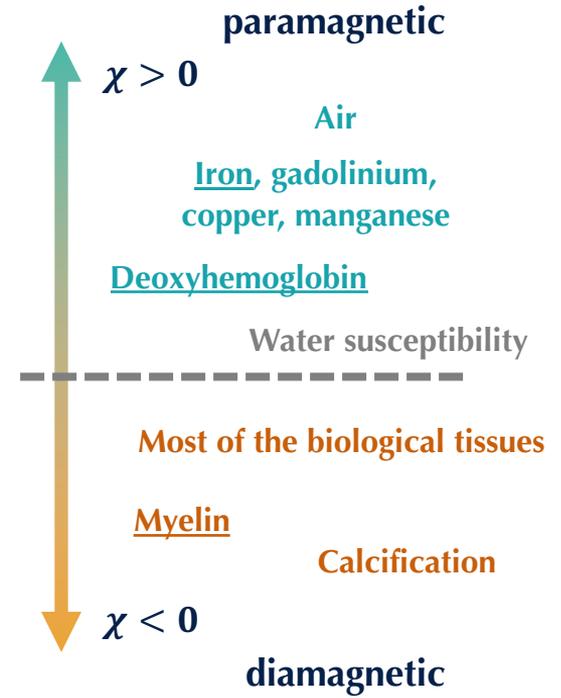
Susceptibility MRI – source of contrast

Susceptibility-weighted imaging

Quantitative susceptibility mapping

paramagnetic

$\chi > 0$

Air

Iron, gadolinium, copper, manganese

Deoxyhemoglobin

Water susceptibility

- - - - - - - - - - - - - - - - - -

Most of the biological tissues

Myelin

Calcification

$\chi < 0$

diamagnetic
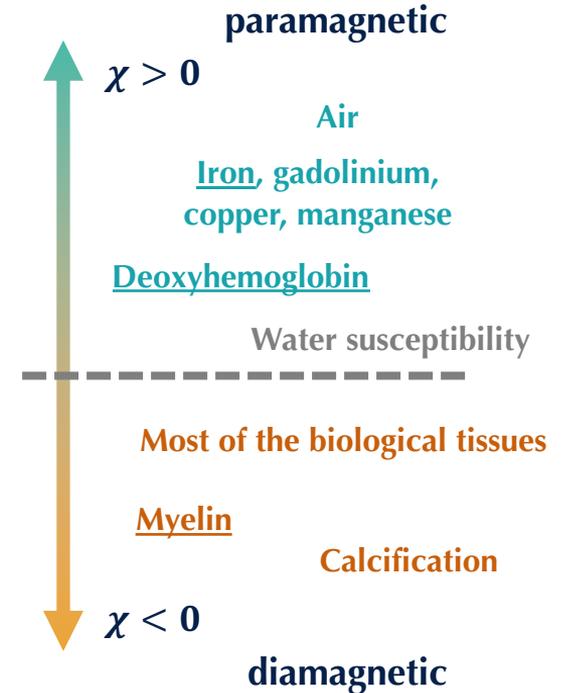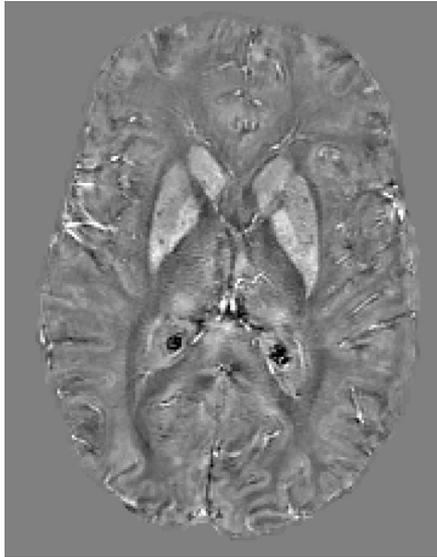
Susceptibility MRI – Deoxyhemoglobin

Deep medullary veins

Septal vein

Caudate vein

Internal cerebral vein

Inf. Ventricular veins

Lateral ventricle

Caudate

Ant. internal capsule

Putamen

Globus pallidus

Internal capsule

Thalamus

Inf. Lateral ventricle

Splenium Of corpus callosum
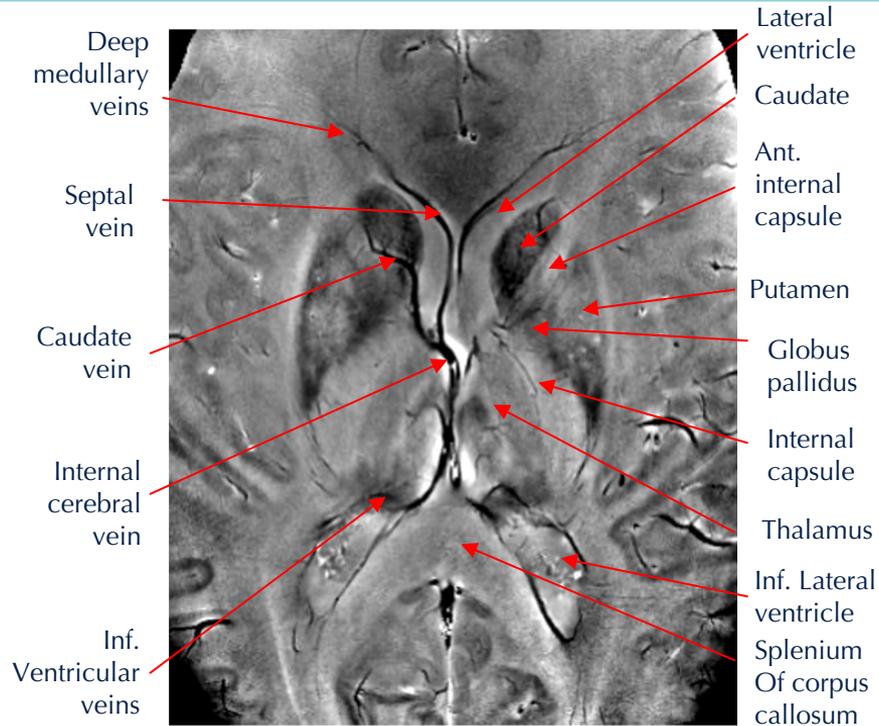
VEINS   **GRE Phase Image**   **STRUCTURES**
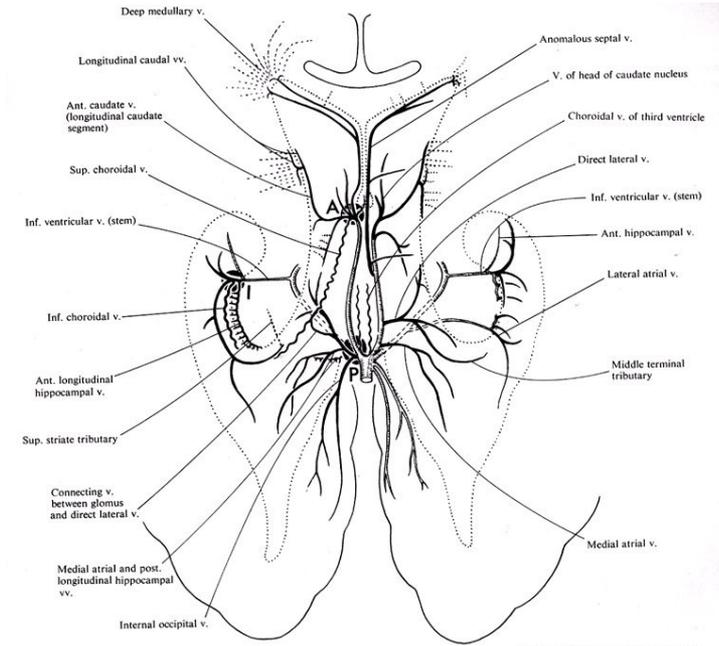
from G.Salamon.Y.P. Huang Radiologic Anatomy of the brain. Springer verlag 1976

Susceptibility MRI – Iron
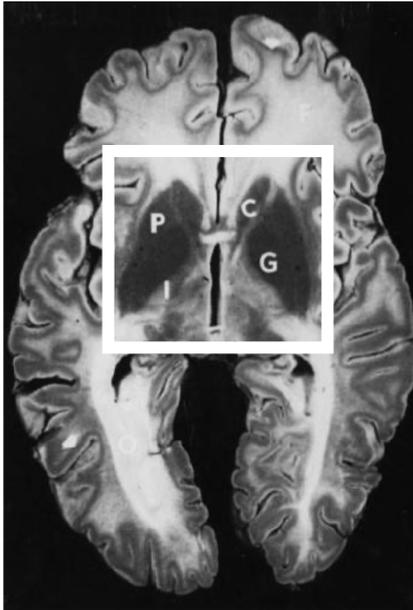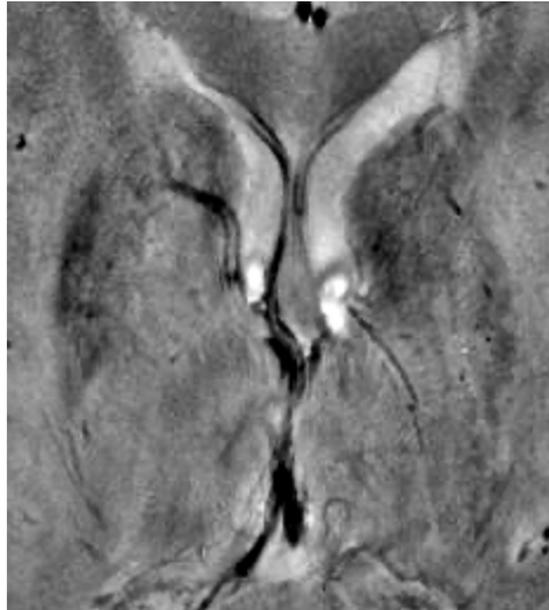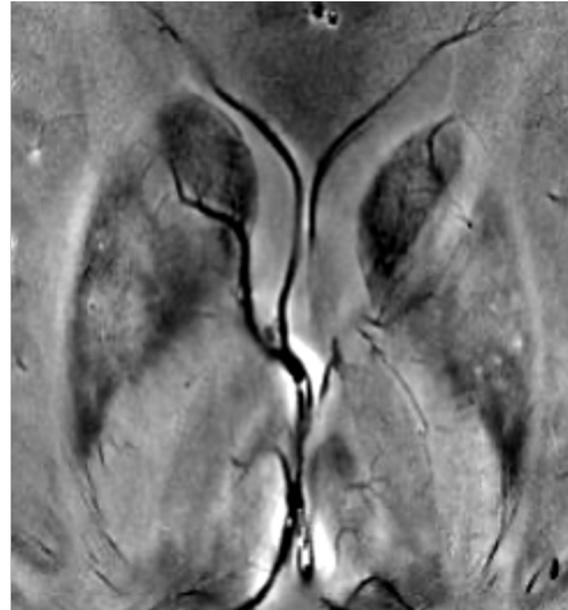
Iron Perl's Stain **GRE Magnitude Image** **GRE Phase Image**

Susceptibility MRI – Iron

Iron Perl's Stain   **GRE Magnitude Image**   **GRE Phase Image**

Susceptibility MRI – Iron

Drayer Am J Roentgenol 1986
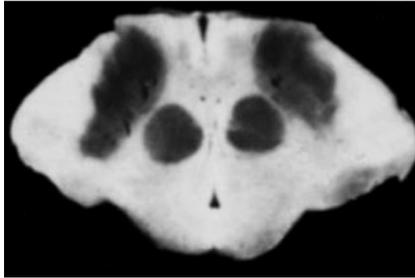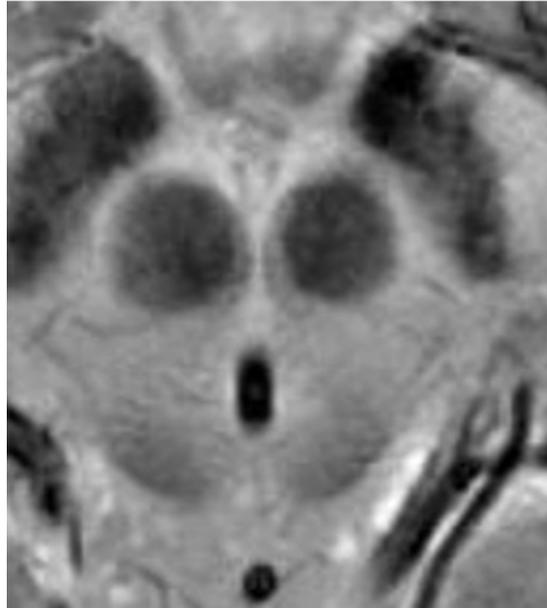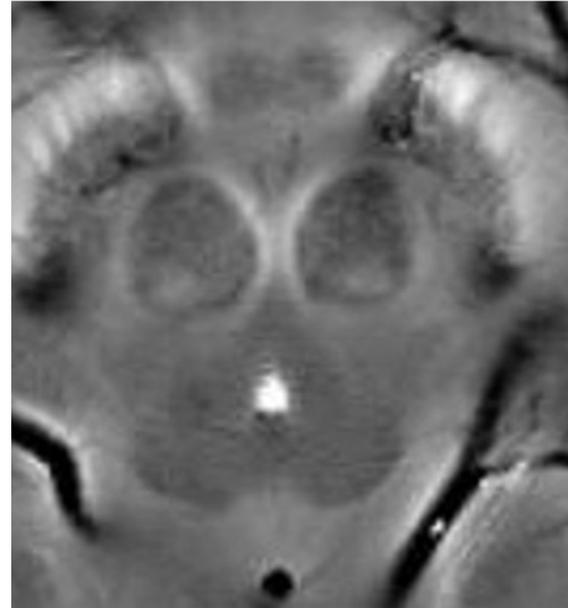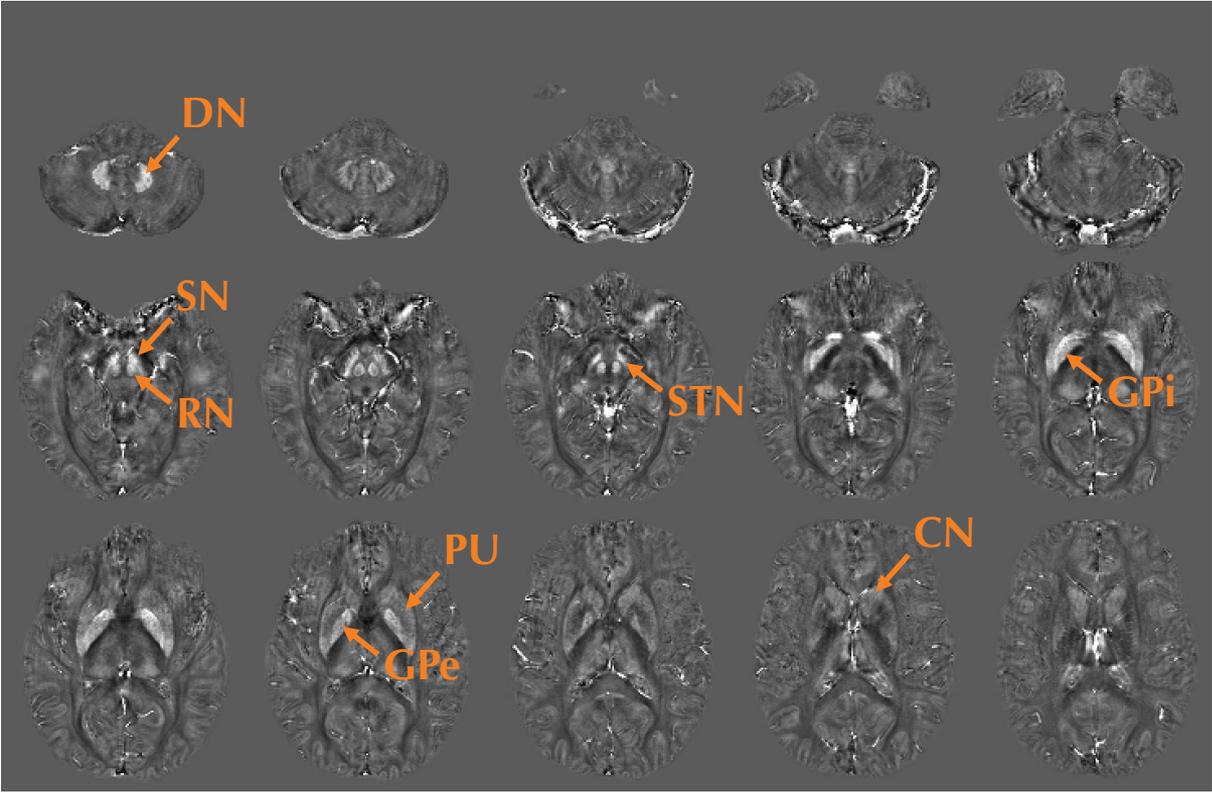
Susceptibility MRI – Calcification

Magnitude   Phase

Magnitude   Phase

Paramagnetic
⇩
Hemorrhage from trauma

Diamagnetic
⇩
Calcification in glioma
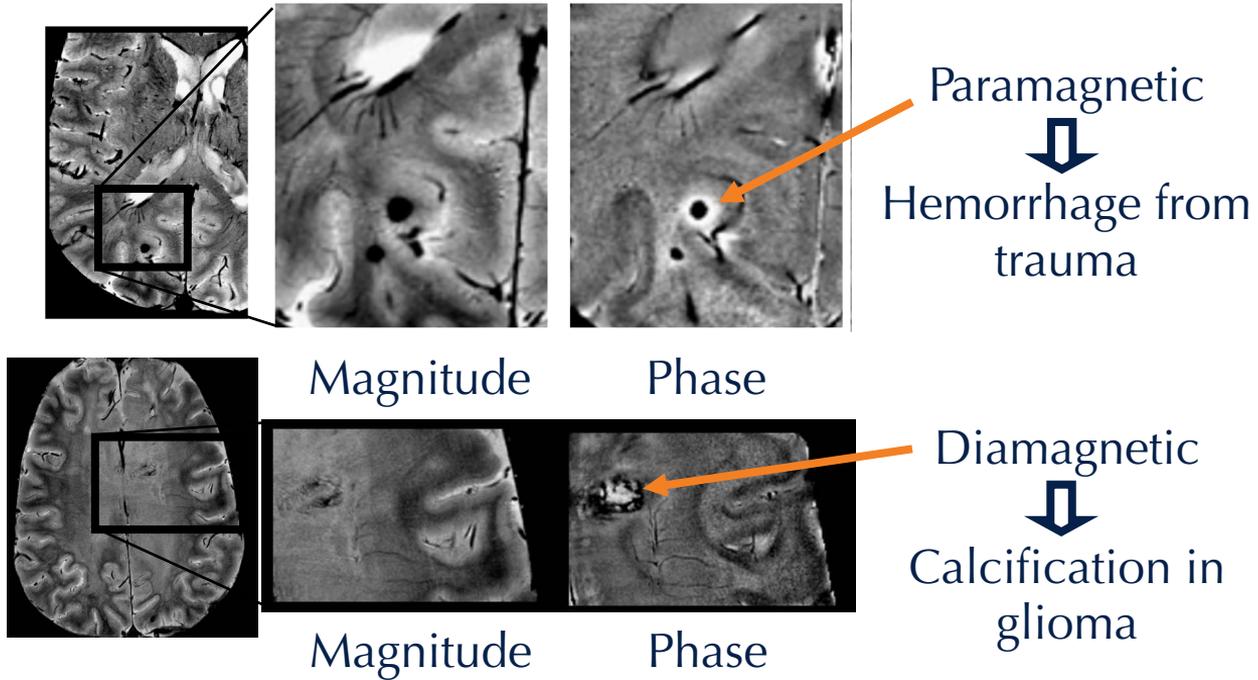
Susceptibility MRI – Iron

Phantom validation

Tissue validation

Deh K et al. Sci Rep 2020
Lankammer C et al. NeuroImage 2012

20   Susceptibility and Conductivity MRI
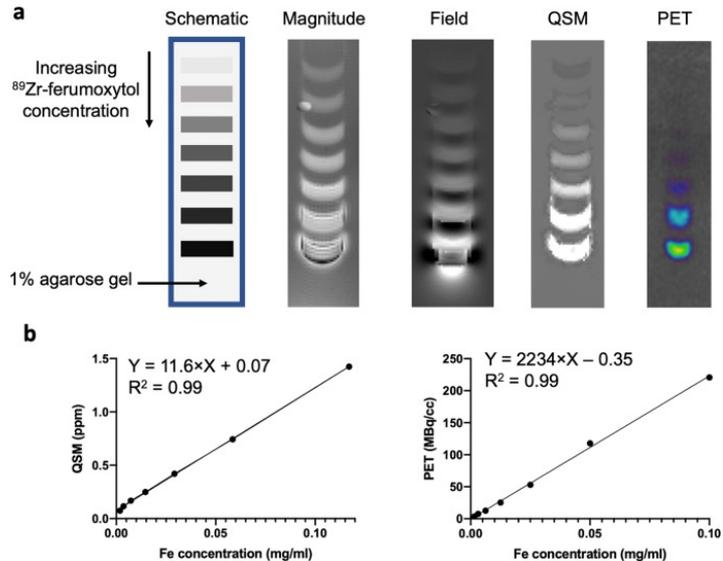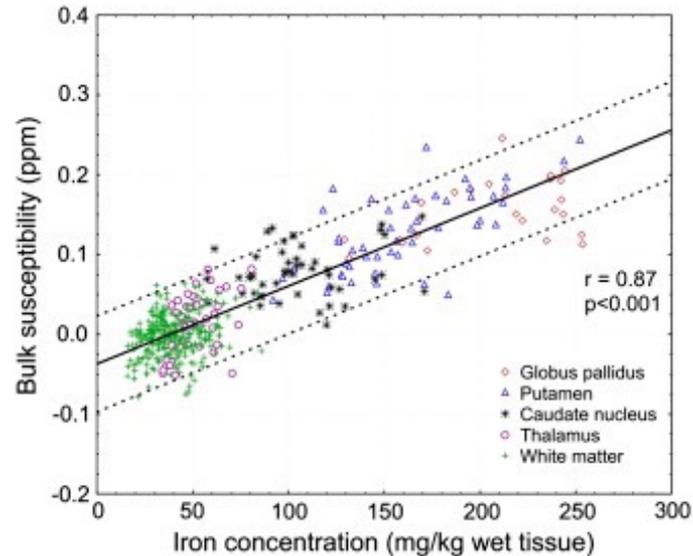
Susceptibility MRI – Signal model
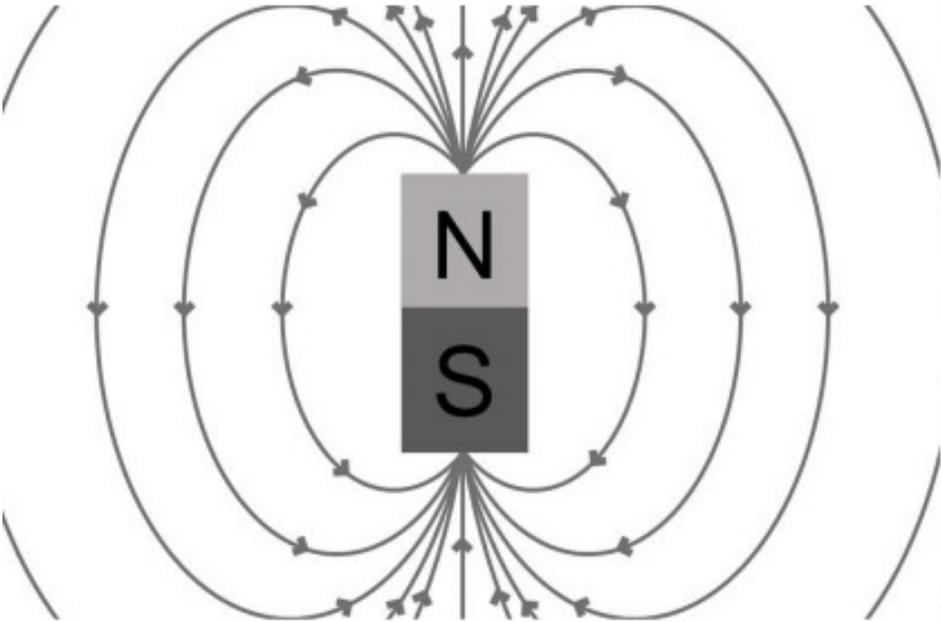
a)
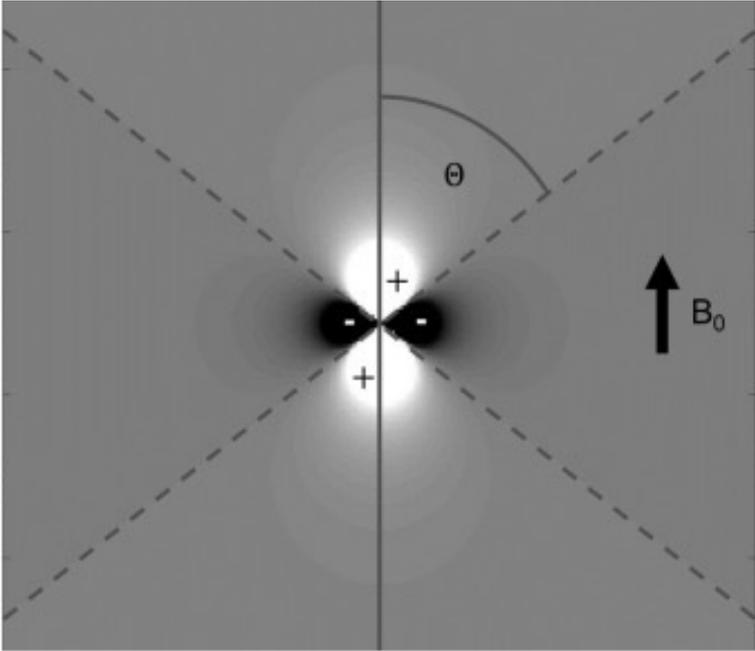
b)

Susceptibility MRI – Signal model

Ferumoxytol phantom
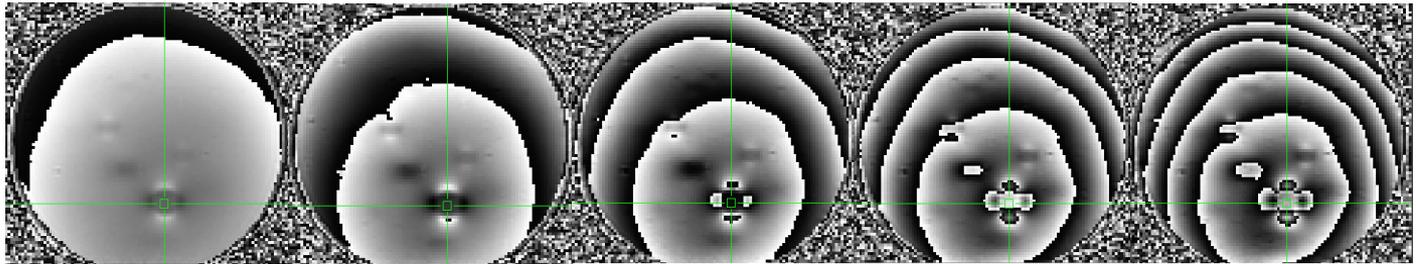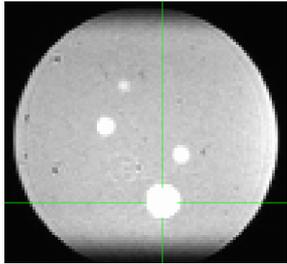
5.84 ms ⟶ 40.56 ms

Susceptibility MRI – Signal model
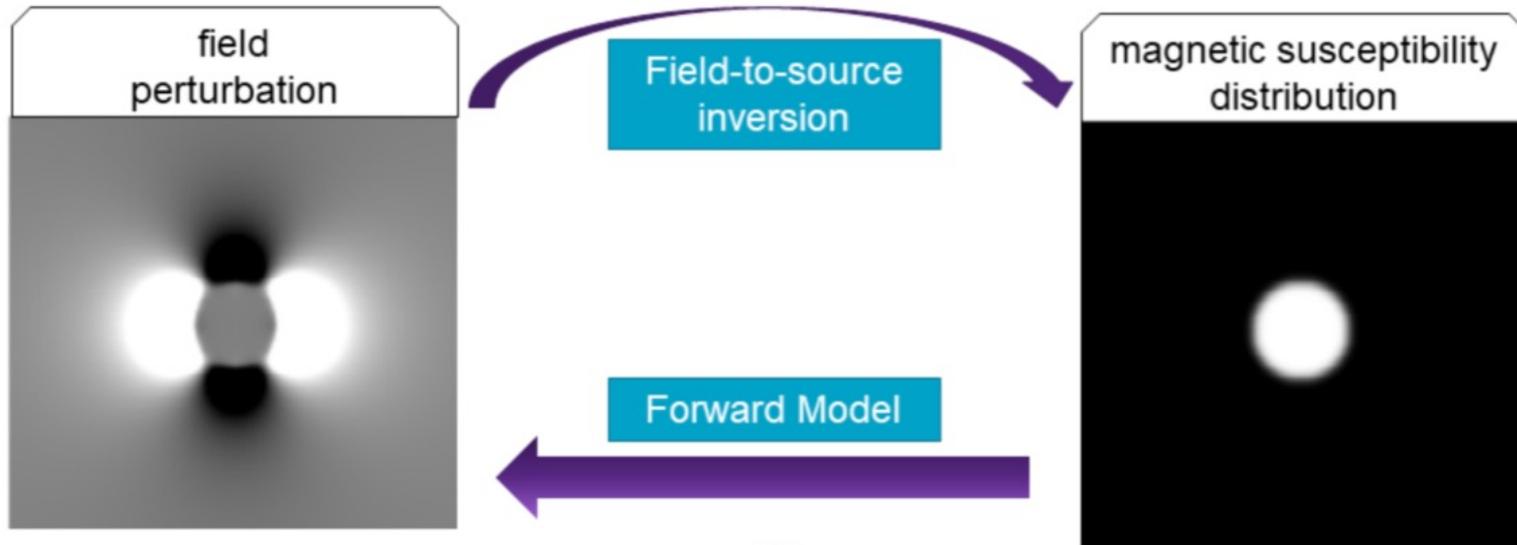
field
perturbation

Field-to-source
inversion

magnetic susceptibility
distribution

Forward Model

$$\Delta B_{\text{int}}(\overrightarrow{r}) = B_0 \cdot \int_{-\infty}^{\infty} \widetilde{\chi}(\overrightarrow{r'}) \cdot d_z(\overrightarrow{r} - \overrightarrow{r'}) d^3 \overrightarrow{r'}$$

Review: Deistung et al. NMR Biomed 2017; Schweser et al. Z Med Phys 2016

Susceptibility MRI – Signal model

$$\Delta B_{\text{int}}(\overrightarrow{r}) = B_0 \cdot \int_{-\infty}^{\infty} \widetilde{\chi}(\overrightarrow{r'}) \cdot d_z(\overrightarrow{r} - \overrightarrow{r'}) d^3\overrightarrow{r'}$$
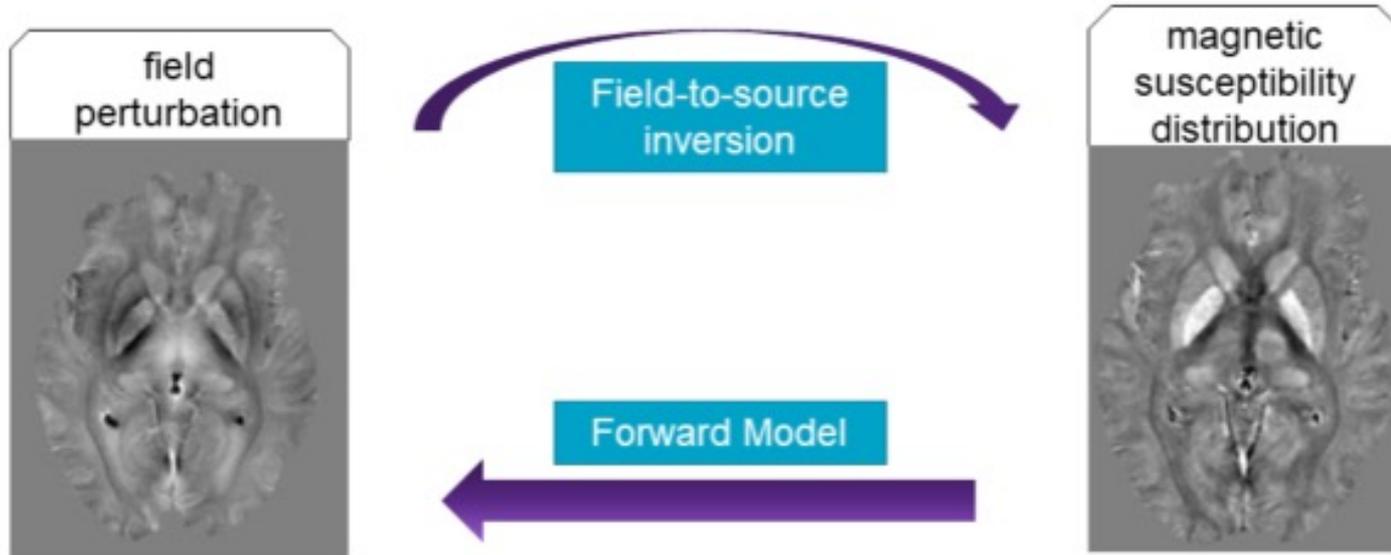
Review: Deistung et al. NMR Biomed 2017; Schweser et al. Z Med Phys 2016

Susceptibility MRI – Signal model

$$\Delta B(\vec{r}) = B_0 \int_{-\infty}^{\infty} \chi(\vec{r'}) d(\vec{r} - \vec{r'}) d^3\vec{r'}$$

$$d(\vec{r}) = \frac{1}{4\pi} \frac{3\cos^2(\theta) - 1}{|\vec{r}|^3}$$

$$\Delta B = B_0(\chi \otimes d)$$

Known

Measured

Unknown

FT
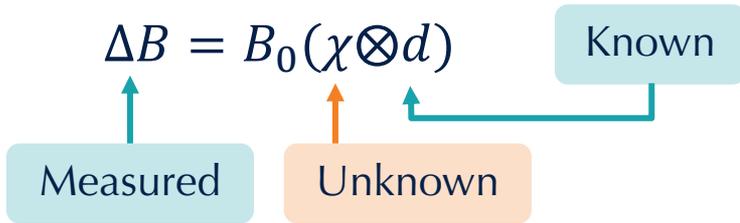
$$\Delta B(\vec{k}) = B_0[\chi(\vec{k}) d(\vec{k})]$$

$$d(\vec{k}) = \frac{1}{3} - \frac{k_z^2}{|\vec{k}|^2}$$

$$\Delta B(\vec{k}) = B_0[\chi(\vec{k})d(\vec{k})]$$

$$d(\vec{k}) = \frac{1}{3} - \frac{k_z^2}{|\vec{k}|^2}$$

$$FT(\quad) = B_0[FT(\quad) * \quad]$$
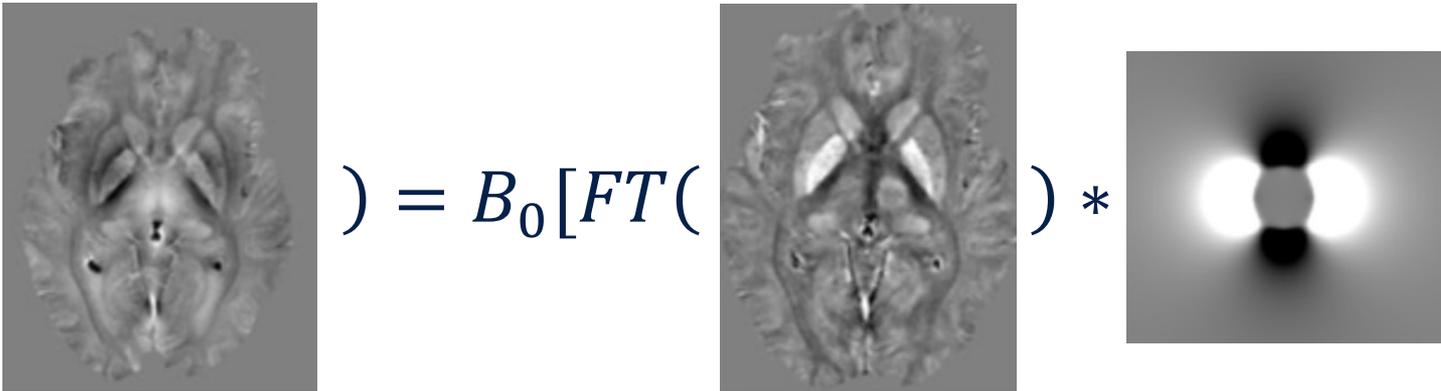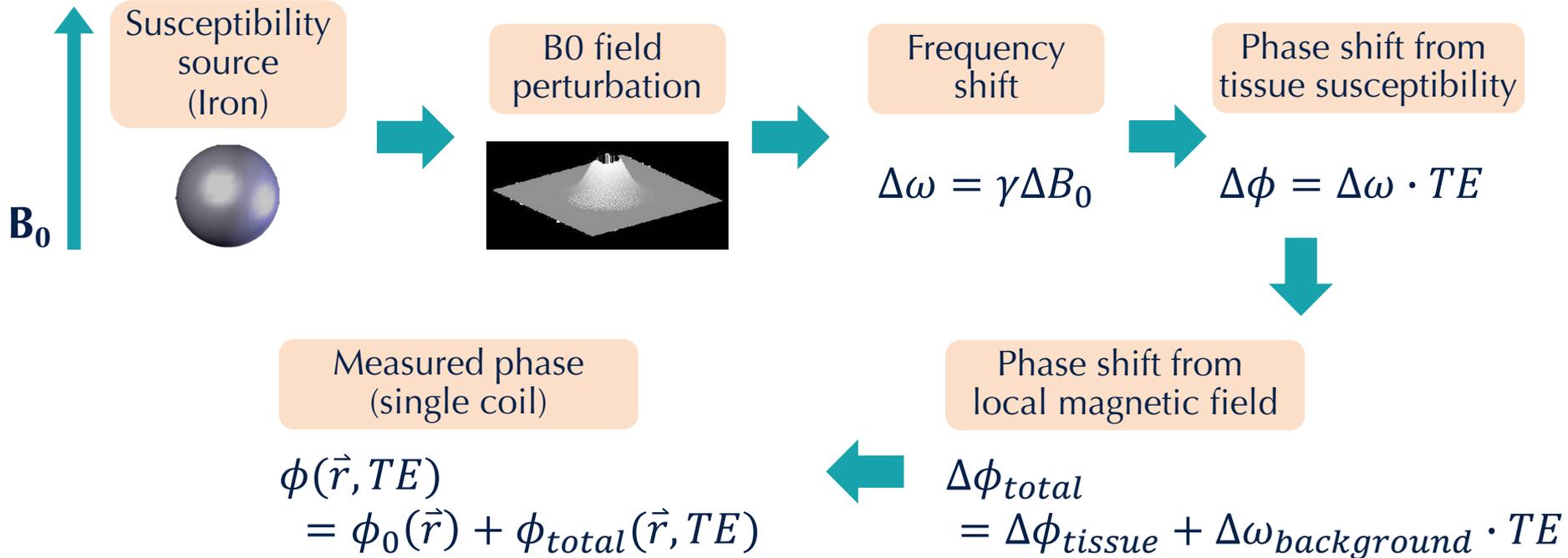
Susceptibility MRI – Signal model

Susceptibility source (Iron) → B0 field perturbation → Frequency shift → Phase shift from tissue susceptibility

B_0

$$\Delta\omega = \gamma\Delta B_0$$

$$\Delta\phi = \Delta\omega \cdot TE$$

Measured phase (single coil) ← Phase shift from local magnetic field

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

$$\Delta\phi_{total} = \Delta\phi_{tissue} + \Delta\omega_{background} \cdot TE$$

Susceptibility MRI – Processing

B_0

Susceptibility source (Iron)

B0 field perturbation

Frequency shift

$$\Delta\omega = \gamma\Delta B_0$$

Phase shift from tissue susceptibility

$$\Delta\phi = \Delta\omega \cdot TE$$

Measured phase (single coil)

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

Phase shift from local magnetic field

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QSM - Processing

1. Image acquisition

T_2^*-weighted sequence

2. Phase unwrapping

3. Background field removal

4. Dipole inversion

QSM - Processing

1. Image acquisition

T_2*-weighted sequence

Coil magnitude

Magnitude

Phase combination approaches

1 echo, No reference scan	• Scalar phase matching [1] • Adaptive combine [2] • Virtual Reference Coil [3]
1 echo, Reference scan	• Roemer/SENSE [4] • COMPOSER [5]
Multiple echoes	• SVD [6] • Solve for ΔB_0 via phase difference [7] • Solve for φ_0^C: ASPIRE [8]

[1] Hammond et al. NI 2008
[2] Walsh et al. MRM 2000
[3] Parker et al. MRM 2014
[4] Roemer et al. NI 1990
[5] Robinson et al. MRM 2017
[6] Khabipova et al. NI 2015
[7] Bernstein et al. MRM 1994
[8] Eckstein et al. MRM 2018
Review: Robinson et al. NMR Biomed 2017
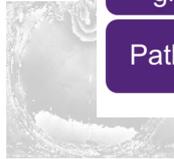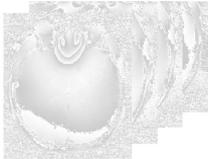
Background field correction methods & assumptions

no sources close to boundaries	• SHARP (Schweser et al. NI 2011) • V-SHARP (Li et al. NI 2011)
no harmonic internal and boundary fields	• LBV (Zhou et al. NMR Biomed 2014)
no implicit boundary assumption	• RESHARP (Sun et al. MRM 2014) • SHARQnet (Bollmann et al. Z Med Phys 2018)

Review: Schweser et al. NMR Biomed 2017

Unwrapping techniques

Laplacian	• differentiable operator applied to the unwrapped phase can produce the same result on the wrapped phase -> Laplacian (Schofield and Zhu, Opt. Lett. 2003) • + fast & robust • - introduces background phase
Region-growing	• Identify discontinuities between regions • PRELUDE (Jenkinson MRM 2003) can take a while to compute for highly wrapped data • SEGUE (Karsa et al. TMI 2019) similar accuracy to PRELUDE, but faster
Path Based	• 3D voxel-by-voxel unwrapping guided by the quality of voxel connections • BEST PATH (Abdul-Rahman et al. AO 2007) • ROMEO (Dymerska et al. MRM 2021)

Review: Robinson et al., NMR Biomed 2017

4. Dipole inversion

Dipole inversion methods & assumptions

multiple orientations	• COSMOS (Liu et al. MRM 2009) • STI (Liu MRM 2010) • analytical solutions, but not practical
inverse filtering	• TKD (Shmueli et al. MRM 2009) • fast, but need parameter tweaking
iterative methods	• LSQR (Li et al. NI 2015) • MEDI (Liu et al. MRM 2013) • slow, need parameter tweaking
agnostic deep learning	• QSMnet (Yoon et al. 2018) • DeepQSM (Bollmann et al. NI 2019) • fast, but fragile
hybrid methods	• FINE (Zhang et al. NI 2020) • Variational Networks (Lai et al. arXiv 2020) • Deep learning priors + data consistency constraints

Reviews: Schweser et al. NMR Biomed 2017; Jung et al. NMR Biomed 2020

3. Background field removal

2. Phase unwrapping

An illustrated comparison of processing methods for MR phase imaging and QSM: combining array coil signals and phase unwrapping

Simon Daniel Robinson[a]*, Kristian Bredies[b], Diana Khabipova[c,d], Barbara Dymerska[a], José P. Marques[c,d] and Ferdinand Schweser[e,f]

QSM - Coil Combination

Coil magnitude

Magnitude

Coil combination

Coil phase

Phase

Single Channel Phase

Single Channel Unwrapped Phase

Single Channel QSM

QSM

QSM - Coil Combination

Coil magnitude

Magnitude
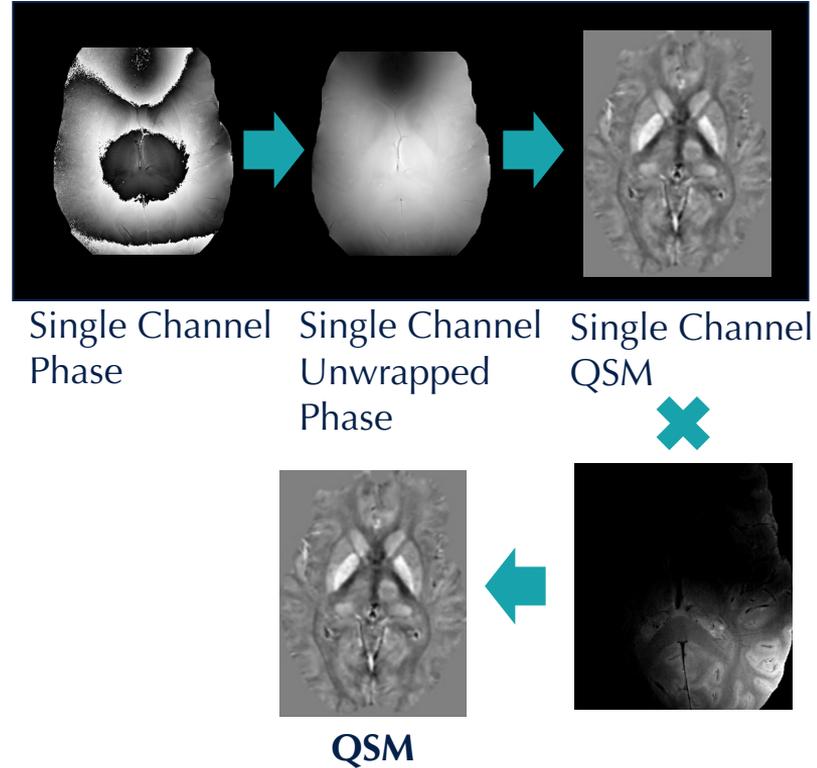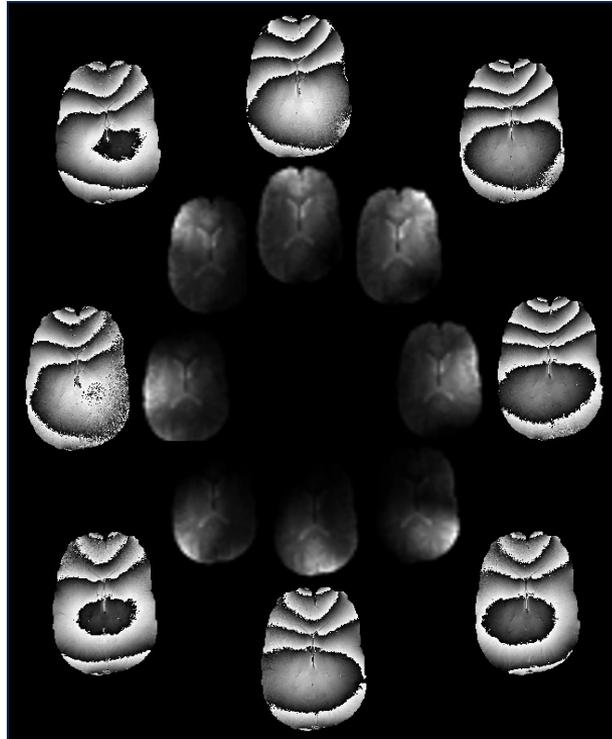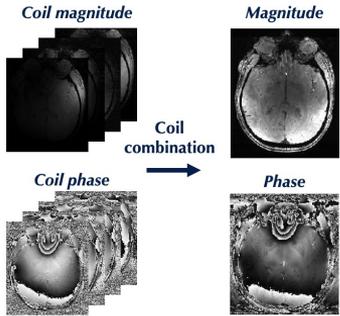
Coil combination
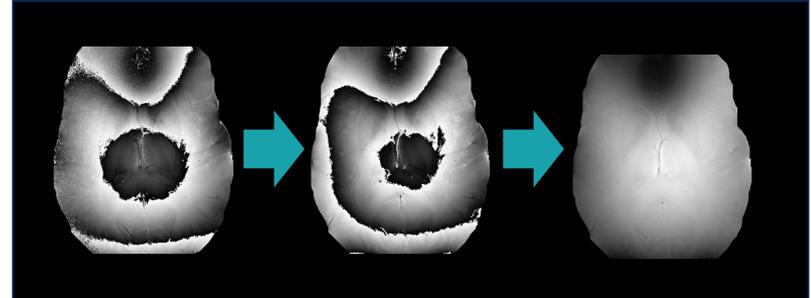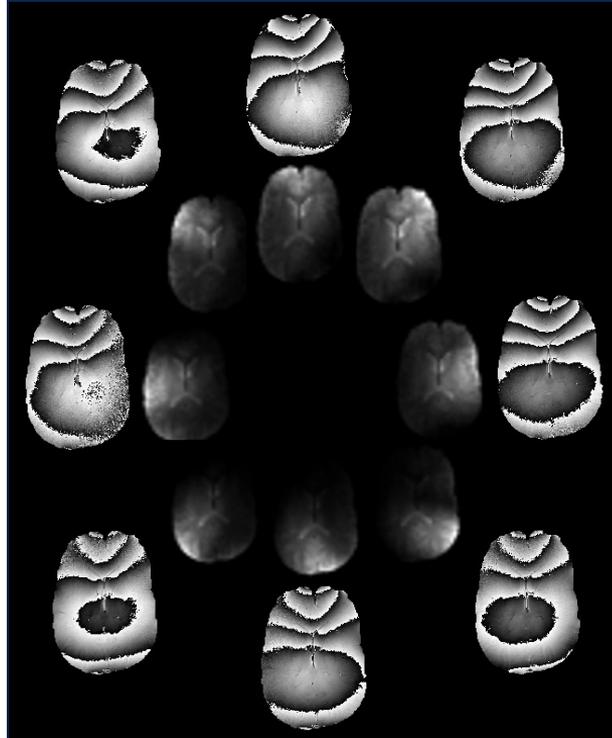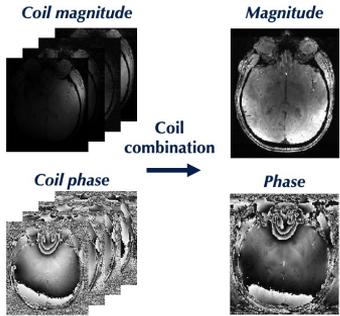
Coil phase

Phase

Single Channel Phase

Multi Channel Phase

Unwrapped Phase

QSM

QSM - Coil Combination

Coil magnitude

Magnitude

Coil combination

Coil phase
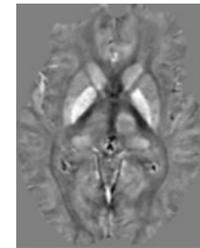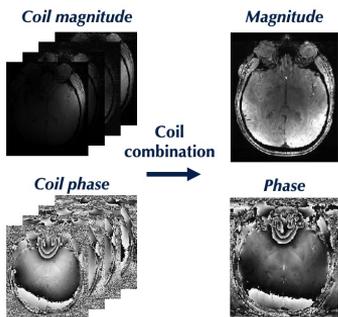
Phase

Measured phase
(single coil)

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

Transceive phase

spatially varying phase
offsets exist between
receive coils

Römer

No Correction

Robinson S et al., NMR Biomed, 2015
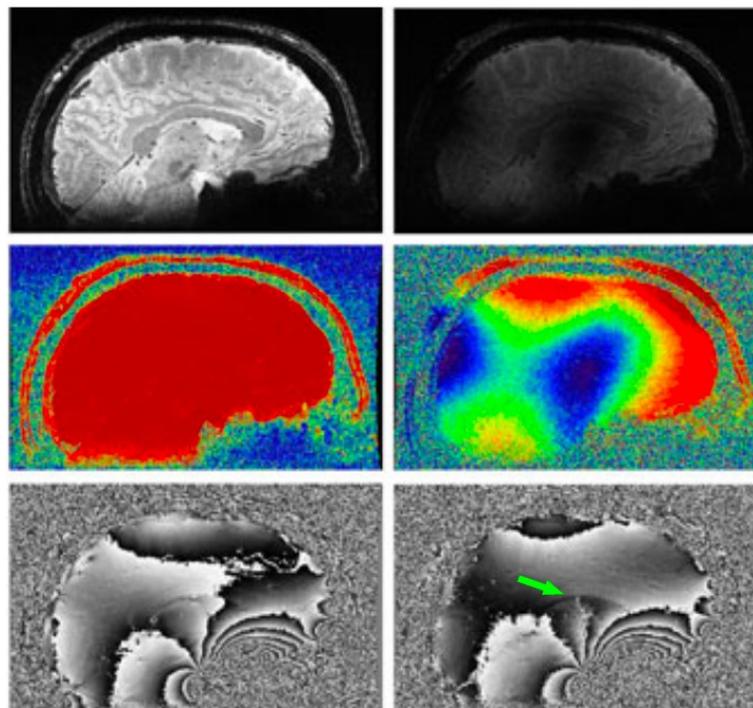
QSM - Coil Combination
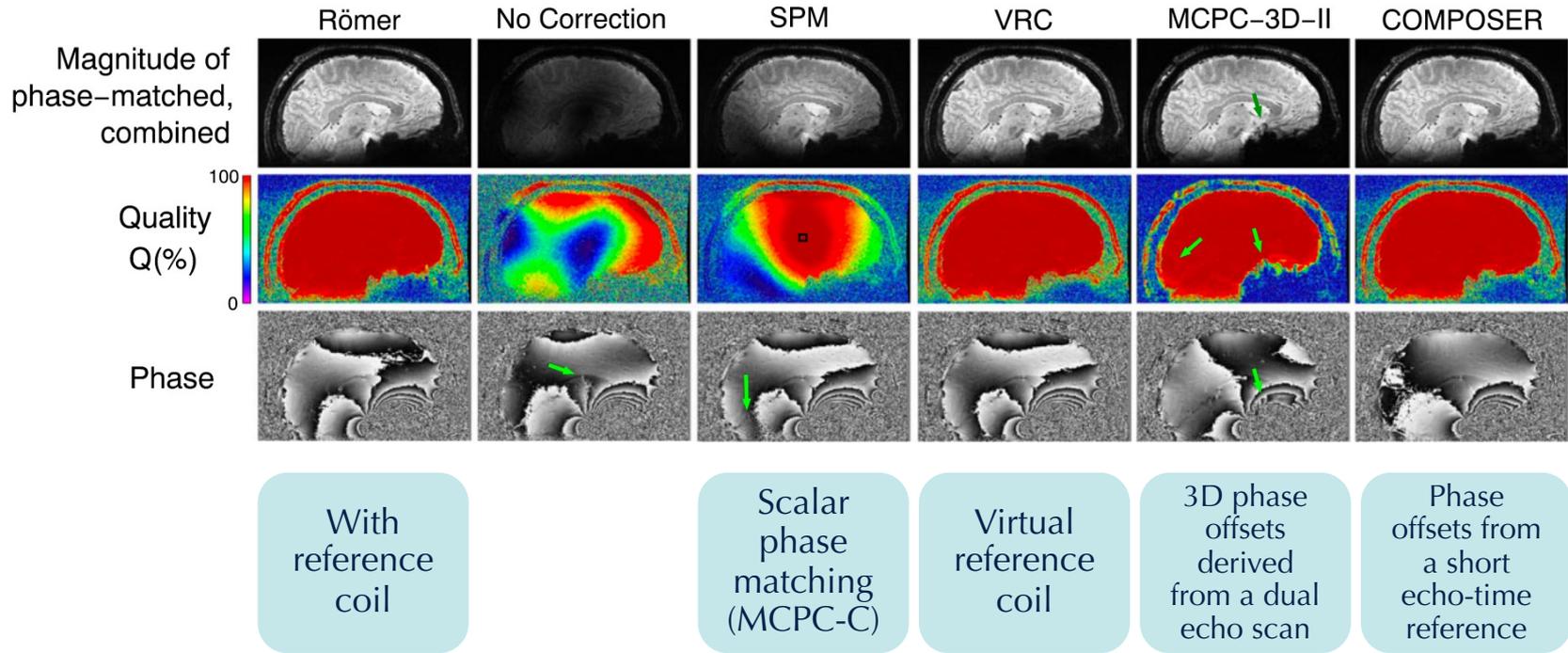
$$\varphi_j^0(\vec{r}) = \frac{\varphi_j(\vec{r}, TE_2)TE_1 - \varphi_j(\vec{r}, TE_1)TE_2}{TE_1 - TE_2}.$$

QSM - MPCP-3D

Steps:

- unwrap each echo phase
- create 3D phase offset map for each coil using each unwrapped echo
- smooth with 5x5 median filter
- subtract 3D phase offset map from phase image of each channel
- weighted mean

$$\varphi_j^0(\vec{r}) = \frac{\varphi_j(\vec{r}, TE_2)TE_1 - \varphi_j(\vec{r}, TE_1)TE_2}{TE_1 - TE_2}.$$

Advantages:

- works where there is no signal overlap between receivers
- no need for reference coil
- also works using a separate low-resolution scan

QSM - Phase unwrapping
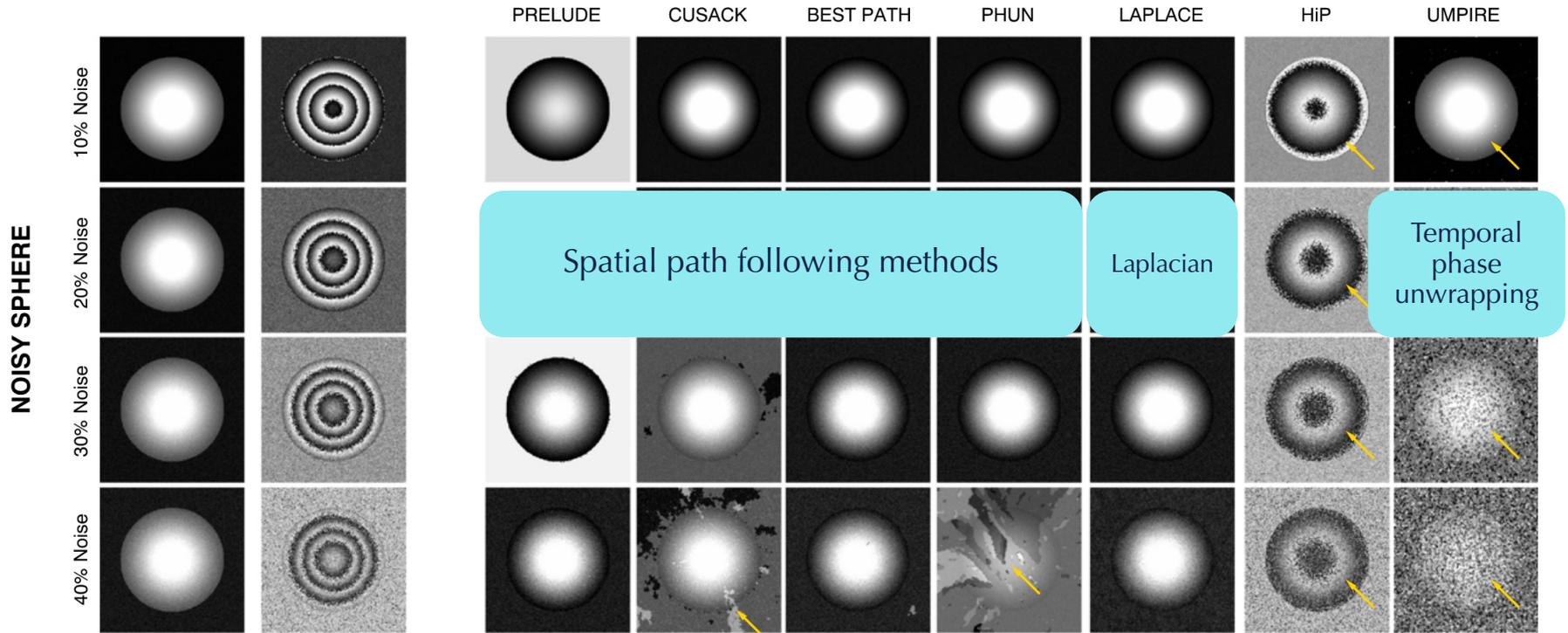
Phase

Total field

Phase unwrapping

5.84 ms ——————————————→ 40.56 ms

Spatial path following methods

Laplacian

Temporal phase unwrapping

QSM - Background field removal
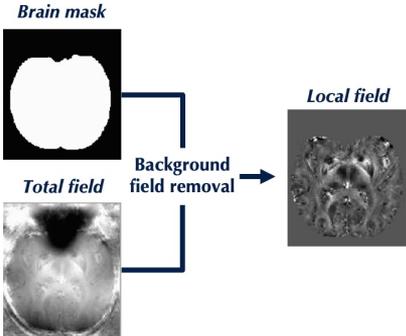
Local field

Background field removal

Total field

High-Precision Mapping of the Magnetic Field Utilizing the Harmonic Function Mean Value Property

Lin Li and John S. Leigh

Department of Biochemistry and Molecular Biophysics, and Metabolic Magnetic Resonance Research & Computing Center, Department of Radiology, University of Pennsylvania, Philadelphia, Pennsylvania 19104

The spatial distributions of the static magnetic field components and MR phase maps in space with homogeneous magnetic susceptibility are shown to be harmonic functions satisfying Laplace's equation. A mean value property is derived and experimentally confirmed on phase maps: the mean value on a spherical surface in space is equal to the value at the center of the sphere. Based on this property, a method is implemented for significantly improving the precision of MR phase or field mapping. Three-dimensional mappings of the static magnetic field with a precision of $10^{-11} \sim 10^{-12}$ T are obtained in phantoms by a 1.5-T clinical MR scanner, with about three-orders-of-magnitude precision improvement over the conventional phase mapping technique. *In vivo* application of the method is also demonstrated on human leg phase maps. © 2001 Academic Press

Key Words: field mapping; harmonic function; mean value property; phase; SMV.

aging, we generate field maps with high precision up to $10^{-11} \sim 10^{-12}$ T. Such a measurement precision is comparable with that of a superconducting quantum interference device (SQUID) for the magnetic field measurement (*17*). Feasibility with *in vivo* applications is also demonstrated.

THEORY

In free space or regions without susceptibility heterogeneity and no macroscopic currents, all the components of the static magnetic field **H** satisfy Laplace's equation, i.e., $\nabla^2 \mathbf{H}_i = 0$, $i = x, y, z$, or $\nabla^2 \mathbf{H} = 0$, which can be easily derived by setting the temporal derivative of the magnetic field in the electromagnetic wave equation (*18*) to zero. Therefore, local magnetic induction (*4, 19*) experienced by a nucleus, $(1 + \chi/3)H$, also satisfies Laplace's equation. Since the spatial

VSHARP

RESHARP

HARPERELLA

PDF

QSM - Dipole Inversion

Dipole inversion
Local field → QSM

$$FT(\quad) = B_0[FT(\quad) * \quad]$$
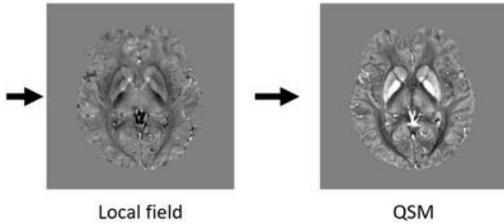
ill-posed inversion problem
Noise amplification near the zero cone surfaces

COSMOS: calculation of susceptibility using multiple orientation sampling

Liu T, MRM, 2008
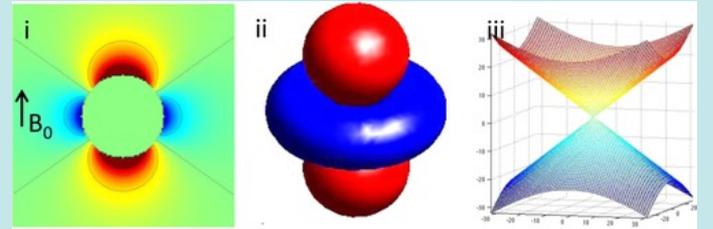
QSM - Dipole Inversion

COSMOS: calculation of susceptibility using multiple orientation sampling

Liu T, MRM, 2008
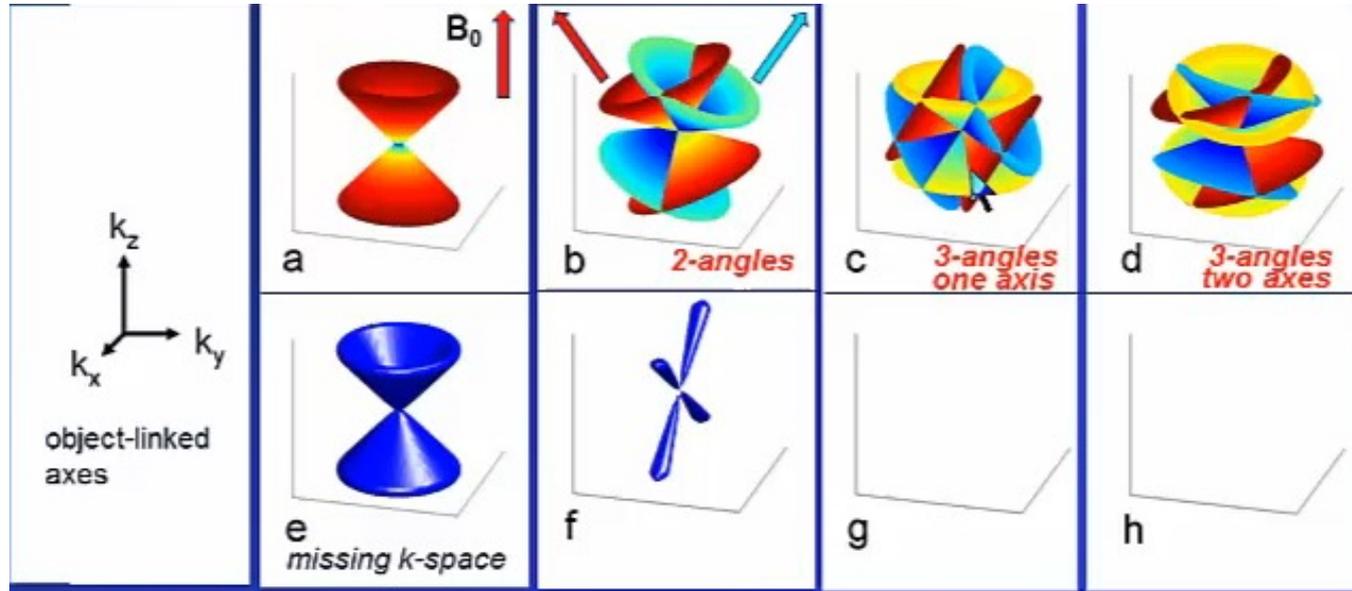
QSM - Dipole Inversion

COSMOS:
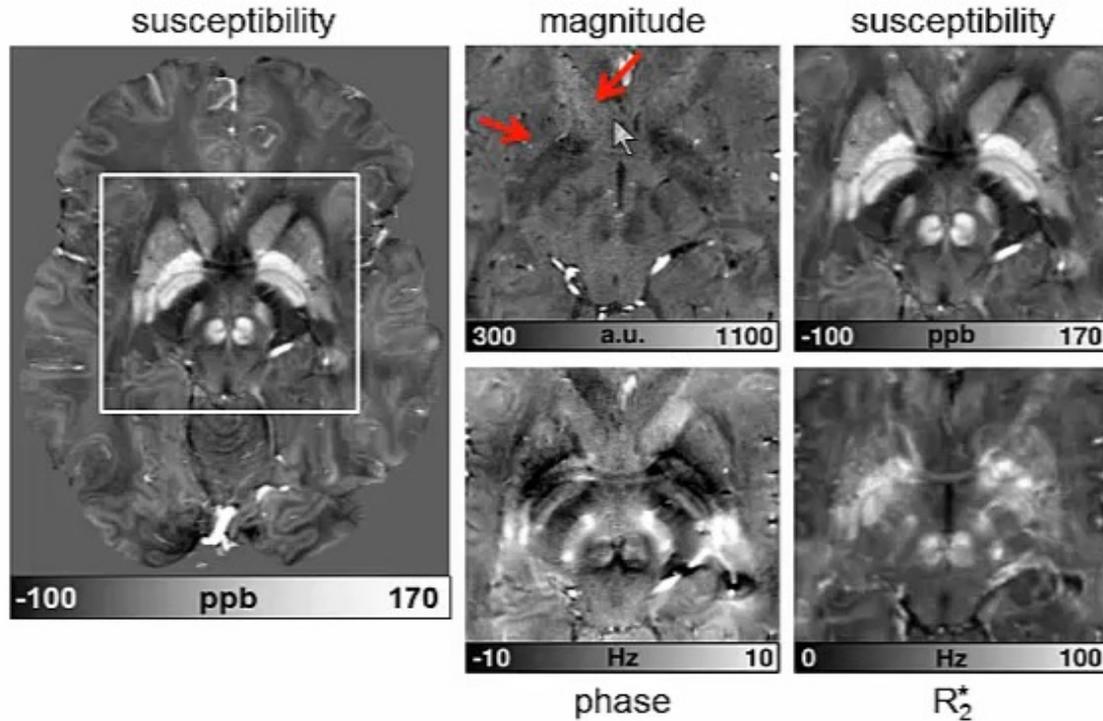
Deistung, A et al. NeuroImage 2013, 65, 299–314.

QSM Dipole Inversion: iLSQR

iLSQR: iterative method solving least square using the orthogonal and right triangular decomposition

Dipole kernel

$$\psi(\mathbf{k}) = D_2(\mathbf{k}) \cdot \chi(\mathbf{k})$$

Field perturbation

Susceptibility distribution

1st order derivative

$$\psi'(\mathbf{k}) + \left[2\left(k_x^2 + k_y^2\right)k_z/k^4\right] \cdot \chi(\mathbf{k}) - D_2(\mathbf{k}) \cdot \chi'(\mathbf{k}) = 0$$

$$D_3(\mathbf{k}) \cdot \chi(\mathbf{k}) + D_2(\mathbf{k}) \cdot FT[i \cdot r_z \chi(\mathbf{r})] = FT[i \cdot r_z \psi(\mathbf{r})]$$

Where: $D_3(\mathbf{k}) = (k_x{}^2 + k_y{}^2)k_z/\pi k^4$

$$\chi\left(\mathbf{k}\right) = D_2(\mathbf{k})^{-1} \cdot \psi\left(\mathbf{k}\right), \quad \text{when } D_2(\mathbf{k}) \geq \varepsilon$$

$$\chi\left(\mathbf{k}\right) \approx D_3(\mathbf{k})^{-1} \cdot FT[ir_z\psi(\mathbf{r})], \quad \text{when } D_2(\mathbf{k}) < \varepsilon$$

QSM Dipole Inversion: iterative inversion methods with regularization

Recon problem

$$\operatorname{argmin}_\chi \frac{1}{2} \left\| W \left(F^H D F \chi - \Phi \right) \right\|_2^2 + \alpha \Omega \left(\chi \right)$$

Data consistency term

Regularization term

Nonlinear variant

$$\operatorname{argmin}_\chi \frac{1}{2} \left\| W \left(e^{iF^H D F \chi} - e^{i\Phi} \right) \right\|_2^2 + \alpha \Omega \left(\chi \right)$$

Method	Data consistency term	Regularization term
STAR-QSM (STreaking Artifact Reduction for QSM)	Linear L2-norm	Total variation
FANSI (FAst Nonlinear Susceptibility Inversion)	Nonlinear L2-norm	Total variation
HD-QSM (Hybrid Data fidelity)	Linear L1+L2-norm	Total variation
MEDI (Morphology Enabled Dipole Inversion)	Linear L1-norm	L1 norm of morphologically weighted gradients

Parameter optimization

(A) $\lambda = 10^{-6}$
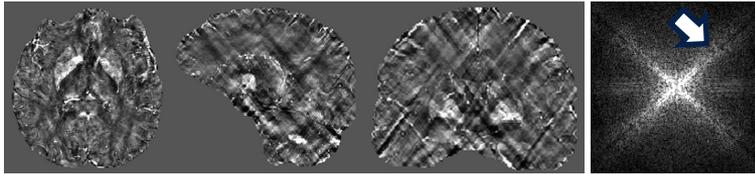
$\lambda = 10^{-4.2}$
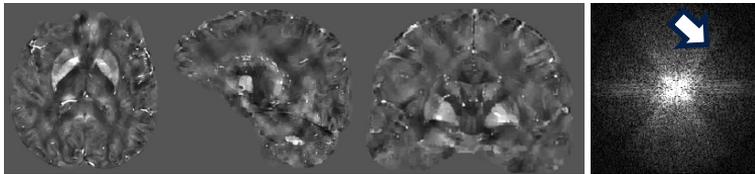
$\lambda = 10^{-3.5}$
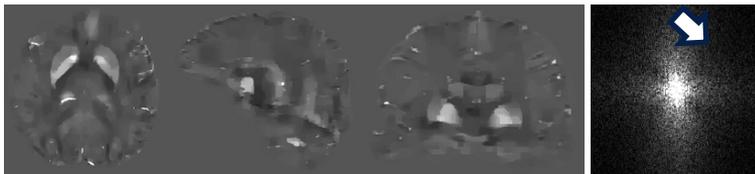
(B)

Frequency Equalization Plot

— ○ — ζ_{23}
♦ Optimal λ

ζ_{23} function

Regularization weight

■ M2
■ M3

$$\xi_{ij} = \left(\frac{A_i - A_j}{A_i + A_j}\right)^2$$

A_i: Mean power in M_i

Yao J et al., NeuroImage, 2022
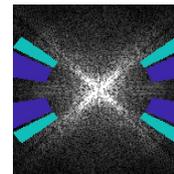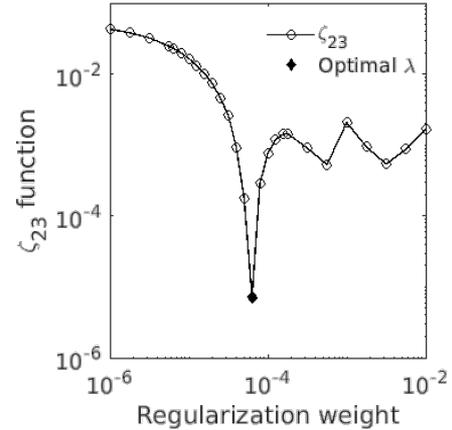
QSM Dipole Inversion: single step methods

QSIP   Quantifying Susceptibility by Inversion of a Perturbation model

$$\chi_1^* = arg\ \min_{\chi_1} \left[\lambda_1 |W \circ (\Delta B - \Delta (K_s * \chi_1))|_1 + \lambda_2 |M \circ (B - (K_s * \chi_1 + B_e))|_2^2 + \lambda_3 |M^C \circ (\chi_1 + \chi_0/\delta)|_2^2 \right]$$

Simultaneously estimating the external susceptibility outside the brain

SSTV, SSTGV   Single Step QSM with Total Variation / Total Generalized Variation penalties

$$\min_{\chi} \frac{1}{2} \sum_i \left| \left| M_i F^{-1} H_i DF\chi - M_i F^{-1} H_i F\Psi(\phi) \right| \right|_2^2 + R(\chi)$$

Perform VSHARP background field removal and dipole inversion in a single step

QSM Dipole Inversion: deep learning-based methods

QSMGAN

xQSM

Single step

iQSM

QSMnet+

Chen Y, NeuroImage, 2020;
Yoon J, NeuroImage, 2018;
Jung W, NeuroImage, 2020

Gao Y, NMR in Biomed, 2020
Gao Y, NeuroImage, 2022

QSM: Anisotropic Susceptibility

$$\mathbf{M} = \underline{\underline{\chi}} \mathbf{H}$$

$$\underline{\underline{\chi}} = \begin{bmatrix} \chi_{11} & 0 & 0 \\ 0 & \chi_{22} & 0 \\ 0 & 0 & \chi_{33} \end{bmatrix}$$
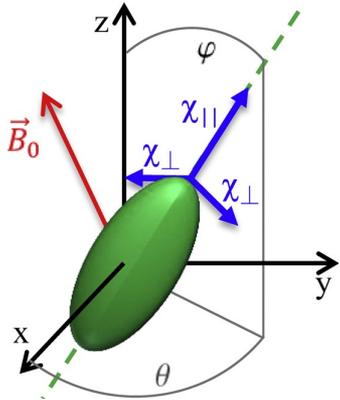
Susceptibility Anisotropy

0.2   -0.2
(ppm)

0.05   -0.05
(ppm)
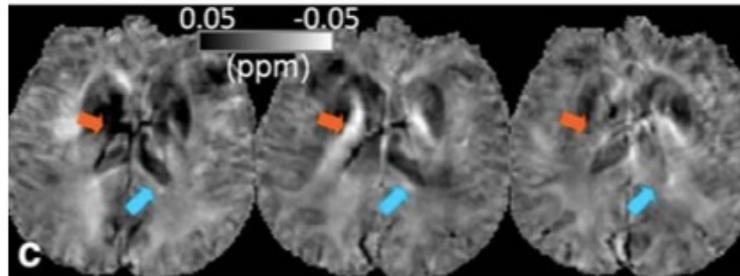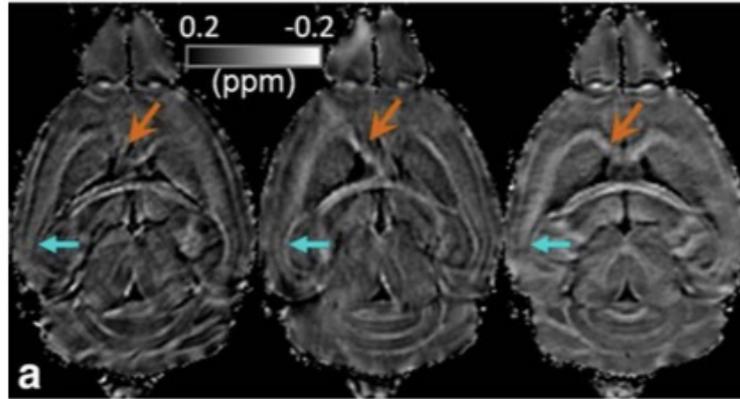
Susceptibility Tensor Imaging

cc
hc
ac

a   b   c   d

Li W et al., NMR Biomed, 2017

QSM: Susceptibility source decomposition

paramagnetic

$\chi > 0$

Air

Iron, gadolinium,
copper, manganese

Deoxyhemoglobin

Water susceptibility

Most of the biological tissues

Myelin

Calcification

$\chi < 0$

diamagnetic

A

Gradient echo signal = Signal of paramagnetic component + Signal of diamagnetic component + Signal of *neutral component

$$S(t,r) = C_+(r)e^{-R_{2,+}^*(r)t}e^{i\phi_+(r)t} + C_-(r)e^{-R_{2,-}^*(r)t}e^{i\phi_-(r)t} + C_0(r)e^{-R_{2,0}^*(r)t}$$

B
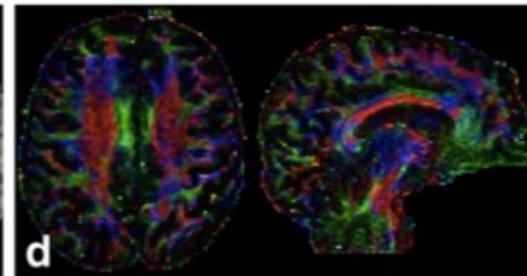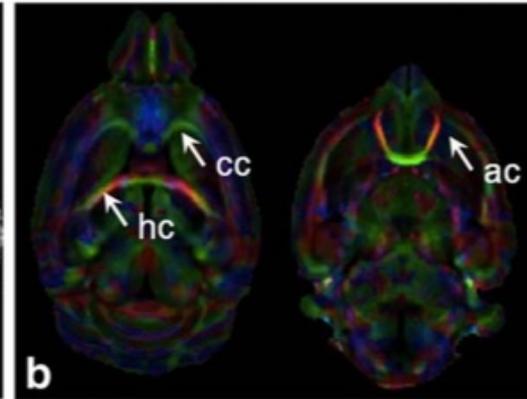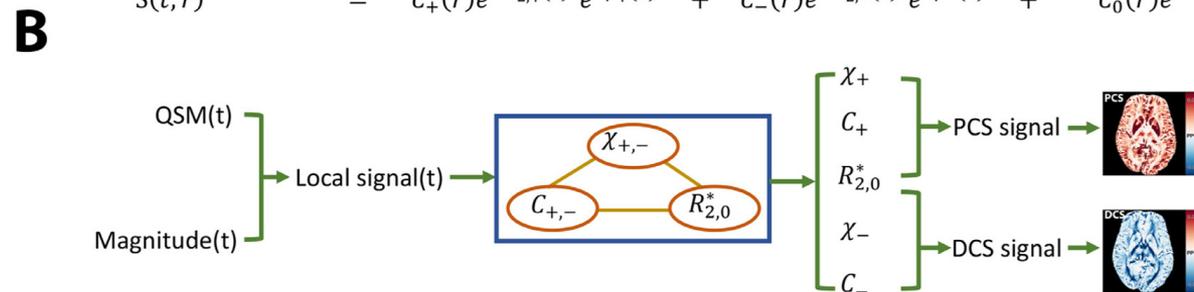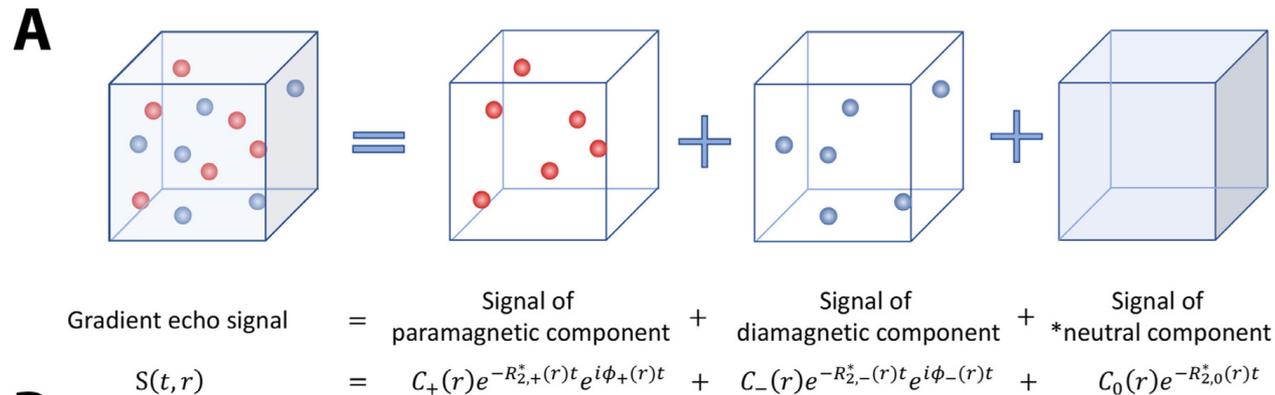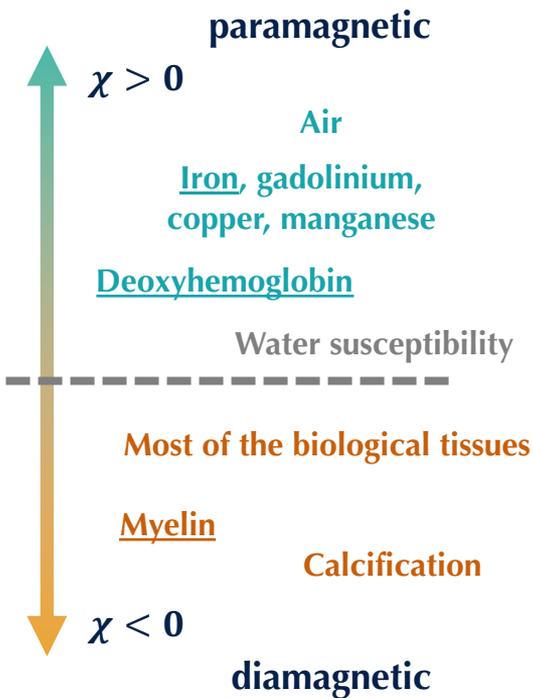
*neutral component means the component is susceptibility neutral relative to the susceptibility reference.
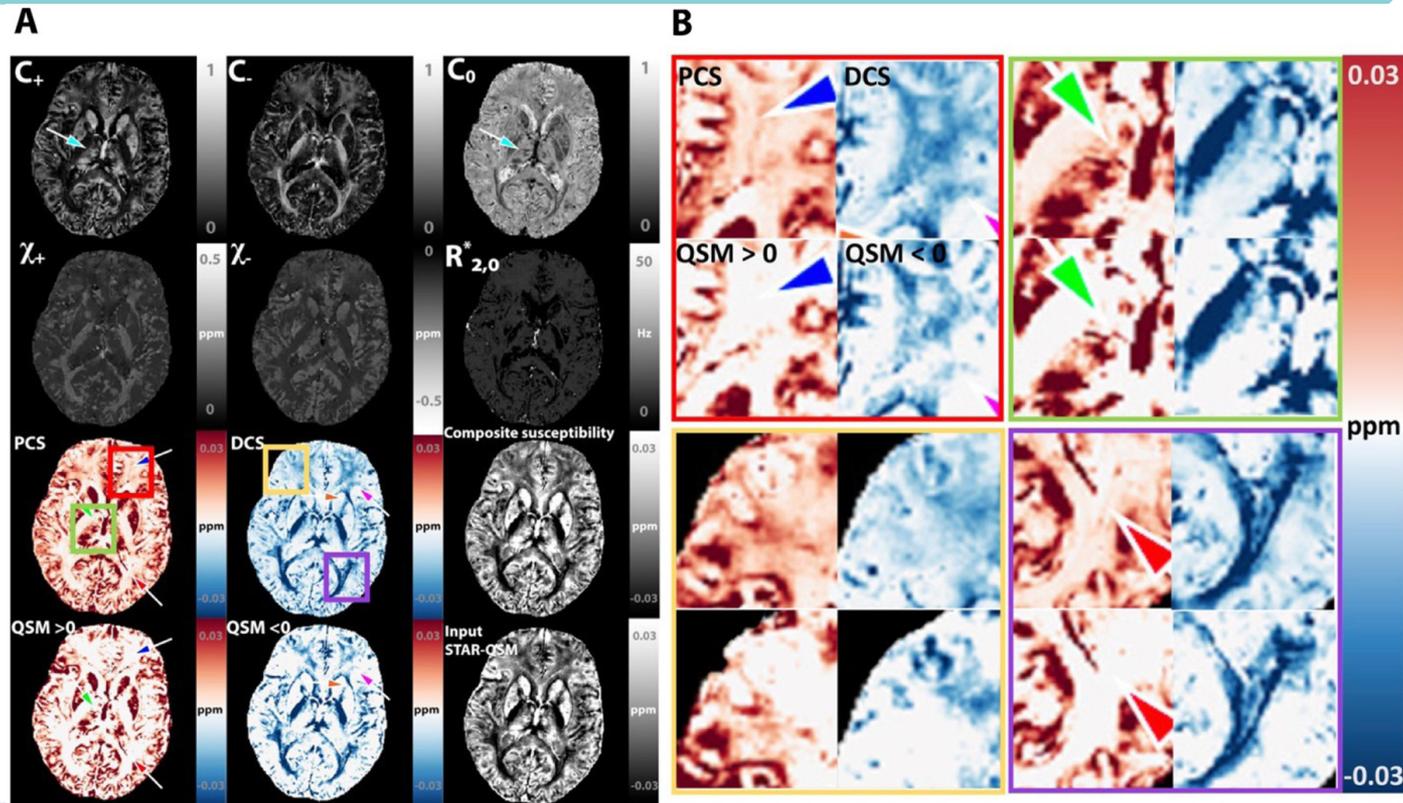
Chen J et al., NeuroImage, 2023

QSM: Susceptibility source decomposition

Chen J et al., NeuroImage, 2023

Susceptibility MRI – SWI

phase-weighted magnitude imaging

paramagnetic

$\chi > 0$

Air

Iron, gadolinium, copper, manganese

Deoxyhemoglobin

Water susceptibility

- - - - - - - - - - - -

Most of the biological tissues

Myelin

Calcification

$\chi < 0$

diamagnetic

Barnes J et al. Magn Reson Imaging Clin N Am 2010

Susceptibility MRI – SWI

phase-weighted magnitude imaging

paramagnetic

$\chi > 0$

Air

Iron, gadolinium, copper, manganese

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- - - - - - - - - - - - - - - - -

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Barnes J et al. Magn Reson Imaging Clin N Am 2010
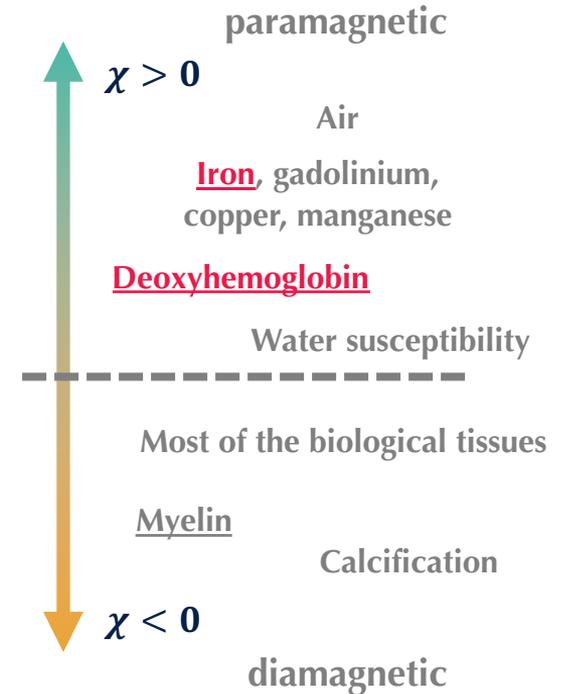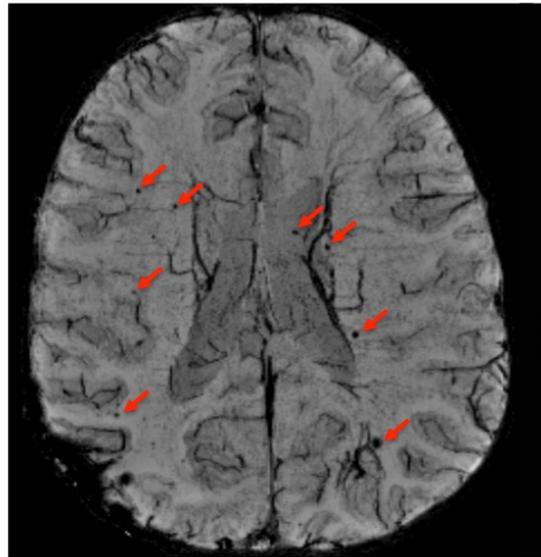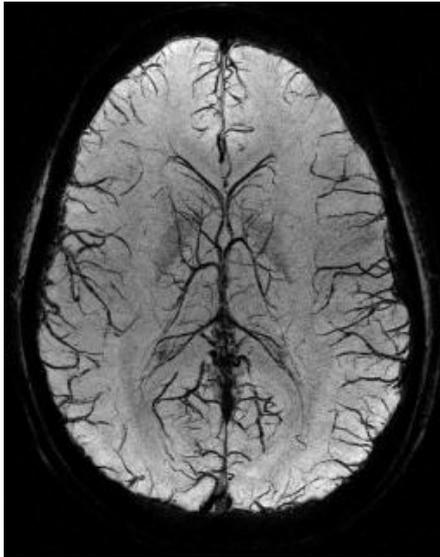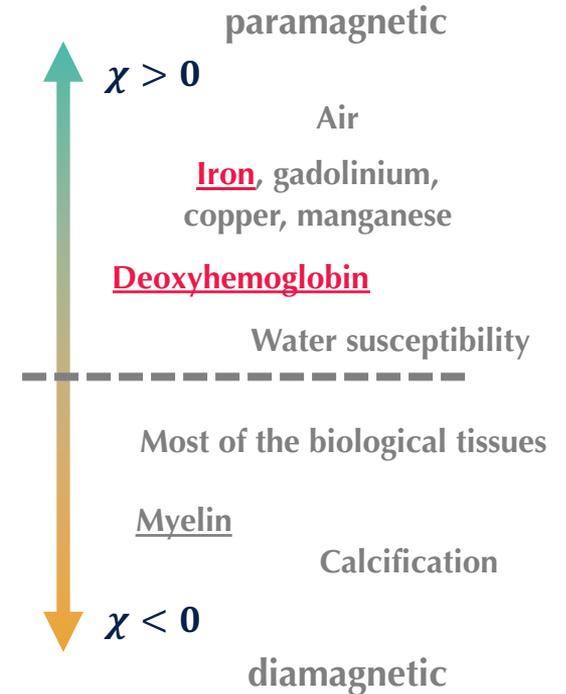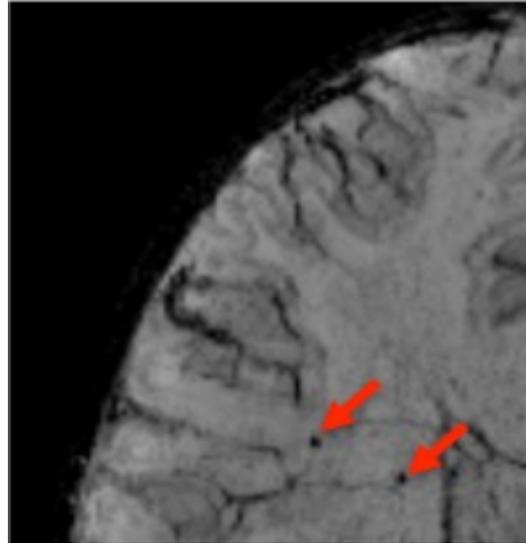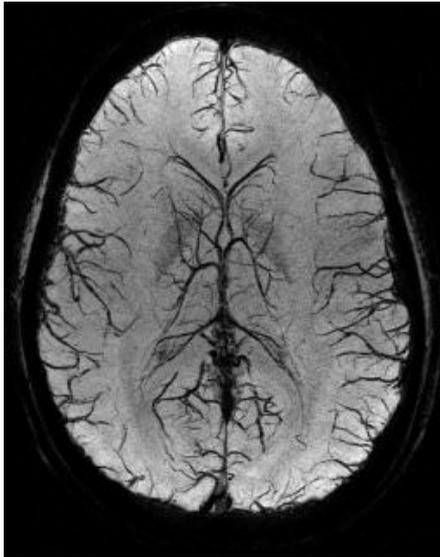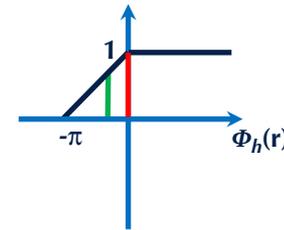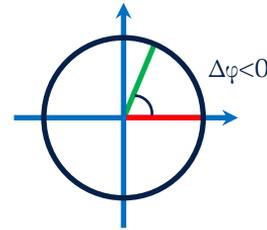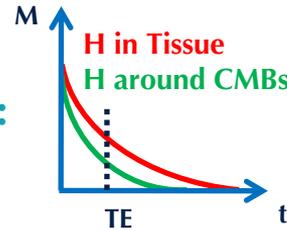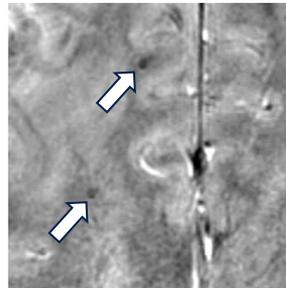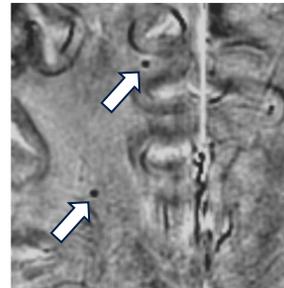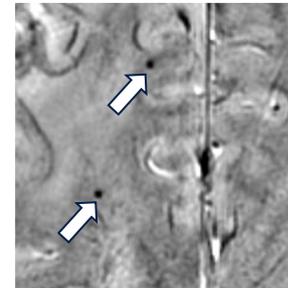
SWI – Processing

SWI – Processing

k-space magnitude phase mask SWI image SWI mIP

raw phase unwrapped phase filtered phase

X

X (4)

High pass filter

$\Phi_h(r)$

-π 1
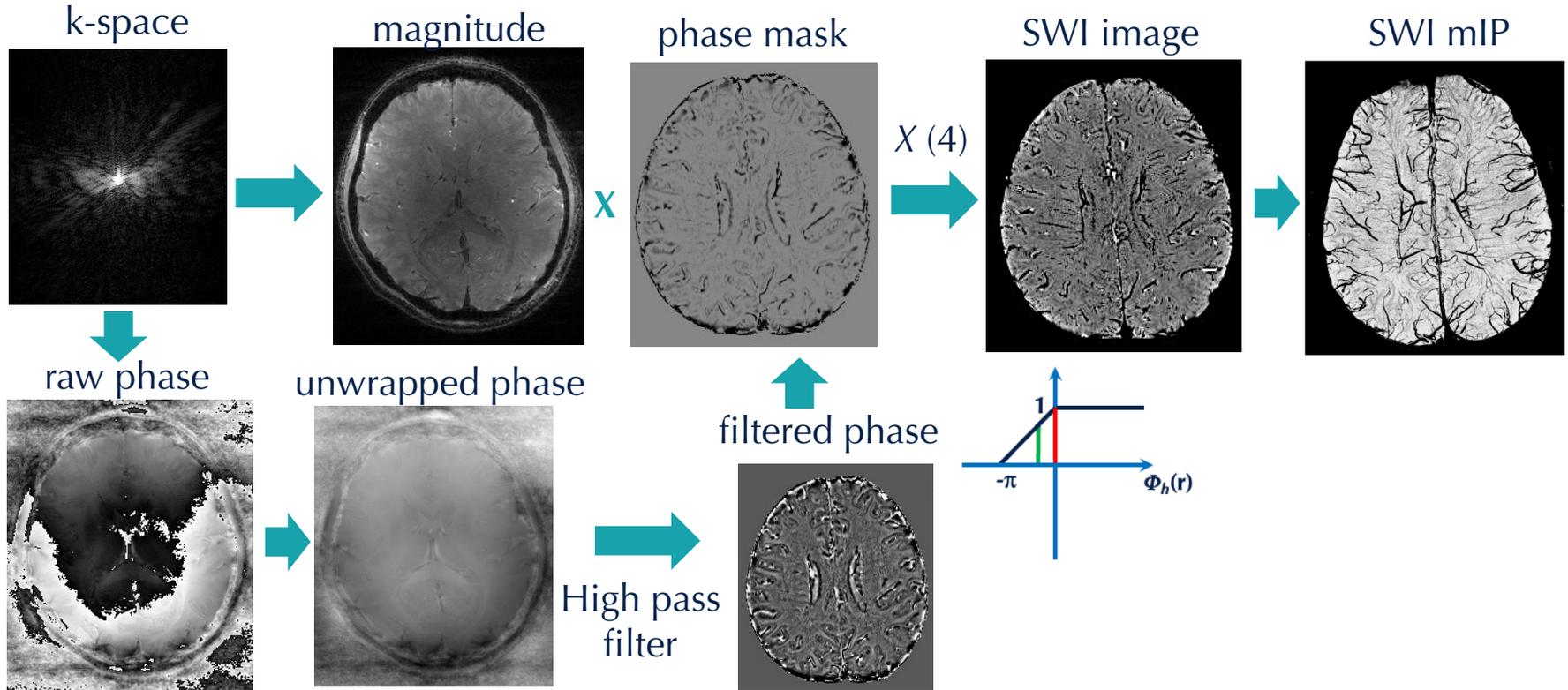
SWI – Processing

unwrapped phase

filtered phase

Hann window

X
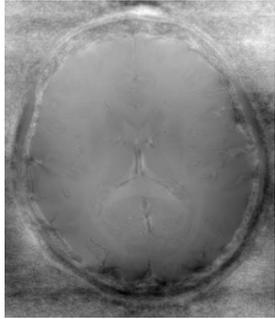
High pass filter
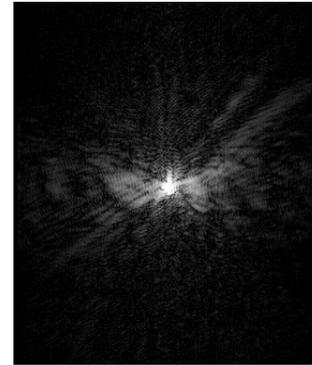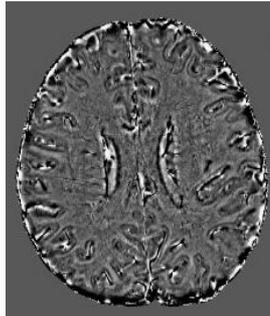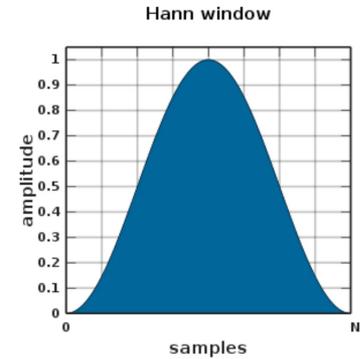
Complex division

Low pass filtered phase

SWI – Processing
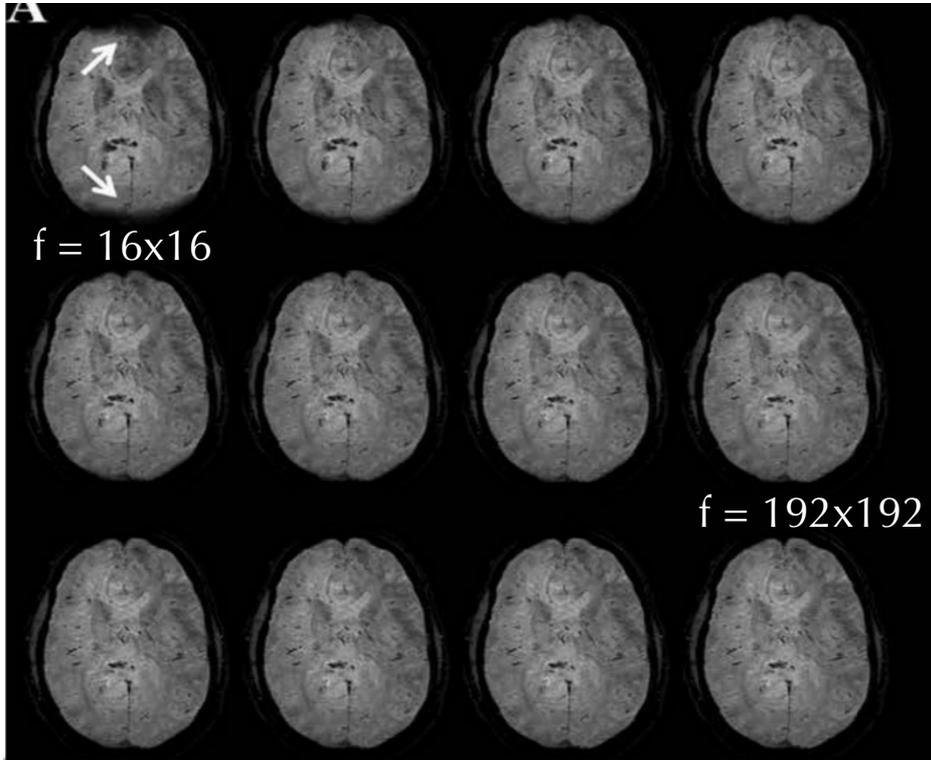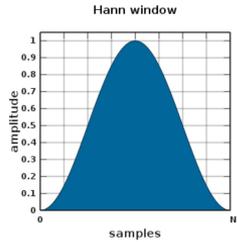
Higher contrast, more artifacts

lower contrast, less artifacts
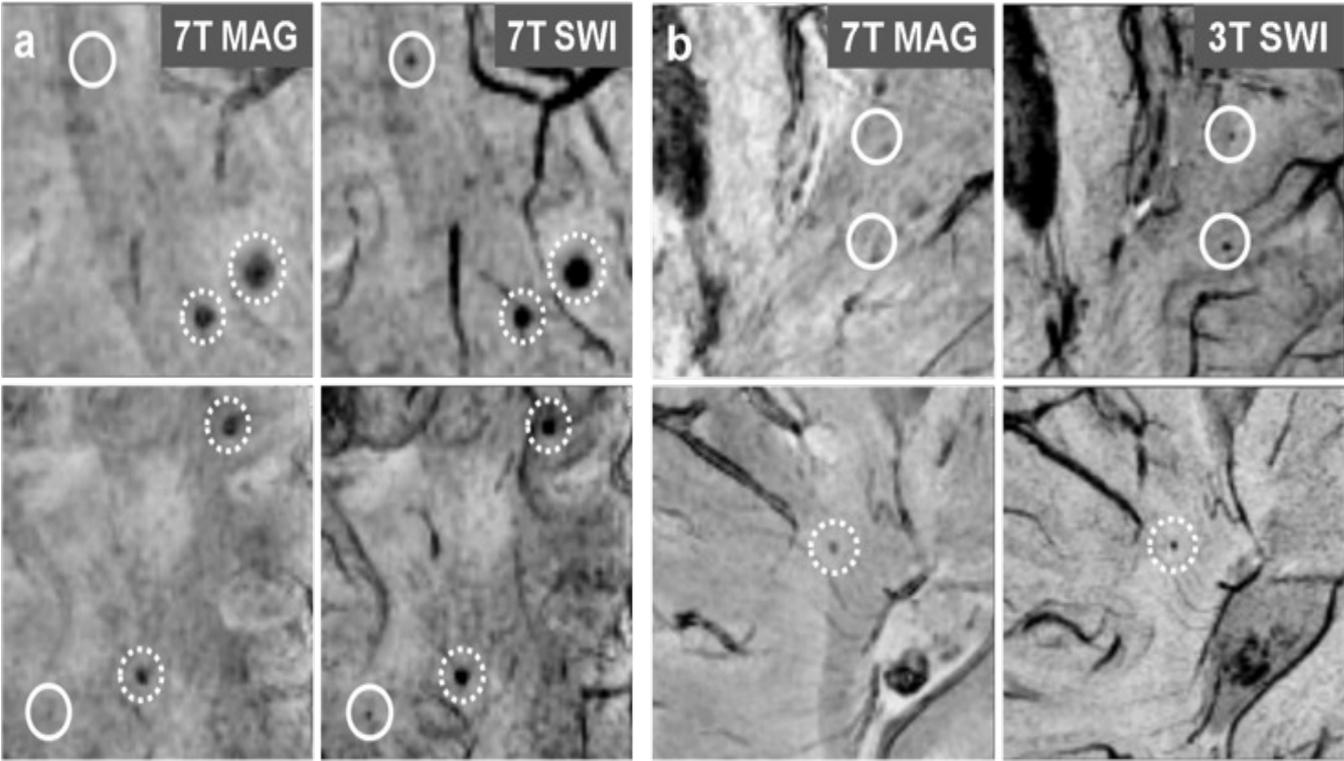
QSM/SWI – Clinical applications

Multiple Sclerosis
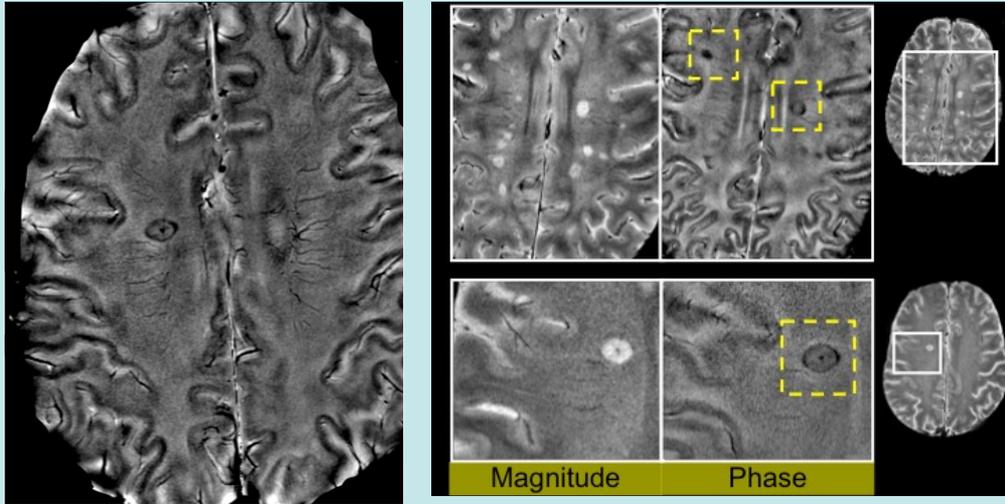
Traumatic Brain Injury
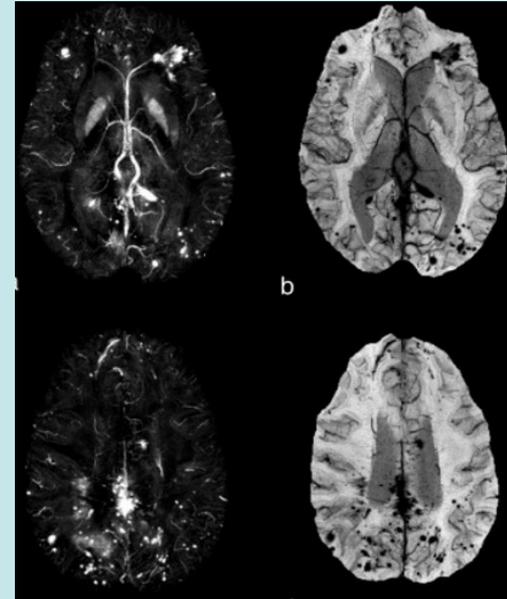
QSM/SWI – Clinical applications

Stroke

Brain Tumor

5 yrs post-RT | 8 yrs post-RT | 9 yrs post-RT | 5 yrs *no*-RT

Liu C, et al JMRI 2015
Kim HS et al AJNR 2009
Lupo et al. IJROBP

QSM/SWI – Clinical applications

Aging

−0.1 ppm   0.25 ppm

−0.1 ppm   0.25 ppm
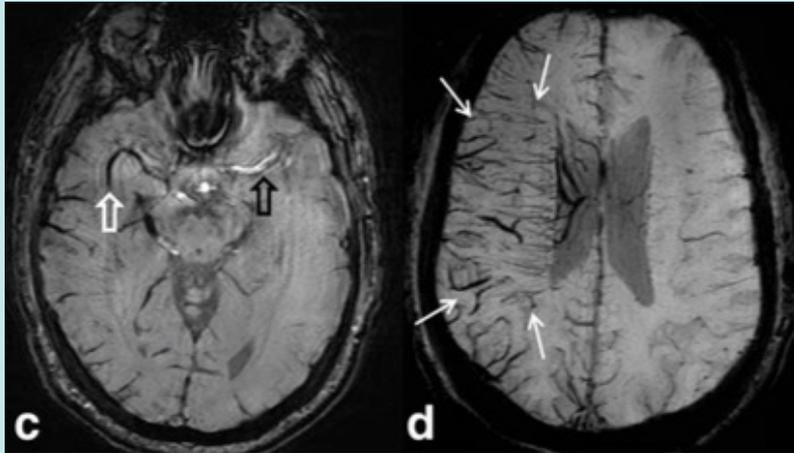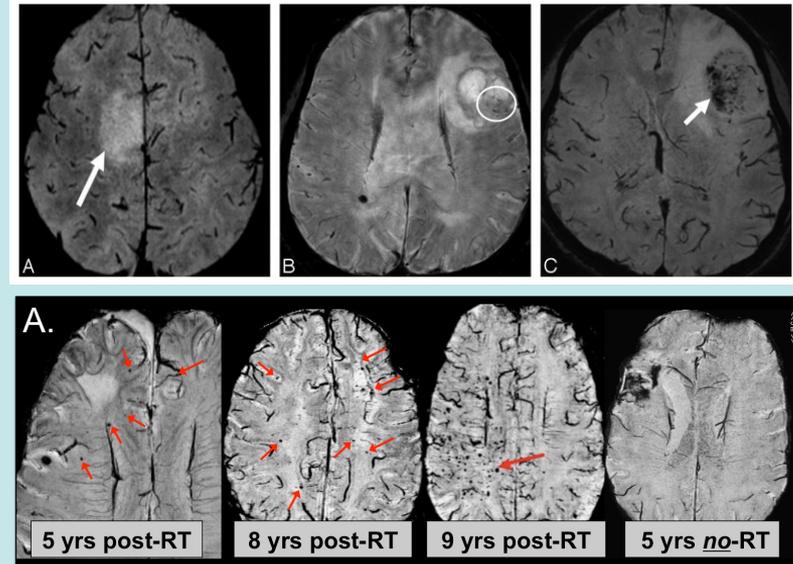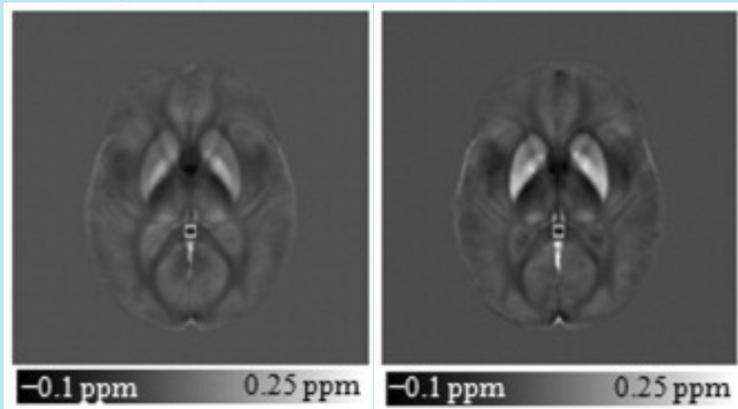
Young   Elderly

Parkinson's Disease

A   CN   PUT   GP   B   SN   RN

C   SN (ipsilateral)   SN (contralateral)

r=0.391 P=0.0094

r=0.347 P=0.0226
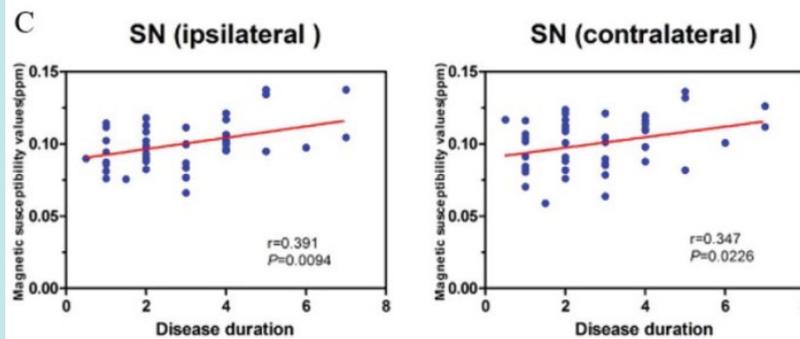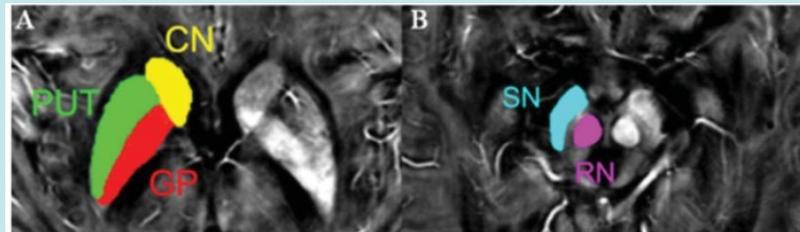
Conductivity imaging (Electrical Properties Tomography)

- **Electrical conductivity (σ)**: the ability of a material to transport charges, or equivalently, to carry an electric current.
- **Electrical permittivity (ε)**: the ability of a material to rotate molecular dipoles and trap/store charge; hence the degree to which a material becomes polarized when placed in an electric field.

Helmholtz equation

$$ -\nabla^2 \mathbf{H} = \frac{\nabla \kappa}{\kappa} \times [\nabla \times \mathbf{H}] + \omega^2 \mu \kappa \mathbf{H}, $$

H: Magnetic Field
κ: complex permittivity
  = $\epsilon - i(\sigma/\omega)$
μ: magnetic permeability

Conductivity imaging

$$- \nabla^2 \mathbf{H} = \frac{\nabla \kappa}{\kappa} \times [\nabla \times \mathbf{H}] + \omega^2 \mu \kappa \mathbf{H},$$

Piecewise constant κ

Constant μ

Simplified H-EPT

$$\sigma = \frac{1}{\omega \mu_0} \nabla^2 \hat{\varphi}^+$$

$$\kappa(\mathbf{r}) = \frac{-1}{\omega^2 \mu_0} \frac{\nabla^2 H^+(\mathbf{r})}{H^+(\mathbf{r})},$$

$$\sigma = \frac{1}{\omega \mu_0} \mathrm{Im} \left\{ \frac{\nabla^2 H^+}{H^+} \right\}, \qquad \epsilon = \frac{-1}{\omega^2 \mu_0} \mathrm{Re} \left\{ \frac{\nabla^2 H^+}{H^+} \right\}$$

Conductivity imaging

Simplified H-EPT/
phase-based EPT

$$\sigma = \frac{1}{\omega \mu_0} \nabla^2 \hat{\varphi}^+$$

Coil magnitude Magnitude

Coil
combination

Coil phase Phase
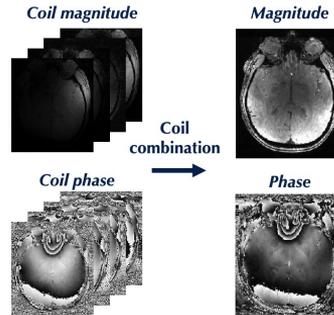
Measured phase
(single coil)

$$\phi(\vec{r}, TE) = \phi_0(\vec{r}) + \phi_{total}(\vec{r}, TE)$$

Transceive phase

$$\phi_0 = \phi^+ + \phi^-$$

$$\phi^+ \approx \tilde{\phi}_0/2 = \left(\phi^+ + \tilde{\phi}^-\right)/2$$

Transceive phase assumption

Conductivity imaging
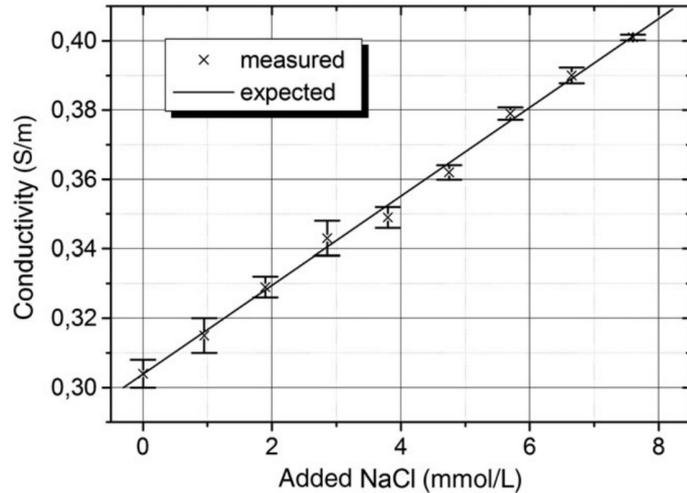
FIGURE 4 Electrical properties tomography (EPT) phantom measurements at different saline concentrations. The plot clearly shows the different steps of added NaCl, corresponding to an accuracy of about 10 mS/m. Error bars indicate standard deviation over the averaged 50 × 50 voxels in the center of the phantom

Conductivity imaging

WILEY **NMR** IN BIOMEDICINE
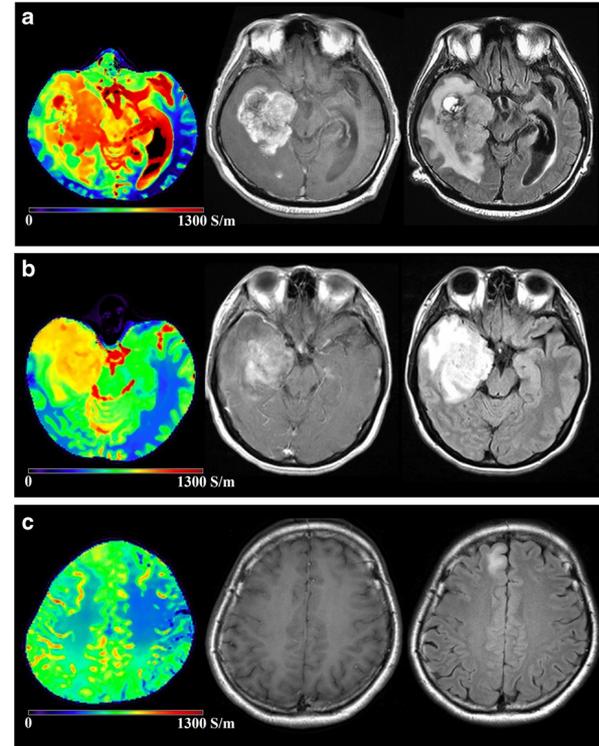
REVIEW ARTICLE

Electric properties tomography: Biochemical, physical and technical background, evaluation and clinical applications

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[2] Department of Radiotherapy, University Medical Center, Utrecht, the Netherlands

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Email: ulrich.katscher@philips.com

Electric properties tomography (EPT) derives the patient's electric properties, i.e. conductivity and permittivity, using standard magnetic resonance (MR) systems and standard MR sequences. Thus, EPT does not apply externally mounted electrodes, currents or radiofrequency (RF) probes, as is the case in competing techniques. EPT is quantitative MR, i.e. it yields absolute values of conductivity and permittivity. This review summarizes the physical equations underlying EPT, the corresponding basic and advanced reconstruction techniques and practical numerical aspects to realize these reconstruction techniques. MR sequences which map the field information required for EPT are outlined, and experiments to validate EPT in phantom and *in vivo* studies are described. Furthermore, the review describes the clinical findings which have been obtained with EPT so far, and attempts to understand the physiologic background of these findings.

KEYWORDS

electric conductivity, EPT, numerical differentiation, permittivity, tumor characterization