Fast Imaging Trajectories: Non-Cartesian Sampling (2)

M229 Advanced Topics in MRI

Holden H. Wu, Ph.D.

2024.04.30

UCLA
Department of Radiological Sciences
David Geffen School of Medicine at UCLA

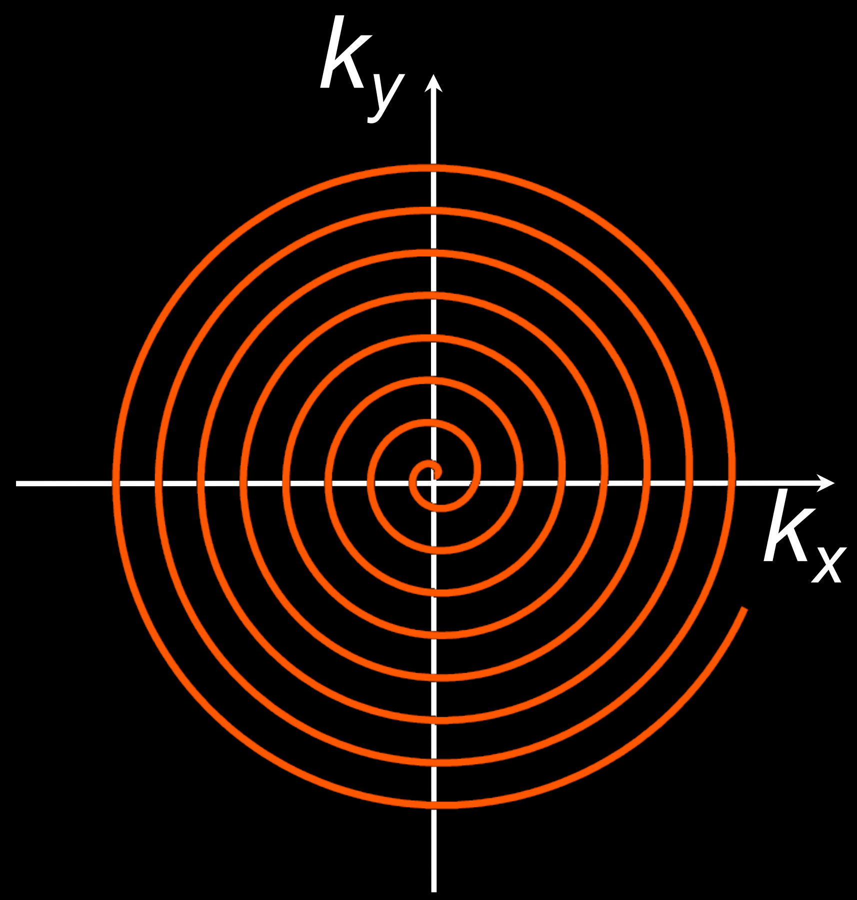
Class Business

- Office hours
- Homework 1 solutions
- Homework 2 being graded
- Final project
  - Discussion on 5/2 Thu
  - Proposal due 5/10 Fri
    can send us a draft to get feedback
  - Abstract due 6/7 Fri
  - Presentations on 6/11 Tue

Outline

- Spiral Trajectory
- Non-Cartesian 3D Trajectories
  - 3D stack of radial
  - 3D radial
  - 3D cones
- Non-Cartesian Image Reconstruction
  - Gridding reconstruction
  - Gradient measurement
  - Off-resonance correction

Spirals



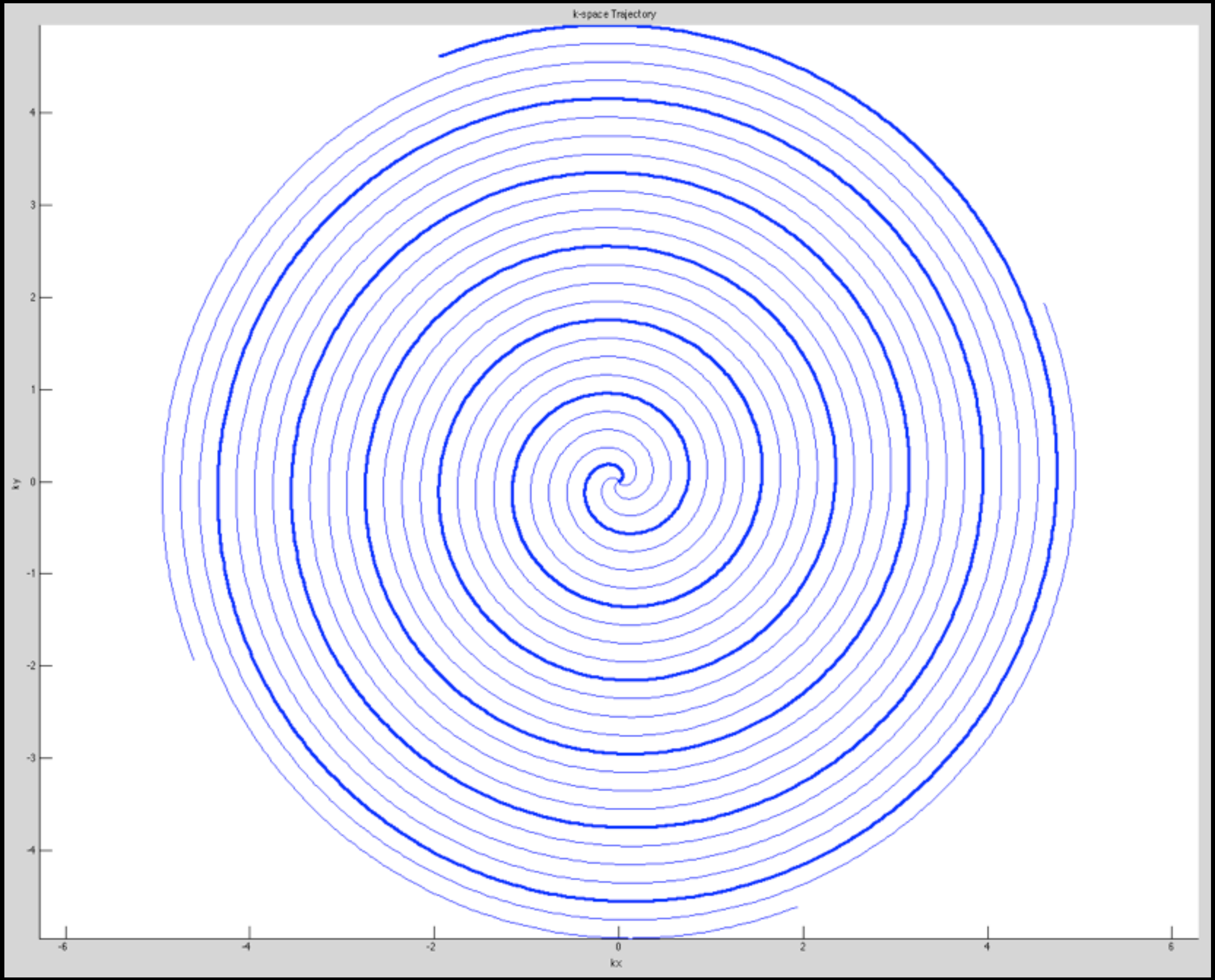
<u>"THE" non-Cartesian trajectory</u>

<u>Highly robust to motion/flow effects</u>

<u>Very fast!</u>

- optimal use of gradients in 2D
- can acquire one image in ~100 ms

Spirals: Sampling Requirements



N interleaves

$2\ k_{r,max} = 1 / dx$

$dk = 1 / FOV$

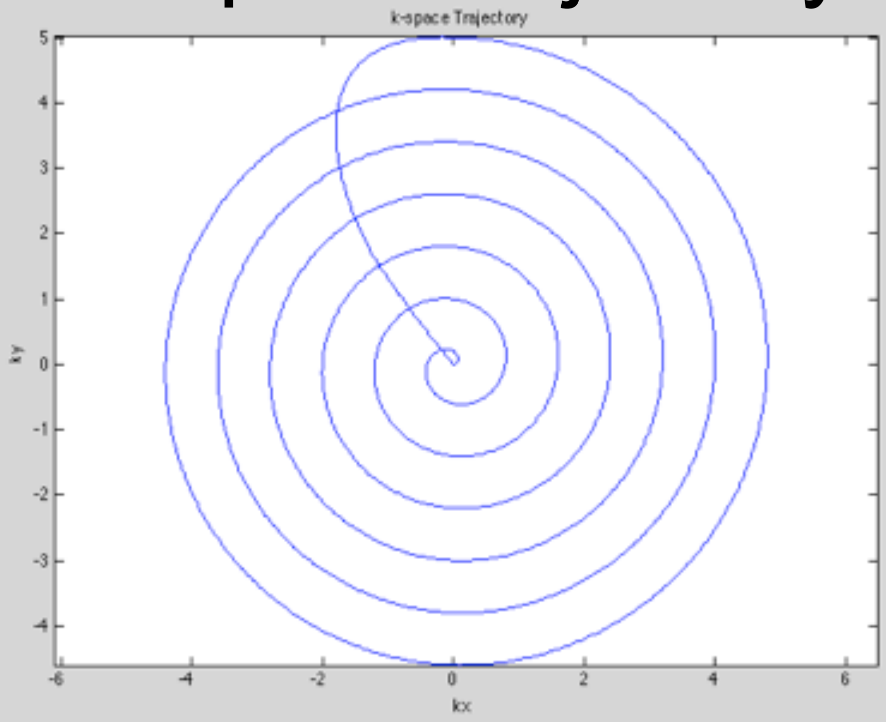
Design 1 interleaf

and rotate

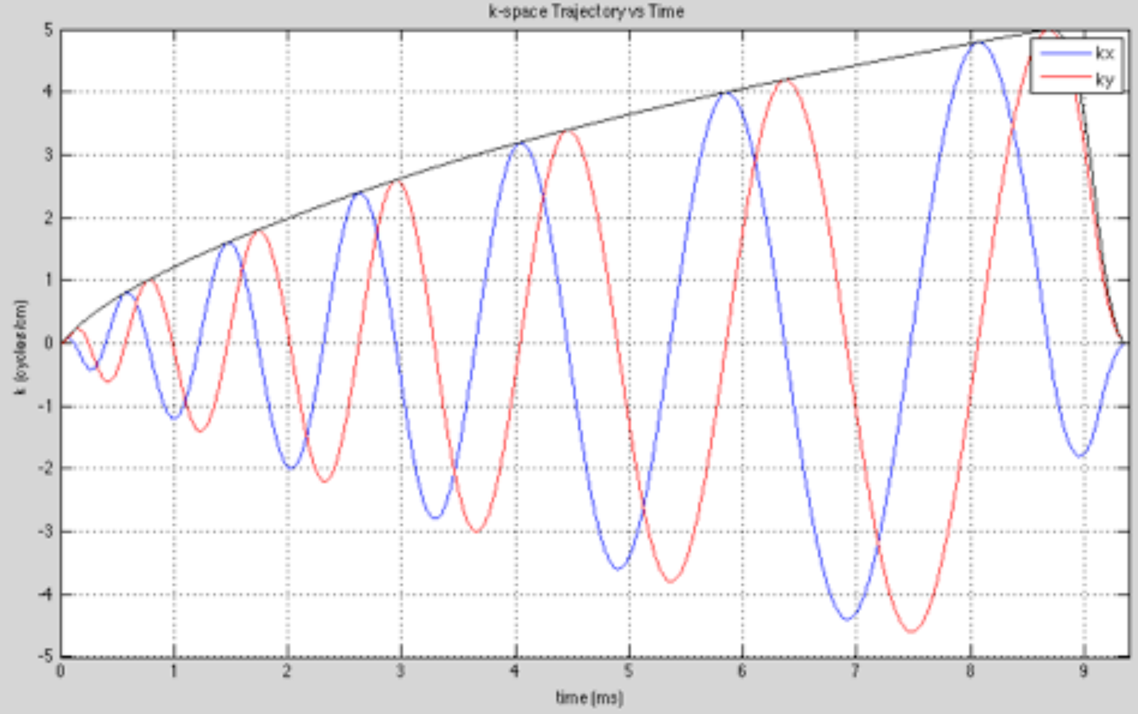
Subject to HW limits

Spirals: Gradient Design

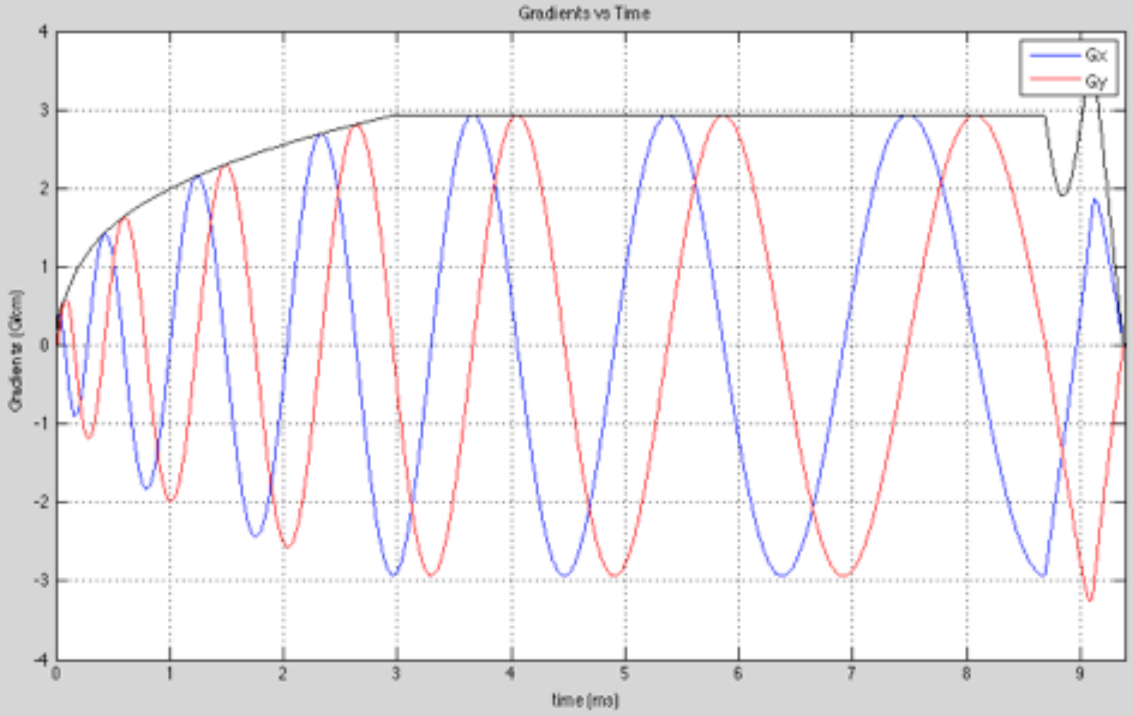
k-space trajectory



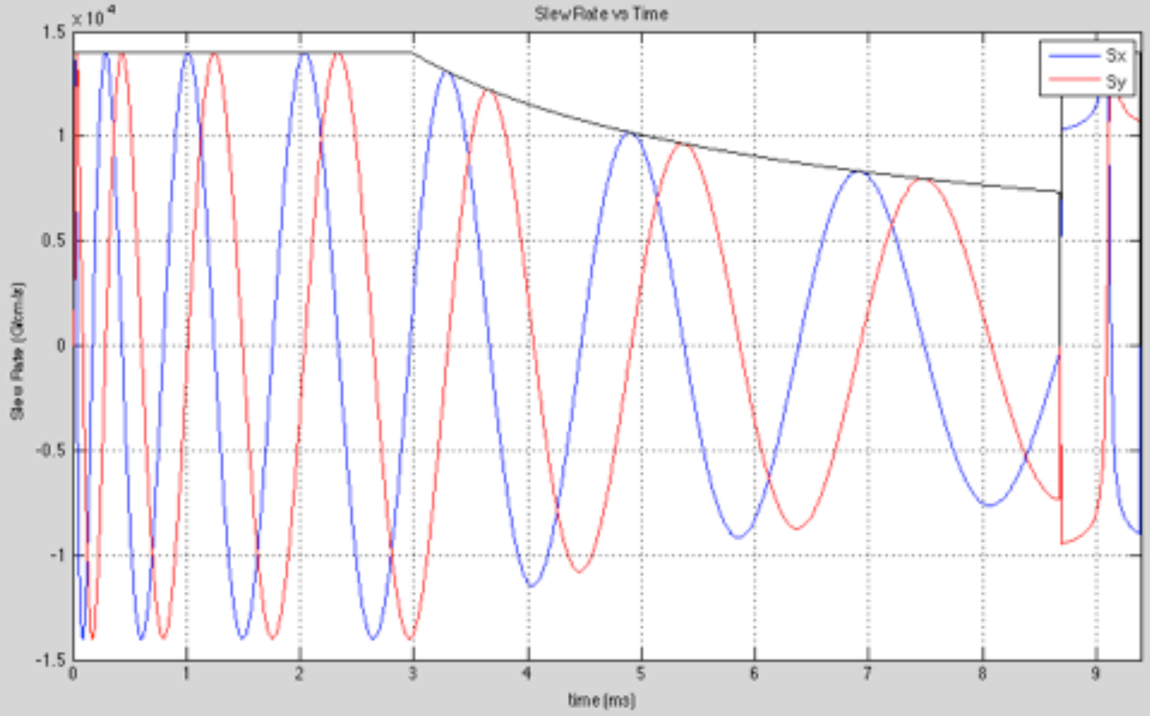
k-space pos vs. time



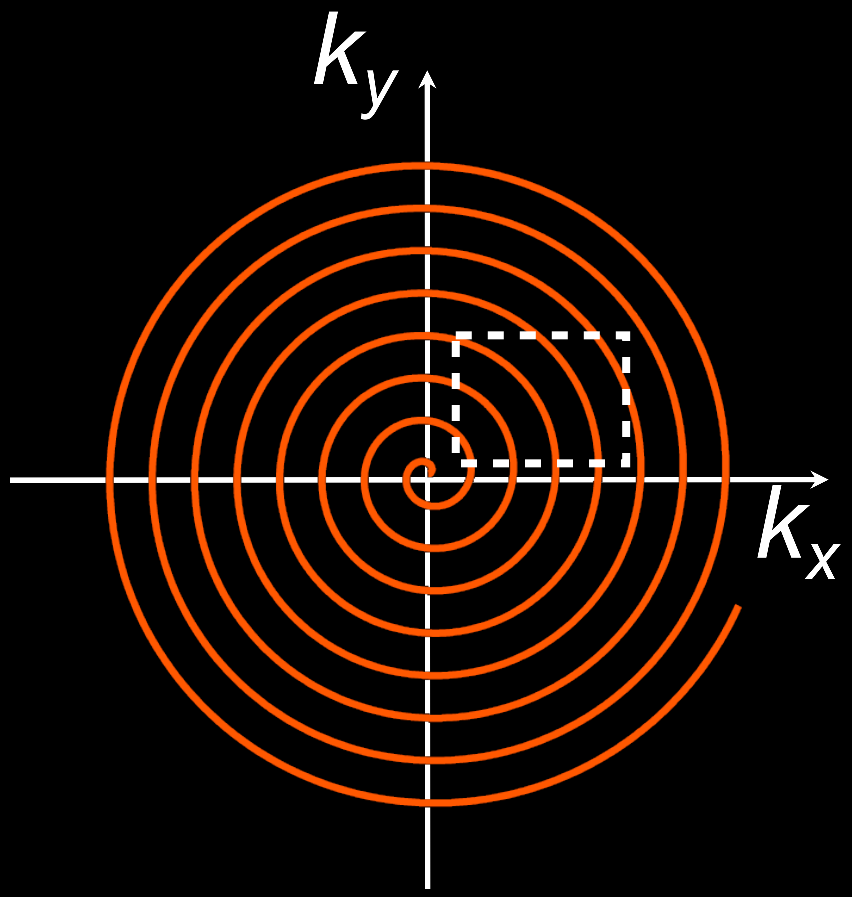
Gradients vs. time



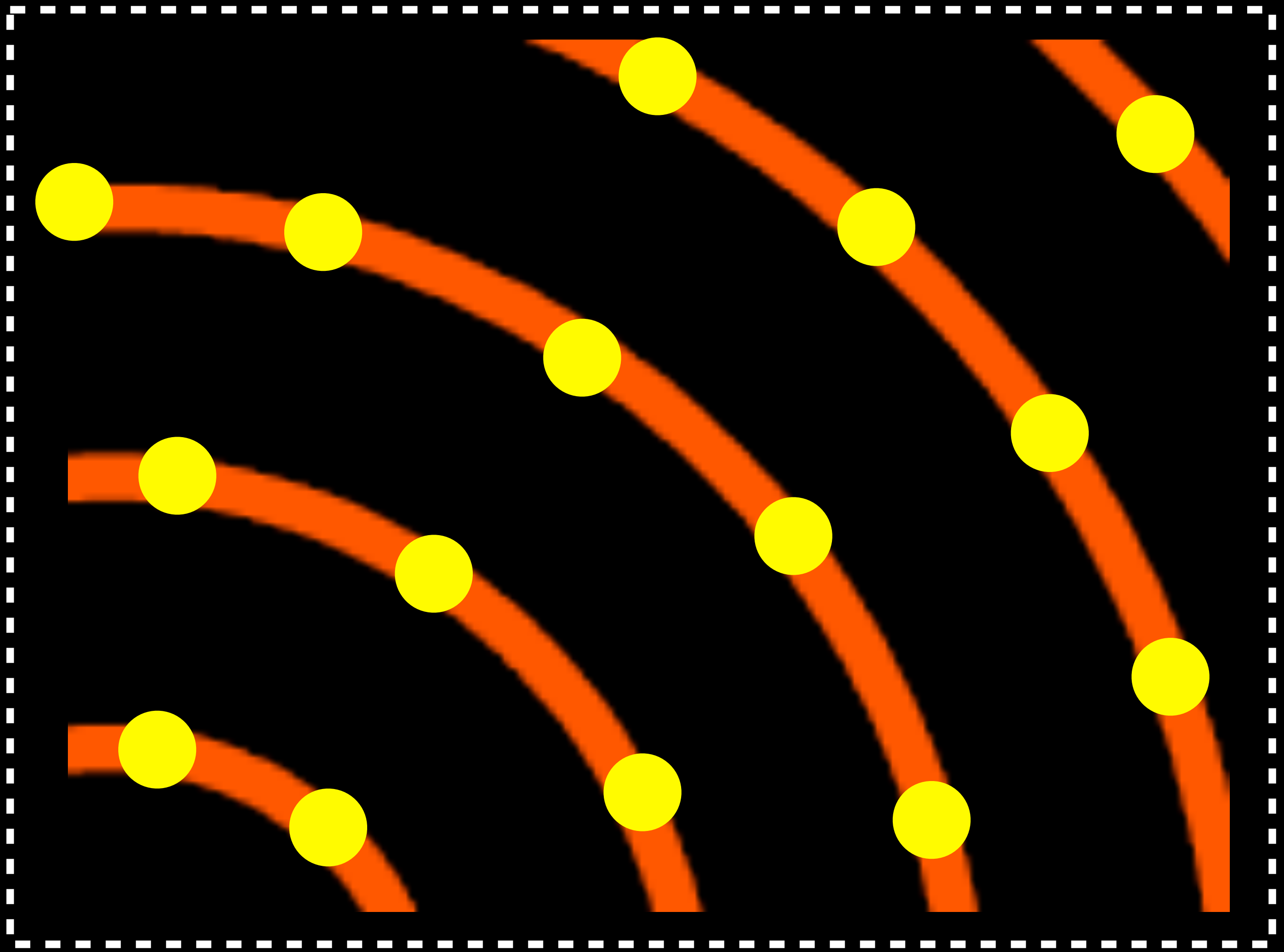
Slew rate vs. time



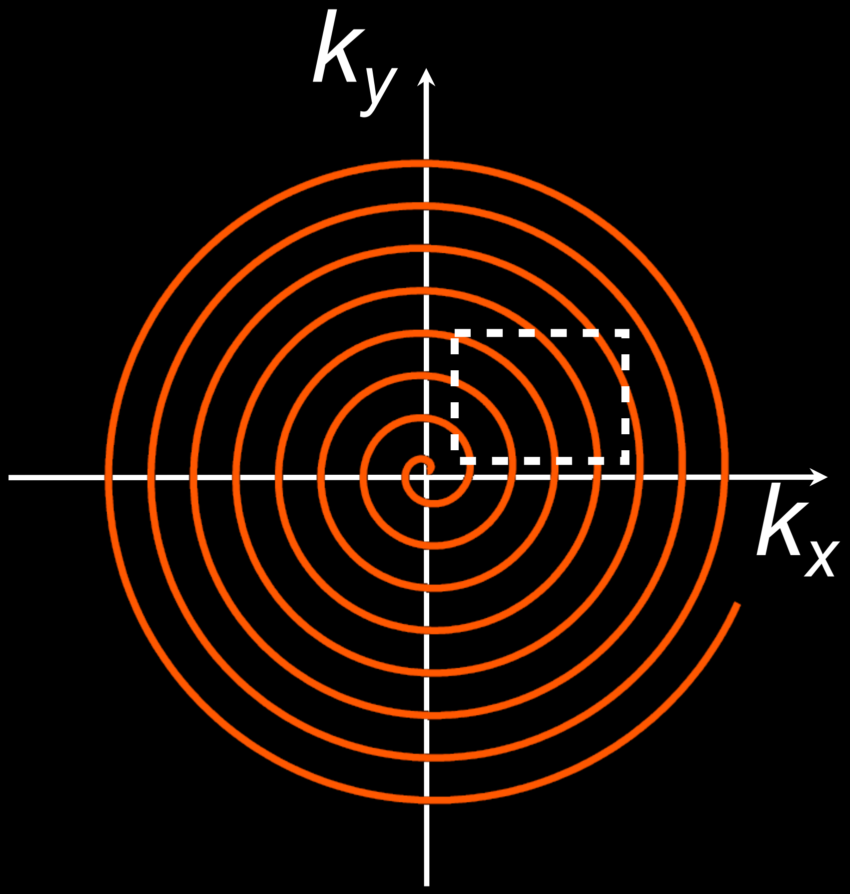
Spirals: Image Reconstruction



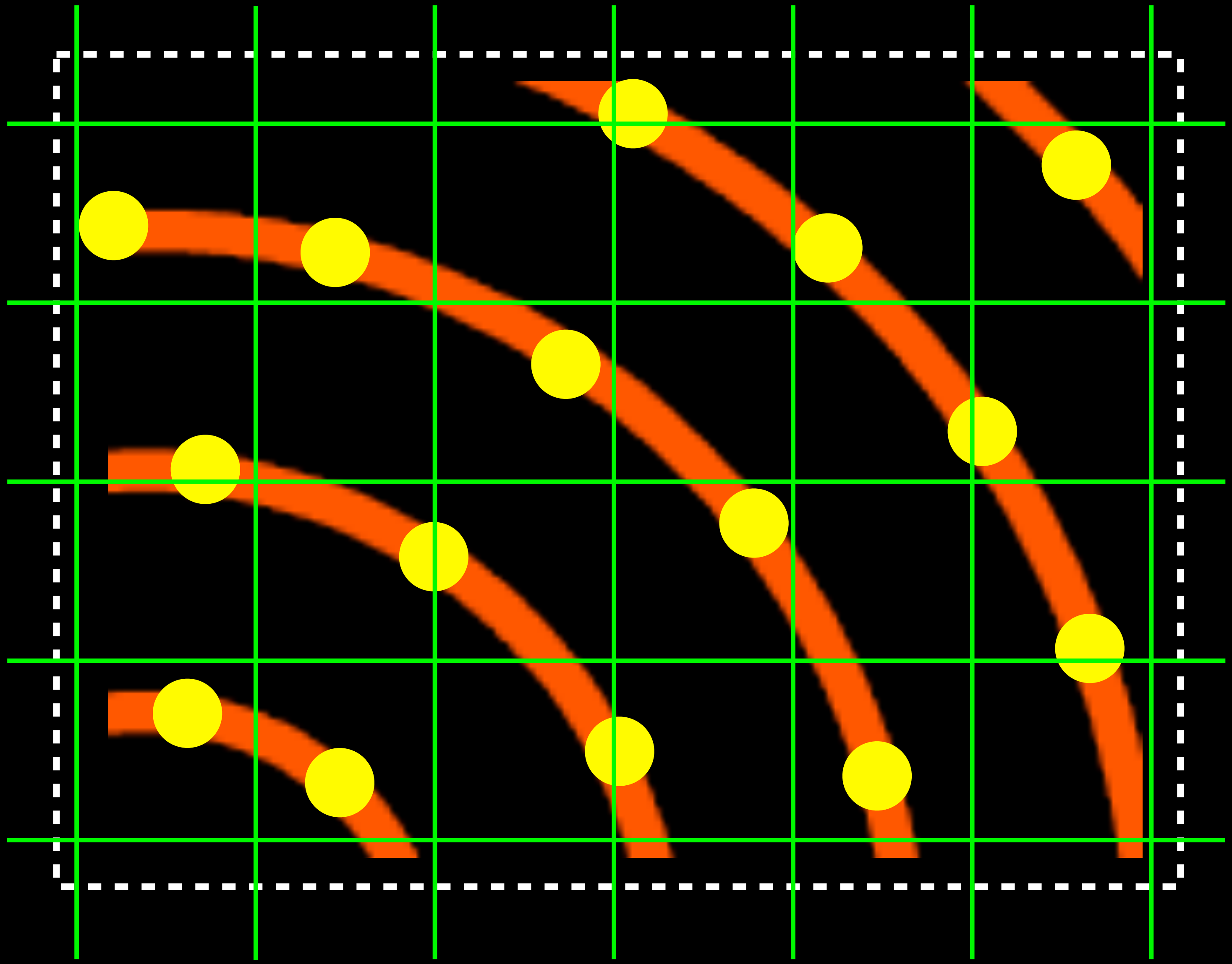
Gridding Algorithm



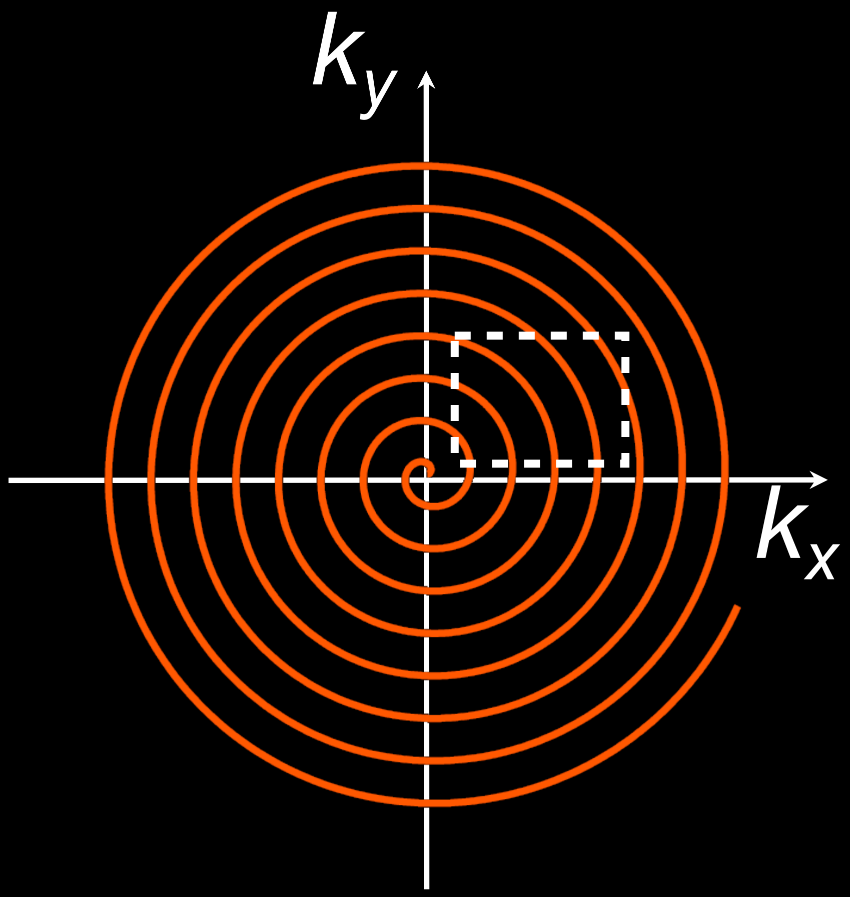
Spirals: Image Reconstruction



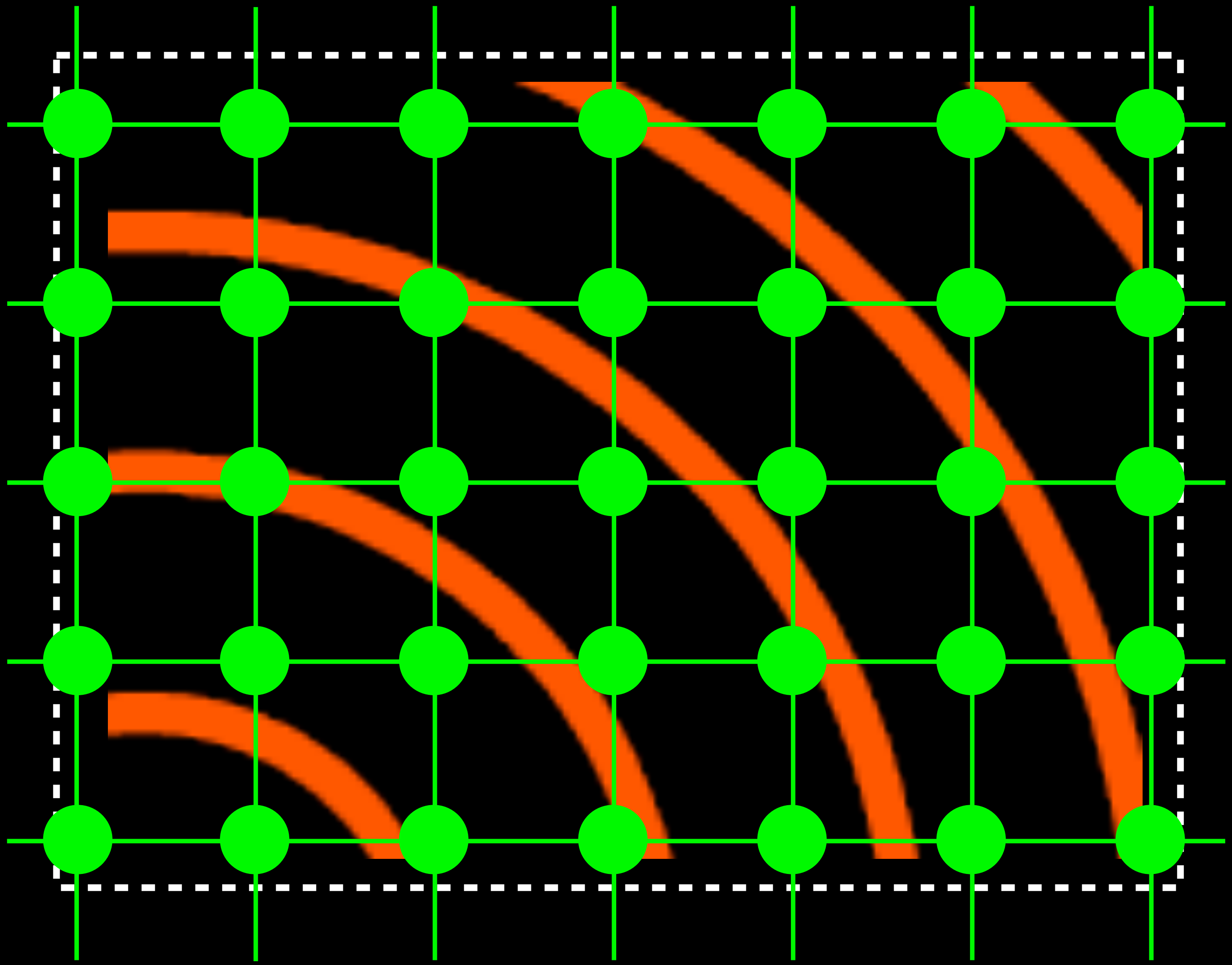
Gridding Algorithm



Spirals: Image Reconstruction

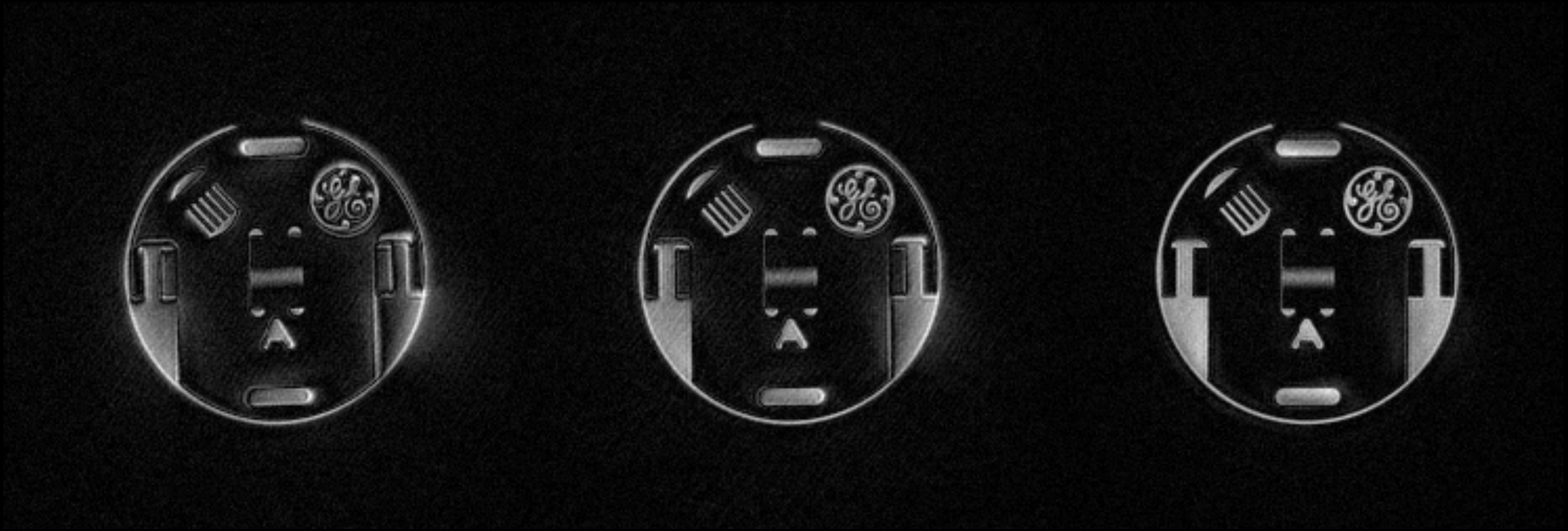


Gridding Algorithm



Follow with 2D Fourier Transform ...

Spirals: Gradient Delays



2 sample delay

1 sample delay

calibrated

Spirals: Off-Resonance Effects

$N_{intlv} = 8$

$T_{rd} = 26.67$ ms

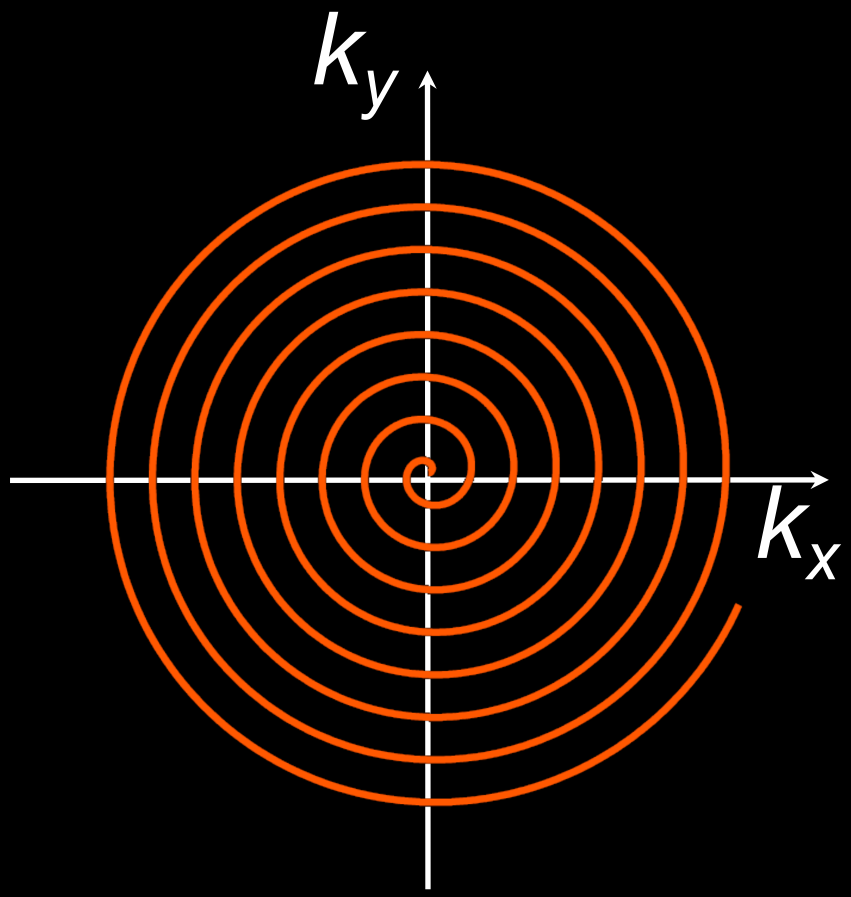
$N_{intlv} = 16$

$T_{rd} = 13.41$ ms

$N_{intlv} = 48$

$T_{rd} = 4.61$ ms

Spirals: Practical Considerations



Trajectory design

Gradient waveform calibration

k-Space density compensation

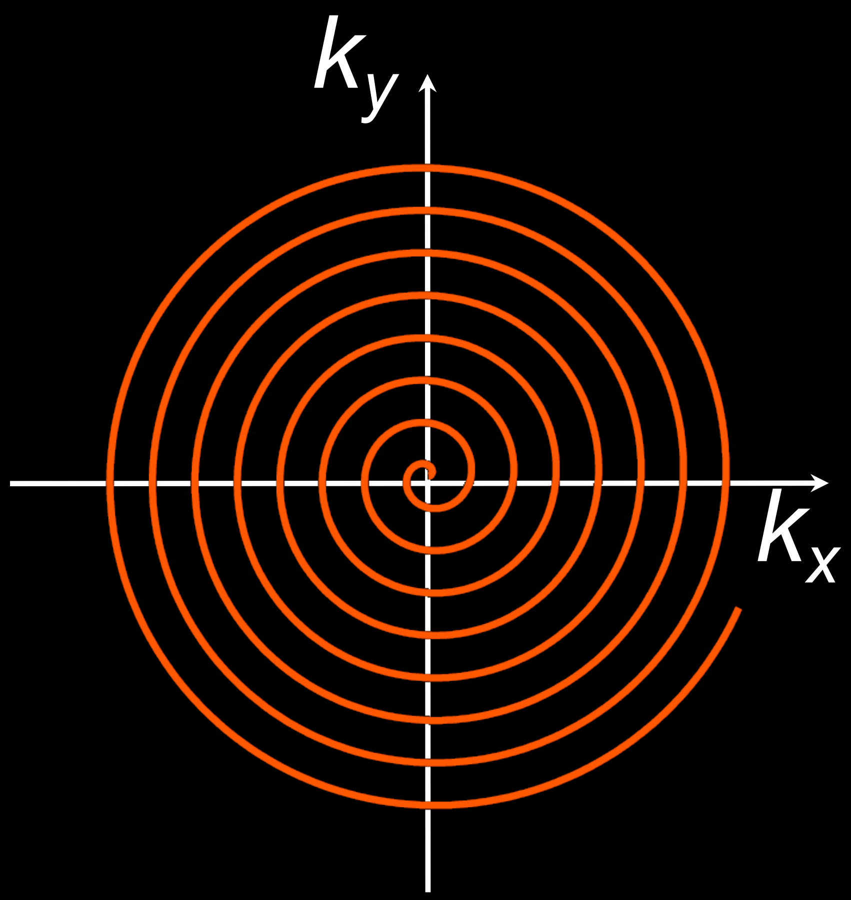
Off-resonance correction

Fat suppression

Gridding reconstruction

applies to non-Cartesian MRI in general

Spirals: Pros and Cons



<u>Pros</u>

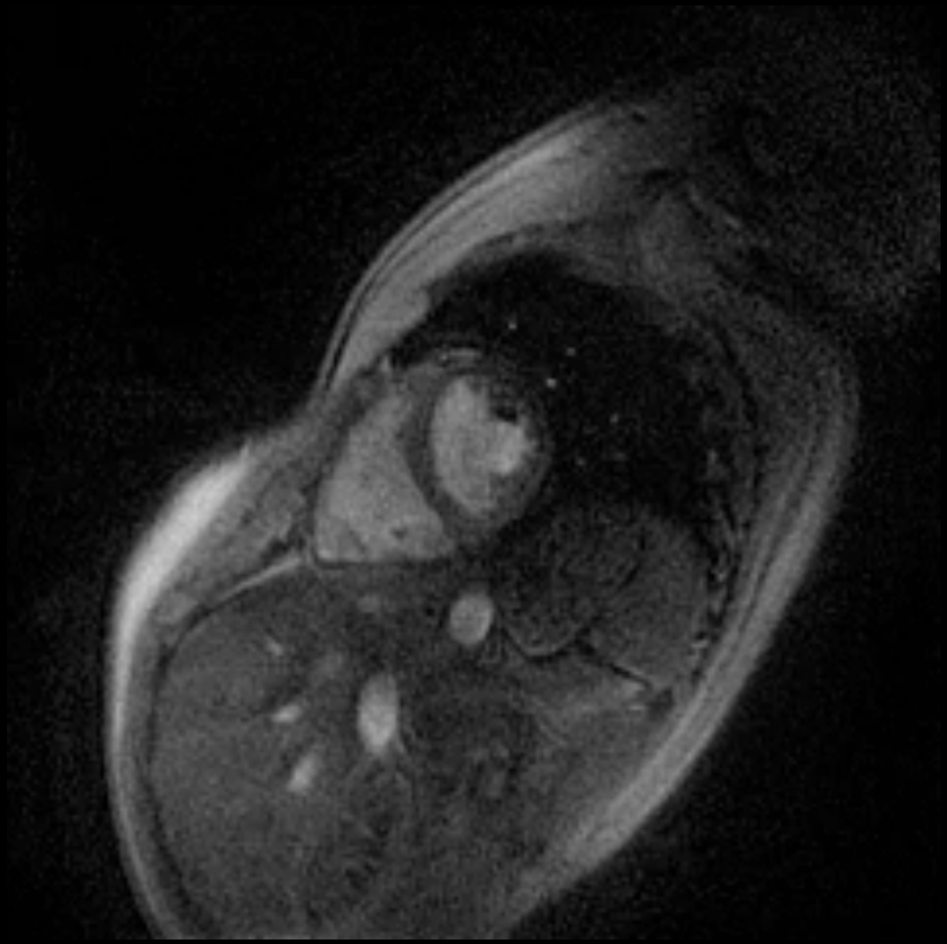
- Very fast (up to single shot)
- Very short TE
- Robust to motion/flow effects

<u>Cons</u>

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

Spirals: Real-Time Cardiac MRI

- Healthy subject; 1.5 T; 8-ch array
- Golden-angle ordering
- Spiral 2D GRE; 8-mm slice
- Spatial resolution = 1.6 mm
- SPIRiT recon with *R* = 2
- 40 cm, 1.6 mm
- 250x250 matrix @ 6 fps
- 12-fold reduction in #TRs (vs. 2DFT)
- 8-TR sliding window display (16 fps)



Wu HH et al., ISMRM 2013, p3828

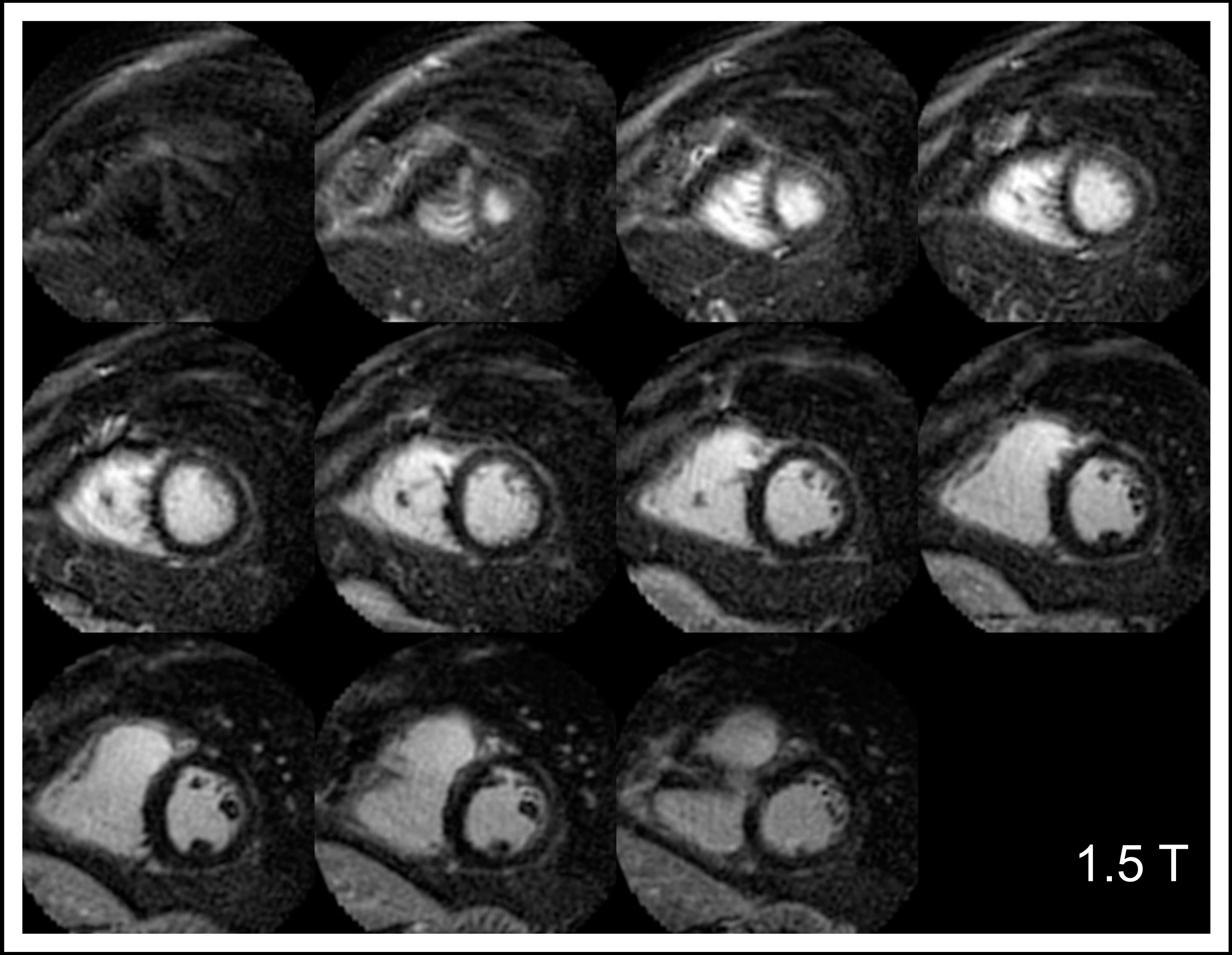
Spirals: 3D LGE MRI

3D Spiral IR-GRE

- 6-interleaf VD spiral
- 7.5-ms readout
- 90 x 90 x 11 matrix
- outer volume suppr
- water-only RF exc
- TR = 15.48 ms
- 8-HB BH scan

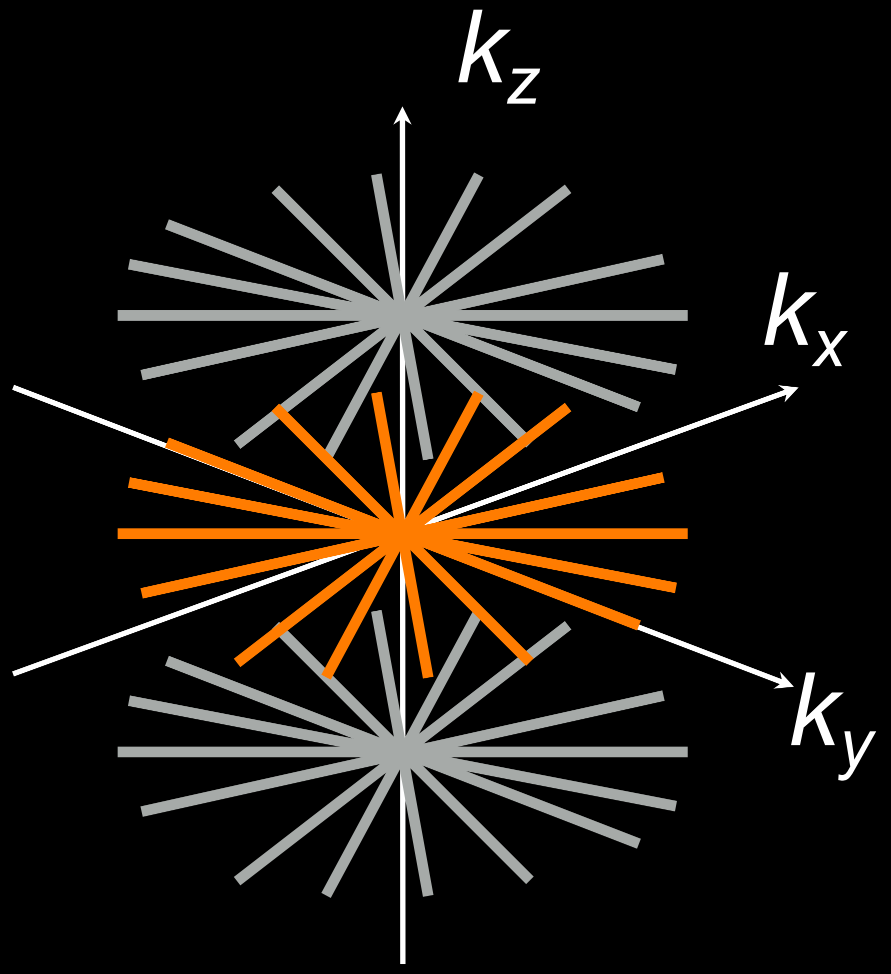
Reconstruction

- SPIRiT (*R* = 2)
- ~5-sec recon

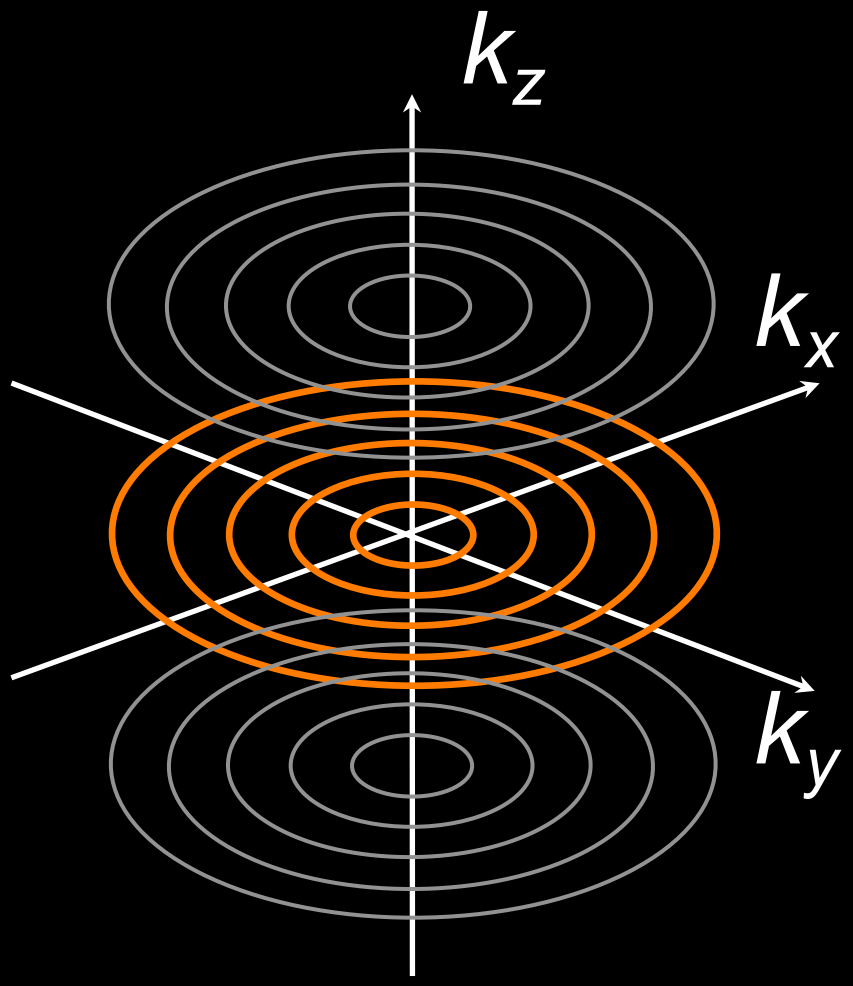


courtesy of Joelle Barral & Juan Santos (HeartVista)

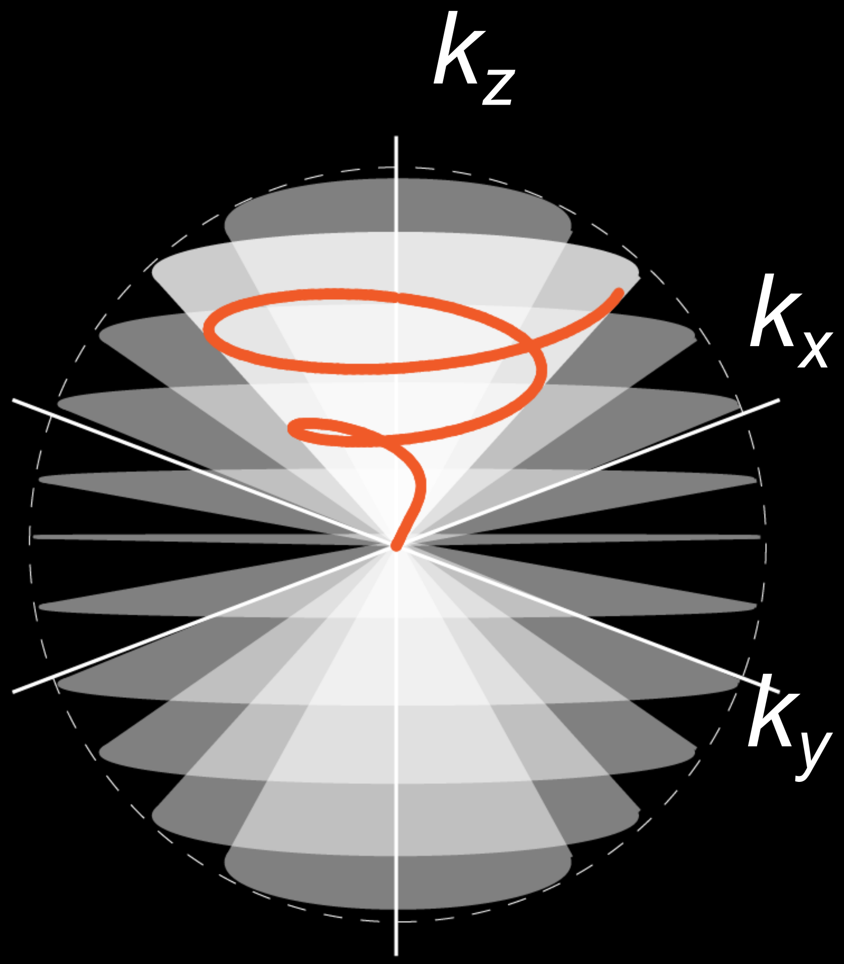
3D Non-Cartesian Sampling



3D Stack of Stars



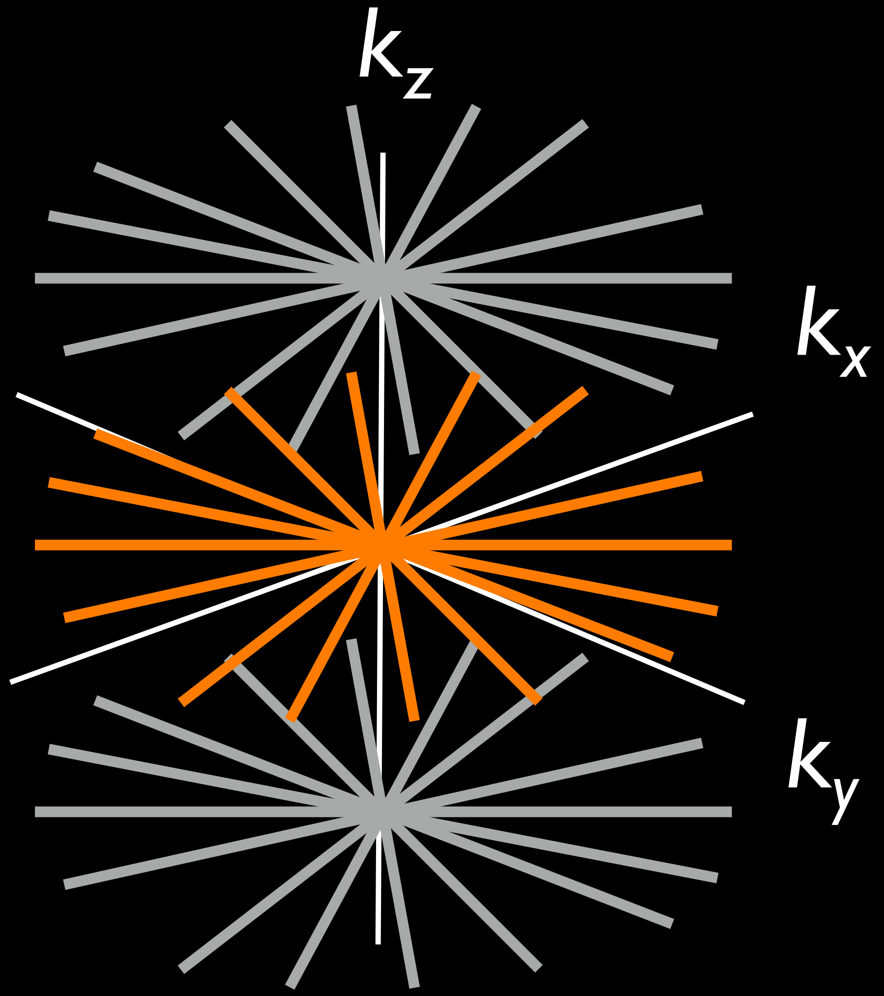
3D Stack of Rings



3D Cones

and much more ...

3D Stack-of-Radial



aka Stack-of-Stars

Pros

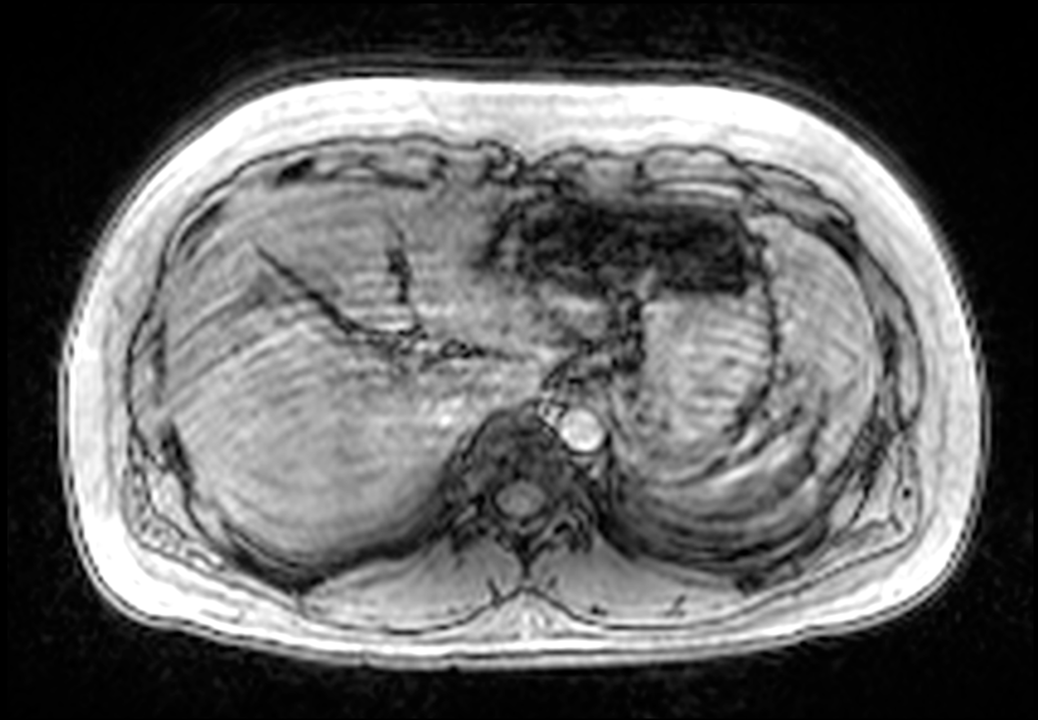
- Straightforward extension of radial
- Robust to motion
- Can tolerate a lot of undersampling

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

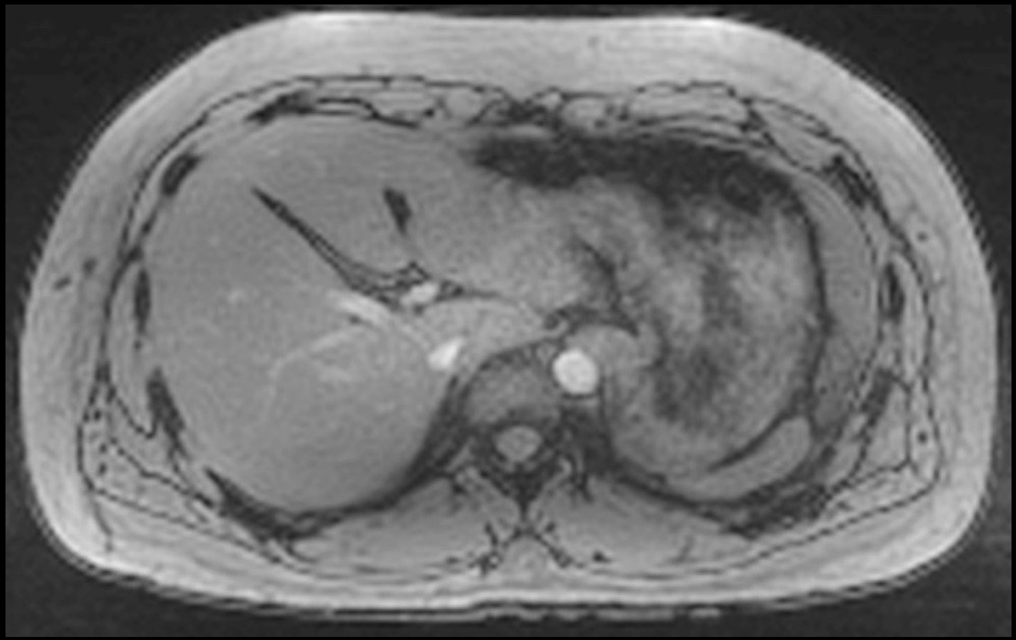
3D Stack-of-Radial: Liver MRI

3D Cartesian MRI

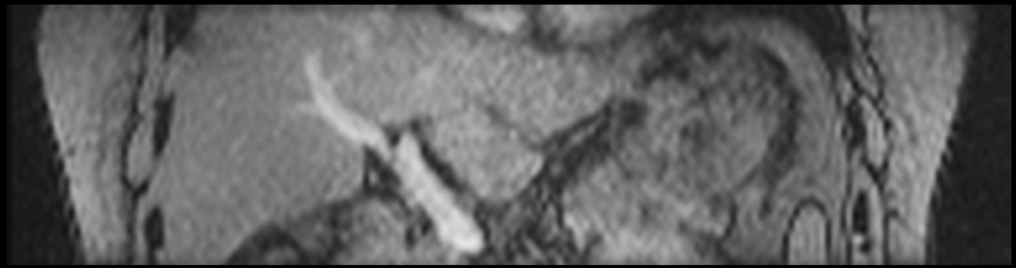


Insufficient breath-holding

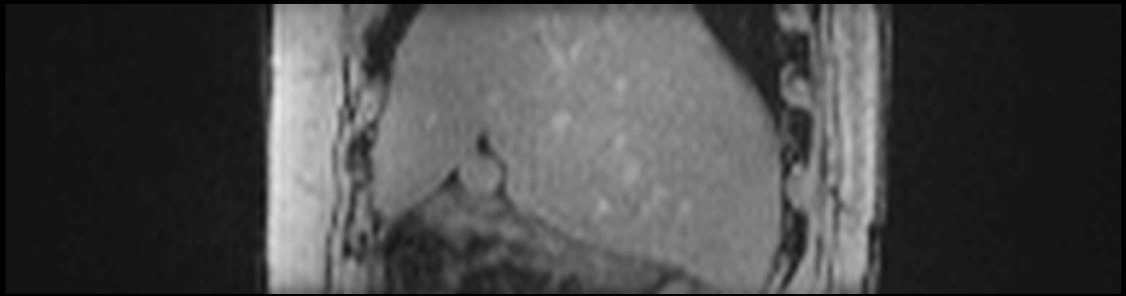
Free-breathing 3D Stack-of-Radial MRI



Axial



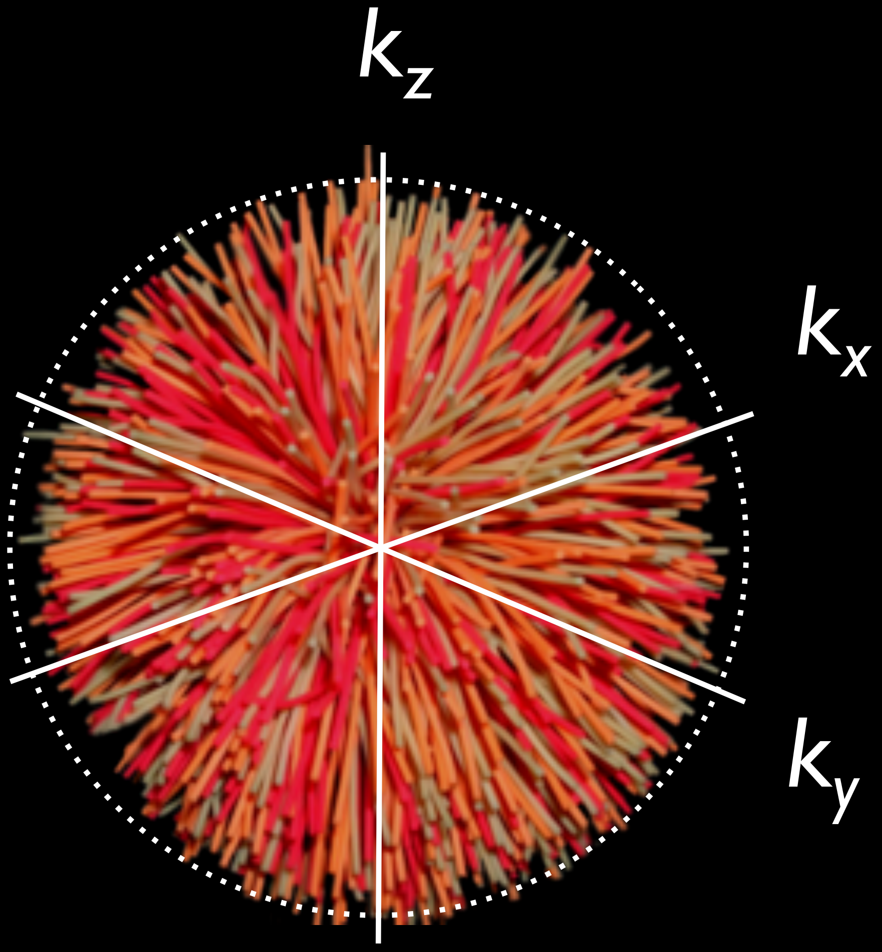
Coronal



Sagittal

courtesy of Tess Armstrong

3D Radial



Pros

- Robust to motion (get DC every TR)
- Can tolerate a lot of undersampling
- Half-spoke PR has very short TE

Cons

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

image from http://en.wikipedia.org/wiki/Koosh_ball

3D Radial: Coronary MRA

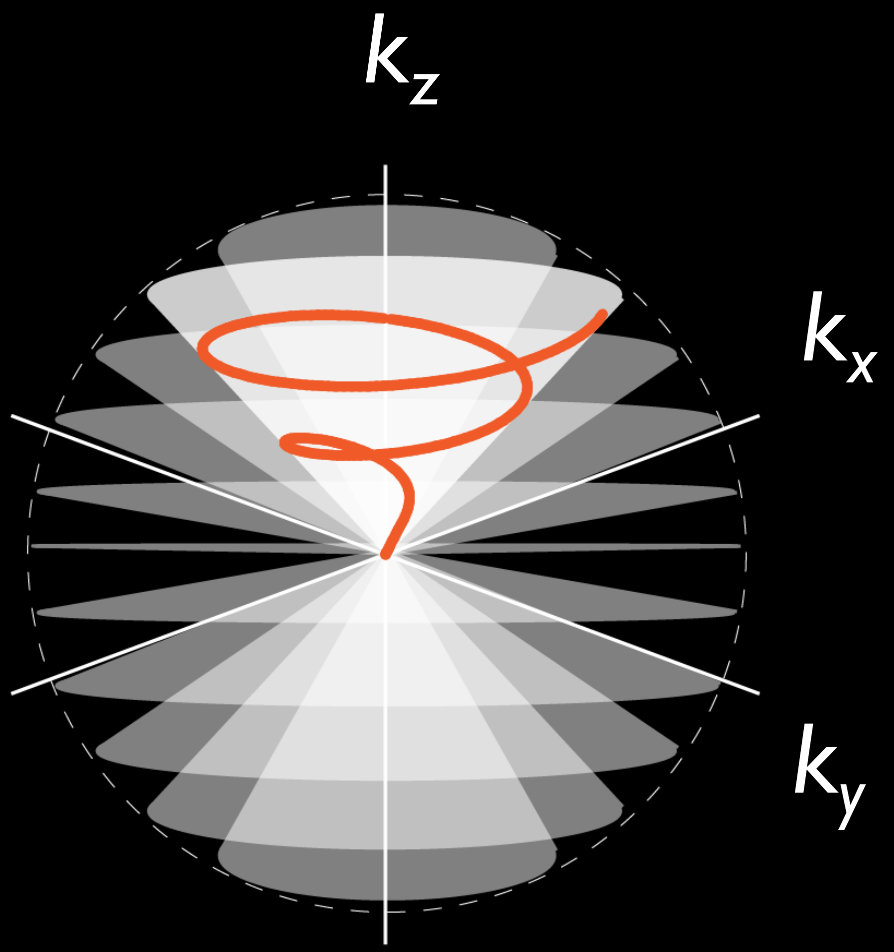
Contrast-Enhanced MRA at 3.0T

ECG-gated, fat-saturated, inversion-recovery prepared spoiled gradient echo sequence

$(1.0\text{ mm})^3$ spatial resolution, 1D self navigation, CG-SENSE recon, 5.4 min scan time

courtesy of Debiao Li and J Pang (Cedars-Sinai)

3D Cones



<u>Pros</u>

- Very fast (3-8x vs. Cartesian)
- Very short TE
- Flexible readout length
- Robust to motion/flow effects

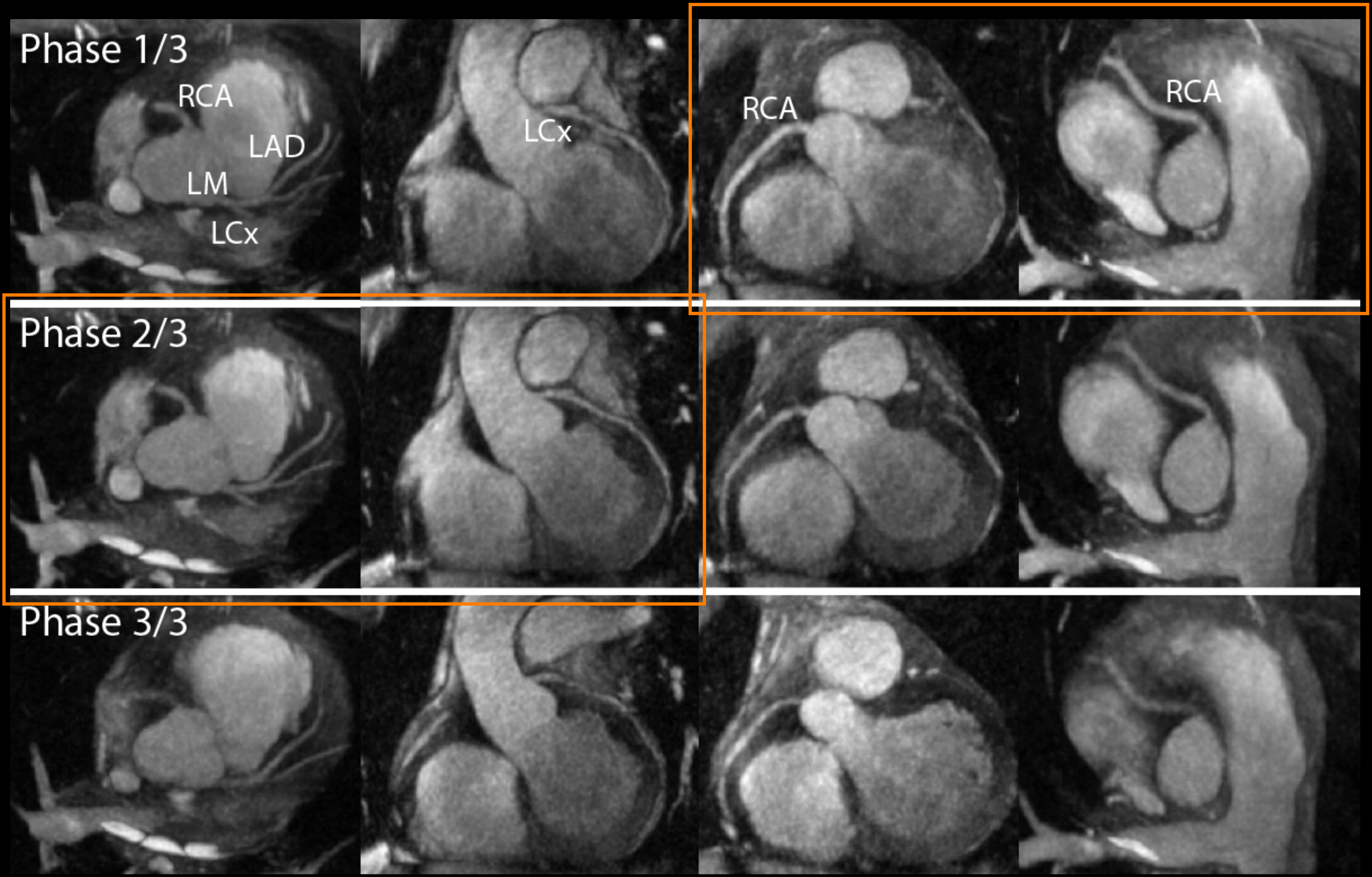
<u>Cons</u>

- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

Gurney PT et al., MRM 2006; 55: 575-82

3D Cones: Coronary MRA

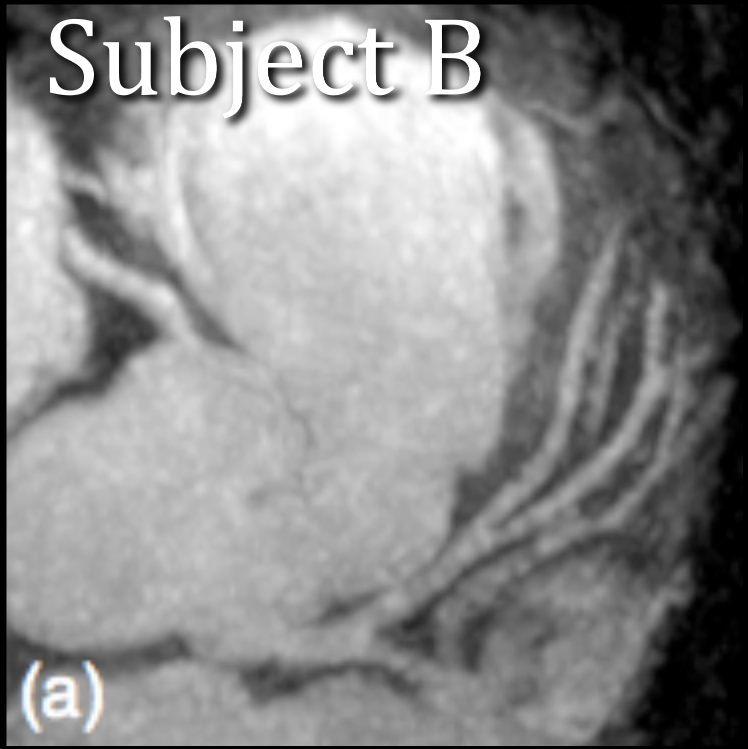
Multi-Phase Thin-Slab MIP Reformats



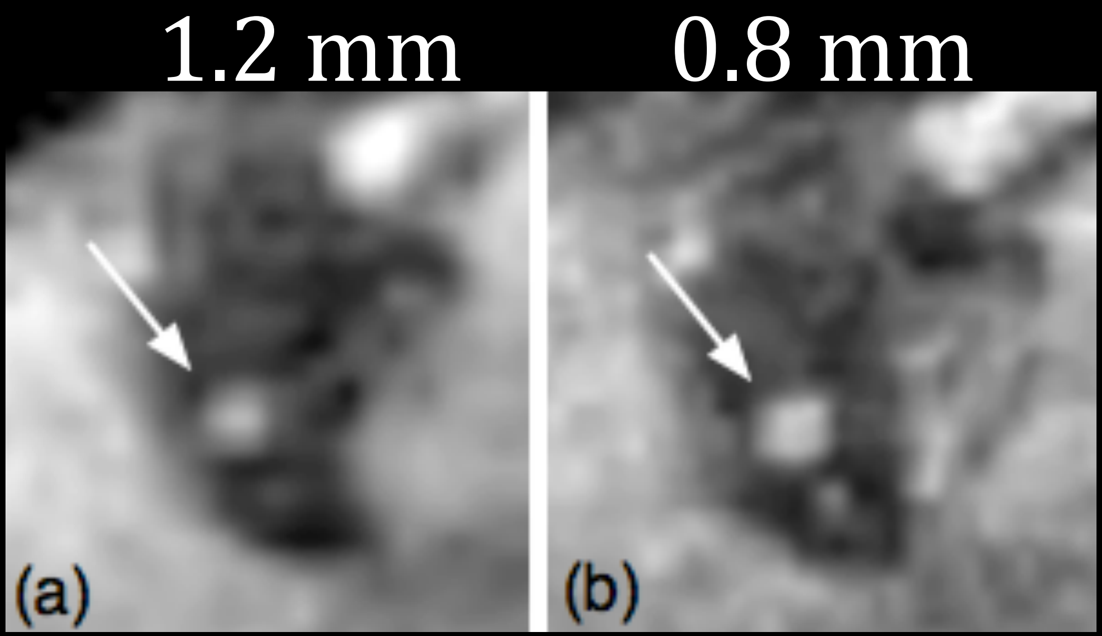
Wu HH et al., MRM 2013; 69: 1083-1093

3D Cones: Hi-res CMRA

Thin-Slab MIP Reformats: 0.8 mm isotropic



Right coronary artery cross section



1.5 T; 8-channel cardiac coil

Addy NO, et al., MRM 2015; 74:614-621

Non-Cartesian Image Reconstruction

- Gridding reconstruction
- Gradient measurement
- Off-resonance correction

MRI Signal Equation

$$s(t) = \iint\limits_{X,Y} m(x,y) \cdot \exp(-i2\pi \cdot [k_x(t)\,x + k_y(t)\,y])\,\mathrm{d}x\,\mathrm{d}y$$

$$= \mathcal{FT}(\,m(x,y)\,) = M(\,k_x(t), k_y(t)\,)$$

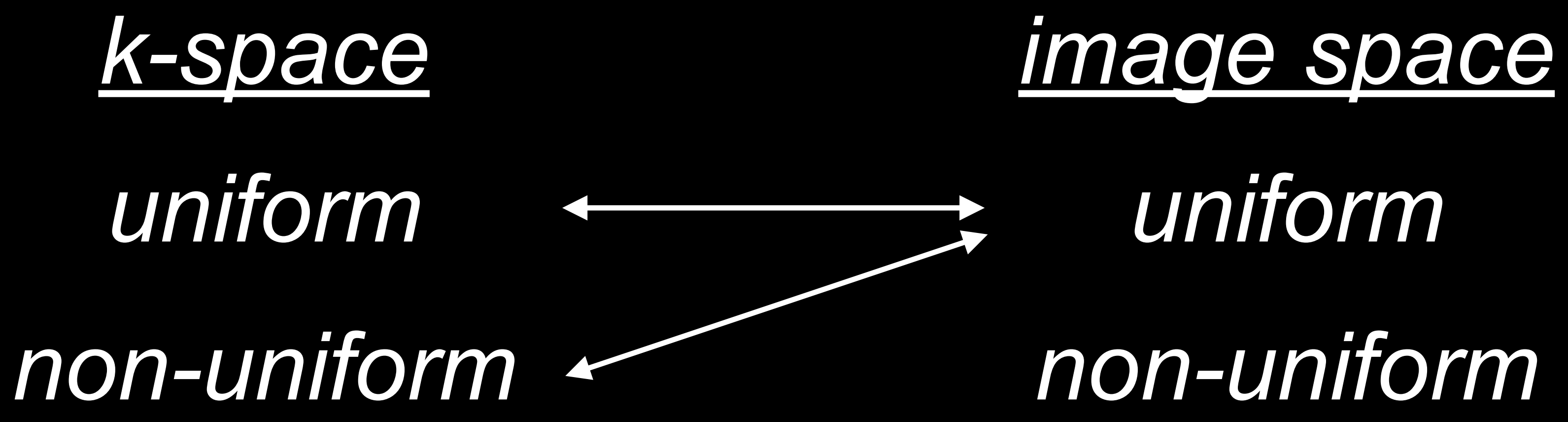
General definition of *k*-space:

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)\,\mathrm{d}\tau, \quad k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)\,\mathrm{d}\tau$$

MRI Reconstruction

$$m(x, y) = \mathcal{FT}^{-1}(\,M(k_x, k_y)\,)$$

$$m(x, y) = \iint_{k_x, k_y} M(k_x, k_y) \cdot \exp(i2\pi \cdot [k_x x + k_y y])\, \mathrm{d}k_x\, \mathrm{d}k_y$$



simple for Cartesian (k_x, k_y) to Cartesian (x, y): 2D FFT

time consuming for non-Cartesian (k_x, k_y) to Cartesian (x, y)

Non-Cartesian Reconstruction

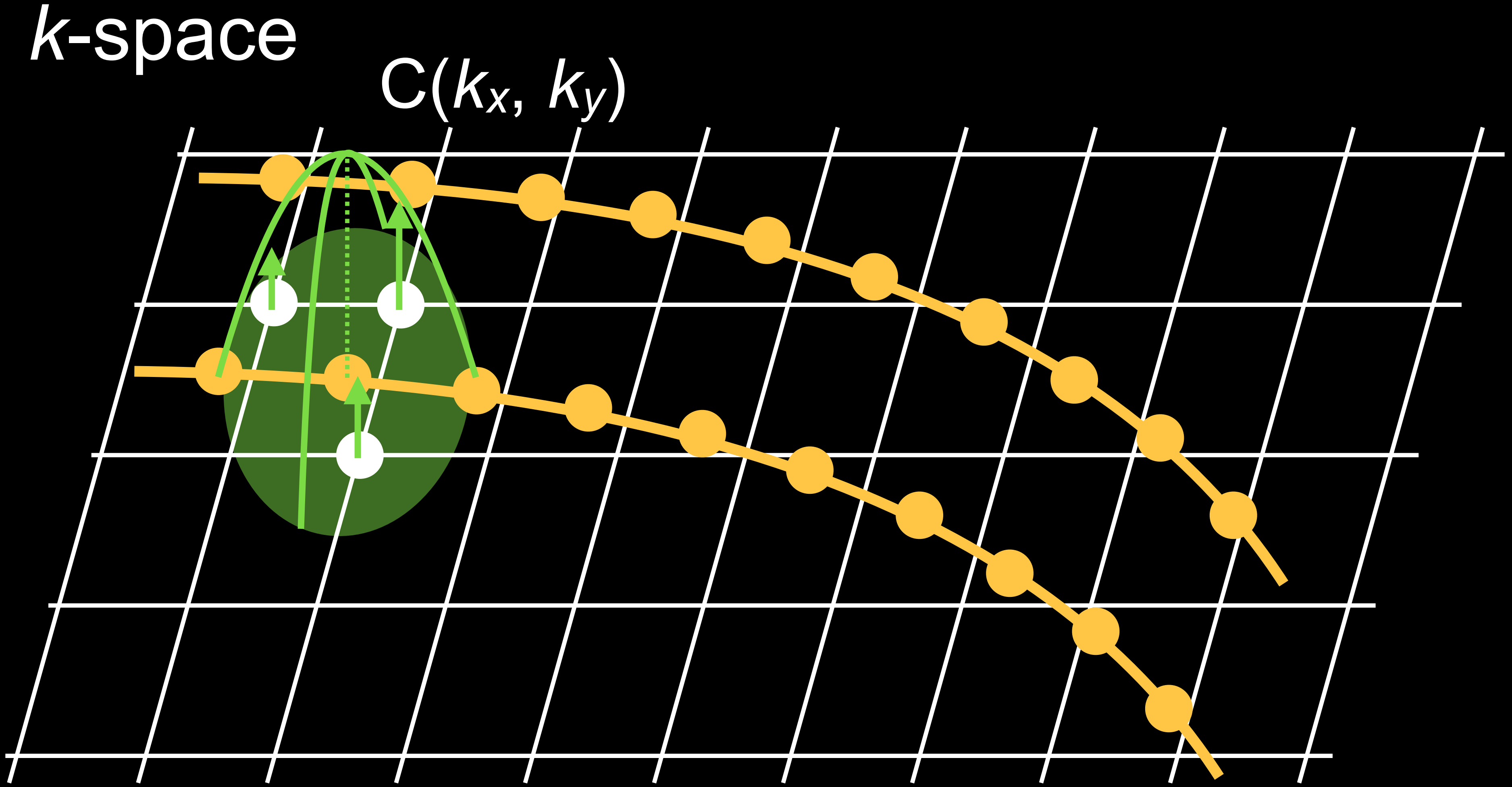
- Inverse Fourier transform
    - aka conjugate phase reconstruction
- Gridding (+FFT)[1]
    - grid driven interpolation
    - data driven interpolation (more popular)
    - forward and reverse (inverse)
- Non-uniform FFT (NUFFT)[2]
- Block Uniform ReSampling (BURS)[3]

[1] *O'Sullivan JD, IEEE TMI 1985; 4: 200-207*

[2] *Fessler JA et al., IEEE TSP 2003; 51: 560-574*

[3] *Rosenfeld D, MRM 2002; 48: 193-202*

Gridding: Basic Idea



convolve each acquired data point with kernel C(k_x, k_y)

resample the convolution onto Cartesian grid points

2D inverse FFT; de-apodization and FOV cropping

Gridding: Basic Math

Sampling pattern: $S(k_x, k_y) = \sum_j^2 \delta(k_x - k_{x,j}, k_y - k_{y,j})$

Convolution kernel: $C(k_x, k_y)$

Grid: $\text{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y})$

Gridding recon:

$$\hat{M}(k_x, k_y) = [\underbrace{(M(k_x, k_y) \cdot S(k_x, k_y))}_{\textit{non-Cartesian dataset}} * \underbrace{C(k_x, k_y)]}_{\textit{interpolation}} \cdot \underbrace{\text{III}(\frac{k_x}{\Delta k_x}, \frac{k_y}{\Delta k_y})}_{\textit{resample to grid}}$$

$$\textit{FFT}\Big\downarrow$$

$$\hat{m}(x, y) = [(m(x, y) * s(x, y)) \cdot \underbrace{c(x, y)]}_{\textit{remove by deap}} * \underbrace{\text{III}(\frac{x}{\text{FOV}_x}, \frac{y}{\text{FOV}_y})}_{\textit{remove by cropping}}$$

$$\rightarrow m(x, y)$$

Gridding: Design Issues

- Convolution kernel
  - apodization; aliasing
- Sampling grid density (Cartesian)
  - aliasing
- Sampling pattern (non-Cartesian)
  - impulse response and side lobes
  - density characterization / compensation

Gridding: Design - Kernel

- Ideal convolution kernel: SINC
    - don’t need de-apodization
    - infinite extent impractical to implement
    - windowed version has limited performance
- Desired kernel characteristics
    - compact support (finite width) in k-space
    - minimal aliasing effects in image (sharp transition)

Gridding: Design - Kernel

Combine with grid oversampling

$$\Delta k_x = \frac{1}{\mathrm{FOV}_x}, \Delta k_y = \frac{1}{\mathrm{FOV}_y}$$

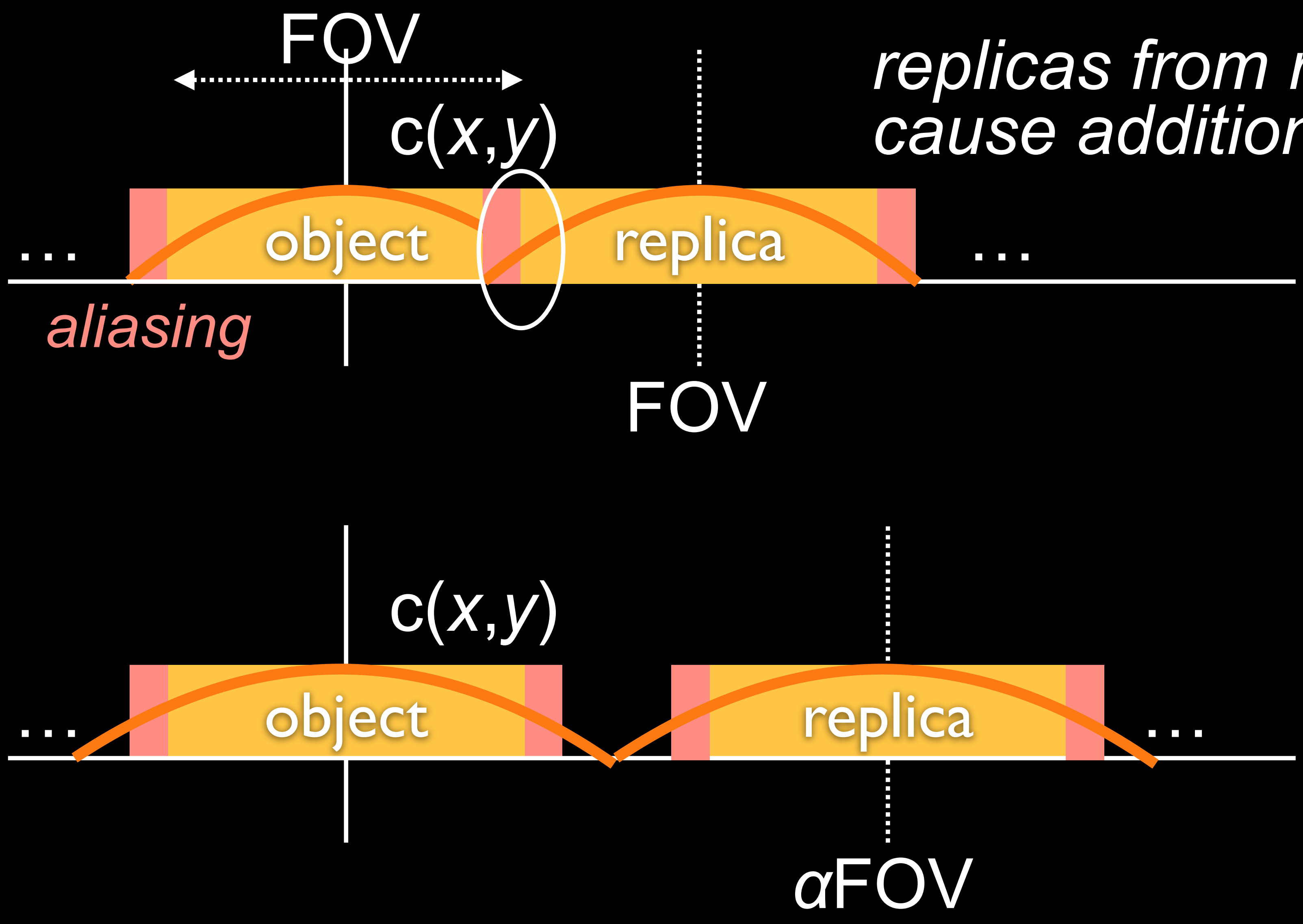
$$\frac{\Delta k_x}{\alpha} = \frac{1}{\alpha \mathrm{FOV}_x}, \frac{\Delta k_y}{\alpha} = \frac{1}{\alpha \mathrm{FOV}_y} \qquad \alpha > 1$$

$$\hat{M}(k_x, k_y) = [(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \mathrm{III}(\frac{k_x}{\Delta k_x/\alpha}, \frac{k_y}{\Delta k_y/\alpha})$$

$$\hat{m}(x, y) = [(m(x, y) * s(x, y)) \cdot c(x, y)] * \mathrm{III}(\frac{x}{\alpha \mathrm{FOV}_x}, \frac{y}{\alpha \mathrm{FOV}_y})$$

Gridding: Design - Kernel

Combine with grid oversampling



replicas from resampling to grid cause additional aliasing

α = 2 very forgiving; many kernels work well; apodization minimal

expensive … especially for 3D gridding

Gridding: Design - Kernel

- Jointly consider *α* and kernel
  - minimize aliasing energy
  - characterize trade-offs
  - numerical designs possible
  - Kaiser-Bessel window works very well, with proper choice of *β* and *kw*[1,2]; precompute a lookup table to speedup calculations[2]

$$C_{KB}(k_x) = \mathrm{I}_0\left(\beta\sqrt{1-(\frac{k_x}{kw/2})^2}\right)$$

[1] *Jackson et al., IEEE TMI 1991; 10: 473-478*

[2] *Beatty et al., IEEE TMI 2005; 24: 799-808*

Gridding: Design - Density

Sampling density of $S(k_x, k_y)$ not uniform: $\rho(k_x, k_y)$

Pre-compensation of sampling density:

$$\hat{M}(k_x, k_y) = [(M(k_x, k_y) \cdot \frac{S(k_x, k_y)}{\rho(k_x, k_y)}) * C(k_x, k_y)] \cdot \mathrm{III}$$

density corrected on a data point basis before convolution

need to know $\rho(k_x, k_y)$

from geometrical analysis, numerical analysis (Voronoi), etc.

inverse of ρ known as the density compensation function (DCF)

Gridding: Design - Density

Post-compensation of sampling density:

$$\hat{M}(k_x, k_y) = \frac{[(M(k_x, k_y) \cdot S(k_x, k_y)) * C(k_x, k_y)] \cdot \mathrm{III}}{\rho(k_x, k_y)}$$

density corrected on a grid point basis after convolution

can estimate ρ along with gridding; grid all 1s:

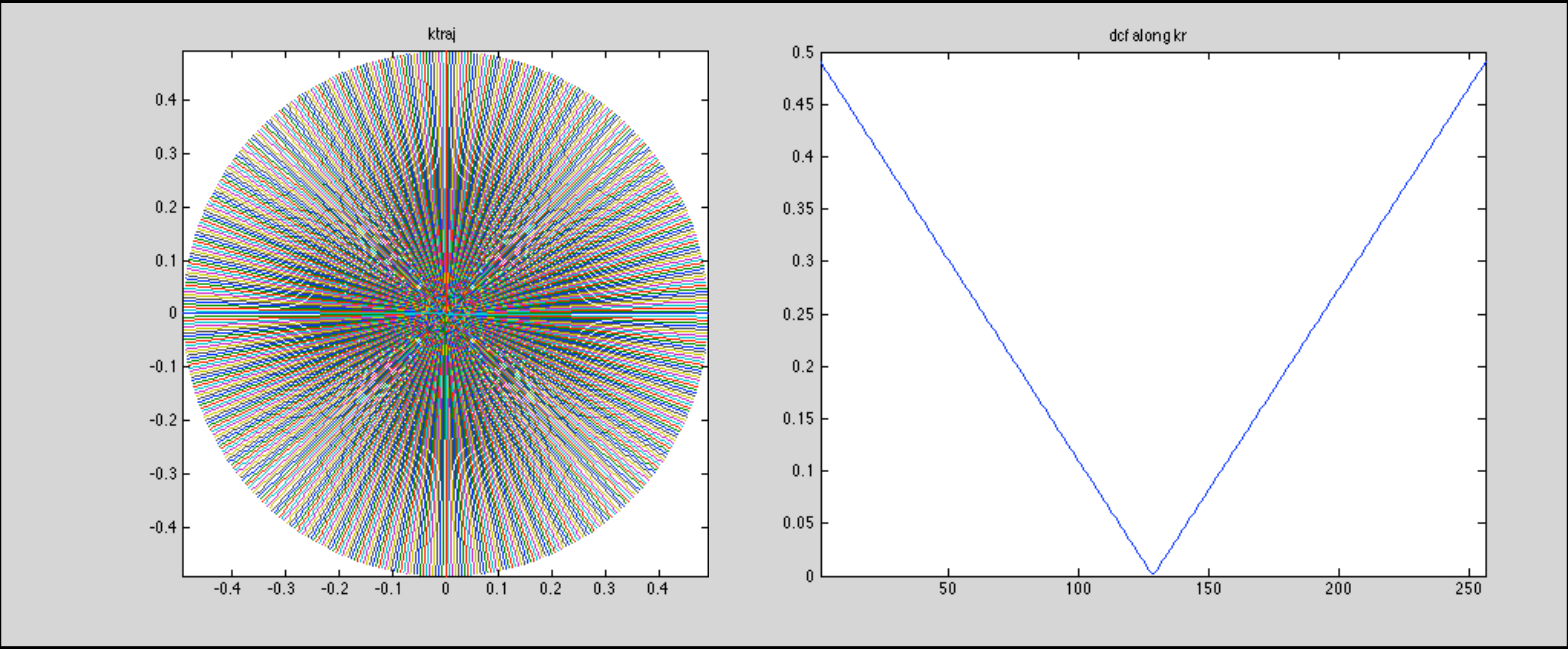
$$\hat{\rho}(k_x, k_y) = [S(k_x, k_y) * C(k_x, k_y)] \cdot \mathrm{III}$$

may be okay if S changes slowly

… but only an approximation and fails when S changes rapidly

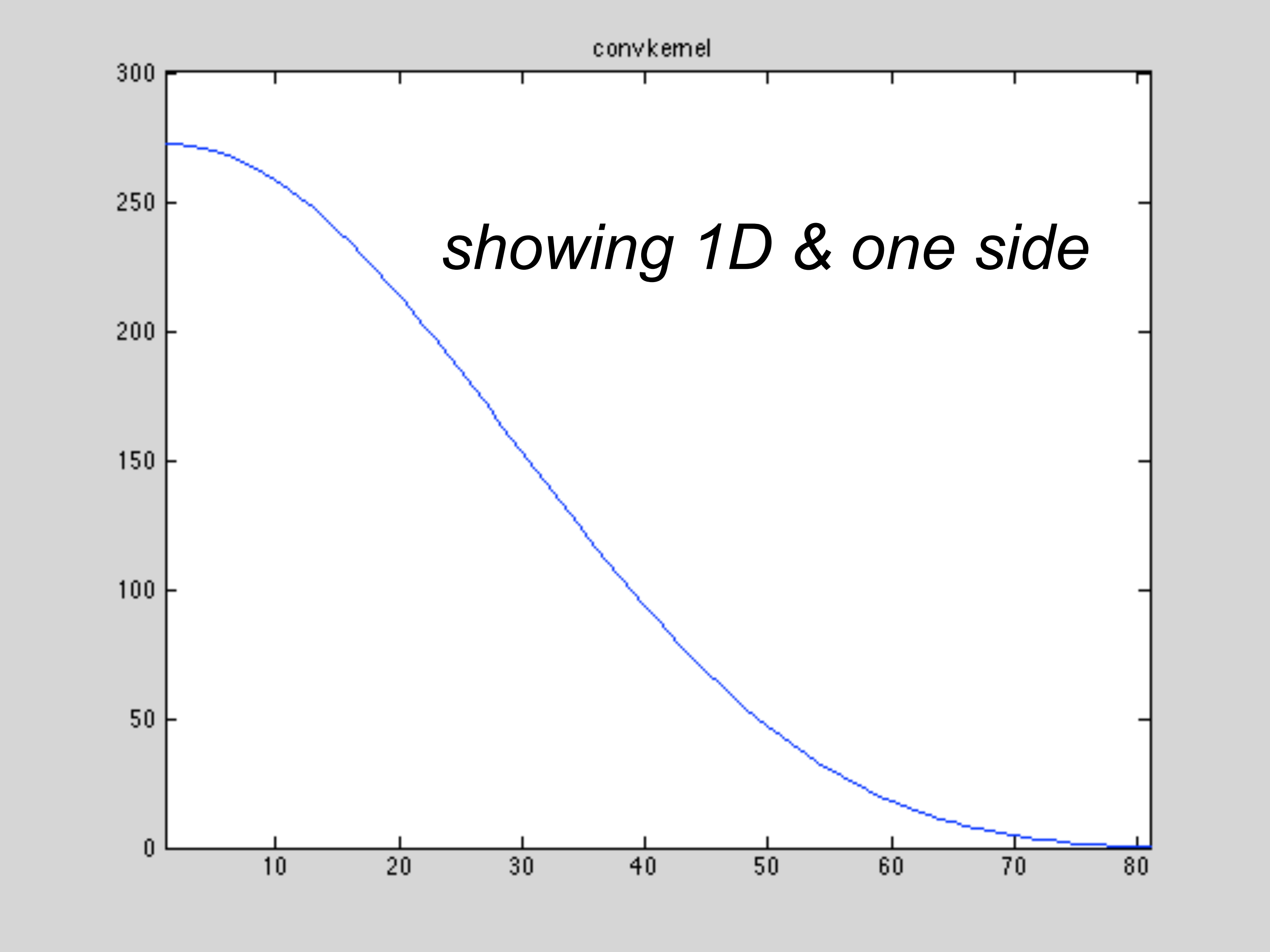
Gridding: 2D Radial Example

Radial trajectory [256x256] with ramp DCF



Gridding: 2D Radial Example

Kaiser-Bessel convolution kernel with linear lookup table[1]



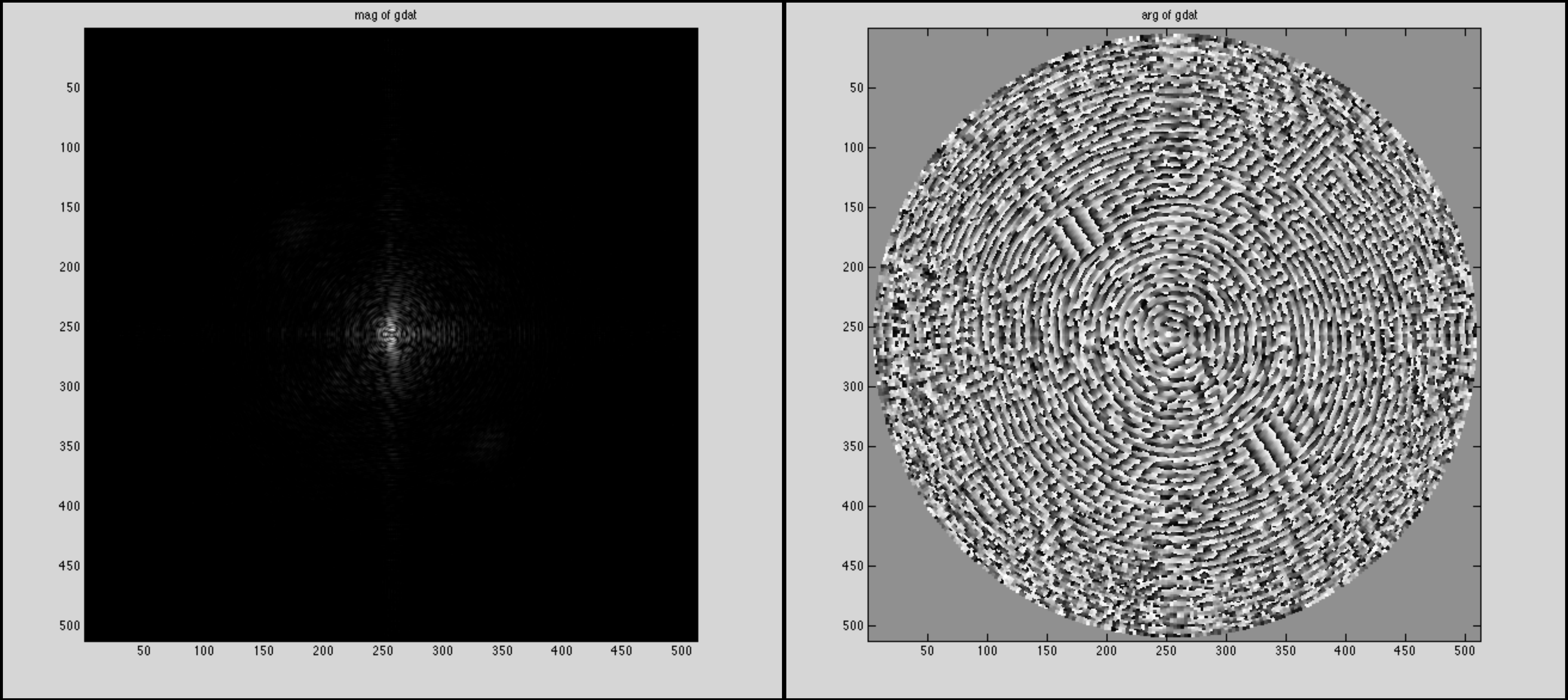
showing 1D & one side

α = 2; grid size = 2x[256 256]; kw = 4;

[1] Beatty et al., IEEE TMI 2005; 24: 799-808

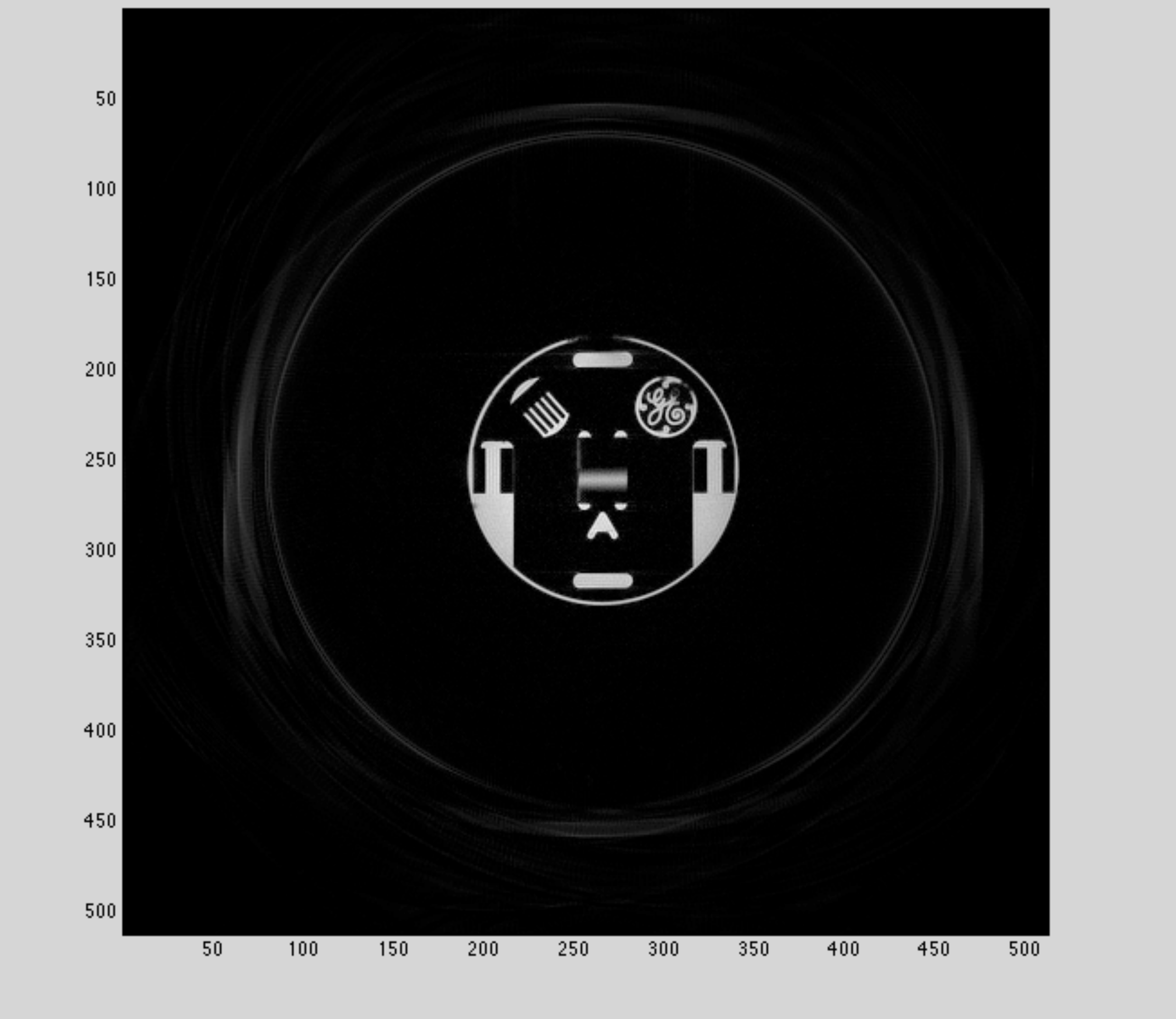
Gridding: 2D Radial Example

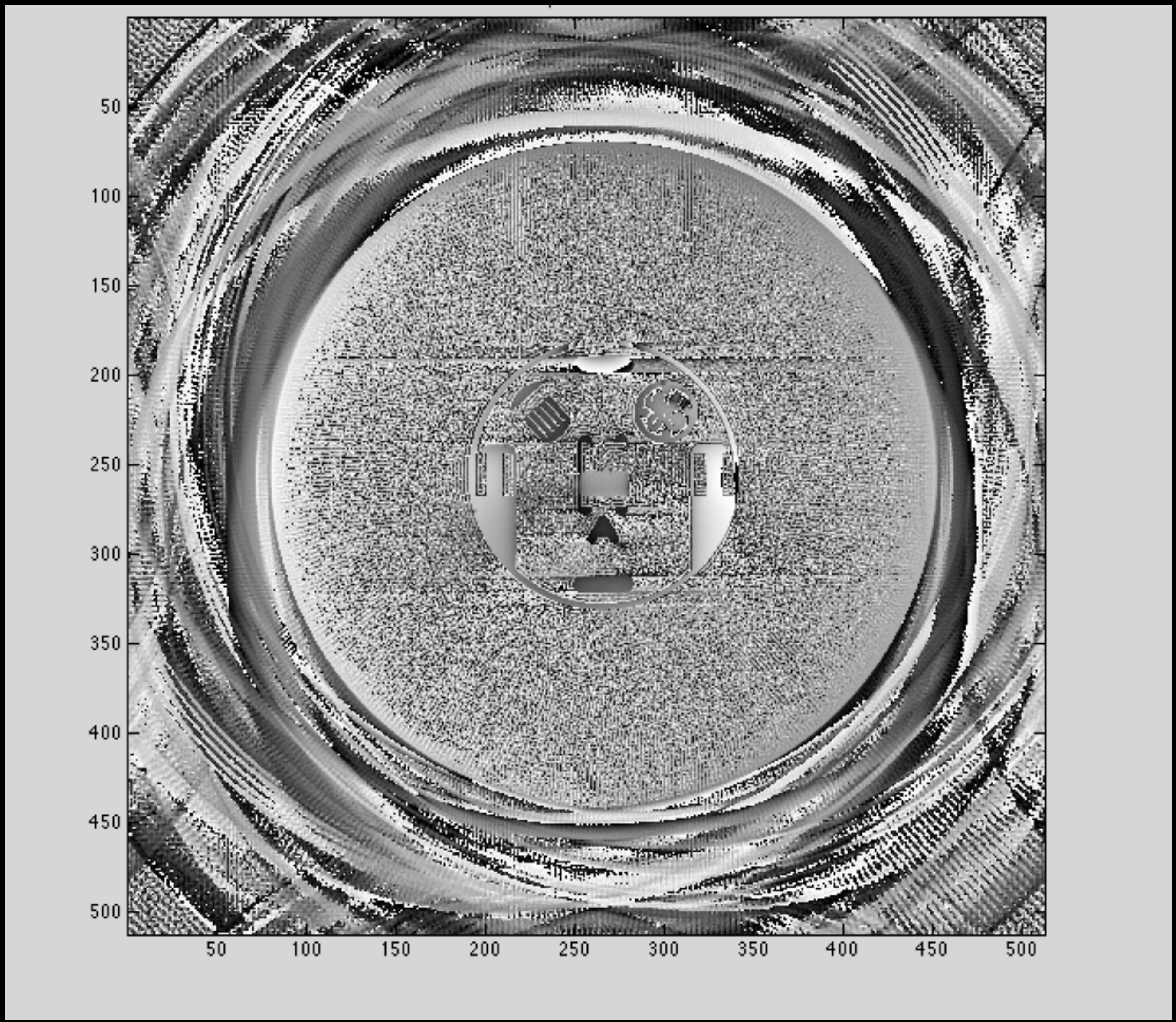
Gridded data on [512x512] grid



Gridding: 2D Radial Example

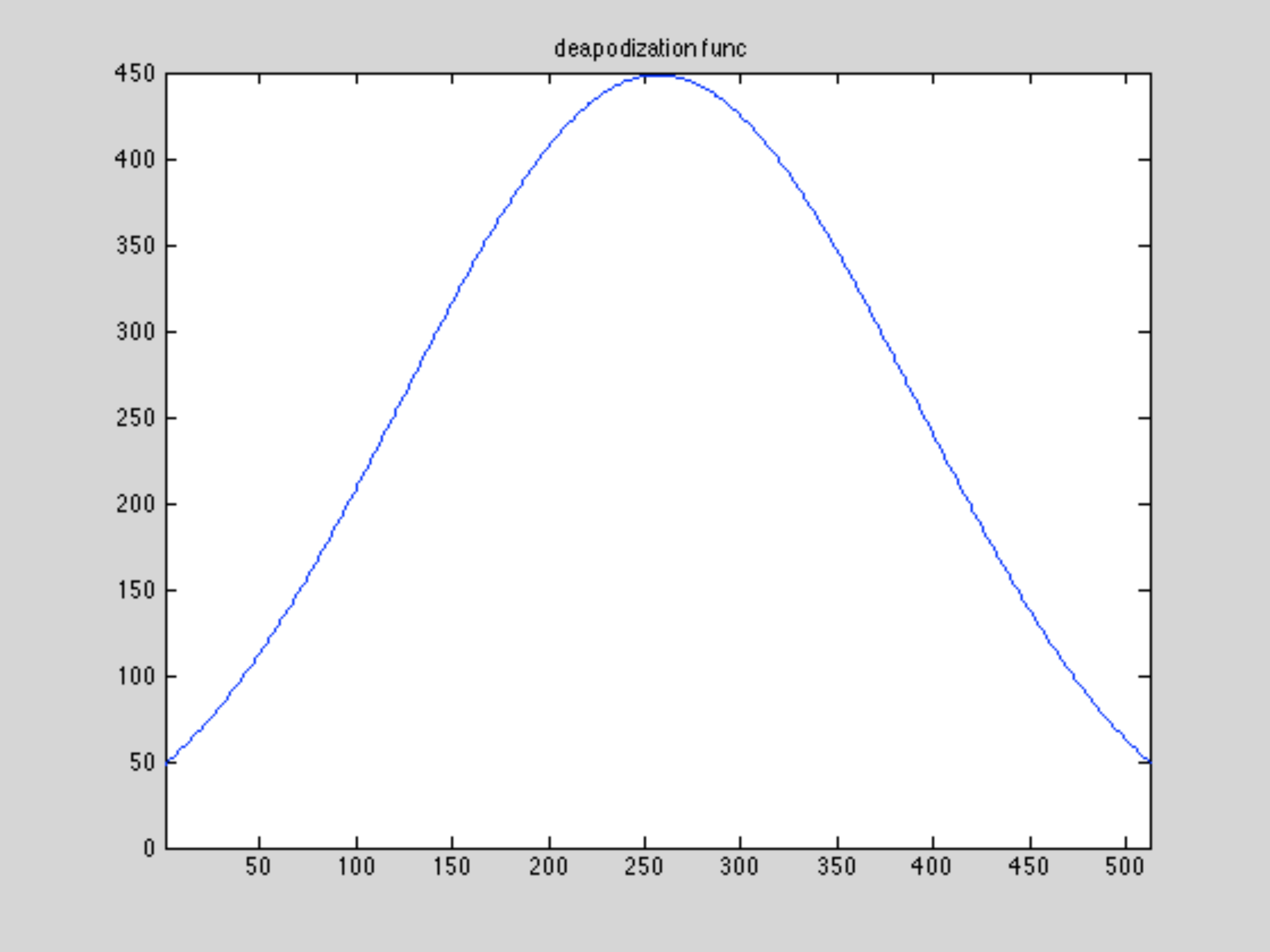
Inverse 2D FFT produces image with 2x FOV

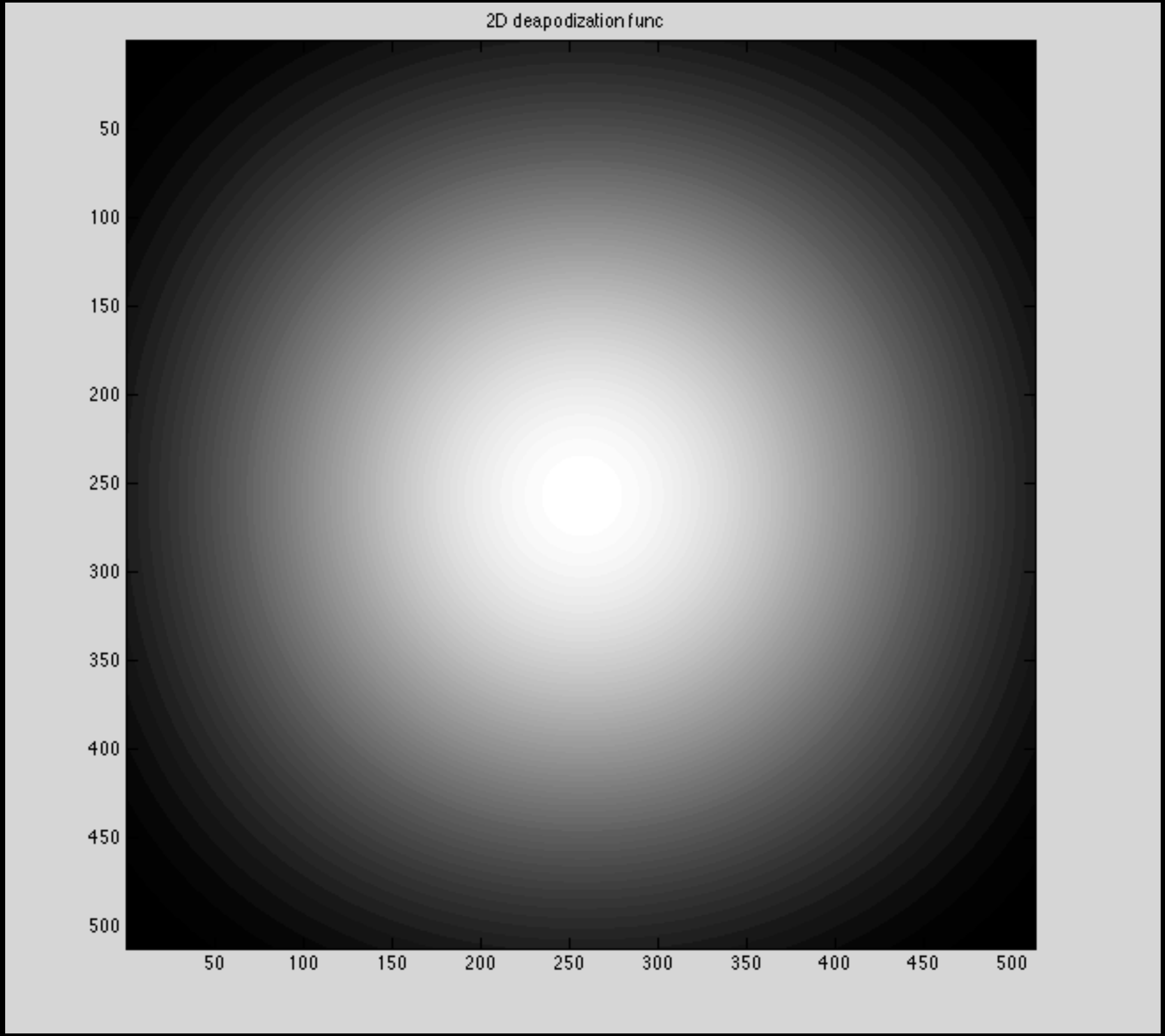




Gridding: 2D Radial Example

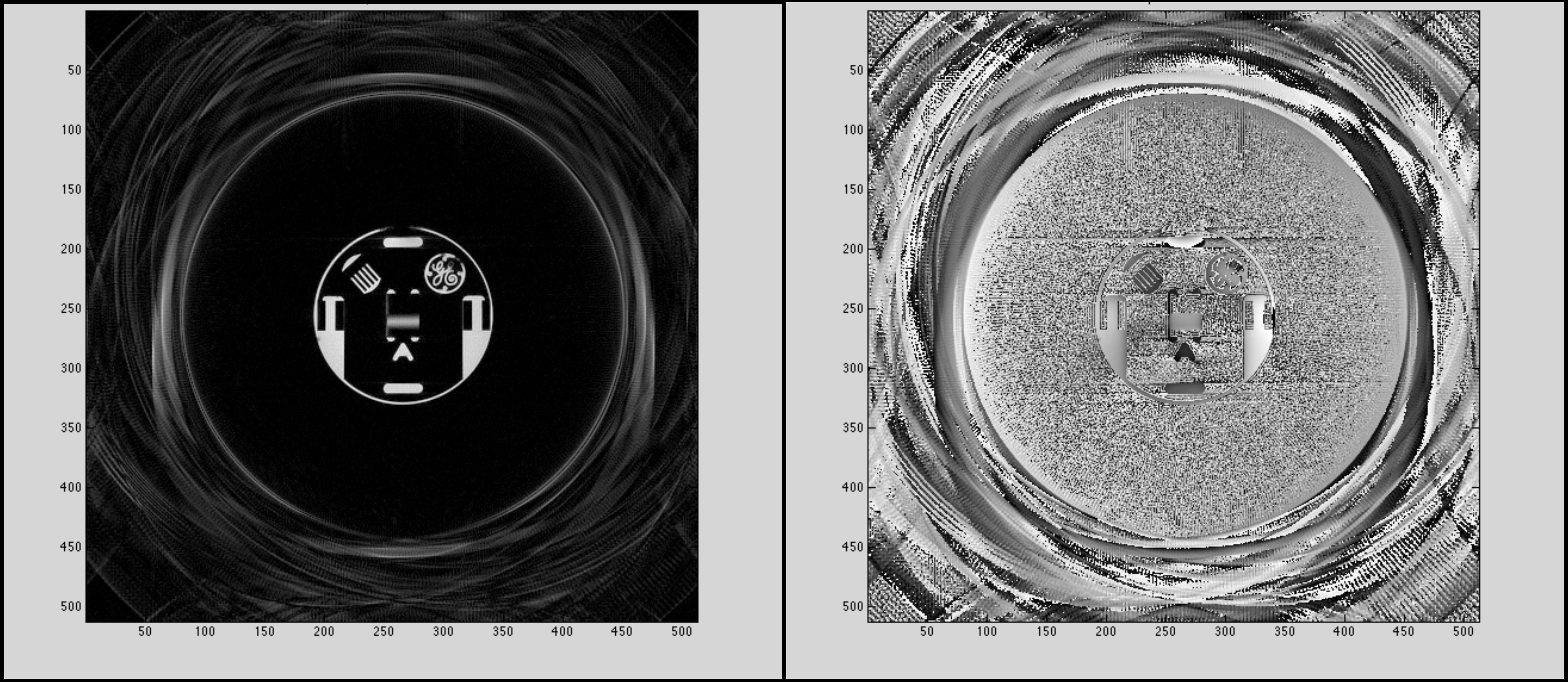
Deapodization function is FT of KB convolution kernel





Gridding: 2D Radial Example

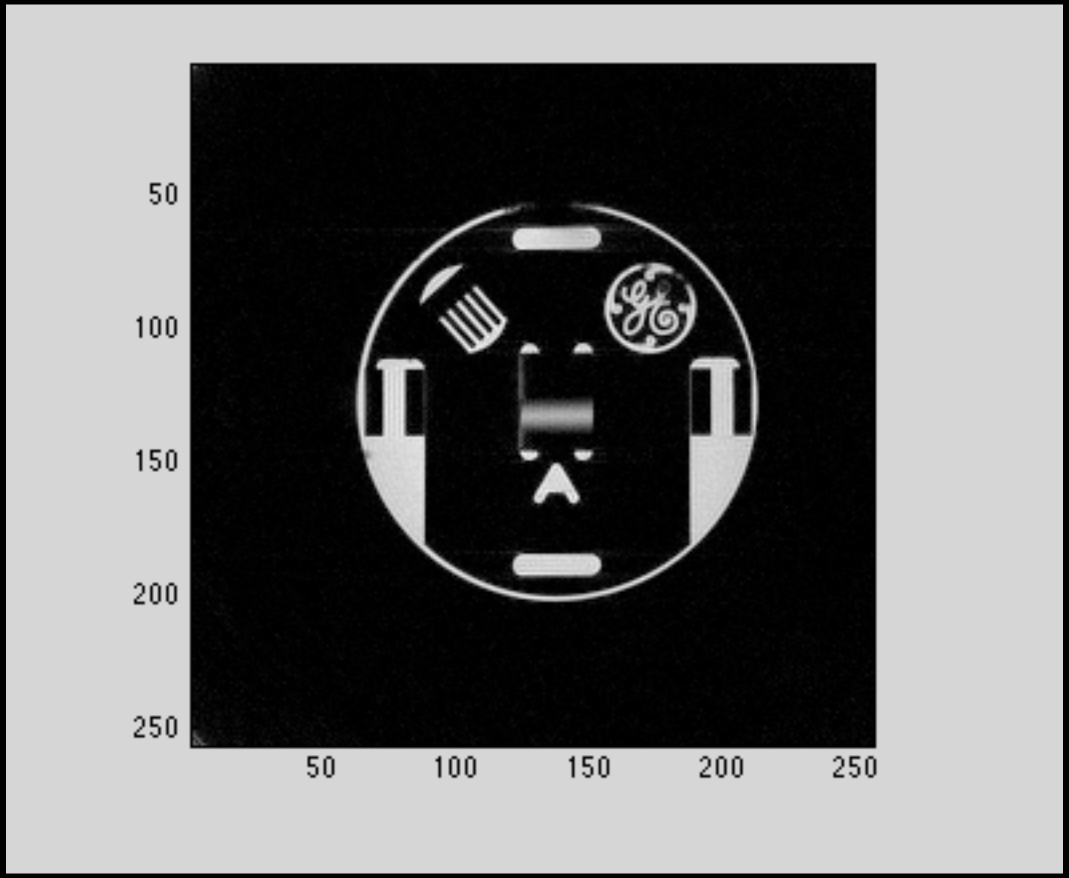
Deapodized image

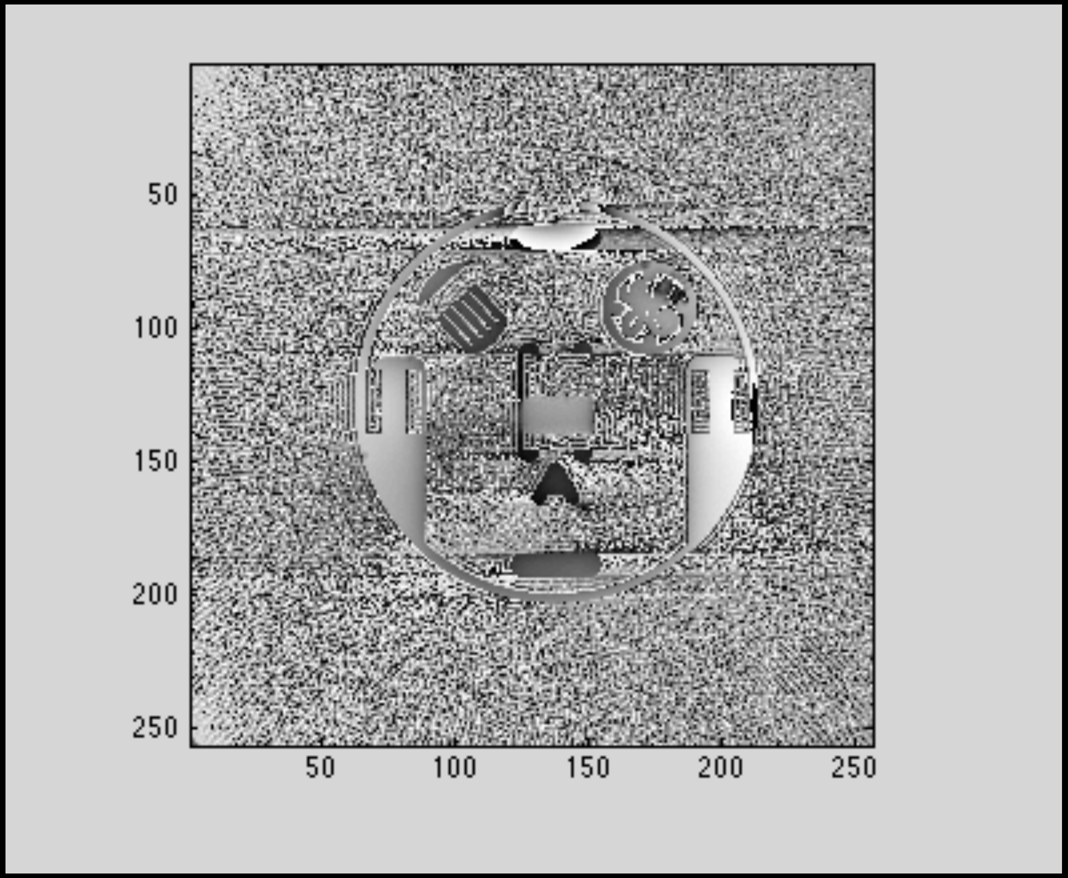


Gridding: 2D Radial Example

FOV cropped to extract desired [256x256] image

$\alpha = 2$, kw = 4

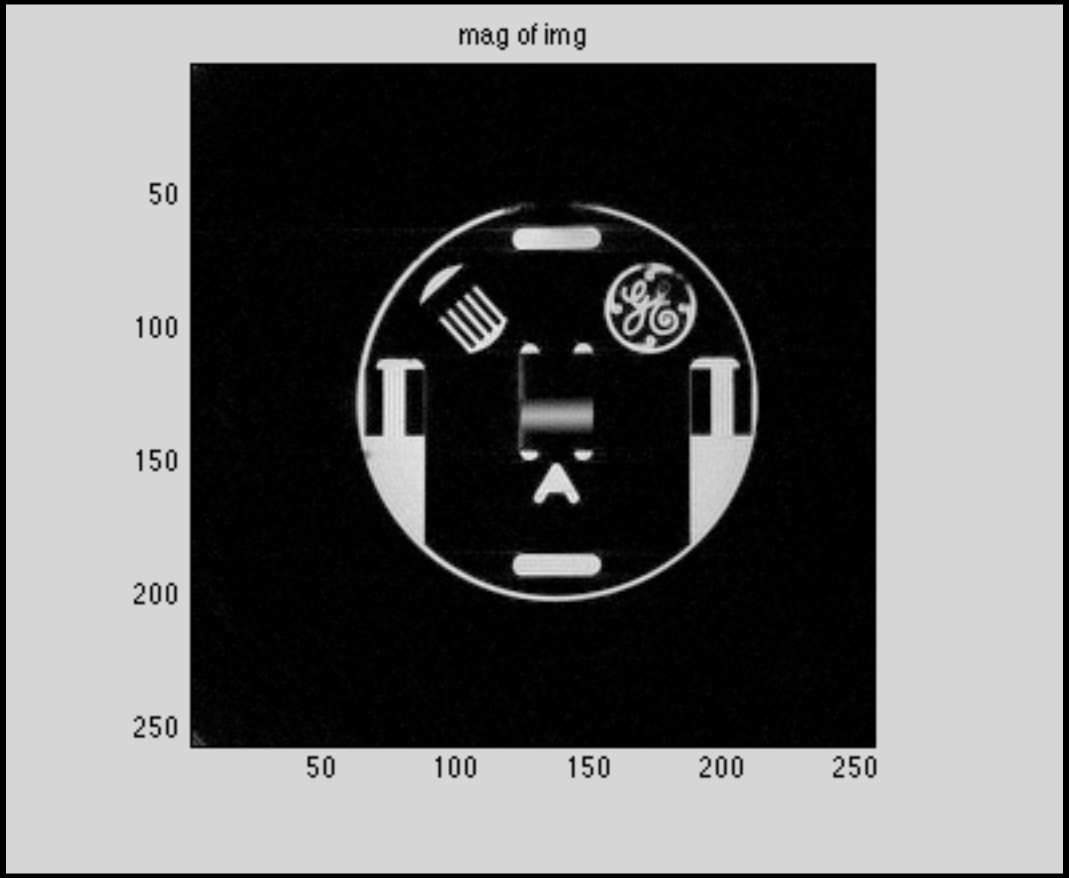




Gridding: 2D Radial Example

FOV cropped to extract desired [256x256] image

α = 1.375, kw = 5[1]



---

[1] *Beatty et al., IEEE TMI 2005; 24: 799-808*

Gridding: Summary

- Data input
  - k-space data
  - k-space traj (usually normalized), DCF
- Gridding params
  - target image dimensions [MxN]
  - grid oversampling factor α
  - kernel type and width
- Data output
  - gridded Cartesian k-space
  - reconstructed image

Gradient Measurement

- Non-Cartesian recon requires
  - k-space trajectory
  - density compensation function
- Both depend on actual gradient waveforms on scanner
  - can deviate from desired
- Knowledge of k-space trajectory also important for RF design

Gradient Measurement

- Gradient imperfections cause artifacts
  - FOV scaling, shifting
  - signal loss, shading
  - image blurring, geometric distortion
- Sources of gradient errors
  - eddy currents (B_0, linear)
  - group delays (RF filters, A/D)
  - amplifier limitations (BW, freq response)
  - gradient warping
  - other ...

Gradient Measurement

- General techniques
  - off-iso slice technique[1,2], and more
- Trajectory-specific techniques
  - radial[3], spiral[4], and more
- Characterize gradient system
  - assume linear time-invariant model[5]

1 Duyn JH et al., JMR 1998; 132: 150-153

2 Beaumont M et al., MRM 2007; 58: 200-205

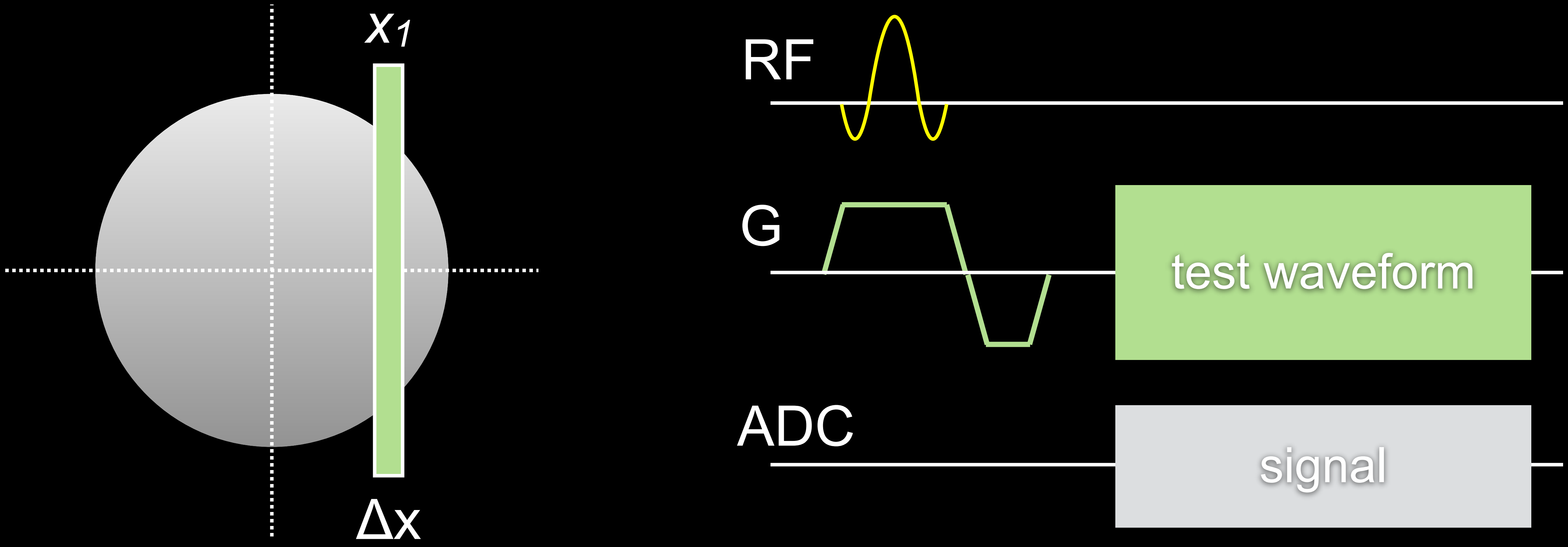
3 Peters DC et al., MRM 2003; 50: 1-6

4 Robison RK et al., MRM 2010; 63: 1683-90

5 Addy NO et al., MRM 2012; 68: 120-129

Gradient Measurement

Off-isocenter slice measurement technique



Can repeat on all three axes G_x, G_y, G_z

Duyn JH et al., JMR 1998; 132: 150-153

Gradient Measurement

Off-isocenter slice measurement technique

Waveform ON:

$$s_{x1,Gon}(t) = \iint\limits_{Y,Z} m(x_1, y, z) e^{-i\phi_0(x_1,y,z,t)} \, . \, e^{-i2\pi\cdot[\frac{\gamma}{2\pi}\int_0^t G(\tau)\mathrm{d}\tau]\cdot x_1} \, \mathrm{d}y \, \mathrm{d}z$$

Waveform OFF:

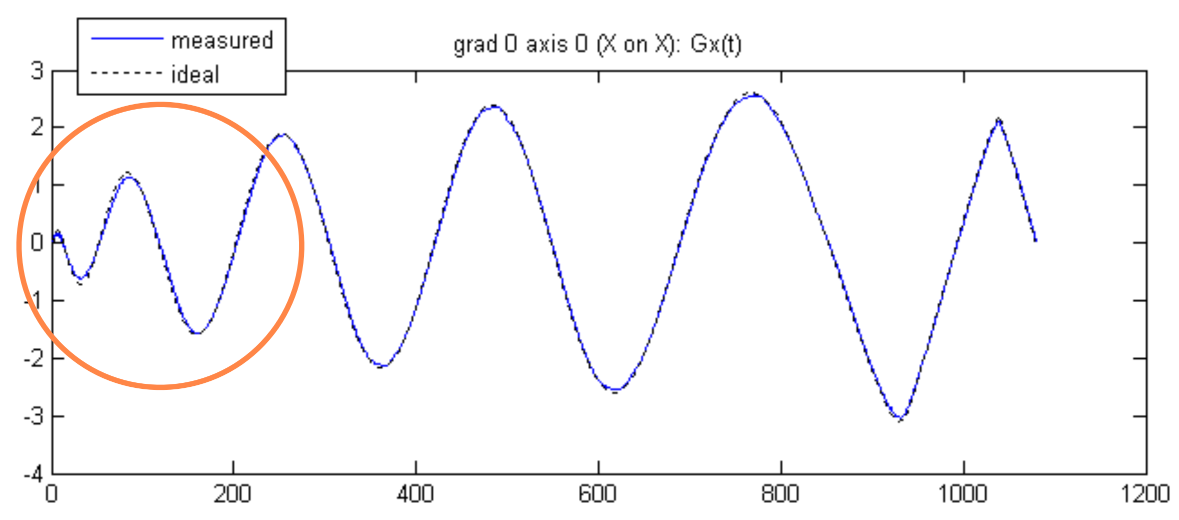
$$s_{x1,Goff}(t) = \iint\limits_{Y,Z} m(x_1, y, z) e^{-i\phi_0(x_1,y,z,t)} \, \mathrm{d}y \, \mathrm{d}z$$

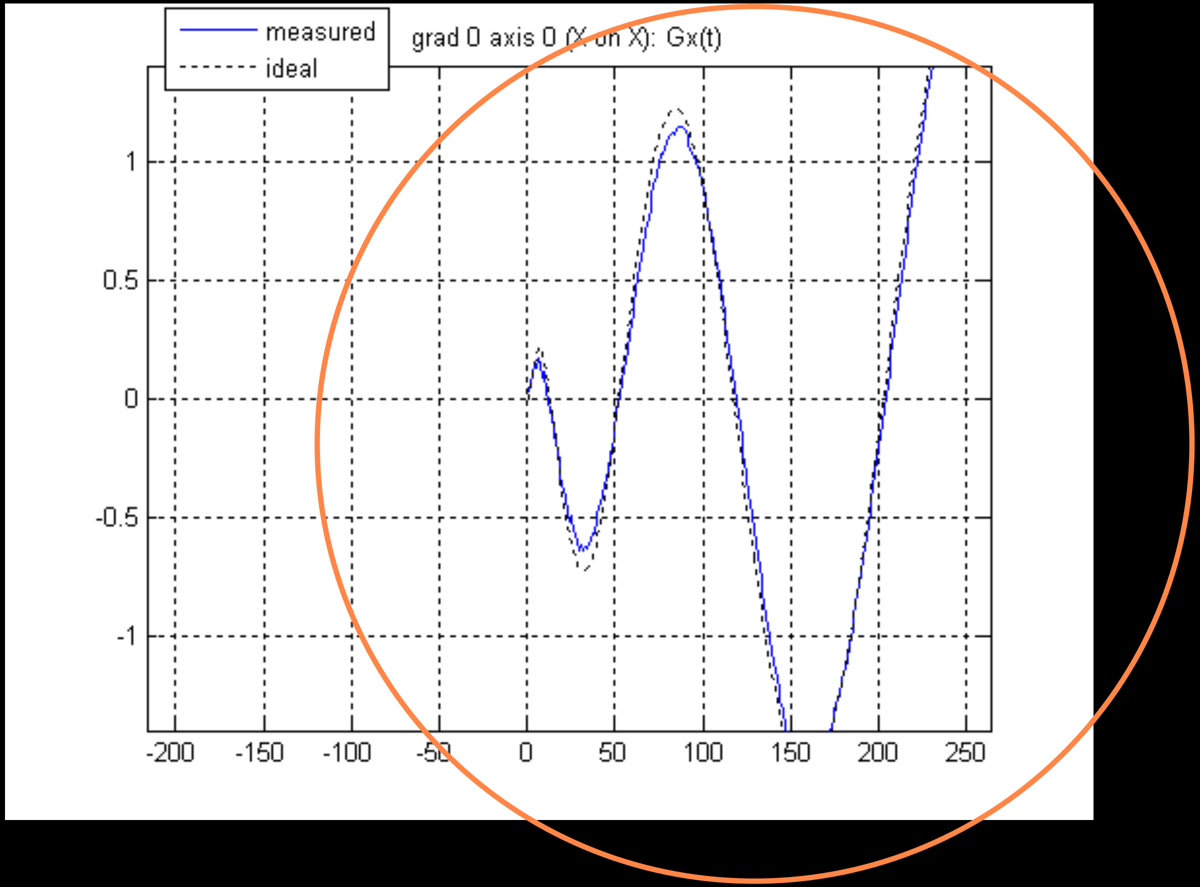
Phase difference:

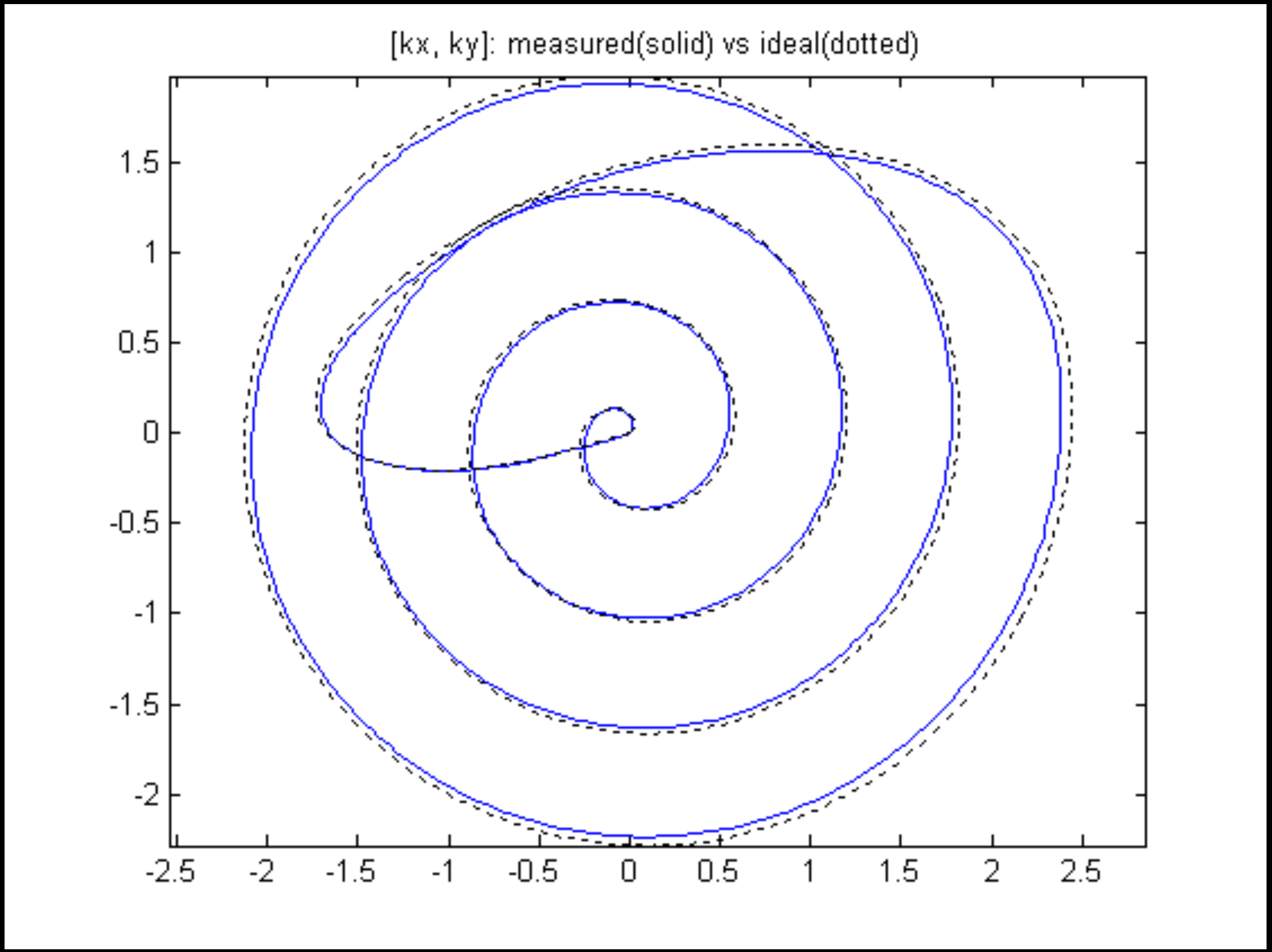
$$\Delta\phi_{x1}(t) = \gamma \int_0^t G(\tau) \cdot x_1 \, \mathrm{d}\tau = x_1 \cdot k(t)$$

Duyn JH et al., JMR 1998; 132: 150-153

Gradient Measurement



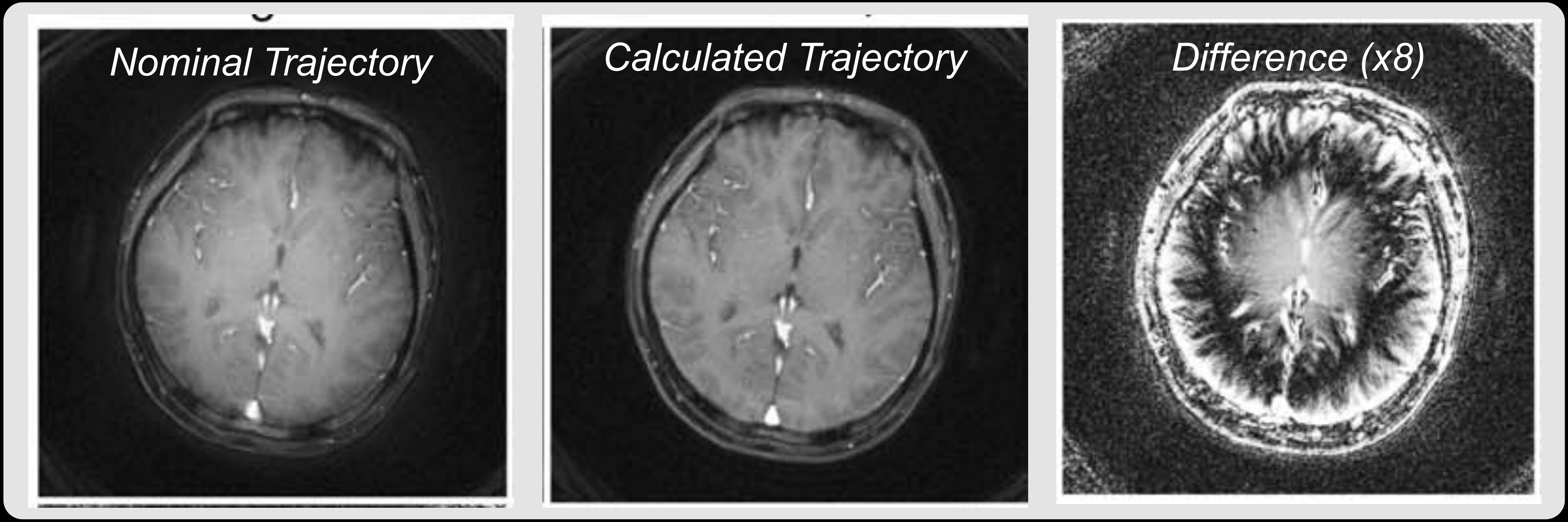




Gradient Measurement

- Gradient (trajectory) correction
  - use actual trajectory for recon
  - pre-tune bulk gradient delay

Example: Axial Spiral at 1.5 T



Addy NO et al., MRM 2012; 68: 120-129

Gradient Measurement

- Off-iso slice measurement technique
  - two measurements per axis
  - can measure X on X, Y on Y, Z on Z, and also cross terms; linearly combine
  - Δx should be small (may need avging)
  - need to account for phase wrapping
  - use spin echo for long waveforms
  - can acquire multiple slice offsets and gradient polarities to model individual gradient error terms

Gradient Measurement

- Delay calibration
  - gradient errors (e.g., linear eddy currents) mainly cause an apparent bulk delay
  - adjust ADC window w.r.t. gradients
  - delays may be different for each axis

Off-resonance Correction

- Off resonance effects (ΔB_0, fat, etc.)

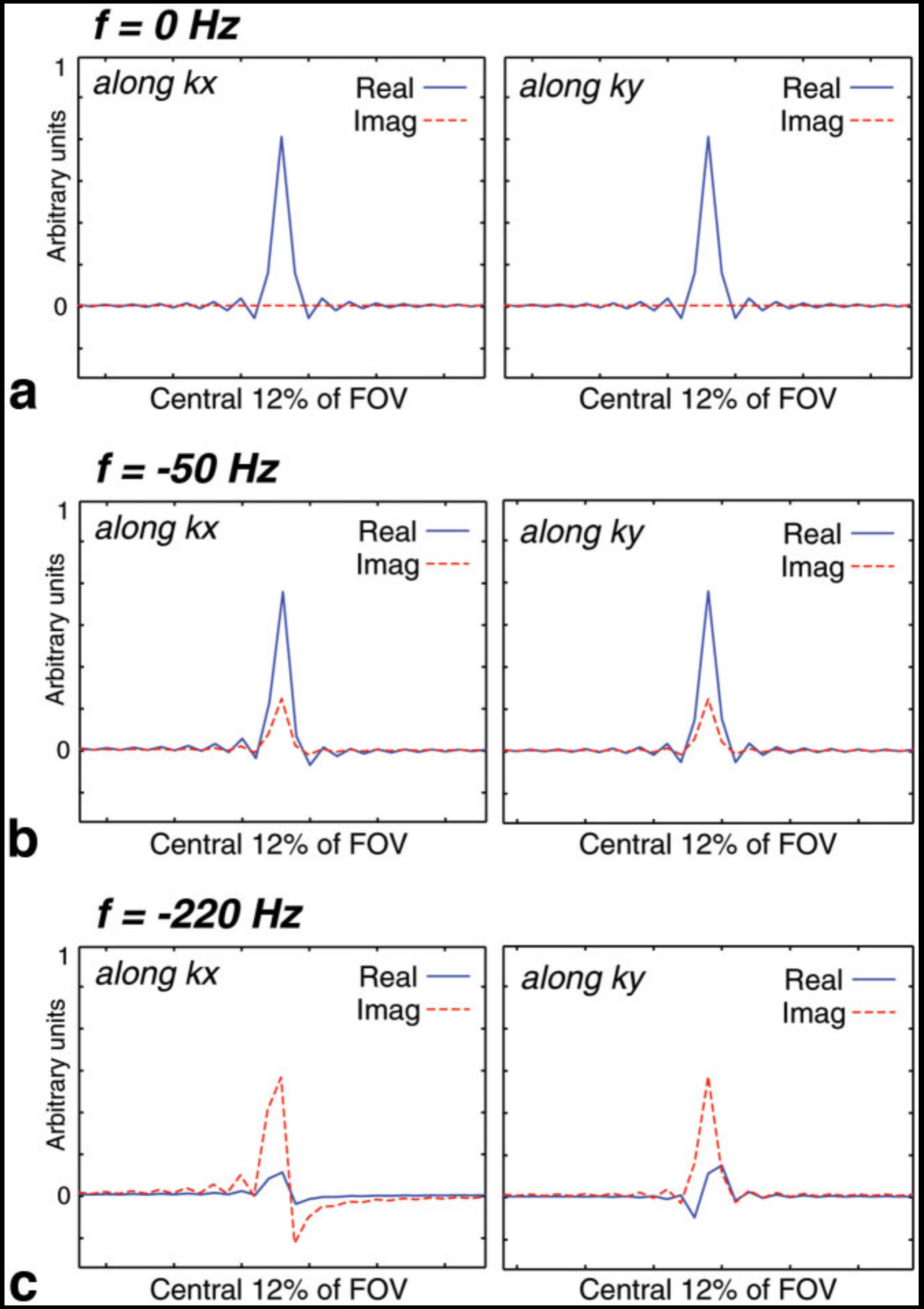
$$s(t) = \iint_{X,Y} m(x,y) \cdot e^{-i\phi(x,y,t)} \cdot e^{-i2\pi\cdot[k_x(t)\,x+k_y(t)\,y]}\,\mathrm{d}x\,\mathrm{d}y$$

$$\phi(x,y,t) = 2\pi\psi(x,y)t$$

  - patient (scan) dependent
  - pre-scan shim calibration helps
  - usually negligible for Cartesian MRI
  - non-Cartesian MRI: signal loss, spatial blurring, geometric distortion

Off-resonance Correction

Effects of off-res for concentric rings: PSF blurring



Off-resonance Correction

- Account for field inhomogeneity
  - use shorter readouts
  - measure/estimate field map

$$s(\mathrm{TE}_1) \longrightarrow I_1 = M'(x,y) \cdot e^{-i2\pi\psi(x,y)\mathrm{TE}_1}$$

$$s(\mathrm{TE}_2) \longrightarrow I_2 = M'(x,y) \cdot e^{-i2\pi\psi(x,y)\mathrm{TE}_2}$$

$$\hat{\psi}(x,y) = \arg(I_1 \cdot I_2^*)/2\pi(\Delta\mathrm{TE}) \quad [\pm 1/2\pi\Delta\mathrm{TE}]$$

and then correct (during recon)[1,2,3]
time-segmented, freq-segmented, etc.

1 Noll DC et al., IEEE TMI 1991; 10: 629-637

2 Noll DC et al., MRM 1992; 25: 319-333

3 Chen JY et al., MRM 2011; 66: 390-401

Off-resonance Correction

Linear Correction

$$\psi(x,y) = f_0 + f_x x + f_y y$$ (can fit to this model)

$$\phi(x,y) = 2\pi f_0 t + 2\pi \Delta k_x(t) x + 2\pi \Delta k_y(t) y$$

$$\Delta k_x(t) = f_x t, \quad \Delta k_y(t) = f_y t$$

$$s(t) = \underbrace{e^{-i2\pi f_0 t}}_{\text{demod}} \iint_{X,Y} m(x,y) \cdot e^{-i2\pi \cdot [\underline{(k_x(t)+\Delta k_x(t))}\, x + \underline{(k_y(t)+\Delta k_y(t))}\, y]} \, \mathrm{d}x \, \mathrm{d}y$$

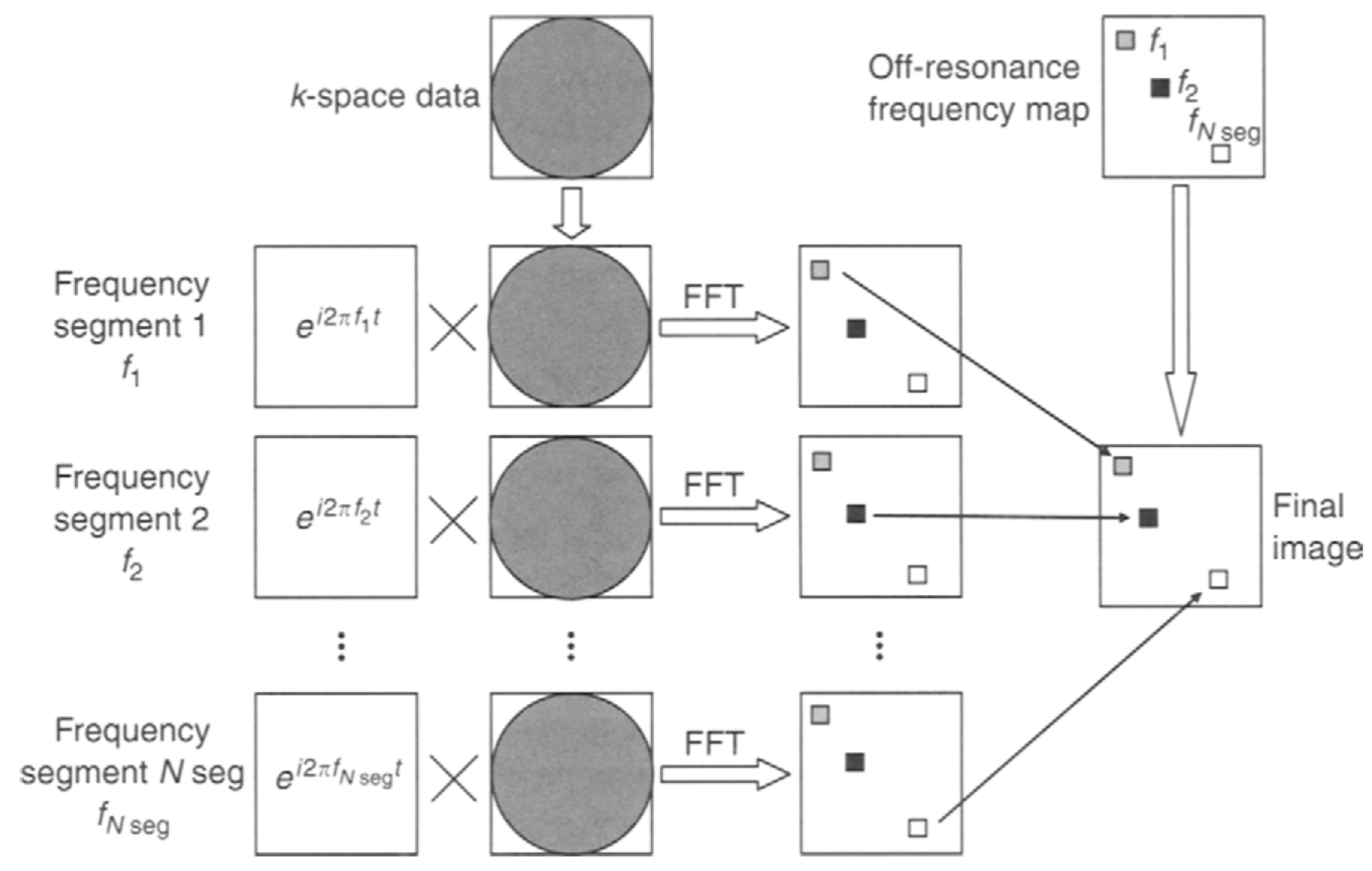
shift k-space trajectory

Can follow with frequency-segmented off-res correction

Irarrazabal P et al., MRM 1996; 35: 278-282

Off-resonance Correction

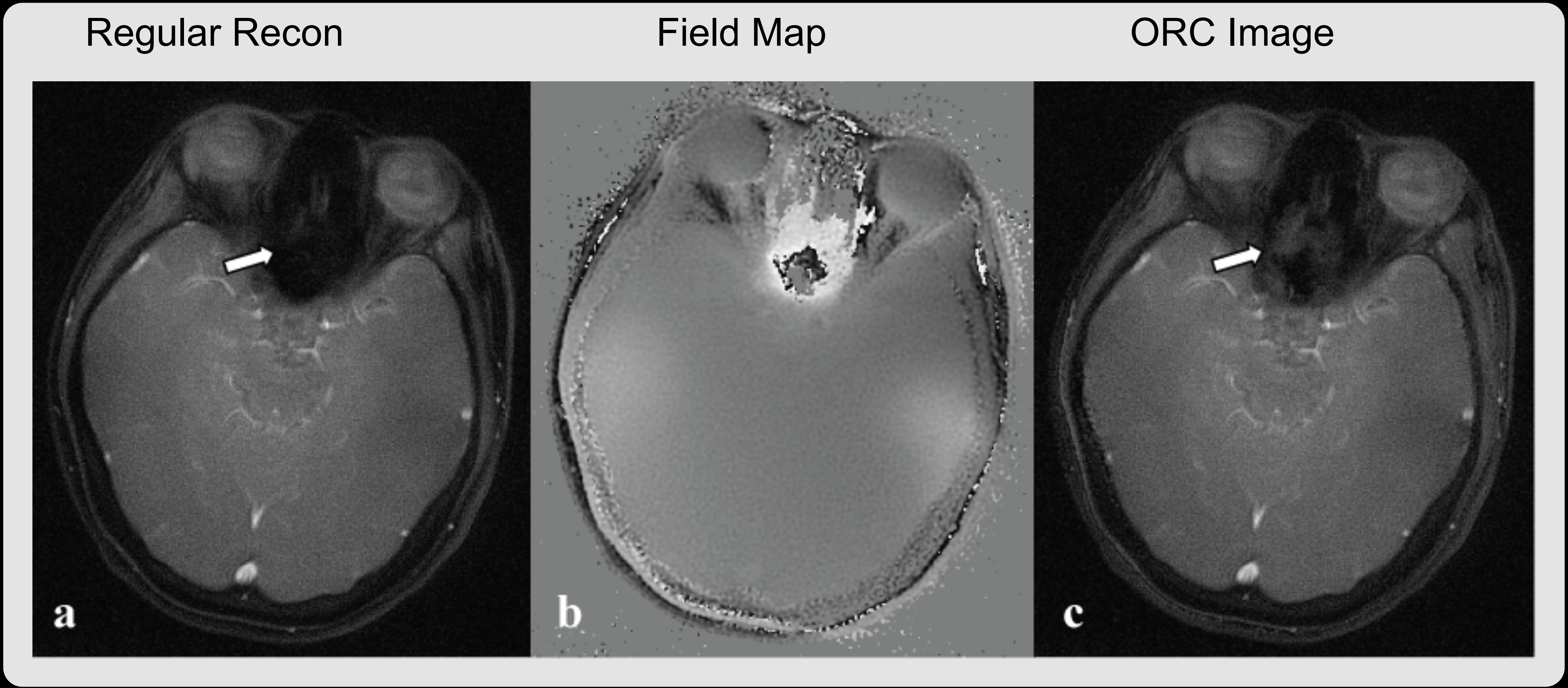
Frequency-segmented correction



Bernstein et al., Handbook of MRI Sequences, Fig. 17.63

Off-resonance Correction

Example: Axial Concentric Rings at 1.5 T



Wu HH et al., MRM 2008; 59: 102-112

Off-resonance Correction

- Field map measurement
- Segmented correction methods
  - Need to recon multiple images, $N_{\mathrm{bins}} \sim 4(f_{\mathrm{max}} - f_{\mathrm{min}})T_{\mathrm{acq}}$
- Other sources of off resonance
  - concomitant gradients
  - chemical shift (*e.g.*, fat)
- Other ORC algorithms
  - autofocusing (field map optional)
  - combine with image reconstruction

Thanks!

- Further reading
  - references on each slide
  - further reading section on website
- Acknowledgments
  - John Pauly's EE369C class notes (Stanford)

Holden H. Wu, Ph.D.

HoldenWu@mednet.ucla.edu

http://mrrl.ucla.edu/wulab