M219 Principles and Applications of MRI (Winter 2023)
Homework Assignment #2 (20 points)
Assigned: 1/30/2023, Due: 2/15/2023 at 5 pm by email

E-mail a PDF (entitled M219_HW01_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

Problem #1. (1 point) True or False (state clearly your reasoning)

T/F: Given a fixed flip angle, the larger the M the stronger the B1 field needs to be because a stronger force is required to flip a larger M.

Problem #2. (4 points)

Suppose you put a water phantom (30×30×30 cm^3) in a 3-Tesla MRI system. The center of the water phantom is located at the Isocenter of the system. Figure 1 shows the YZ views of the water phantom. The B0 field is along the +Z direction.

a) You want to excite the yellow slice shown in Fig. 1, which is located 5cm off isocenter in the +Z direction and has a thickness of 1cm. To achieve this, you want to use a slice selection gradient of 40 mT/m. What center frequency and bandwidth should you use for the excitation RF pulse? (2 points)

b) However, when you set up the phantom MRI experiment, you inadvertently changed the shimming parameters of the MRI system, such that the main field within the phantom is severely distorted. Instead of a nice homogeneous field, the main static field as a function of space now follows:

$$B_z = 3T + 20\frac{uT}{cm^2}(z^2 + y^2)$$

However, since you are not aware of the change in shimming parameters, you still used the same RF pulse and slice selection gradient as in A). Using Matlab, plot the shape of the excited "slice" in the Y-Z plane. Label the coordinators of important boundary points the "slice". (2 points)

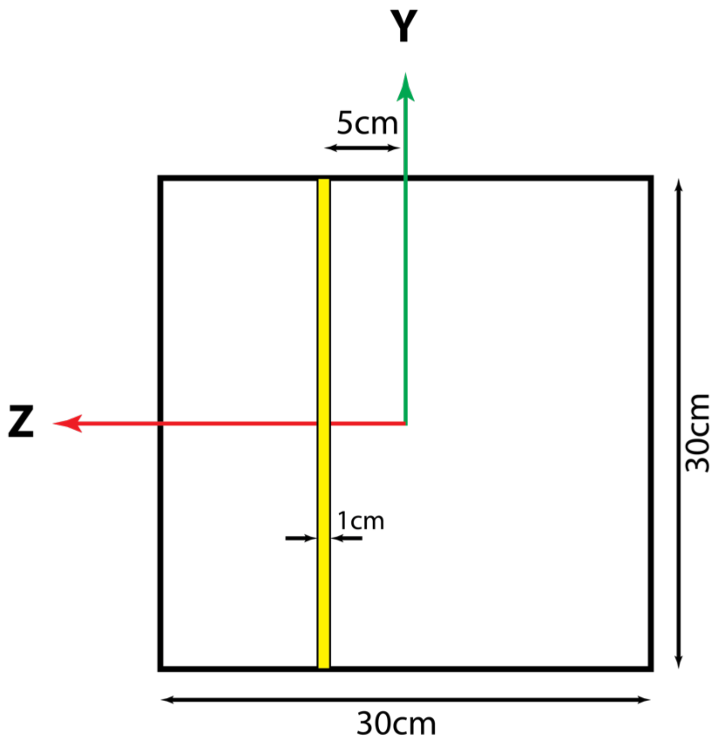


Figure. 1: *The Y-Z view of the MRI phantom in Problem #2.*

Problem #3. (4 points)

Using Matlab, perform a Bloch equation simulation of a hard RF pulse of 90° flip angle with phase 0 (i.e. along the X' axis) using a max B1 amplitude of 25 uT. The simulation should include spins with an off-resonance range of -2000Hz to 2000Hz. Plot the simulated transverse magnetization (magnitude only) immediately after the hard RF pulse as a function of off-resonance frequency. Compare with the results obtained from the Fourier relationship based on small tip angle approximation. What conclusion can you draw?

Problem #4. (2 points)

Repeat Problem #3, but for a hard RF pulse of 150° flip angle using a max B1 amplitude of 25 uT.

Problem #5. (6 points)

Consider a special type of RF pulse that has 0° RF phase (i.e. B1 along X' axis in the rotating frame) and variable B1 amplitude. However, it is different from a typical RF pulse in that its carrier frequency also varies. It has a duration of 10ms, starting from $t = 0\ ms$. Its B1 field strength varies according to:

$$B_1^e = 80 \sin\left(\frac{\pi}{10ms} t\right)\ uT$$

Its carrier frequency (in Hz) at any time point t varies according to:

$$f = f_0 - 80uT\frac{\gamma}{2\pi}\cos\left(\frac{\pi}{10ms}t\right)$$

where $f_0 = \omega_0/2\pi$, and ω_0 is the Larmor frequency of a sample that is placed in a perfectly uniform B0 field.

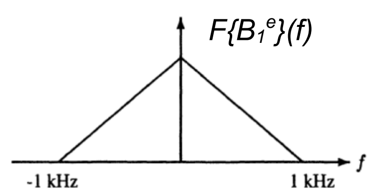
a) Write an expression for the effective B1 field B_{eff} vector as a function of time from $t = 0 - 10\ ms$. Plot the B_{eff} field vector's Z and X' component as a function of time during the 10ms RF pulse. Describe the motion of B_{eff} during the RF pulse. (2 points)

b) Assuming the initial magnetization vector of the sample is [0 0 1], which is along the Z axis. Perform a Bloch simulation for the duration of the RF pulse in step size of a microsecond. Plot the magnetization vector's Z and X' component as a function of time during the 10ms RF pulse. (2 points)

c) Compare results from a) and b). What conclusion can you draw? (2 points)

Problem #6 (3 points)

A spin systems has three isochromats with resonance frequencies at $w_0 - \Delta$, w_0, $w_0 + \Delta$, where $w_0 = 42\text{MHz}$, and $\Delta = 250\text{ Hz}$. We assume that the RF pulse that is applied to the spin system is defined by $B_1(t) = B_1^e(t)e^{-i2\pi w_0 t}$, where the Fourier transform of $B_1^e(t)$ is given in the figure to the right. It is known that the RF pulse will flip the three isochromats by 67.5°, 90°and 67.5°, respectively. Calculate the flip angles of all the isochromats for the following RF pulses based on the Fourier theory.



a) $2B_1^e(2t)e^{-i2\pi w_0 t}$ (1 points)

b) $B_1^e(2t)e^{-i2\pi(w_0+\Delta)t}$ (1 points)

c) $0.5B_1^e(t/2)e^{-i2\pi(w_0-\Delta)t}$ (1 points)