Homework 1: RF Pulse Design
M229 Advanced Topics in MRI (Spring 2019)
Assigned: 4/5/2019, Due: 4/26/2019 at 5 pm by email

1. Consider a nonselective excitation in which a constant pulse of amplitude B_1 and duration τ is applied in the presence of B_0. If the carrier frequency ω is not exactly tuned to ω_0 (i.e., $\omega \neq \omega_0$), determine the resultant transverse magnetization output at time τ. Use the small tip angle solution based in the rotating frame at the carrier frequency ω.

Solution: $B_1(t) = B_1 \cdot \sqcap\left(\frac{t-\tau/2}{\tau}\right)$

From small tip-angle solution for selective excitation,

$$m_r(\tau, z) = iM_0 e^{-\frac{i\omega(z)\tau}{2}} \cdot \mathcal{F}\left\{\sqcap\left(\frac{t}{\tau}\right)\right\}\Big|_{f=-\frac{\omega(z)}{2\pi}}$$

$$= iM_0 e^{-\frac{i\omega(z)\tau}{2}} \cdot B_1 \cdot \tau \cdot sinc\left(\frac{\omega(z)\tau}{2\pi}\right)$$

where $\omega(z)$ = amount of offset frequency. In this case, we are off-resonance because the carrier frequency was not tuned to ω_c, the resultant magnetization output m_r can be expressed as,

$$= iM_0 e^{-\frac{i\Delta\omega\tau}{2}} \cdot B_1 \cdot \tau \cdot sinc\left(\frac{\Delta\omega\tau}{2\pi}\right)$$

where $\Delta\omega = \omega_c - \omega$, the specific amount of off in frequency.

2. Adiabatic Full Passage Pulse Design using Hyperbolic Secant Pulse Equations

Design an adiabatic RF pulse using the method outlined in class:

$$B_1(t) = A(t)e^{-i\int \omega(t')t'}$$
$$A(t) = A_0\,\mathrm{sech}(\beta t)$$
$$\omega_1(t) = -\mu\beta\,\tanh(\beta t)$$

a) amplitude modulation function

Design an amplitude modulation function, A_t, with *sech*. Use β = 672 [rad/s] and A_0 = 0.12 [G]. Use 512 samples for the pulse duration of 10.24 ms.

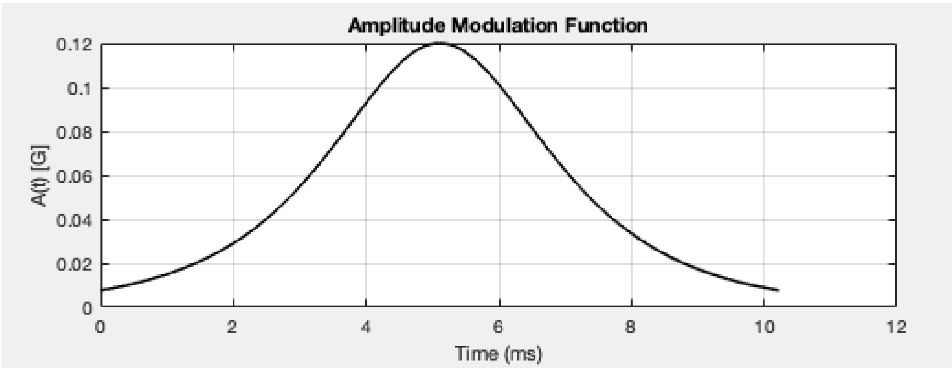
```
>> beta = 672;
>> pulseWidth = 10.24; % in [ms]
>> A0 = 0.12 % in [G]
>> nSamples = 512; % the number of samples
>> dt = pulseWidth/nSamples/1000; % time step in [sec]
```

Plot the amplitude modulation function in *Gauss*.

```
>> plot(time, A_t); grid on;
```

>> title('Amplitude Modulation Function'); xlabel('Time (ms)'); ylabel('A(t) (G)');

Solution:



b) frequency modulation function

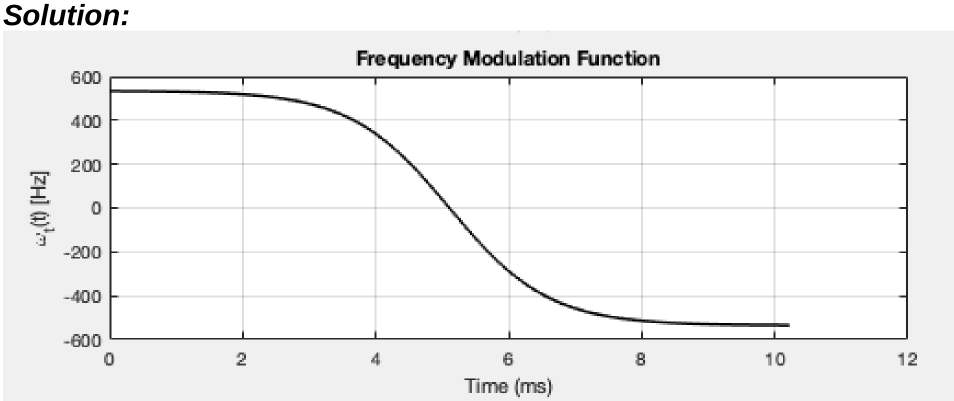
Design a frequency modulation function, *w1_t*, with *tanh*. Use μ = 5 [dimensionless].

>> mu = 5; % [dimensionless]

Plot the frequency modulation function in *Hz*.

>> plot(time,w1_t); grid on;
>> title('Frequency Modulation Function'); xlabel('Time (ms)'); ylabel('\omega_1(t) (Hz)');

Solution:



c) inversion profile using Bloch simulation

Combine amplitude and frequency modulation functions:

>> rf_pulse = A_t .* exp(1i .* cumsum(w1_t)*dt));

Or, you can use phase modulation function instead:

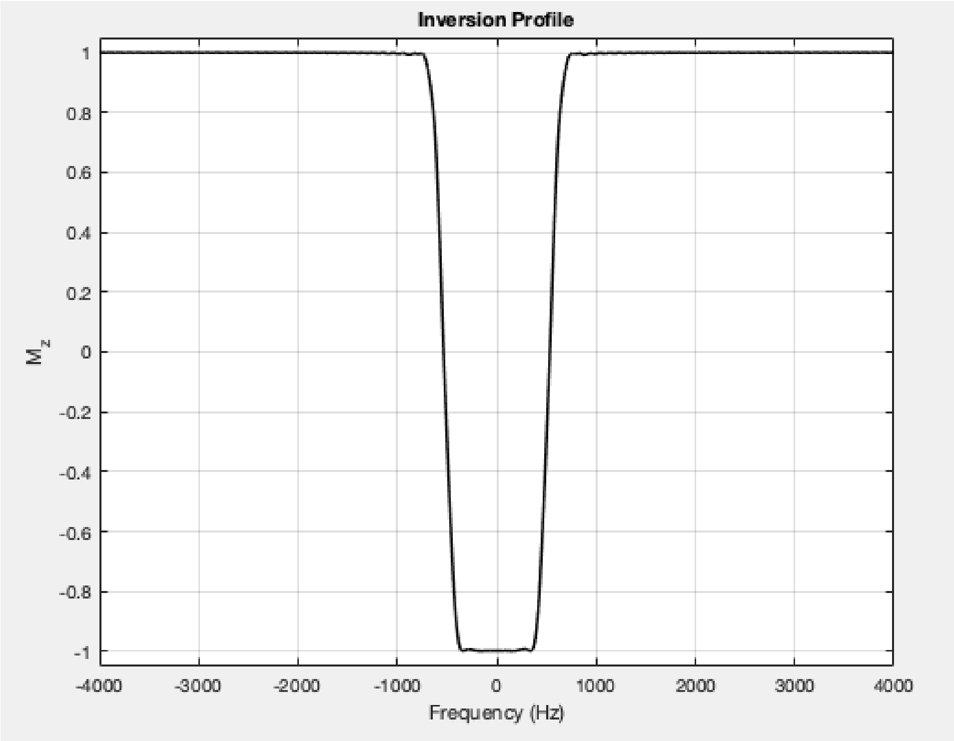
$$\phi(t) = \mu \ln(\operatorname{sech} \beta t)$$

>> rf_pulse = A_t .* exp(1i .* phi);

Simulate the inversion profile over a sufficient range of frequency (e.g. -4,000 Hz to 4,000 Hz) using Bloch simulation. Plot the inversion profile.

```
>> plot(freq_range, mz);
>> title('Inversion Profile'); xlabel('Frequency (Hz)'); ylabel('M_z'); grid on;
```

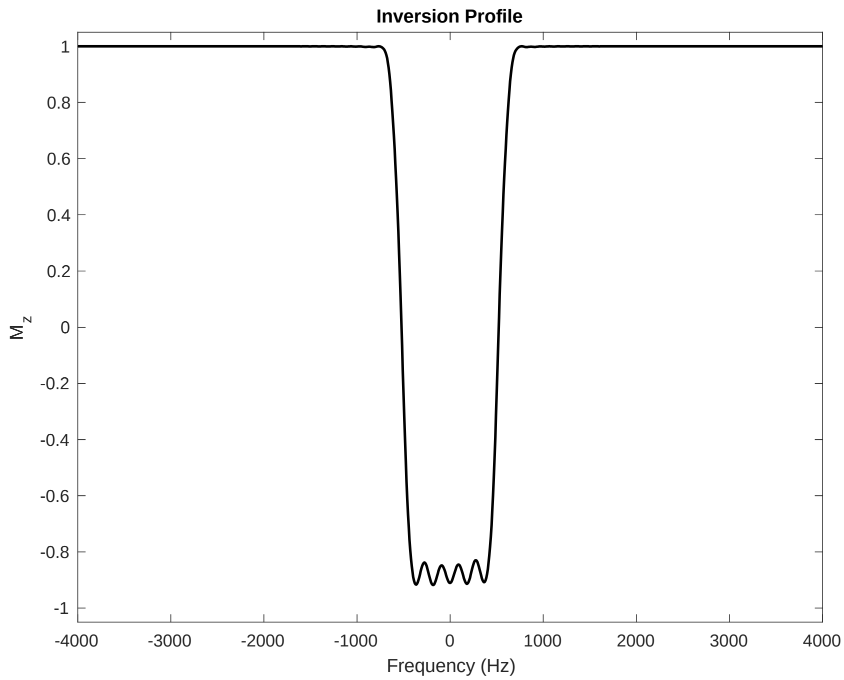
Solution:



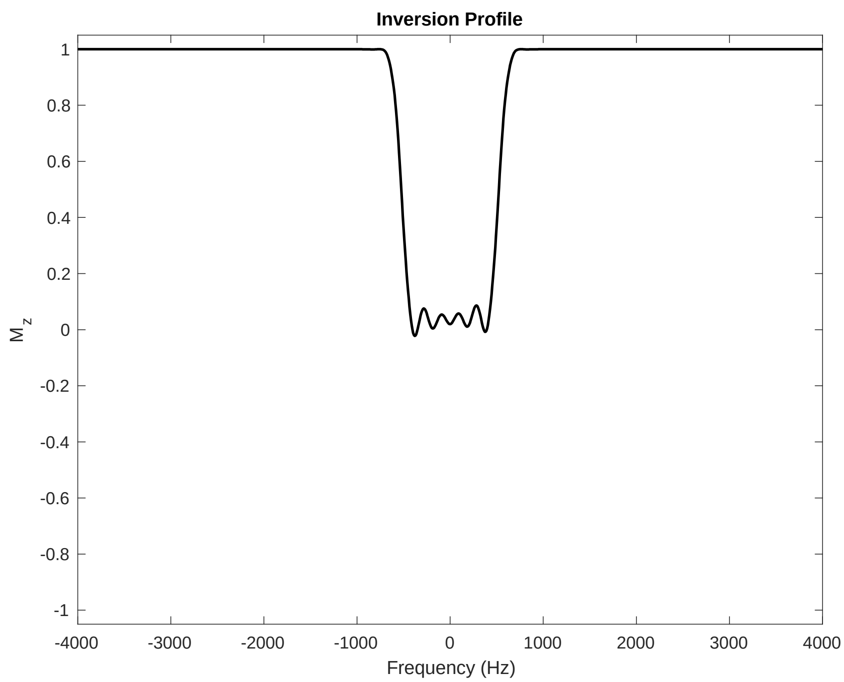
d) inversion profiles with different B1+ variation

Simulate the inversion profiles with different B1+ variations (60% attenuated, 30% attenuated, and 150% amplified pulses). Plot the inversion profiles.

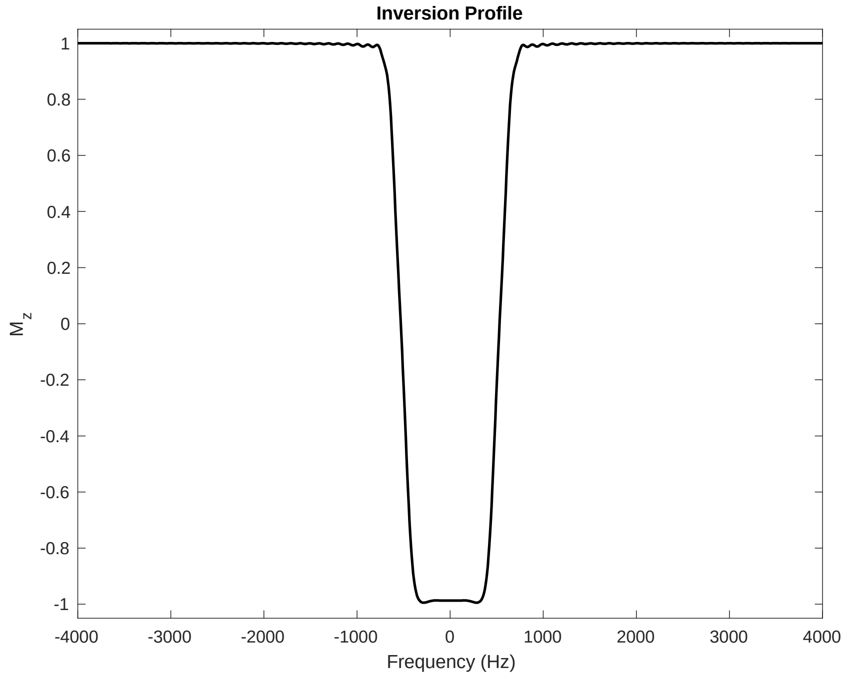
Solution:
With 60% attenuated,



With 30% attenuated,



With 150% amplified,



3. 2D EPI Pulse Design

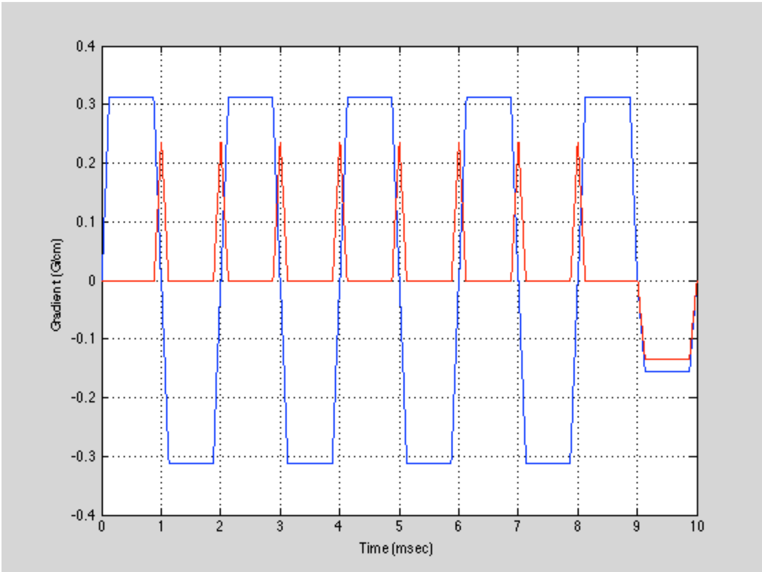
Design a 2D separable EPI RF pulse using the method outlined in class.

a) Gradient Design

Design a blipped EPI trajectory with $k_{x,max} = k_{y,max} = 0.5$ *cycles/cm*, and nine lines ($L=9$). Assume the trapezoid ramps (τ_R) are 1/8 ms long, and the trapezoids themselves (τ) are 1 ms long. The blips are 1/4 ms long. What is the maximum amplitude of G_x and G_y?

Solution: *$G_{x,max} = 0.31$ G/cm and $G_{y,max} = 0.24$ G/cm*

Sample the RF and gradient waveforms at 5 us (200 samples per trapezoid and 25 samples per trapezoid ramp). Include a refocusing lobe at the end to bring the trajectory back to $k_x = k_y = 0$, using 1 ms trapezoids on x and y. Plot the gradient waveforms, G_x and G_y, with the axes labeled.

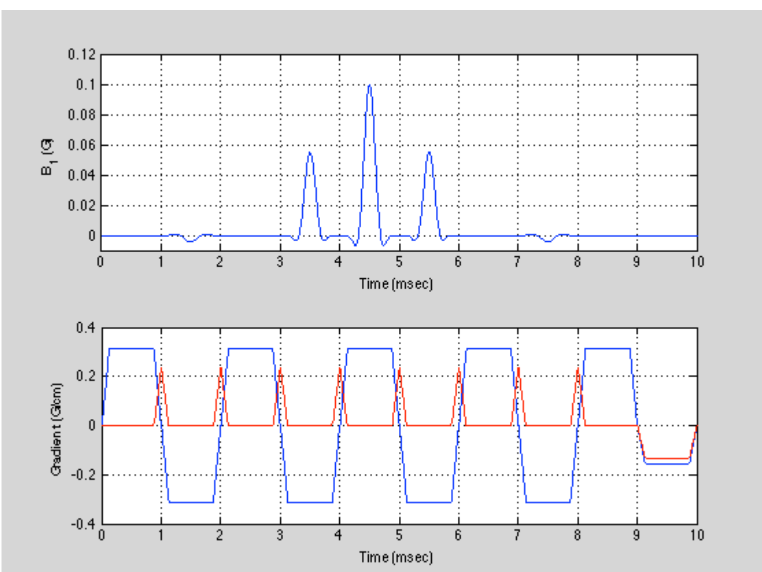


b) RF Pulse Design

Design the RF waveform with TBW = 4 for the subpulses, and the envelope. This will produce a 4 cm by 4 cm excited volume.

>> tbw = 4;
>> rf_fast = wsinc(tbw,samples);
>> rf_slow = wsinc(tbw,L);

Apply the “flat-top only design” (RF only played flat part), and use the RF waveform to be zero during the refocusing gradient. Scale the RF to a flip angle of 1 radian (i.e. sum(rf) = 1). Plot the RF waveform in Gauss.



c) 2D Bloch Simulation

Simulate the pulse over a sufficient range (e.g., -12cm to 12cm in x and y). Plot the profile as an image using

>> imshow(abs(mxy),[]);

and cross-section plots along x (M_{xy} vs. x) and y (M_{xy} vs. y)

>> subplot(211); plot(x,abs(mxy(:,round(length(y)/2))));
>> subplot(212); plot(y,abs(mxy(round(length(x)/2),:)));