The MRI Signal Equation

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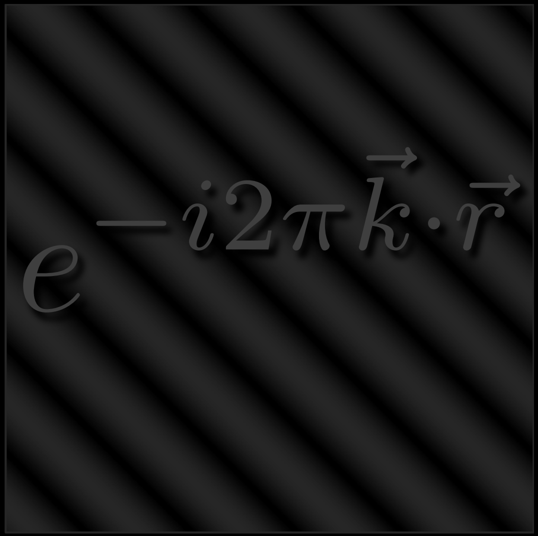

$$S(\vec{k}) = \int_{object} M_{xy}(\vec{r}, 0) \; e^{-i2\pi\vec{k}\cdot\vec{r}} \; d\vec{r}$$

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Lecture #8 Learning Objectives

- Describe the pros and cons of a GRE acquisition, especially in comparison to a spin-echo sequence.
- Understand why GRE can't acquire true T2 contrast.
- Explain why spoilers are typically used with GRE.
- Understand how to calculate scan time.
- Be able to derive the optimal flip angle for a GRE sequence, and understand why we might not use that value (contrast).
- Describe methods of fat-water separation and their utility.

Spin Echo Contrast

$$A_{Echo} \propto \rho \left(1 - 2e^{-(TR - TE/2)/T_1} + e^{-TR/T_1}\right) e^{-TE/T_2}$$

If $TE \ll TR$, then

$$A_{Echo} \propto \rho \left(1 - e^{-TR/T_1}\right) e^{-TE/T_2}$$

Gradient Echo Contrast

$$A_{echo} \propto \frac{\rho \left(1 - e^{-TR/T_1}\right)}{1 - \cos \alpha e^{-TR/T_1}} \sin \alpha e^{-TE/T_2^*}$$

RF pulse and gradient timing encode image contrast in the echo (M_{xy}).

A major challenge in MRI is encoding spatial information in the echo.

Lecture #9 Learning Objectives

- Understand that SE and GRE control image contrast at the echo time.
- Appreciate that gradients move us through k-space.
- Describe how to calculate scan time.
- Explain the concept of “coil sensitivity.”
- Explain why MRI is not directly sensitive to M_z.
- Understand the role of phase sensitive detection.
- Describe the importance of quadrature detection.
- Be able to define the MRI signal equation and each term.

Dipoles to Images

$\vec{\mu}$ Magnetic Moment

$\downarrow$ $\Big\}$ B_0

$\vec{M}$ Bulk Magnetization

$\downarrow$ $\Big\}$ B_1

$\vec{M}_{xy}(t)$ Transverse Magnetization

$\downarrow$ $\Big\}$ $G_{Encoding}$

$\vec{M}_{xy}(\vec{r}, t)$ Spatially Encoded Magnetization

$\downarrow$ $\Big\}$ Coil

$V\left(t\right)$ Received Voltage

$\downarrow$ $\Big\}$ PSD

$S\left(t\right)$ Complex Signal

$\downarrow$ $\Big\}$ $G_{Decoding}$

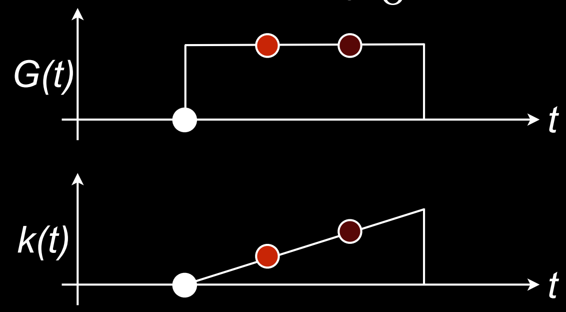
$S\left(\vec{k}\right)$ k-space signal

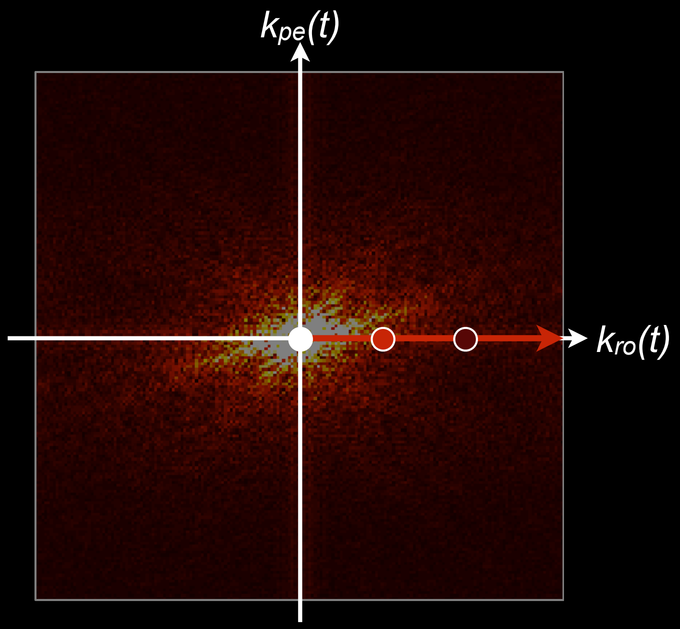
$\downarrow$ $\Big\}$ Reconstruction

$I\left(\vec{r}\right)$ Image

What is *k*-space?

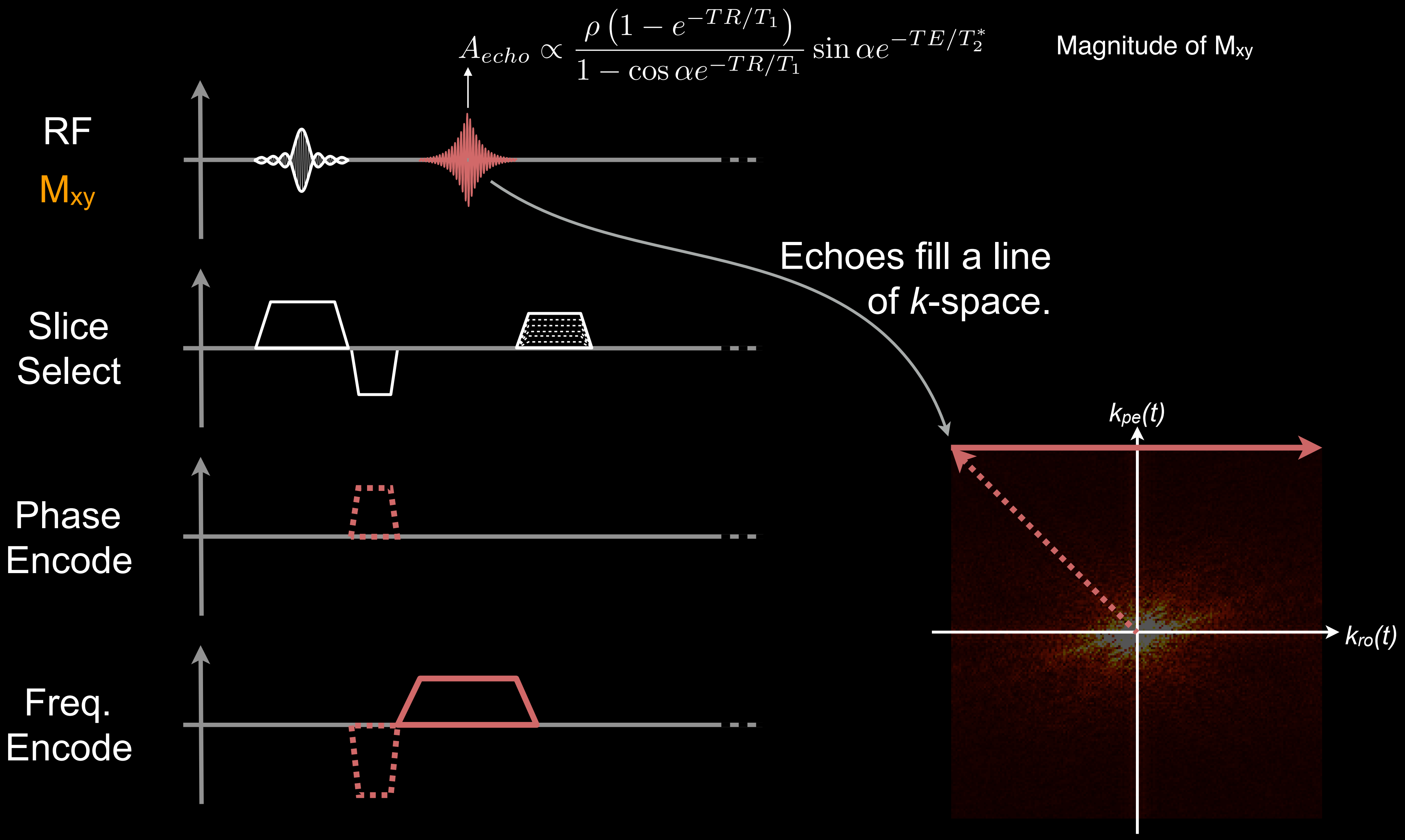
$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)d\tau$$





Gradients move us through k-space.

Spoiled Gradient Echo



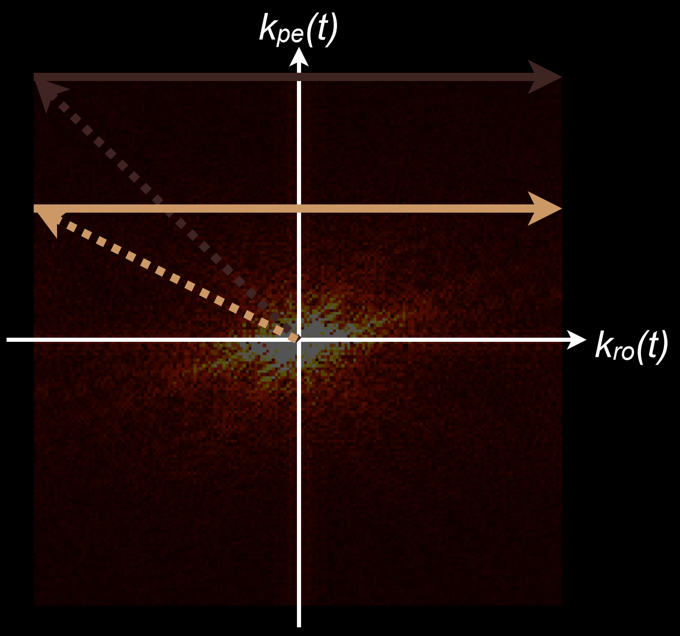
$$A_{echo} \propto \frac{\rho\left(1-e^{-TR/T_1}\right)}{1-\cos\alpha e^{-TR/T_1}} \sin\alpha e^{-TE/T_2^*}$$

Magnitude of M_{xy}

Echoes fill a line of *k*-space.

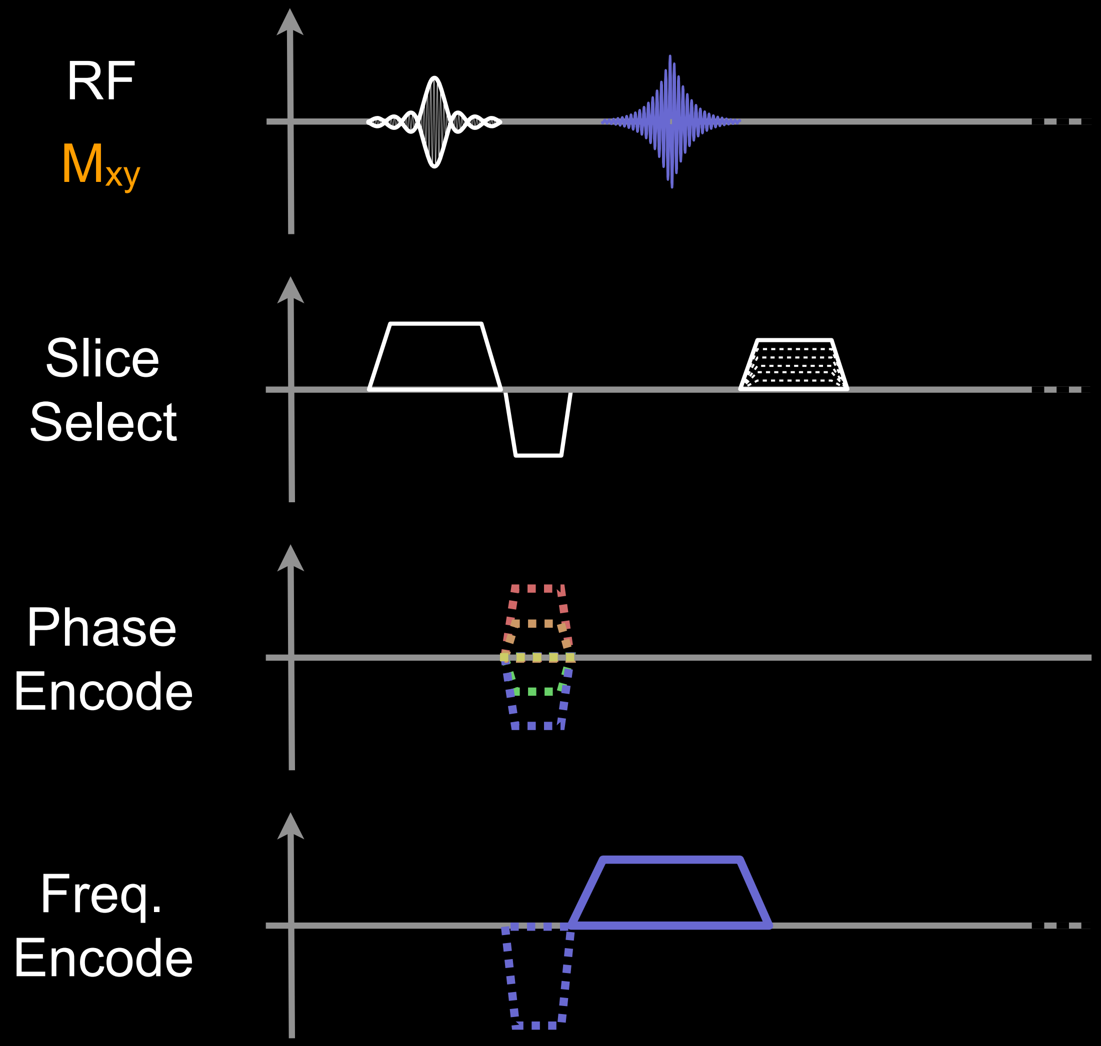
Gradients move us through *k*-space!

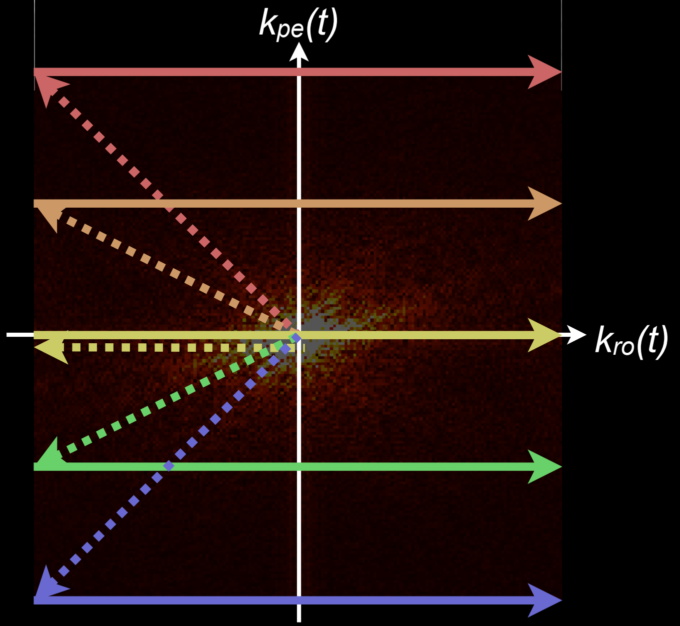
Spoiled Gradient Echo



Gradients move us through *k*-space!

Spoiled Gradient Echo



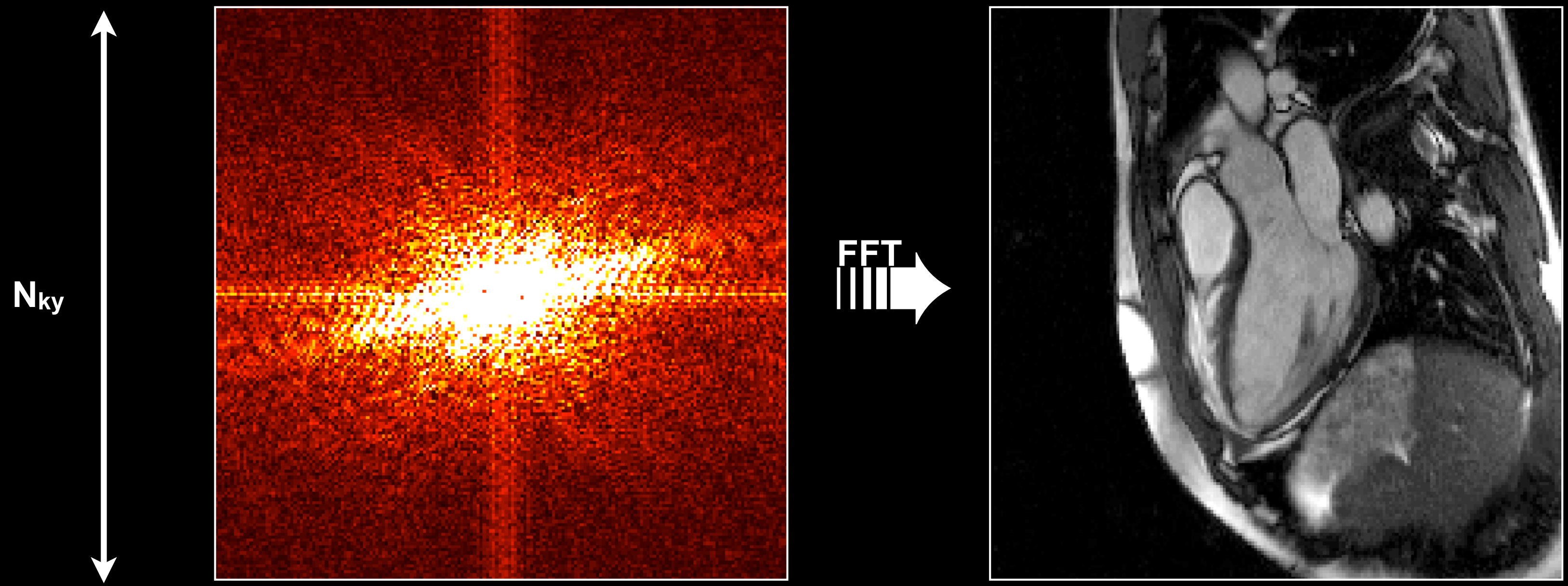


Gradients move us through *k*-space!

MRI is slow...

$$T_{Scan} = TR \cdot N_{PE} \cdot N_{Avg}$$

- One phase encode step per TR.
  - Each phase encode step acquires one echo.
- ~128 echoes (N_{ky}=# phase encodes) per image.
- $T_{Scan}=TR \cdot N_{ky}$
  - T_{Scan}=2500ms•128=5:20 (MM:SS)



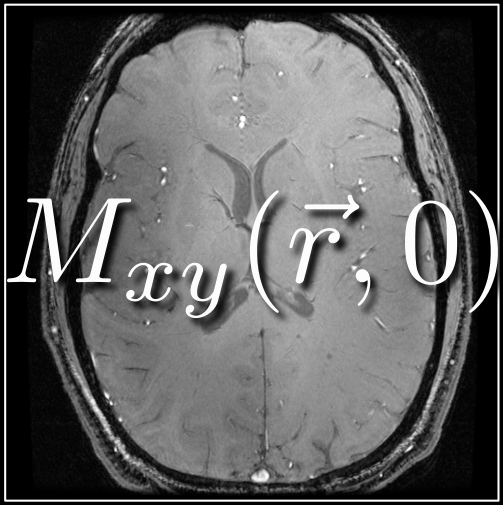
What is *k*-space?

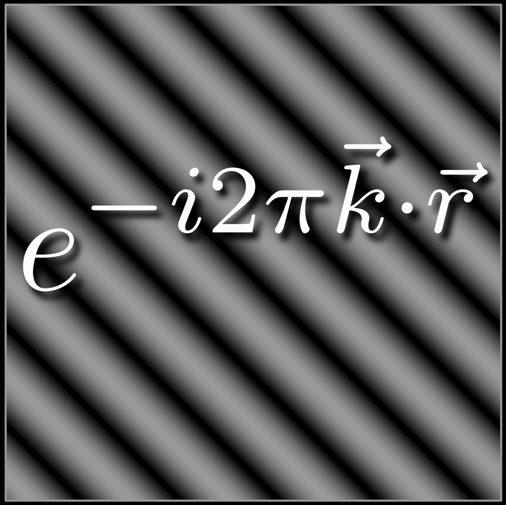
Signal at a point in k-space

Faraday’s Law of Induction

State of M_{xy} in the *object*

k-space sampling function



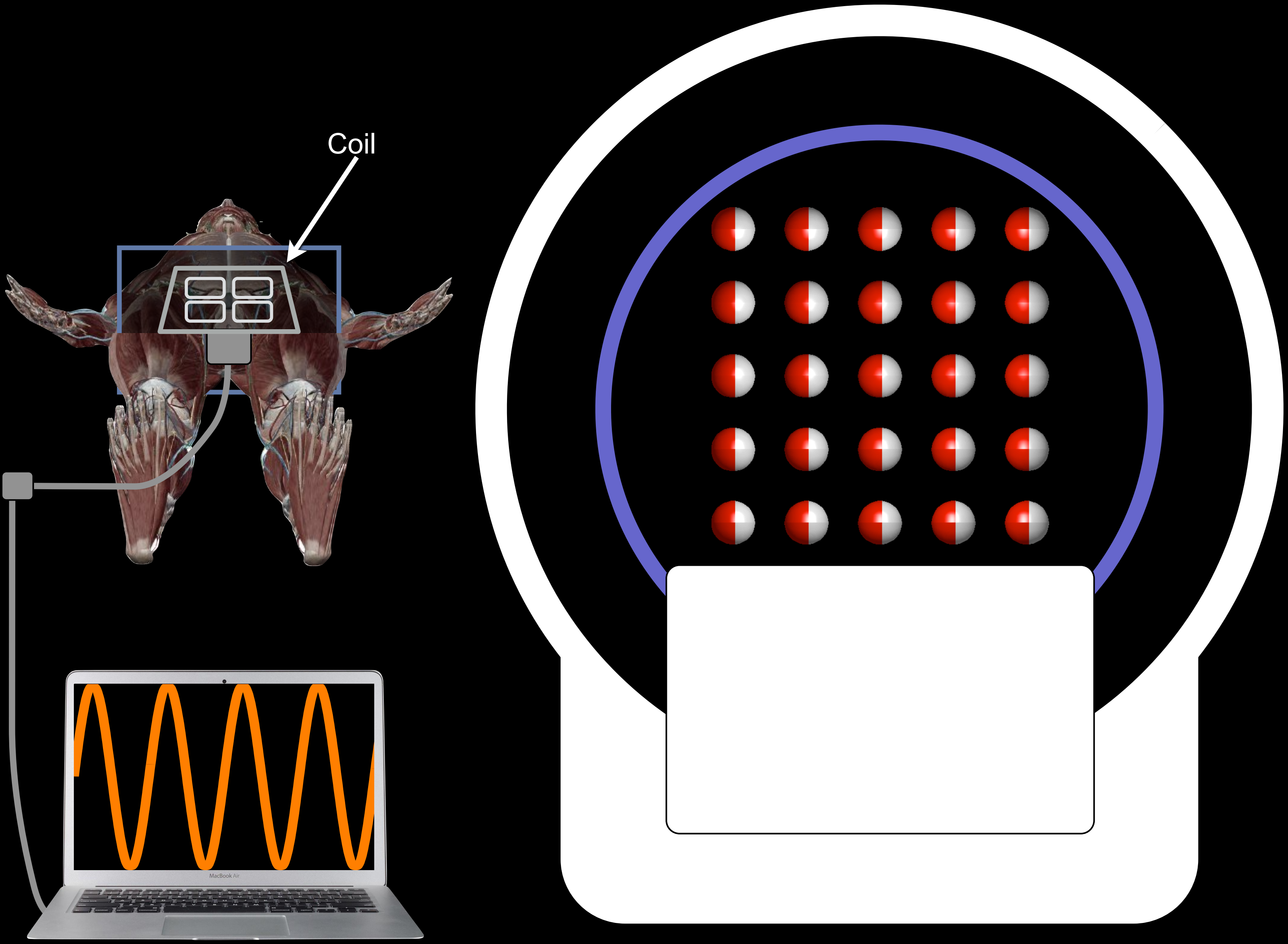


$$S(\vec{k}) = \int M_{xy}(\vec{r},0)\; e^{-i2\pi\vec{k}\cdot\vec{r}}\; d\vec{r}$$

MRI *acquires* point-wise the Fourier Transform of the object.

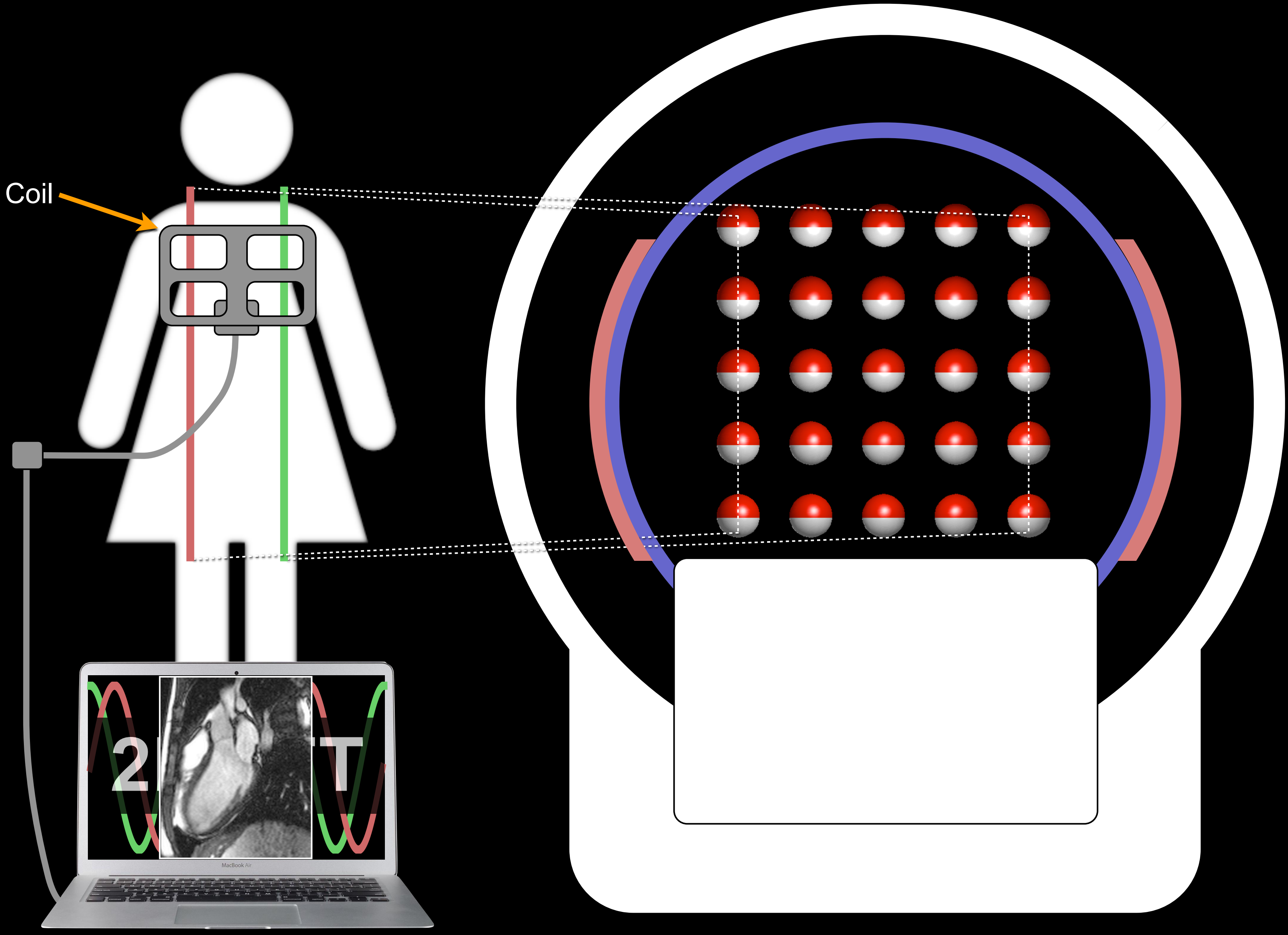
How do we measure M_{xy}?

Faraday’s Law of Induction



Precessing spins *induce* a current in a nearby coil.

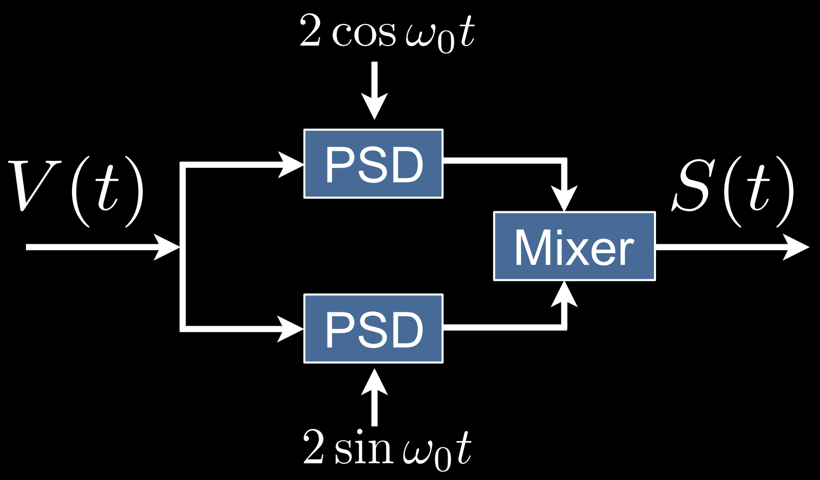
Faraday's Law of Induction



The trick is to encode spatial information and image contrast in the echo.

Signals in MRI

$$V(t)$$



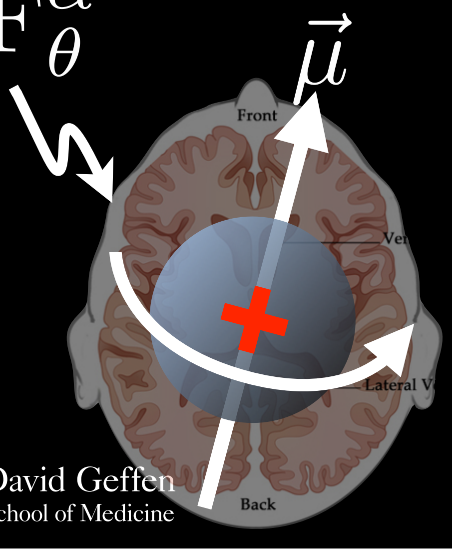
$$\vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe}t \\ \bar{\gamma}\,\vec{G}_{fe}(t-TE) \end{cases}$$

$$S(\vec{k})$$

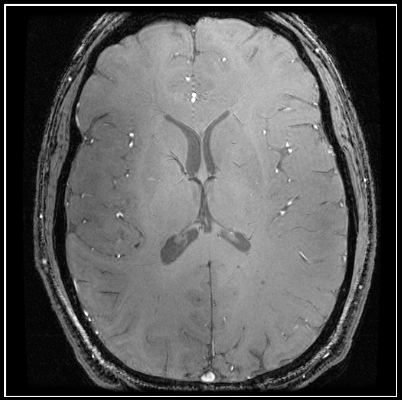
We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *k*-space?

$$M_{xy}(\vec{r},t)$$

$$\mathrm{RF}_\theta^\alpha$$

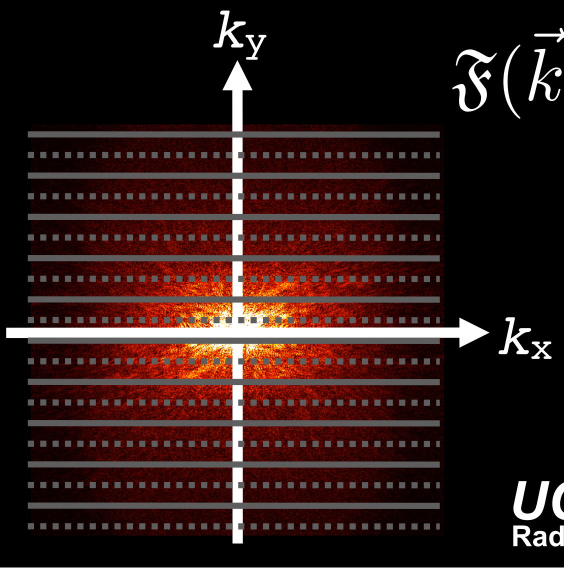


$$I(\vec{r})$$



FFT

$$\mathfrak{F}(\vec{k})$$



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Signal Detection

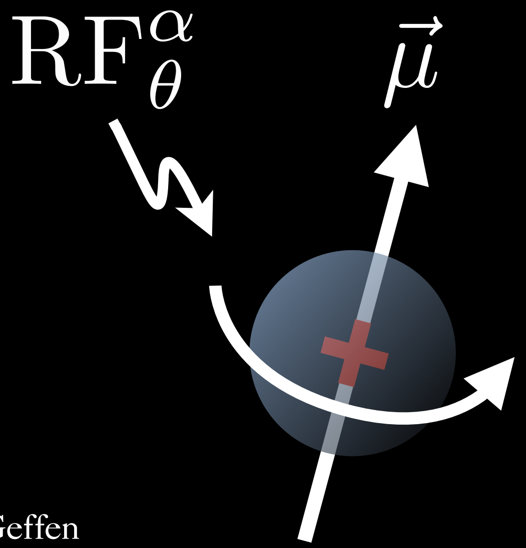
$$M_{xy}\left(\vec{r},t\right) \text{ to } V\left(t\right)$$

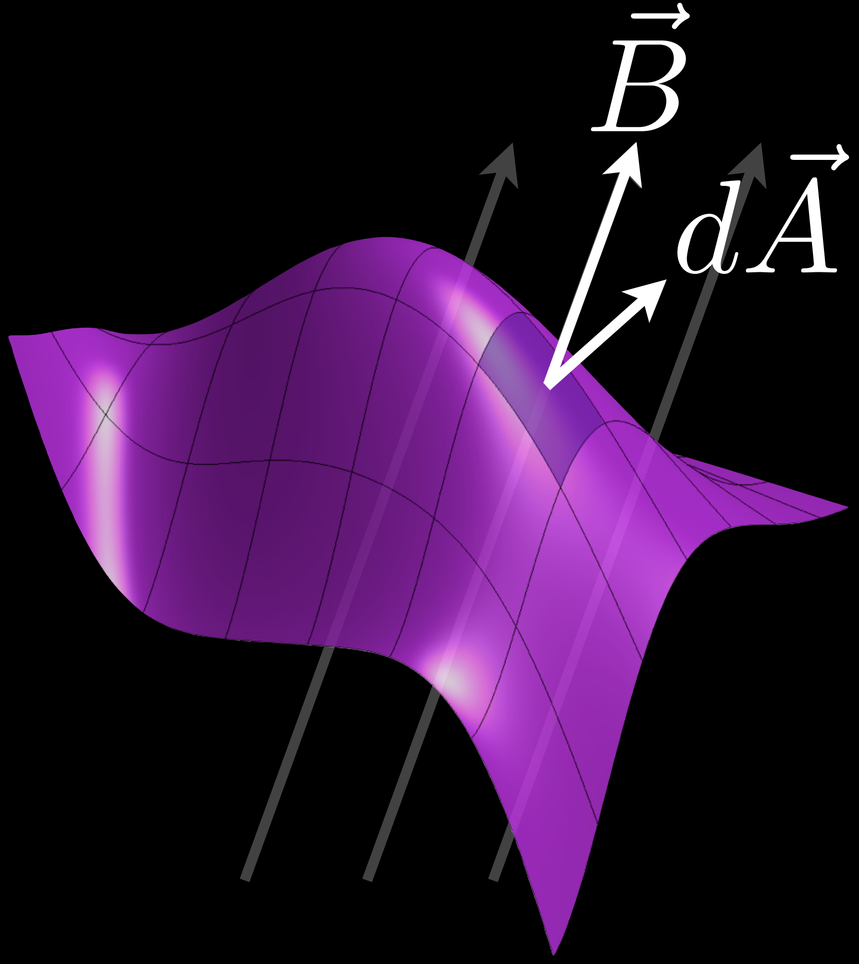
Signal Detection

$$\Phi = \int_{\text{Coil Area}} \vec{B} \cdot d\vec{A}$$

$$V(t) = -\frac{\partial \Phi(t)}{\partial t}$$

$$M_{xy}(\vec{r}, t)$$





Basic Detection Principles

Magnetic Flux Through The Coil – *Reciprocity*

$$\Phi\left(t\right) = \int_{object} \vec{B}_r\left(\vec{r}\right) \cdot \vec{M}\left(\vec{r}, t\right) d\vec{r}$$ Eqn. 3.126

Magnetic Flux

Coil Sensitivity

Bulk Magnetization

What happens if the coil has poor sensitivity?

What happens if the coil's sensitivity is perpendicular to the bulk magnetization? How would that happen?

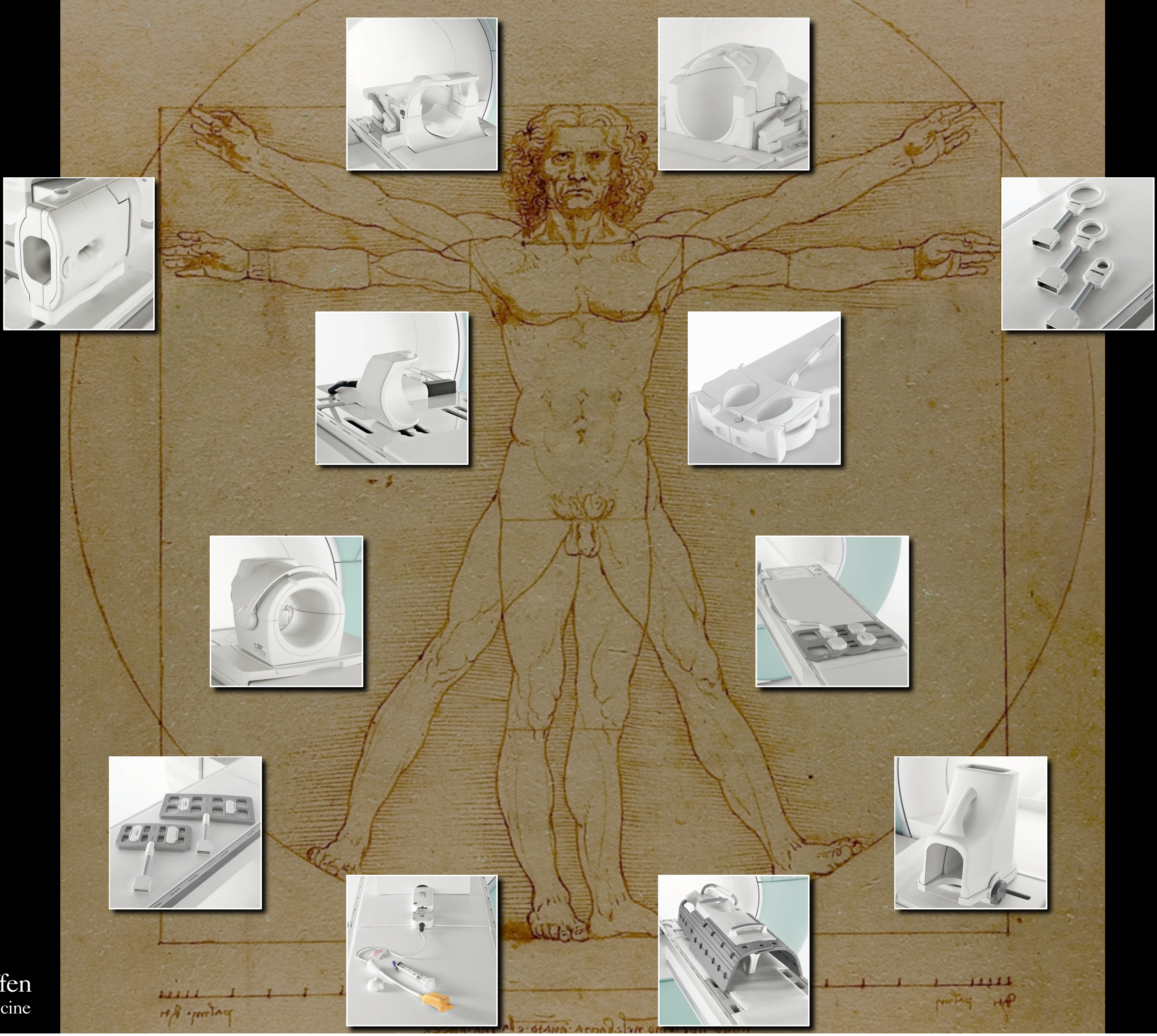
Coil Sensitivity & Phase

$$\|\vec{B}_r\left(\vec{r},t\right)\| \quad \angle\vec{B}_r\left(\vec{r},t\right)$$

Sensitivity
(Magnitude)

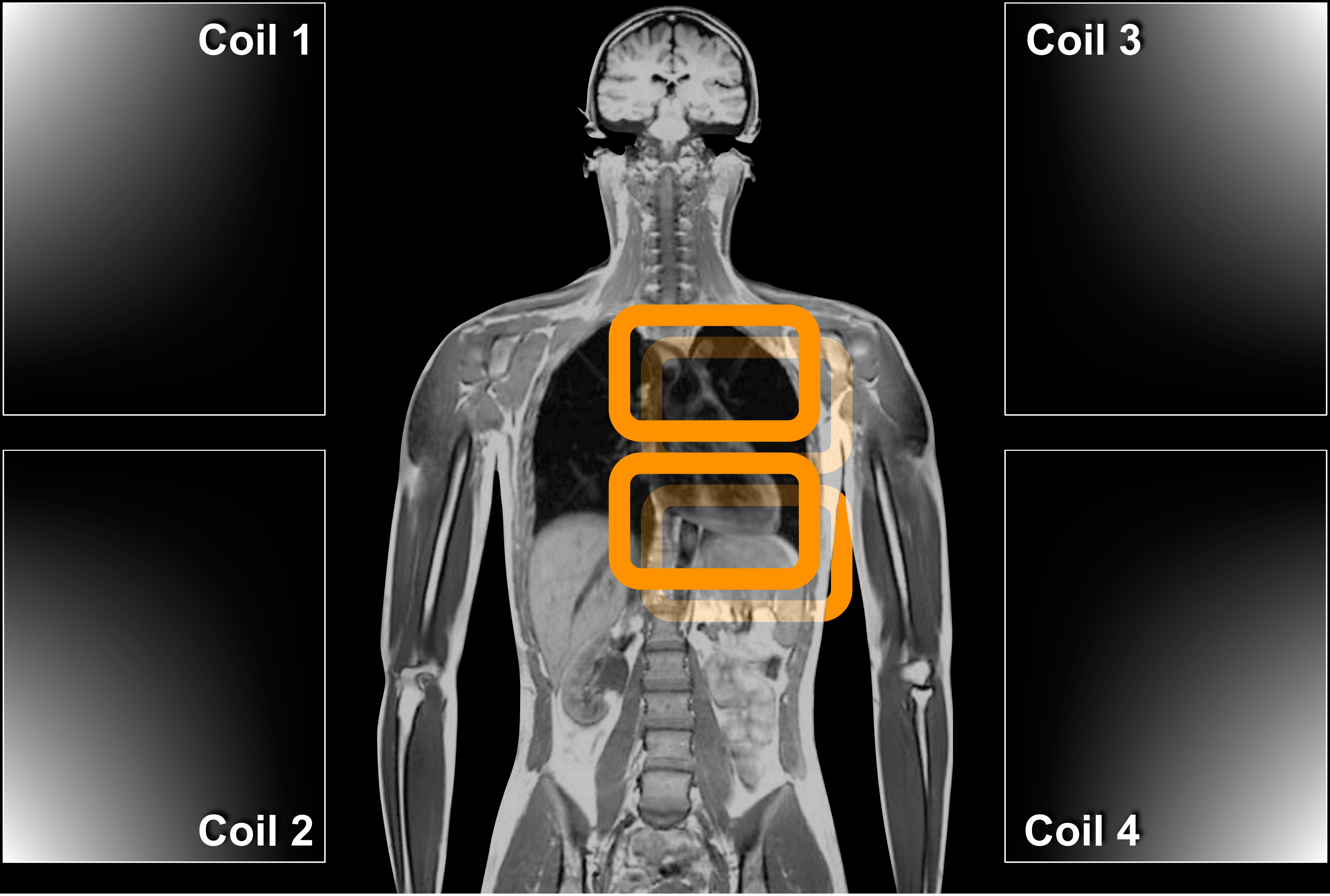
Phase

MRI Coils



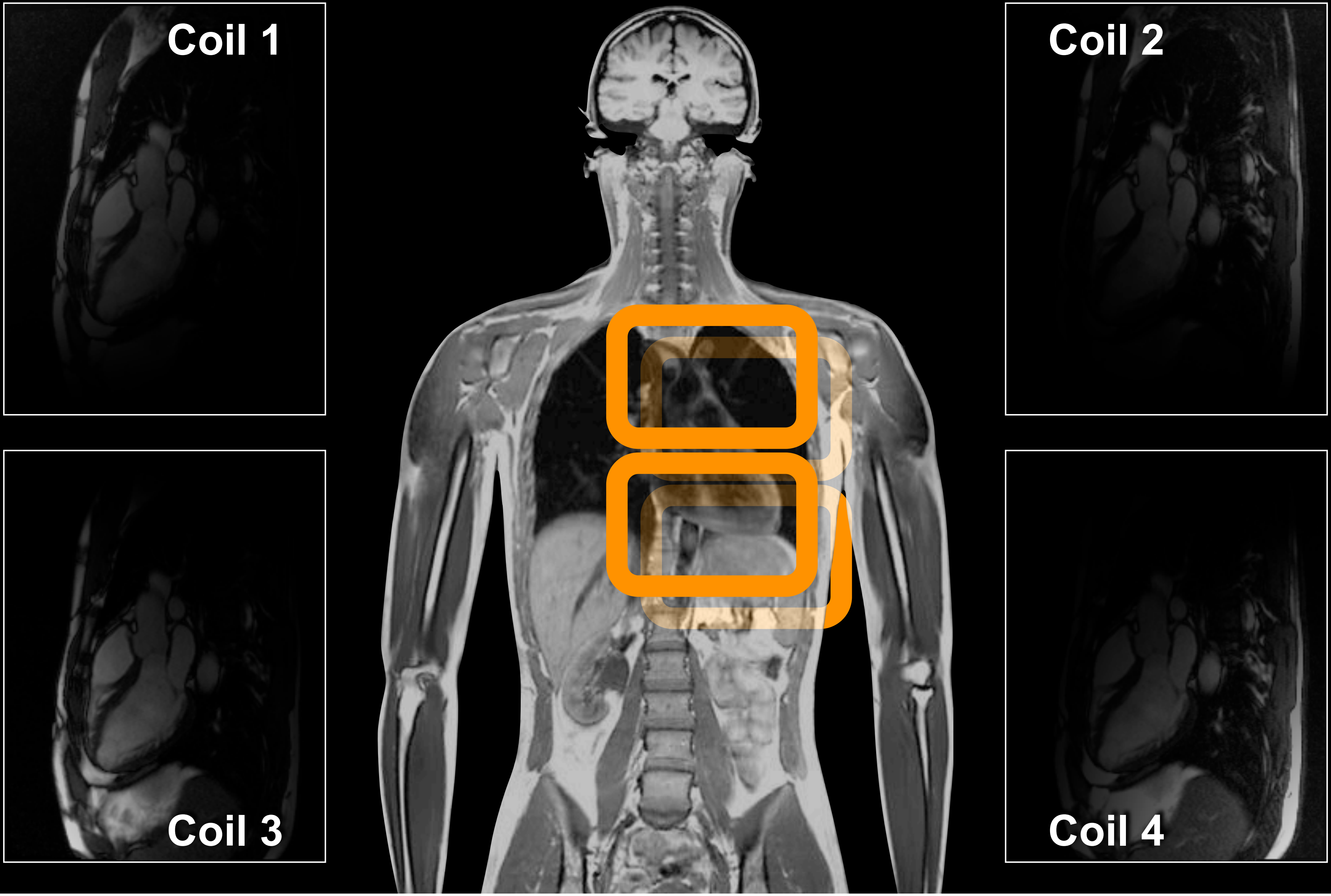
4-Channel Cardiac Coil

Each coil element has a unique sensitivity profile.

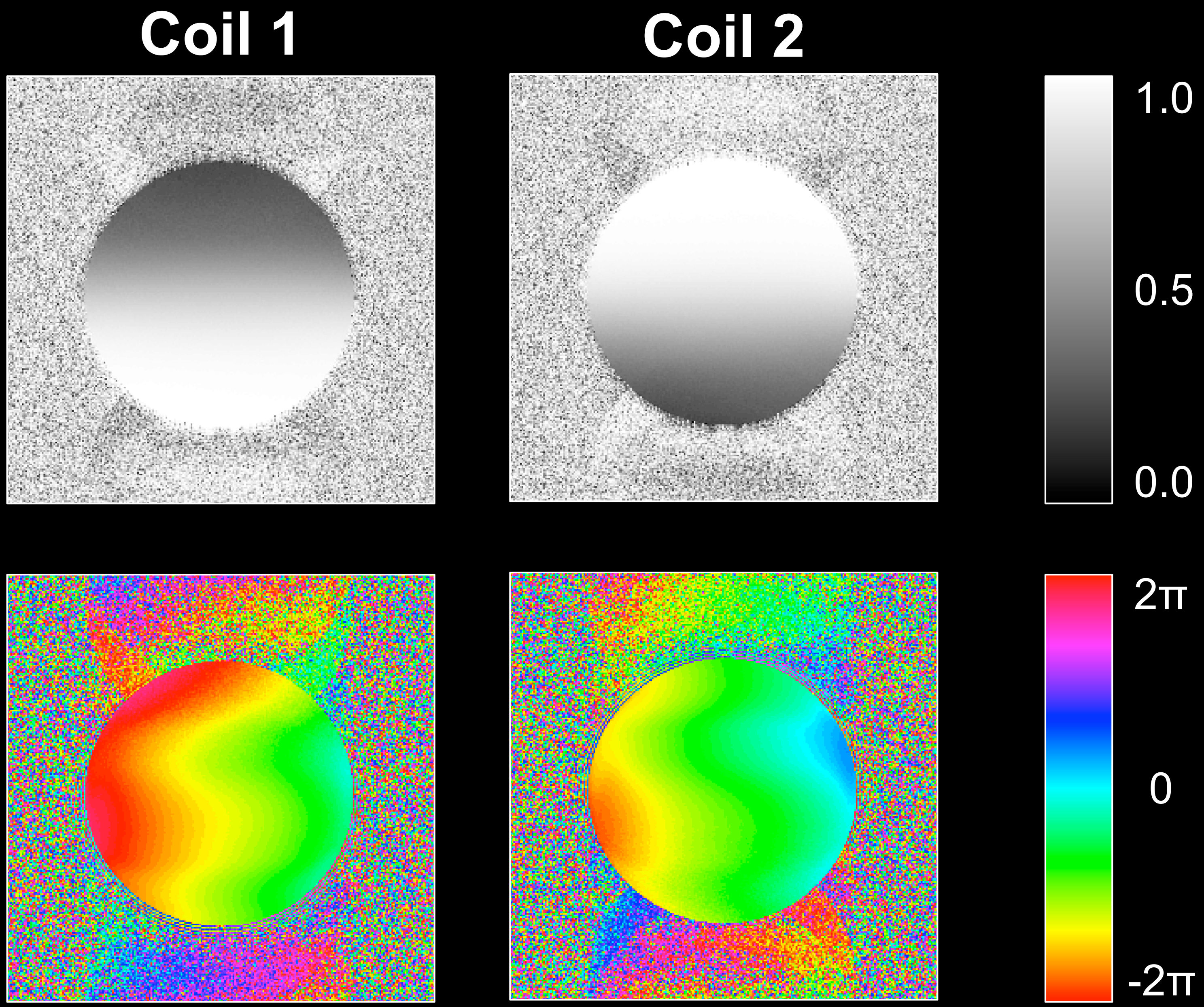


4-Channel Cardiac Coil

Each coil element has a unique sensitivity profile.



Multi-coil Magnitude & Phase



$$\left\|\vec{B}\left(\vec{r}\right)\right\|$$

$$\angle\vec{B}\left(\vec{r}\right)$$

Basic Detection Principles

Magnetic Flux Through The Coil - Reciprocity

$$\Phi\left(t\right) = \int_{object} \vec{B}_r\left(\vec{r}\right) \cdot \vec{M}\left(\vec{r}, t\right) d\vec{r}$$ Eqn. 3.126

Magnetic
Flux

Coil
Sensitivity

Bulk
Magnetization

Faraday’s Law of Induction & The Principle of Reciprocity

$$V\left(t\right) = -\frac{\partial \Phi\left(t\right)}{\partial t} = -\frac{\partial}{\partial t} \int_{object} \vec{B}\left(\vec{r}\right) \cdot \vec{M}\left(\vec{r}, t\right) d\vec{r}$$

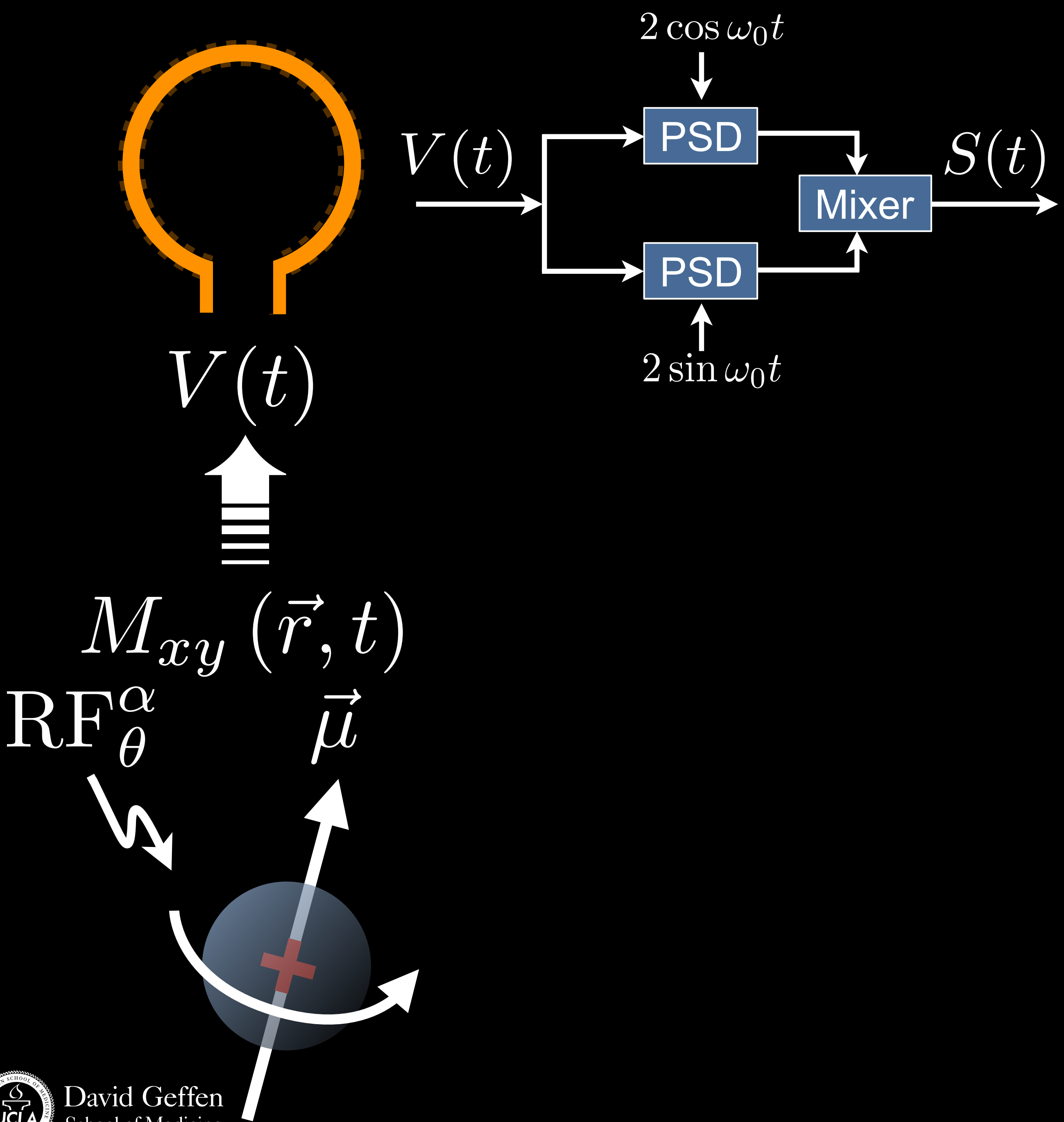
To The Board...

$$M_{xy}\left(\vec{r},t\right) \text{ to } V\left(t\right)$$

Phase Sensitive Detection

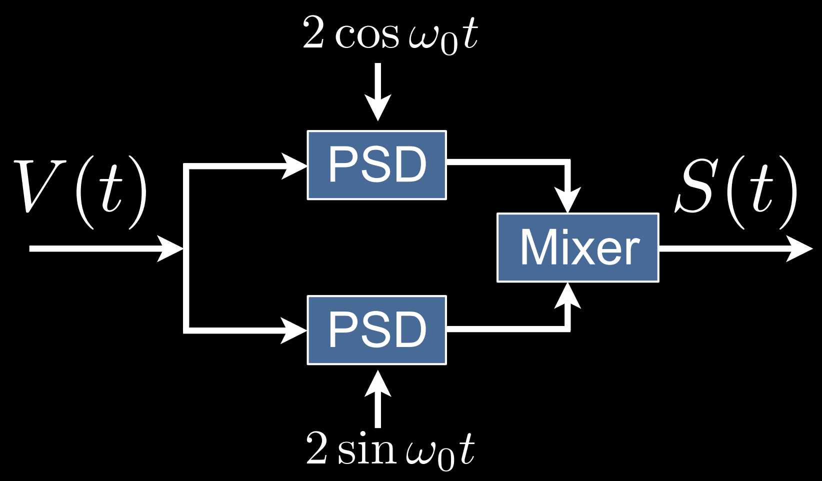
$$V\left(t\right) \rightarrow V_{psd}^{c}\left(t\right) \text{ and } V_{psd}^{s}\left(t\right)$$

Signals in MRI



Why PSD?

- PSD can extract a signal with a known carrier frequency from an extremely noisy environment
- Consists of multiplying V(t) by a reference signal, then low pass filtering to remove high frequencies
- Reduces “unnecessary problems” associated with high frequency signals at later processing stages
- Use of two “orthogonal” PSDs comprises “quadrature detection” and results in the complex signal, S(t).



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http://en.wikipedia.org/wiki/Lock-in_amplifier

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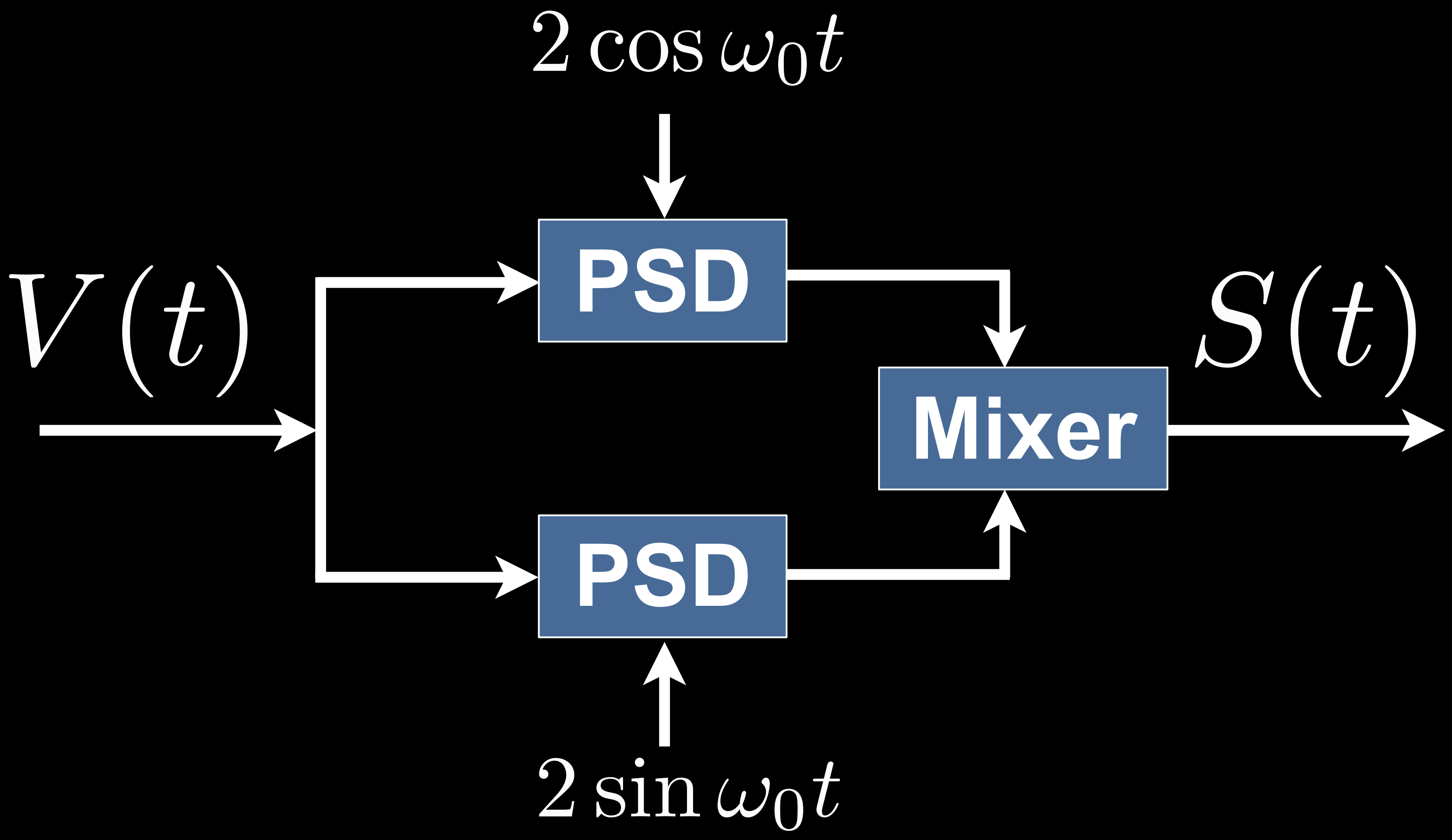
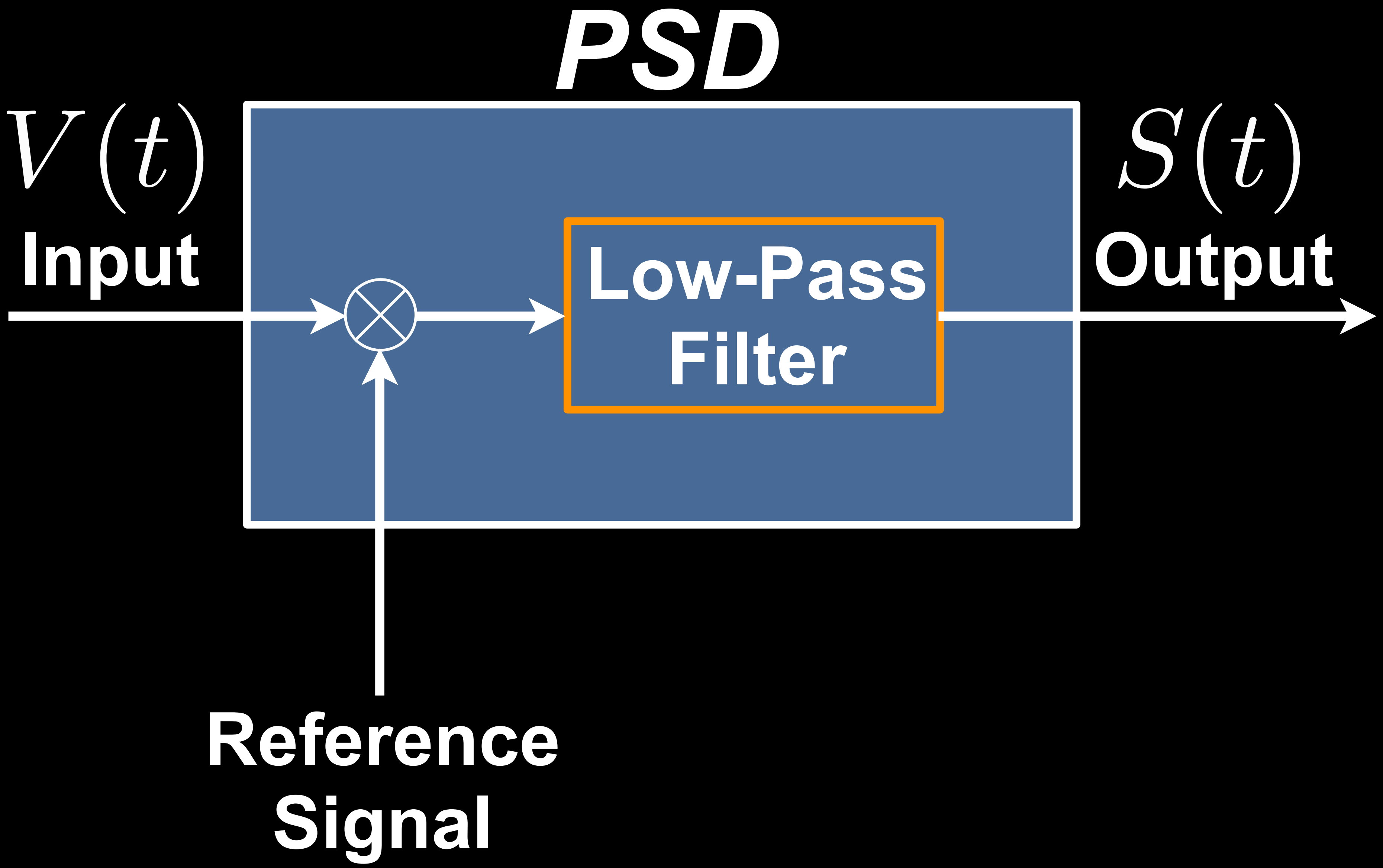
Signal Mixing



Phase Sensitive Detection



PSD

$V(t)$

Input

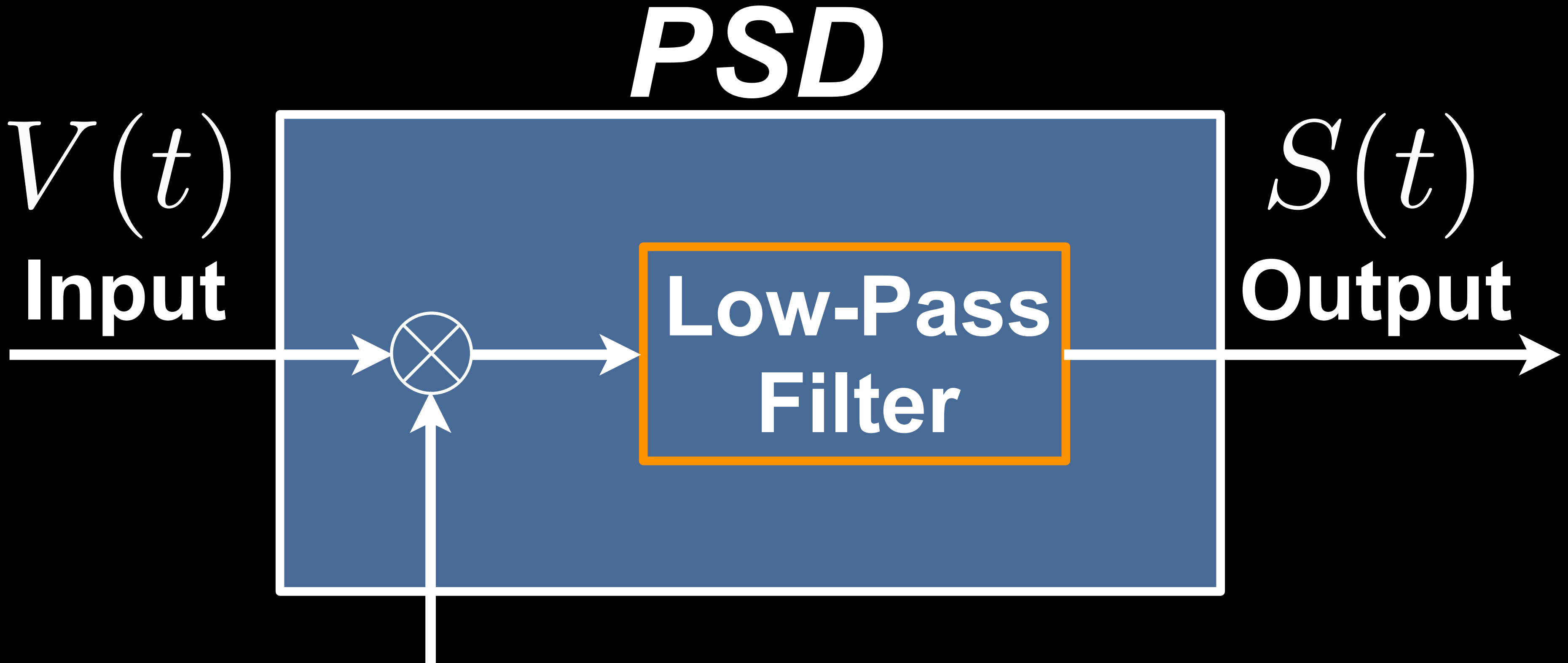
$S(t)$

Output

Low-Pass Filter

Reference Signal

Phase Sensitive Detection



PSD

$V(t)$

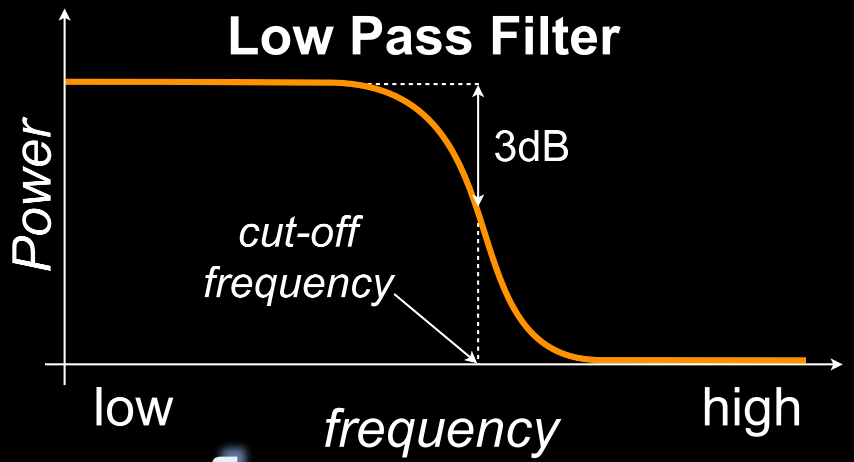
Input

Low-Pass Filter

$S(t)$

Output

Reference Signal

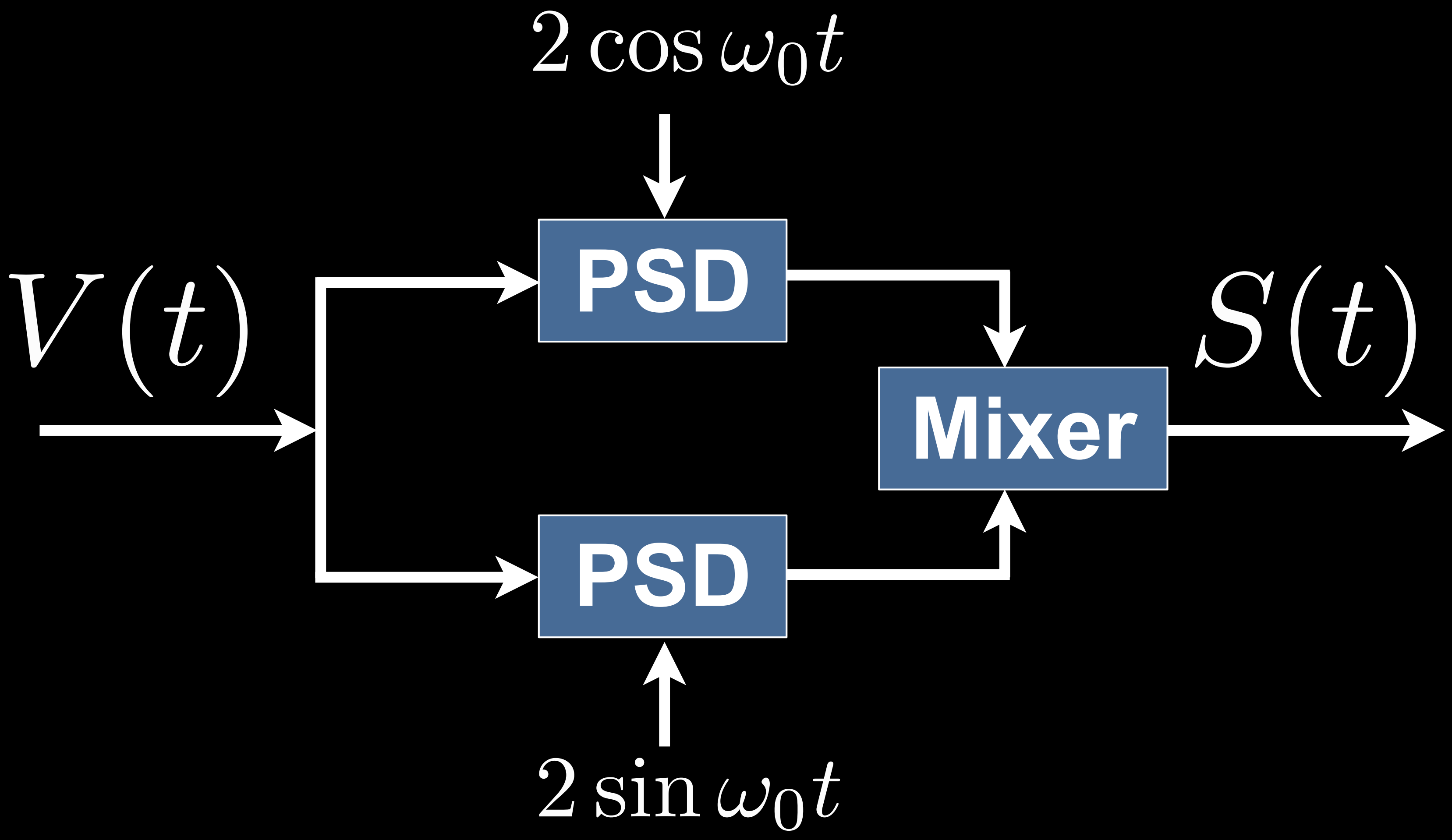


To The Board...

Quadrature Detection

$$V_{psd}^{c}(t) \text{ and } V_{psd}^{s}(t) \rightarrow S(t)$$

Quadrature Detection

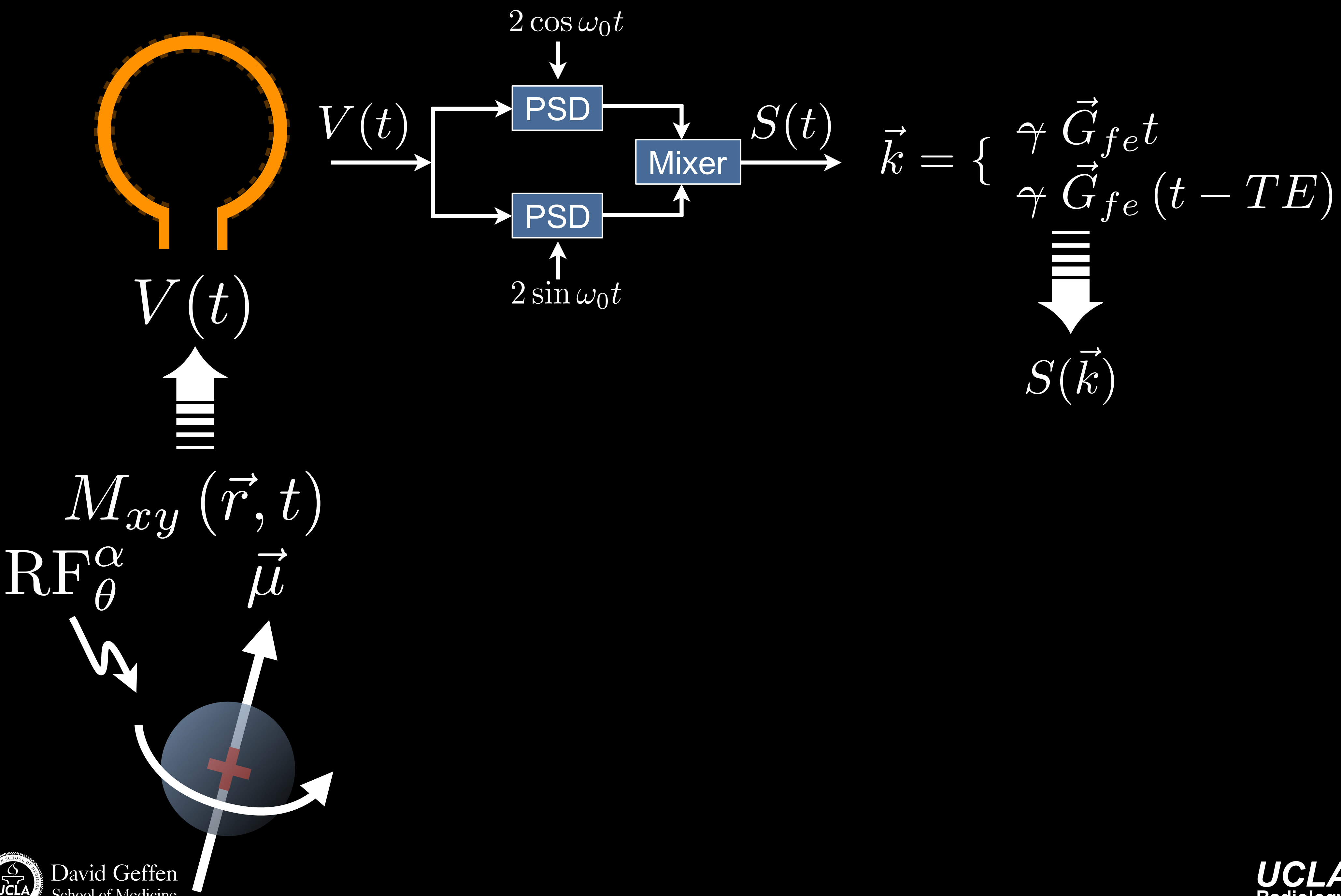


To The Board...

Phase Sensitive Detection

$$S\left(t\right) \text{ to } S\left(\vec{k}\right)$$

Signals in MRI



$$\vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe}t \\ \bar{\gamma}\,\vec{G}_{fe}\,(t - TE) \end{cases}$$

$$S(\vec{k})$$

$$V(t)$$

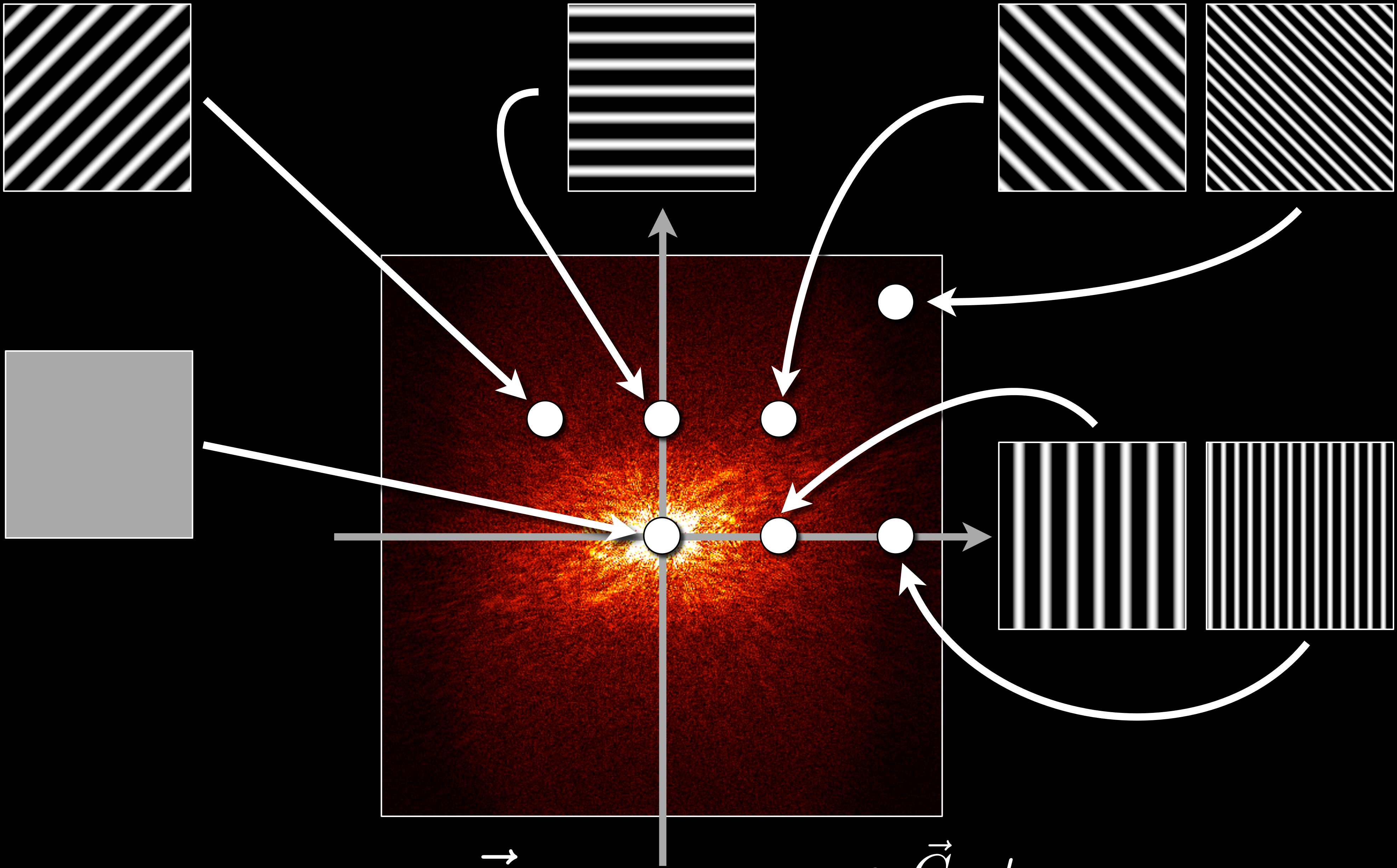
$$M_{xy}\,(\vec{r}, t)$$

$$\mathrm{RF}_{\theta}^{\alpha} \qquad \vec{\mu}$$

How does S(t) relate to S(k)?

To The Board...

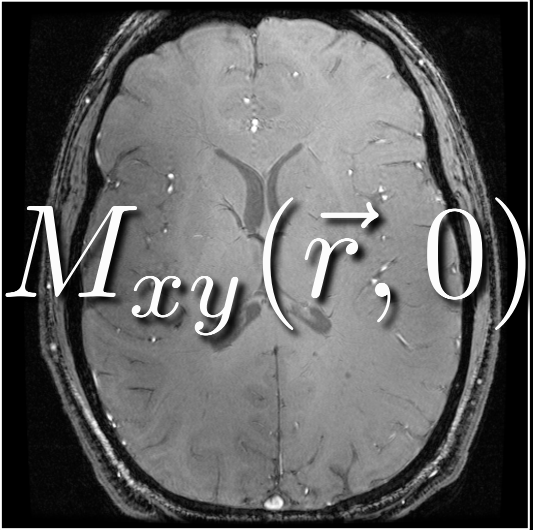
k-space

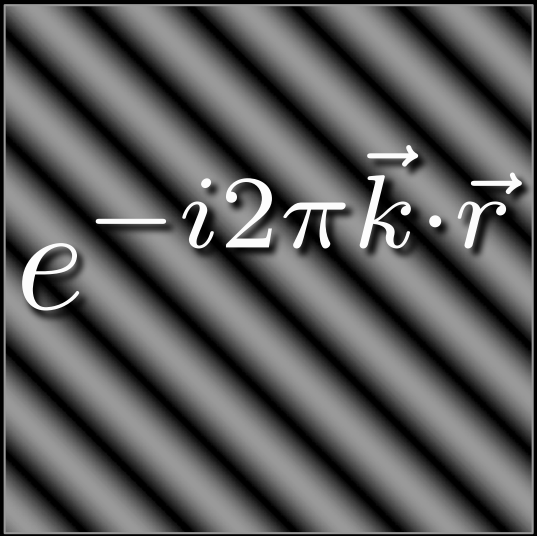


$$e^{-i2\pi\vec{k}\cdot\vec{r}} \qquad \vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe}t \\ \bar{\gamma}\,\vec{G}_{fe}\,(t-TE) \end{cases}$$

k-space Signal

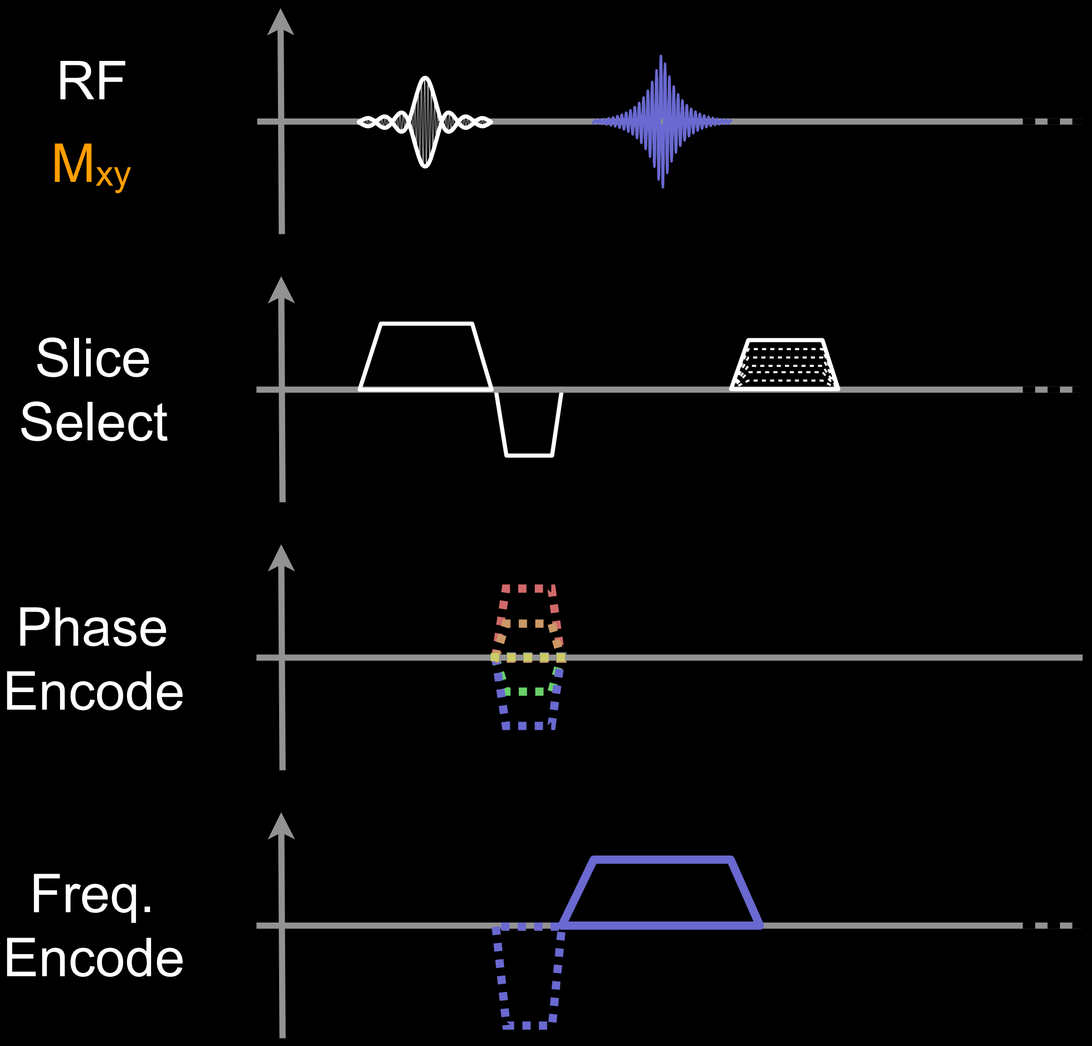
$$S(\vec{k}) = \int_{object} M_{xy}(\vec{r},0)e^{-i2\pi\vec{k}\cdot\vec{r}}d\vec{r}$$

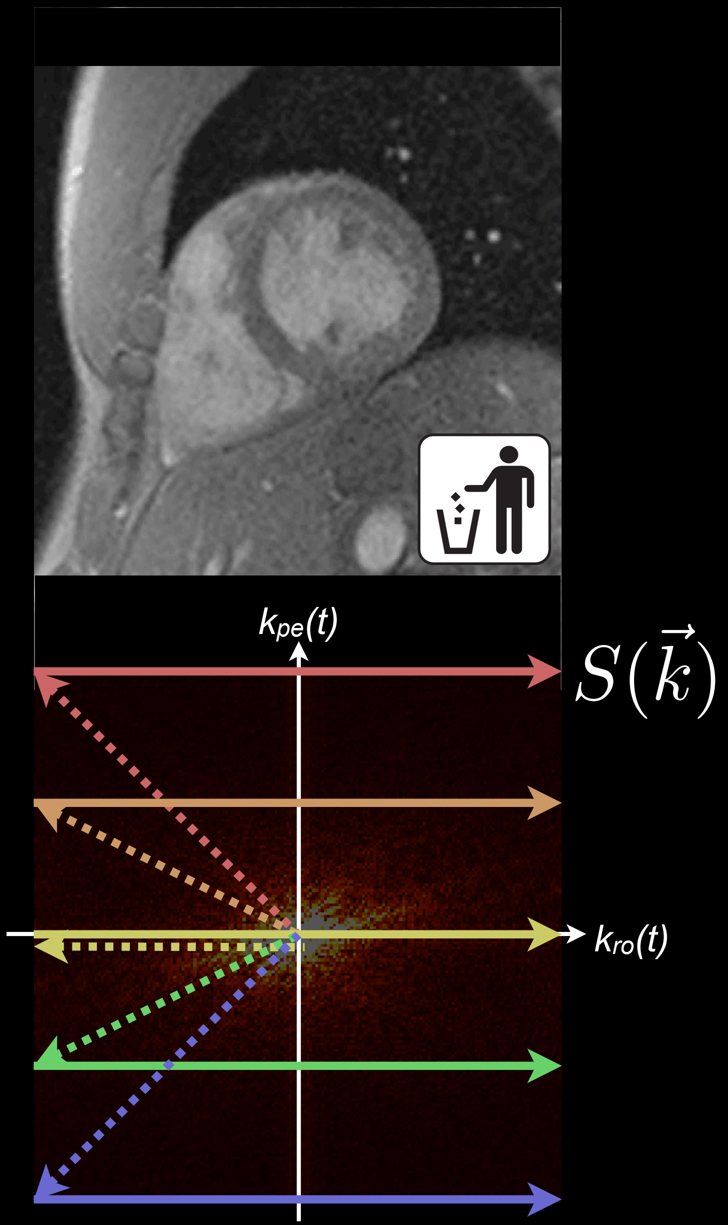




$$S(\vec{k}) = \int_{object} M_{xy}(\vec{r},0)\ e^{-i2\pi\vec{k}\cdot\vec{r}}\ d\vec{r}$$

S(*k*) into *k*-space





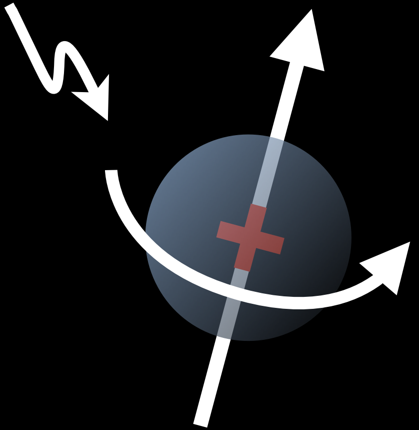
Gradients move us through *k*-space!

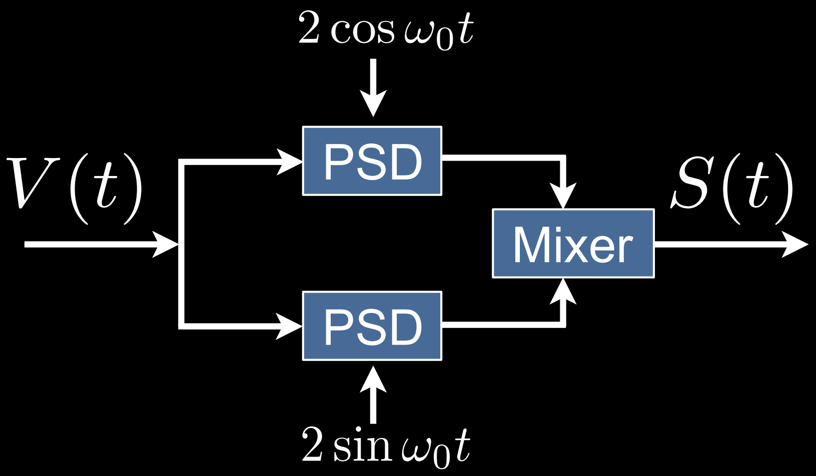
Signals in MRI

$V(t)$

$M_{xy}(\vec{r}, t)$

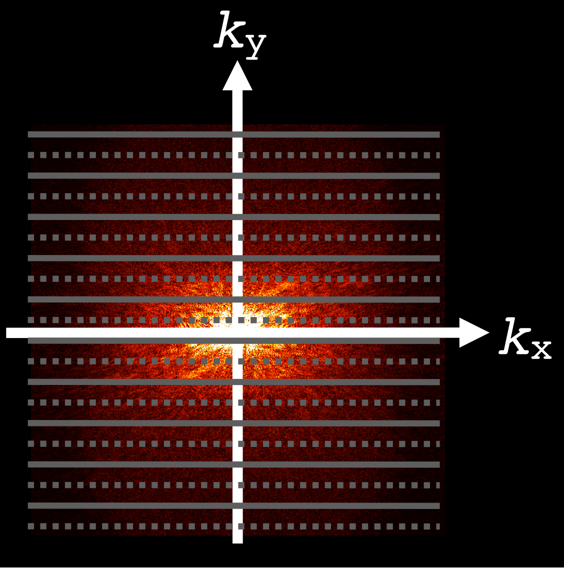
$\mathrm{RF}_{\theta}^{\alpha}$ $\vec{\mu}$





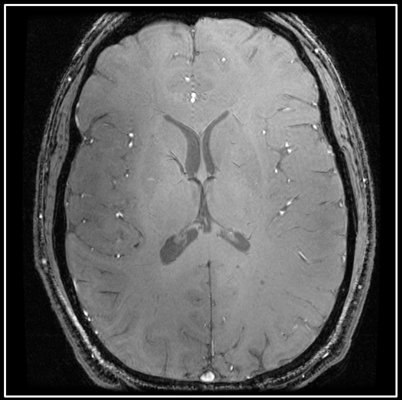
$$\vec{k} = \begin{cases} \bar{\gamma}\,\vec{G}_{fe} t \\ \bar{\gamma}\,\vec{G}_{fe}(t - TE) \end{cases}$$

$S(\vec{k})$



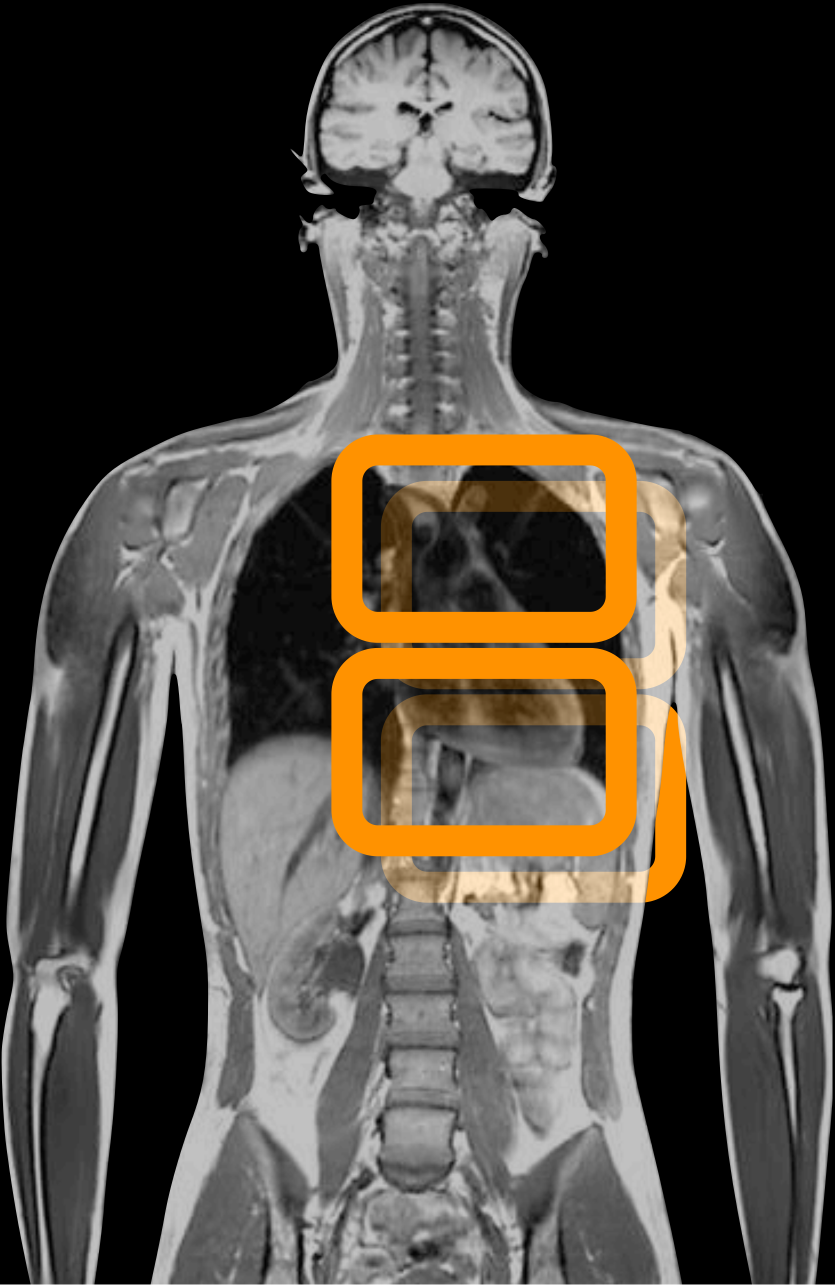
FFT

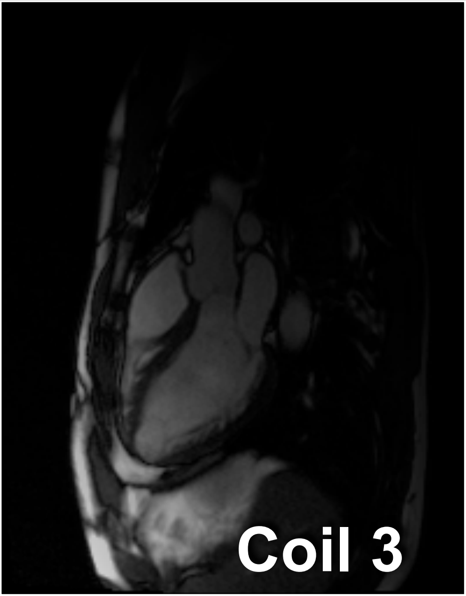
$I(\vec{r})$



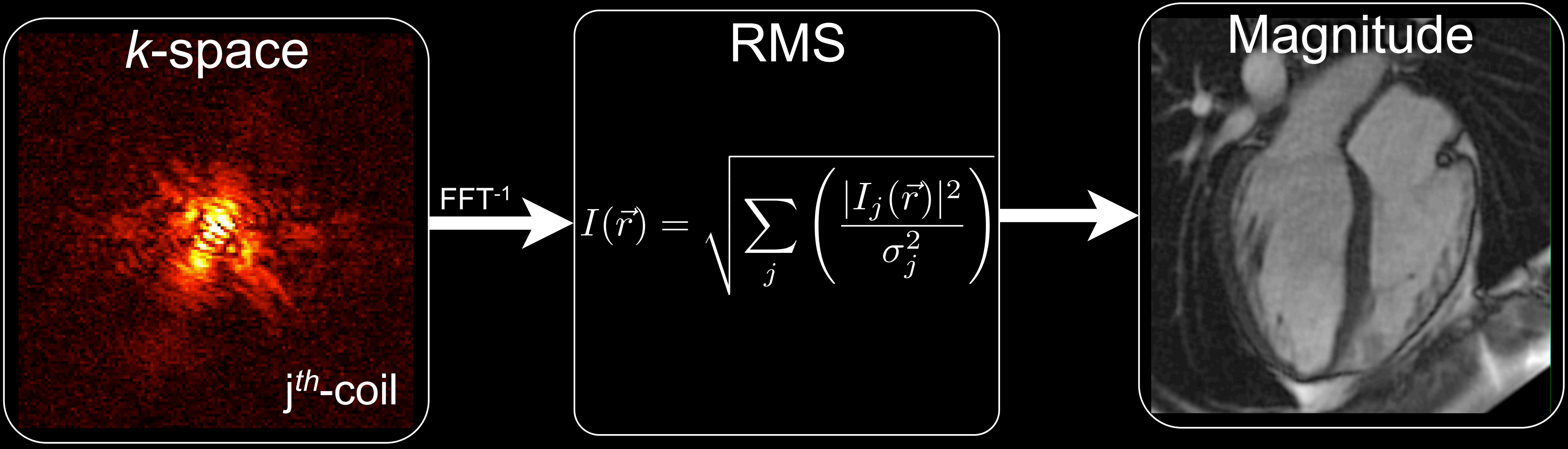
Multiple Coil Reconstruction

Each coil element has a unique sensitivity profile.





Multiple Coil Reconstruction



$$I(\vec{r}) = \sqrt{\sum_j \left(\frac{|I_j(\vec{r})|^2}{\sigma_j^2} \right)}$$

$I(\vec{r}) \rightarrow$ Final ***magnitude*** image

$I_j \rightarrow$ Image from j^{th} coil

$\sigma_j^2 \rightarrow$ Noise variance

- Depends on coil loading
- Proximity to patient
- Measured with “noise scan”
- Weights each coil’s contribution

Thanks

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