Spatial Localization / Imaging Sequences

M219 - Principles and Applications of MRI

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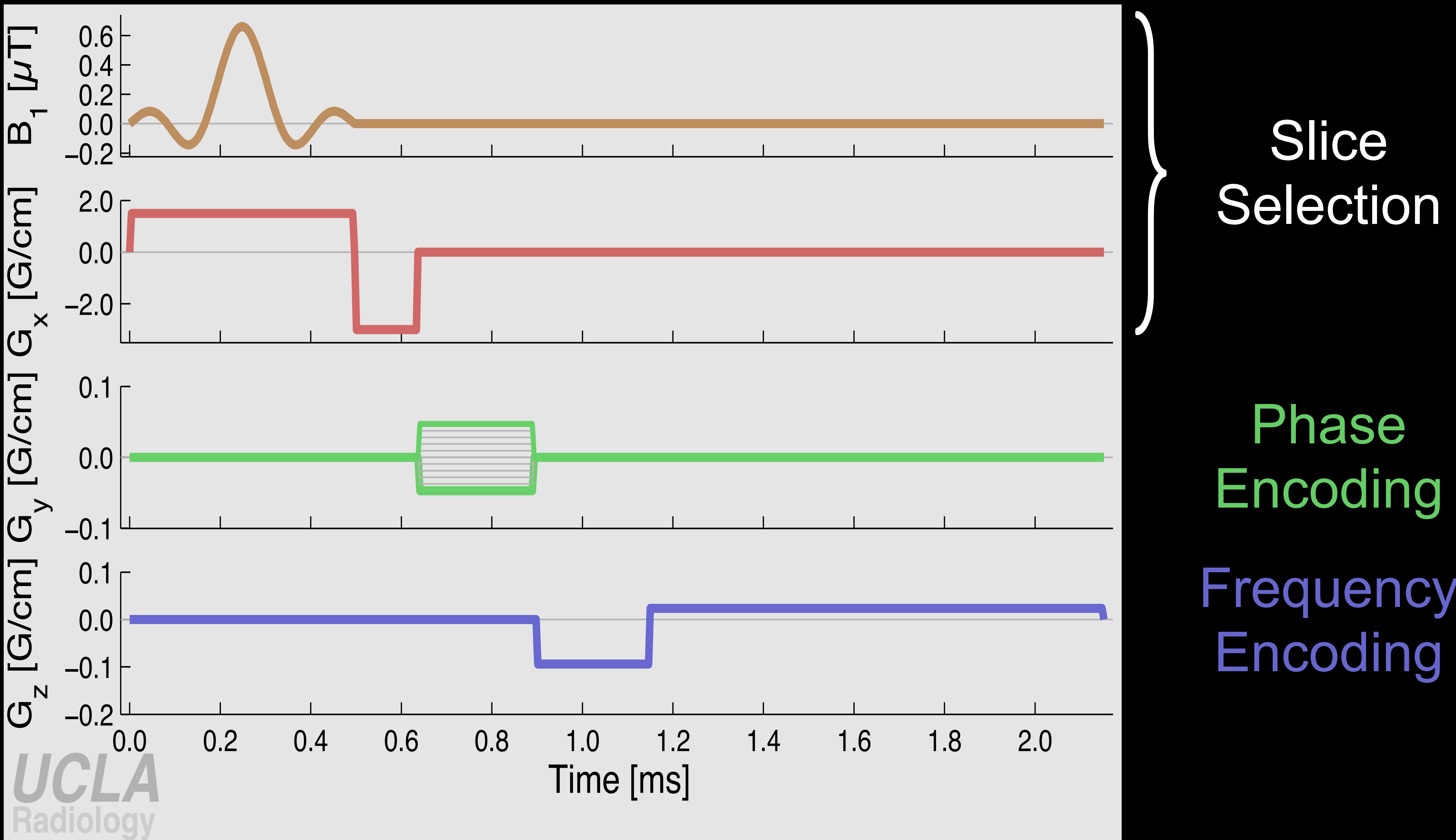
2/22/2023

Course Overview

- 2023 course schedule
  - https://mrrl.ucla.edu/pages/m219_2023
- Assignments
  - Homework #3 is due on 3/8
- Office hours, Fridays 10-12pm
  - In-person (Ueberroth, 1417B)
  - Zoom is also available (https://uclahs.zoom.us/j/98066349714?pwd=cnVmV1J5QjR1d3I3cmJkQnVLSFZVZz09)

Spatial Localization

3 Steps for Spatial Localization

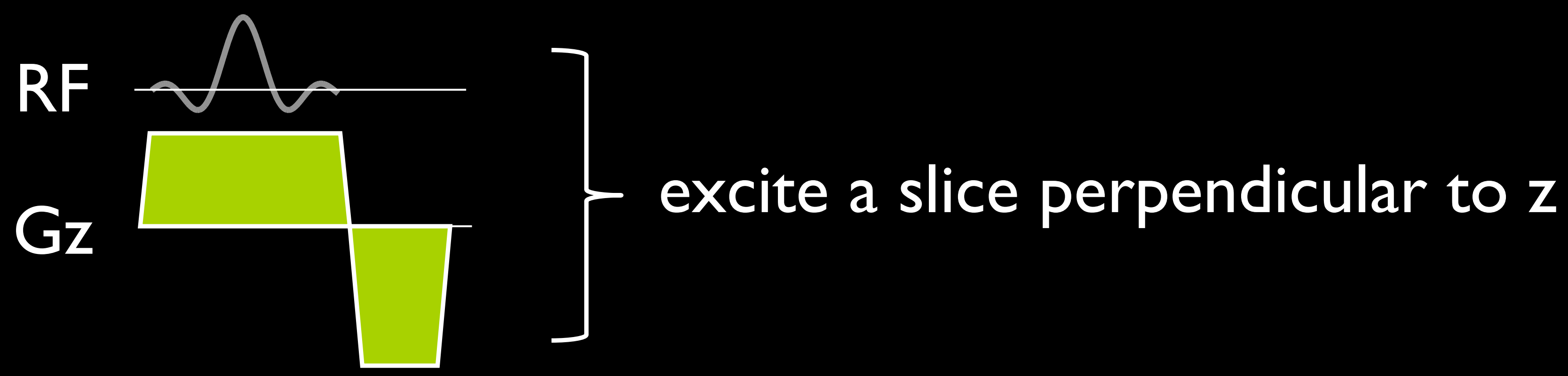


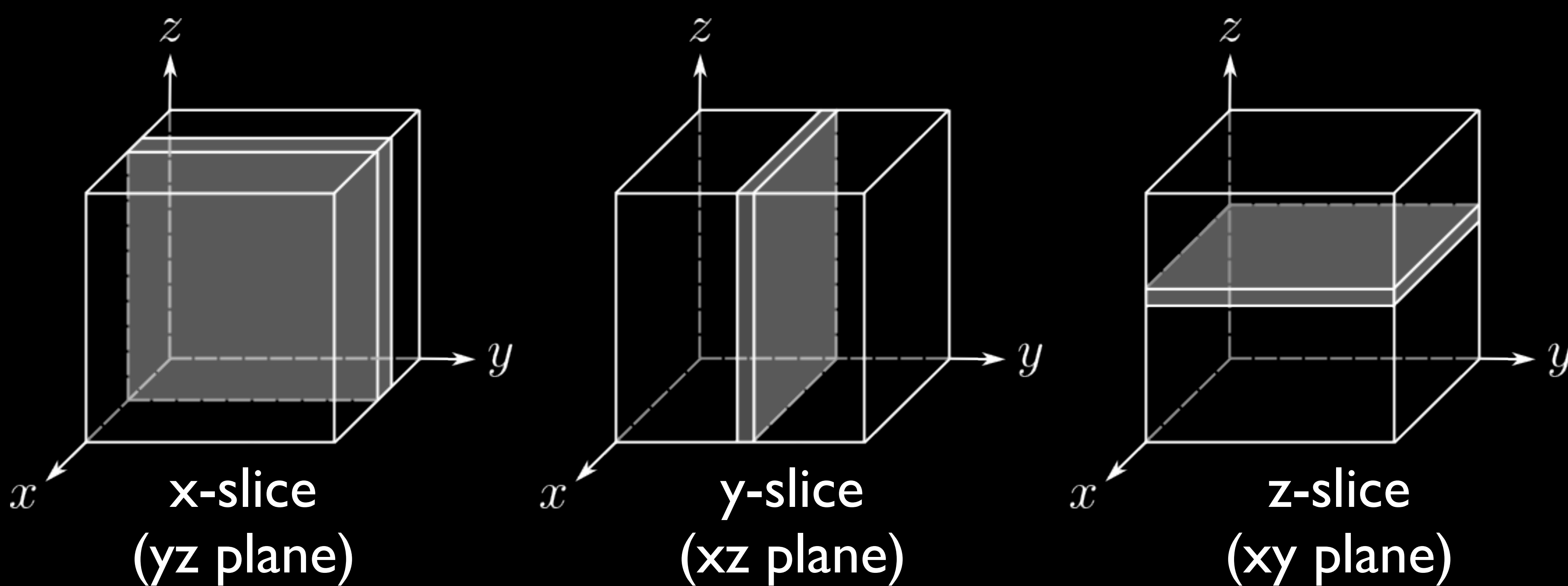
Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

David Geffen School of Medicine

UCLA Radiology

Selective Excitation





Bloch Equation with Gradient

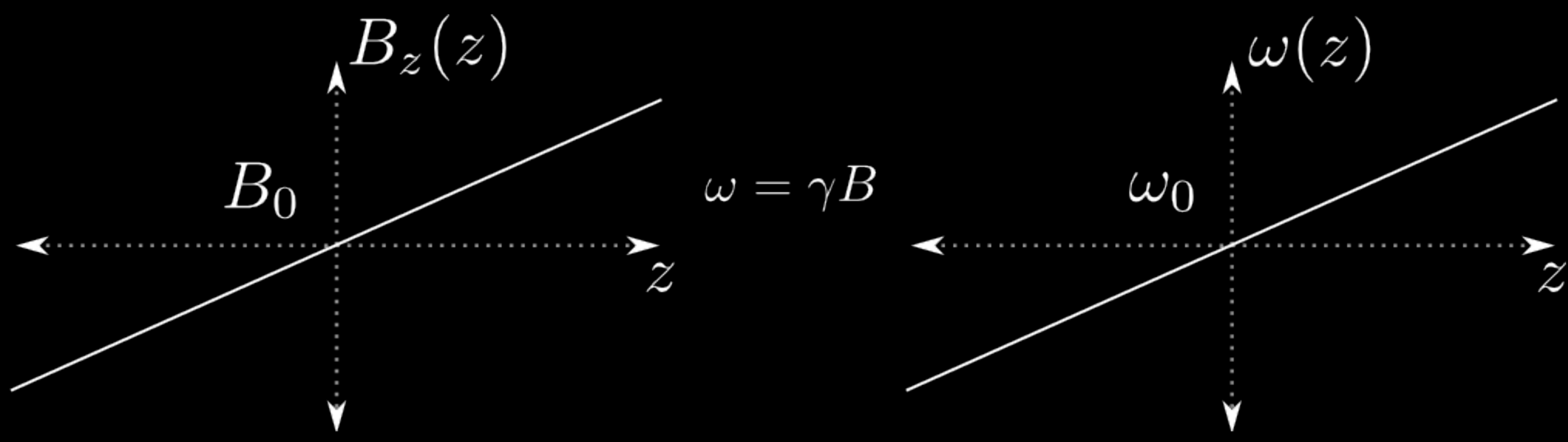
$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \left(\begin{array}{c} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{array} \right) \Rightarrow \vec{B}_{eff} = \left(\begin{array}{c} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} + G_z z \end{array} \right)$$

Gradients?

gradients produce a spatial distribution of frequencies

$$B_z(z) = B_0 + G_z \cdot z$$

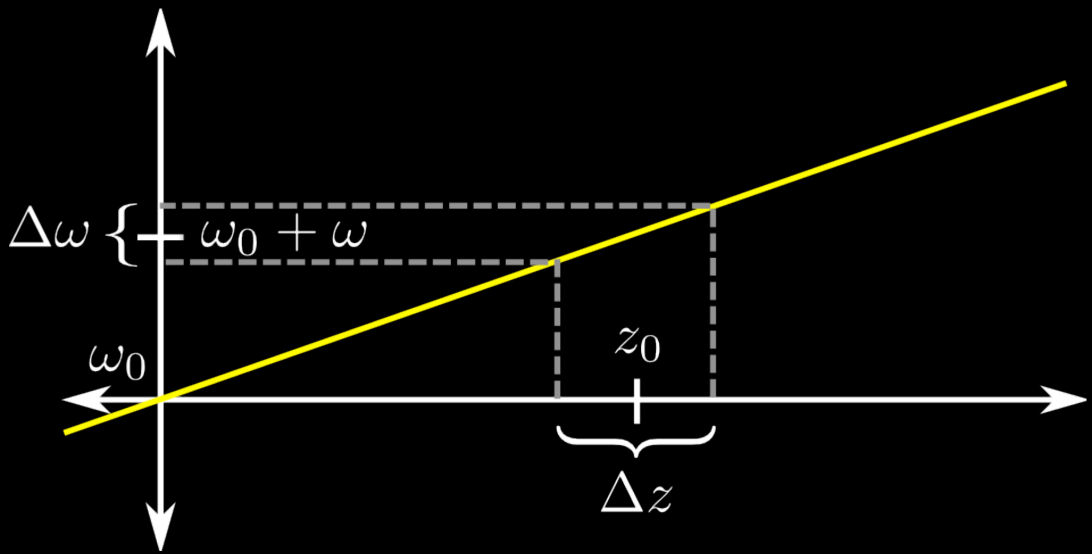


$$\omega(z) = \omega_0 + \gamma G_z \cdot z$$

there is a direct correspondence between frequency and spatial position

Slice Selection

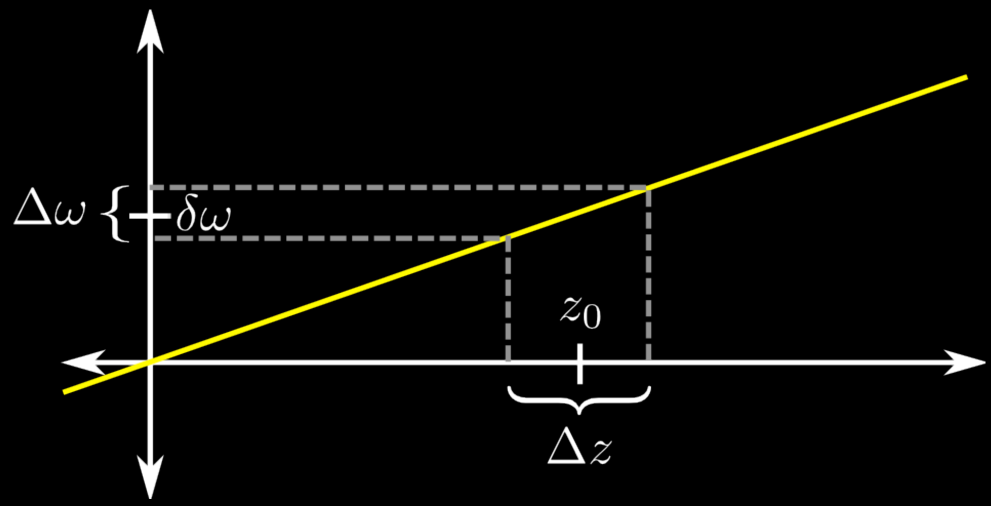
how do we physically set the parameters?

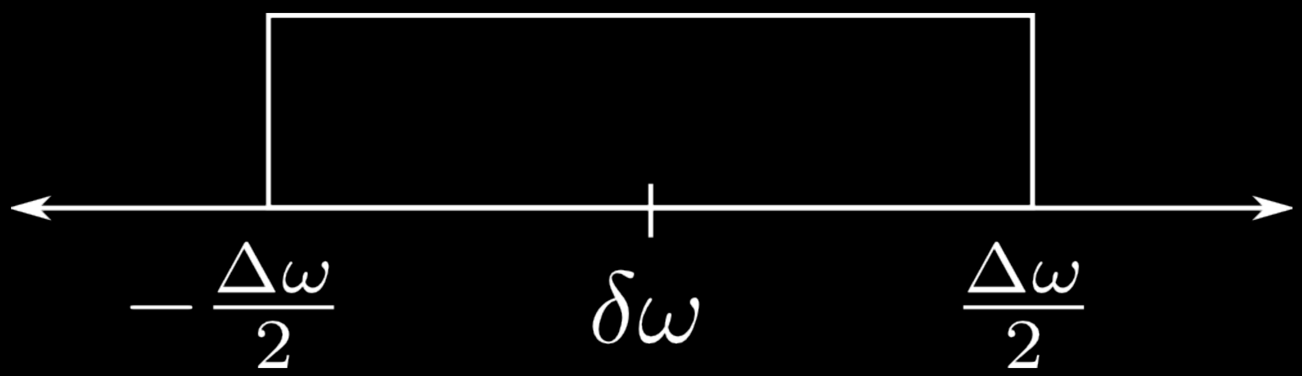


ω - the carrier frequency of the RF pulse

$\Delta\omega$ - frequency bandwidth of the RF pulse

Slice Selection

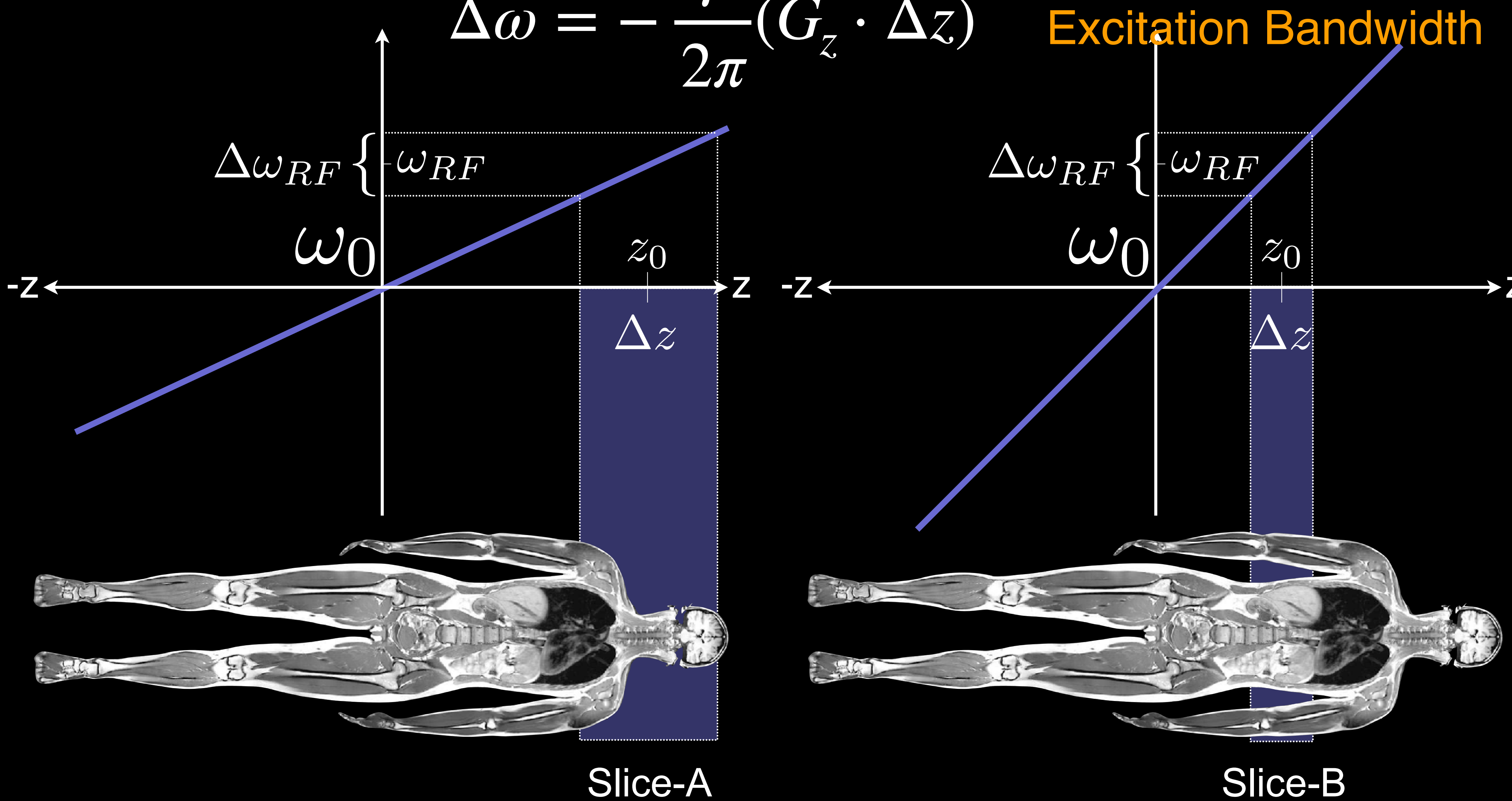




we want a pulse with as rectangular of an slice profile as possible

Slice Selective Excitation

$$\Delta\omega = -\frac{\gamma}{2\pi}(G_z \cdot \Delta z)$$



How do you move the slice along ±z?

Compare $\Delta\omega$ and ω_{RF} for Slice-A and Slice-B.

Do we usually acquire $\omega_{RF}>\omega_0$?

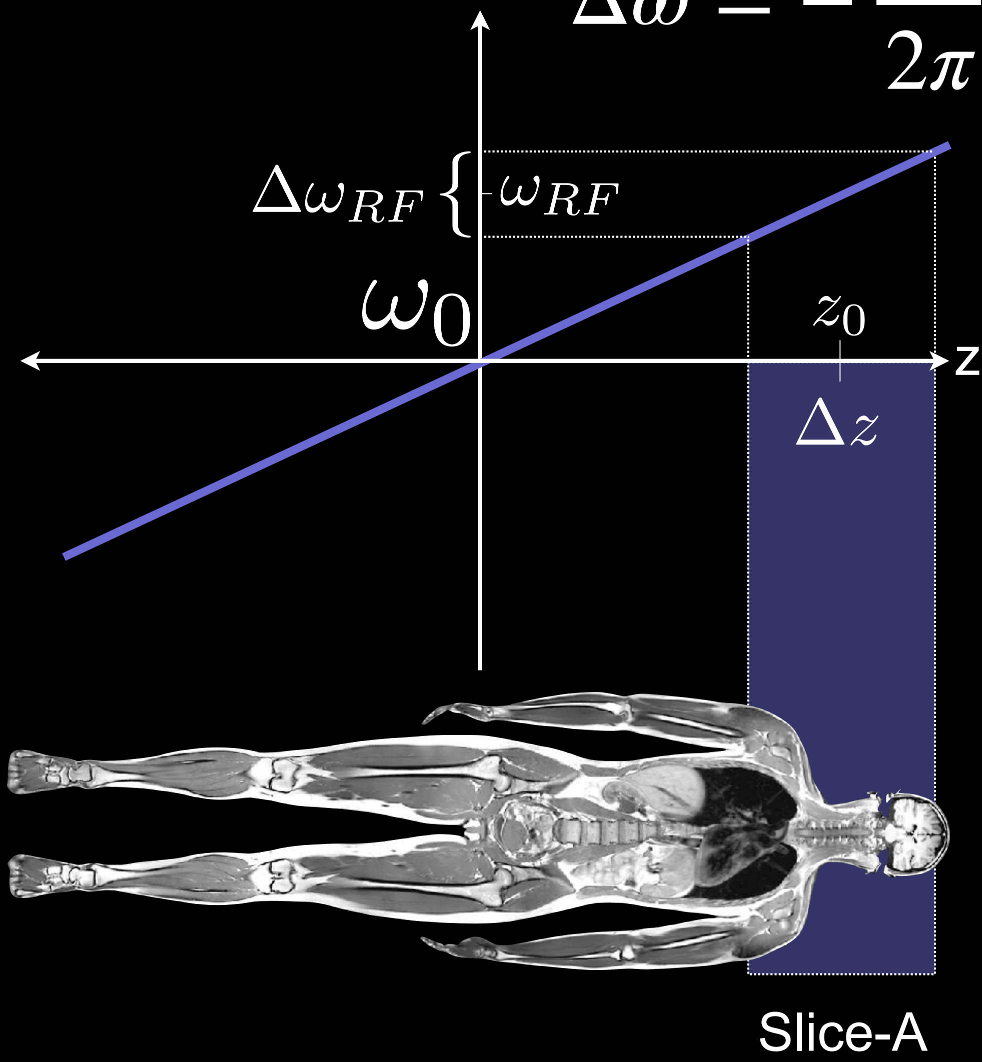
Time Bandwidth Product (TBW)

- **Time bandwidth (TBW) product:**
  - **Pulse Duration [s] x Pulse Bandwidth [Hz]**
  - **Unitless**
  - **# of zero crossings**
  - **High TBW**
    - Large # of zero crossings ∴ fewer truncation artifacts
    - Longer duration pulse
- **Examples:**
  - **TBW = 4, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?
  - **TBW = 16, RF = 1ms**
    - Excitation (RF) bandwidth?
    - Required G_z for 1cm slice?

Slice Selective Excitation - Example

$$\Delta\omega = -\frac{\gamma}{2\pi}(G_z \cdot \Delta z)$$

Excitation Bandwidth



$$\mathrm{TBW} = \tau_{RF} \cdot \Delta\omega_{RF}$$

$$\Delta\omega_{RF} = \frac{\mathrm{TBW}}{\tau_{RF}} = \frac{4}{1\mathrm{ms}} = 4\mathrm{kHz}$$

$$G_z = \frac{\Delta\omega_{RF}}{\gamma\Delta z} = \frac{4000\mathrm{Hz}}{42.57e6\,\frac{\mathrm{Hz}}{\mathrm{T}}\,\frac{1\mathrm{T}}{10000\mathrm{G}} \cdot 10\mathrm{mm}} = 0.94\,\frac{\mathrm{G}}{\mathrm{cm}}$$

RF Pulse Bandwidth and Slice Profile:

Small Tip Angle Approximation

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

When we simplify the cross product,

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

Small Tip Approximation

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

sin θ ≈ θ
cos θ ≈ 1
$M_z \approx M_0 \rightarrow$ constant

$$\frac{dM_z}{dt} = 0$$

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

$$M_{xy} = M_x + iM_y$$

First order linear differential equation. Easily solved.

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

Solving a first order linear differential equation:

$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma G_z z \cdot (t-s)} ds$$

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\mathcal{T}}{2})\} \mid_{f=-(\gamma/2\pi)G_z z}$$

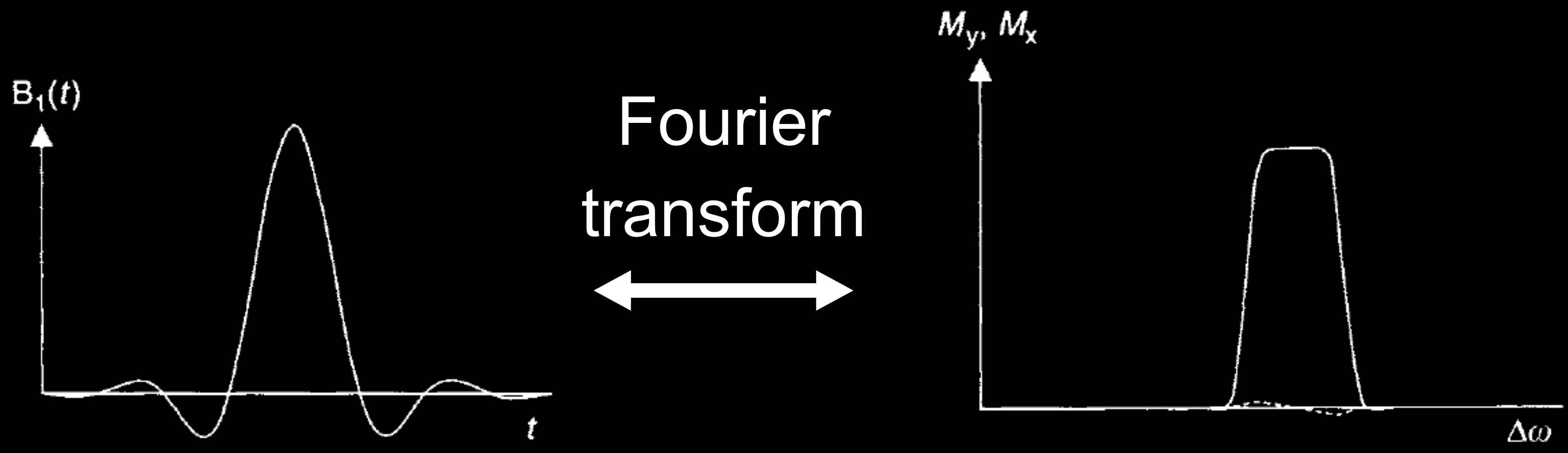
(See the note for complete derivation)

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

To the Board

Small Tip Approximation

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$



- For small tip angles, “the slice or frequency profile is well approximated by the Fourier transform of B1(t)”
- The approximation works surprisingly well even for flip angles up to 90°

Small Tip Approximation

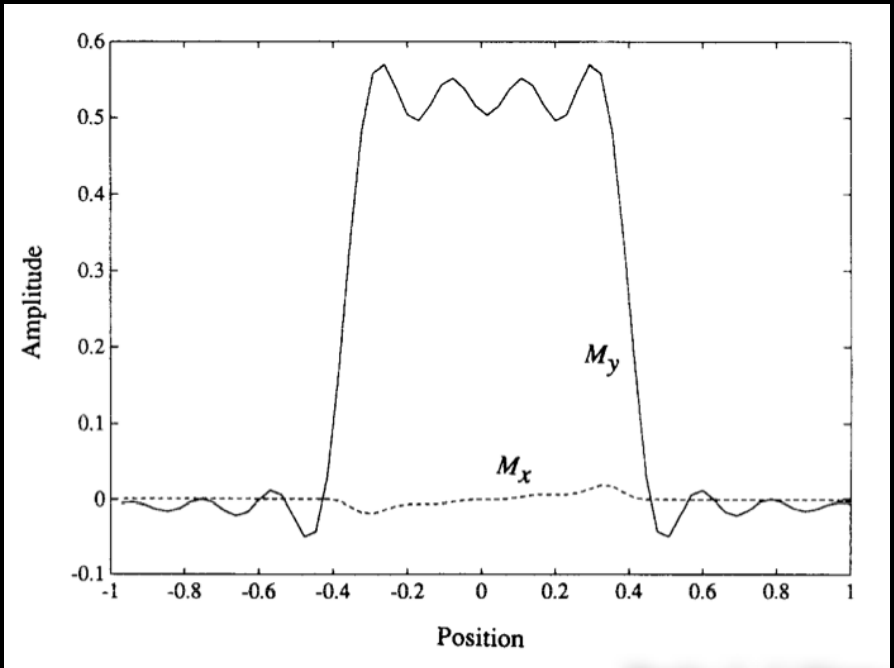
the excitation profile, within the small angle approximation, is just the Fourier transform of the pulse

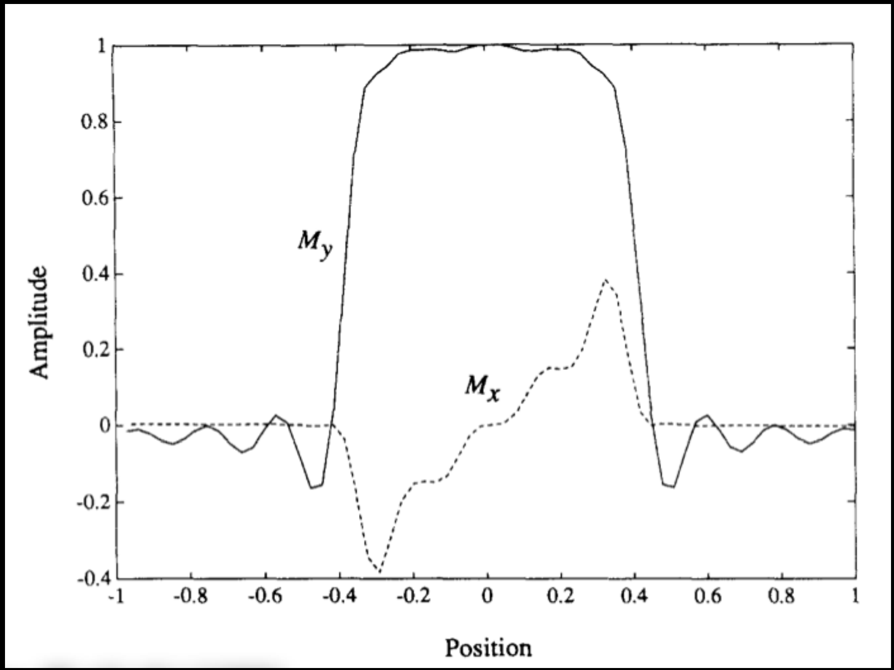
remember that the Bloch equations are non-linear and thus cannot be expected to behave linearly

the approximation works surprisingly well even for flip angles up to 90^0

Shaped Pulses

30° 90°

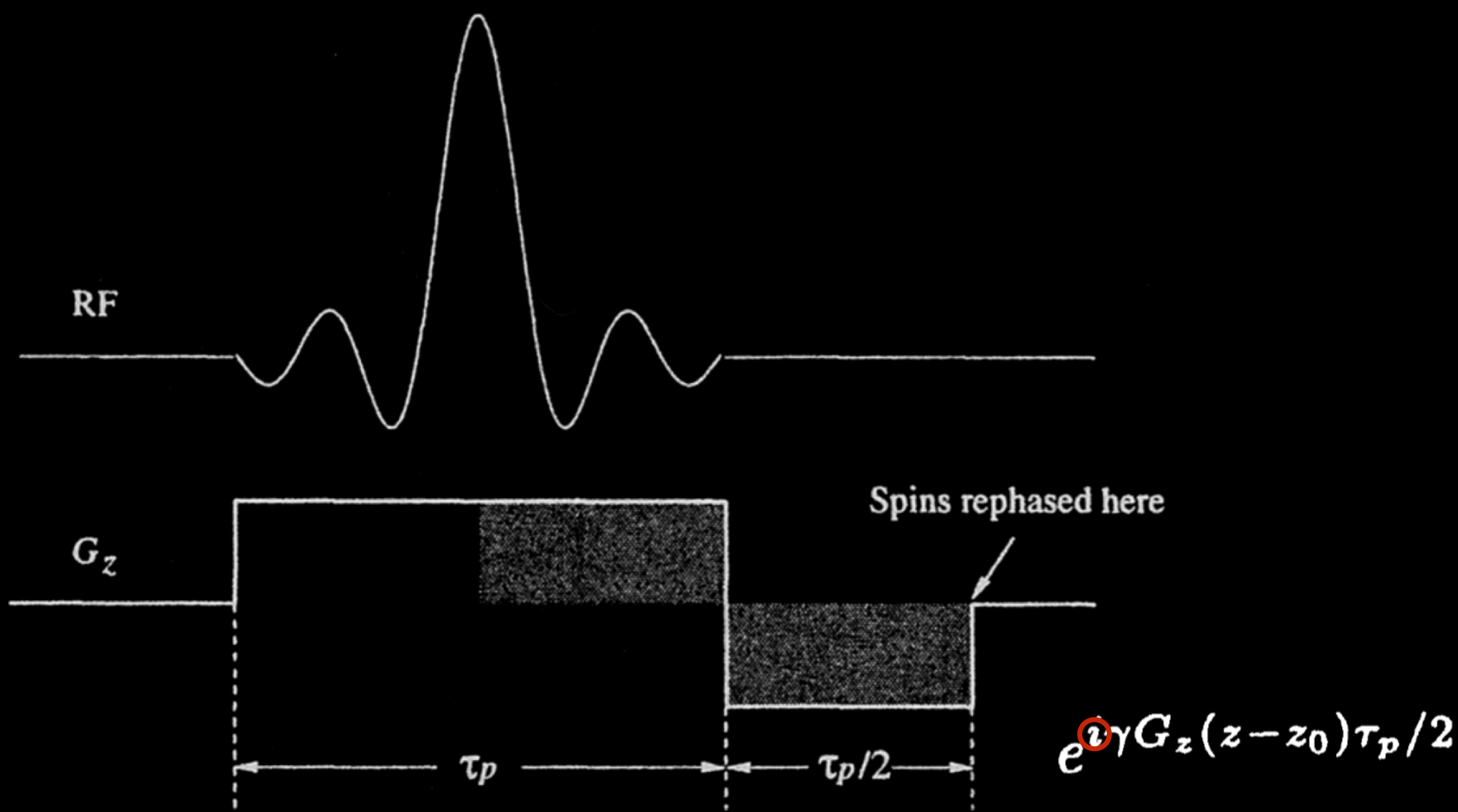




Pauly, J. *J. Magn. Reson.* **81** 43-56 (1989)

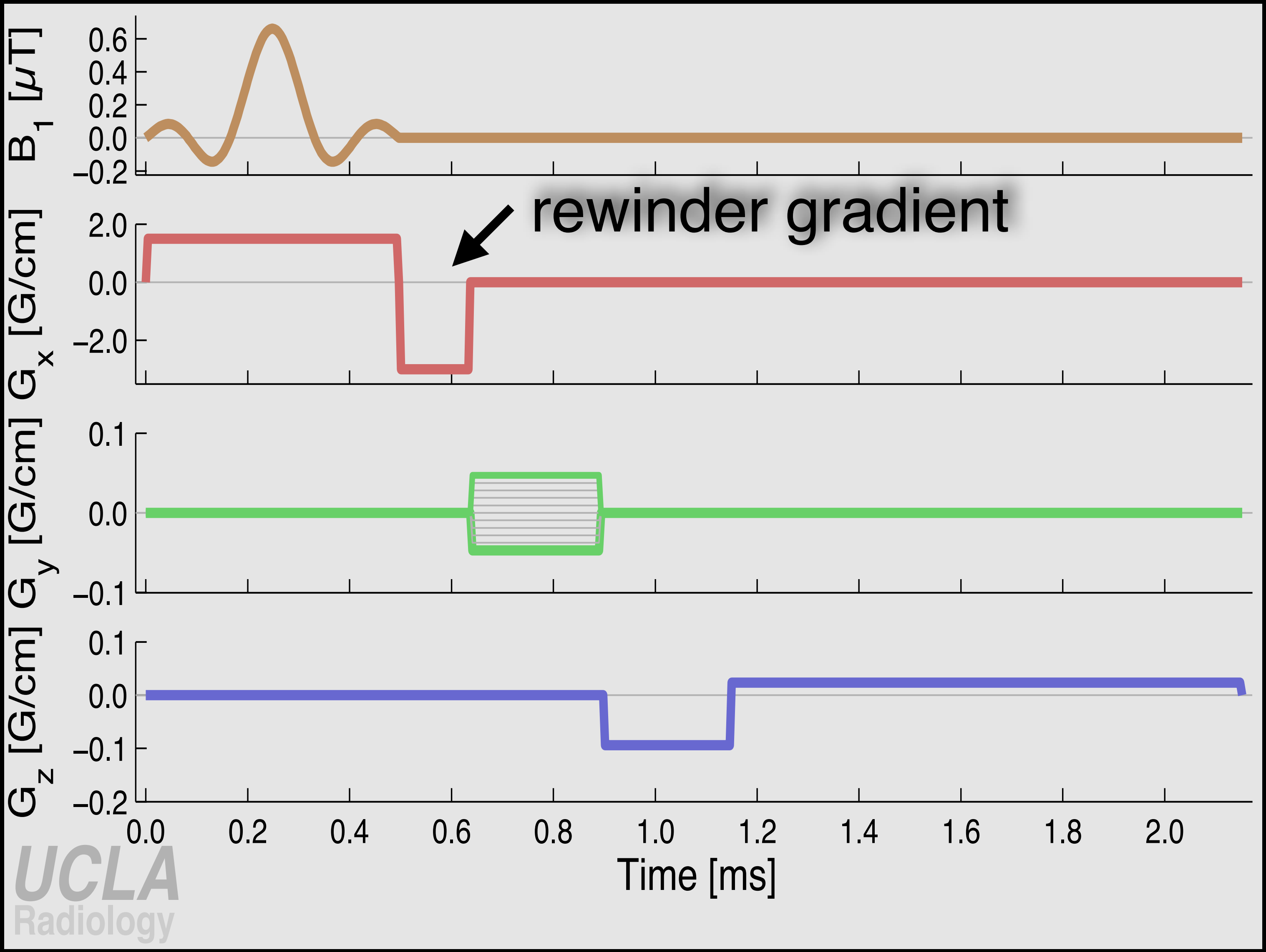
small-angle approximation still works reasonably well for flip angles that aren't necessarily "small"

Slice Rewinder



Opposite Polarity

Slice Selective Excitation Example



slice select gradient rewinder eliminates the linear phase ramp

Truncation Artifacts

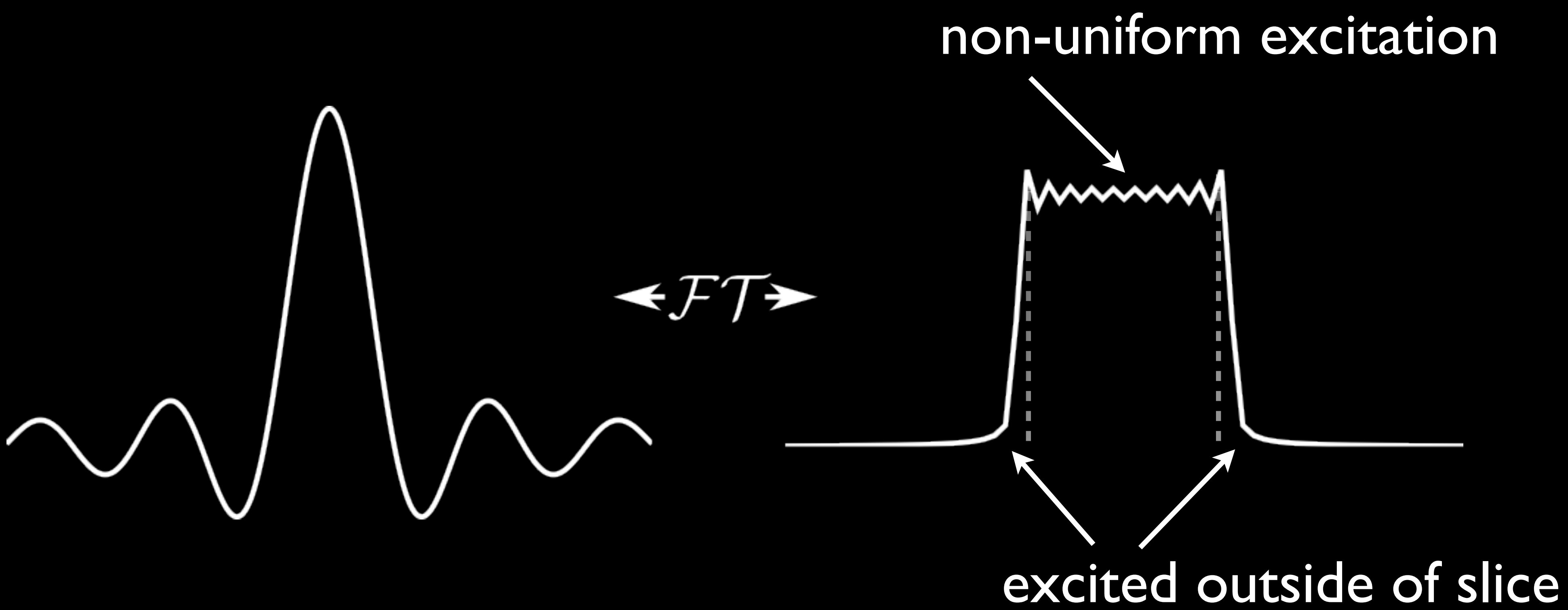
in MRI we want pulses to be as short as possible to avoid relaxation effects

the sinc function is defined over all time which is impractical in any experiment

the sinc pulse needs to be truncated to be appropriate for clinical scans

Truncation Artifacts

what happens when we truncate our pulses?



these deviations from the ideal are known as truncation artifacts

Truncation Artifacts

alternative Pulse Shapes

gaussian

$$B_x(t) = A \exp\left[-a(t - \tau/2)^2\right]$$

reduced side-lobes, but not as flat of a profile

Window Functions

Hamming, Hanning, …

MATLAB Demo

```
%% Design of Windowed Sinc RF Pulses
tbw = 4;
samples = 512;
rf = wsinc(tbw, samples);
```

```
%% Plot RF Amplitude
flip_angle = pi/2;
rf = flip_angle*rf;

pulseduration = 1;          % in msec
dt  = pulseduration/samples;
rfs = rf/(gamma*dt);        % Scaled to Gauss

bw = tbw/pulseduration; % in kHz
gmax = bw/gamma_2pi;

b1     = [rfs zeros(1,samples/2)];                      % in Gauss
g      = [ones(1,samples) -ones(1,samples/2)]*gmax;     % in G/cm
t_all = (1:length(g))*dt;  % in msec
```

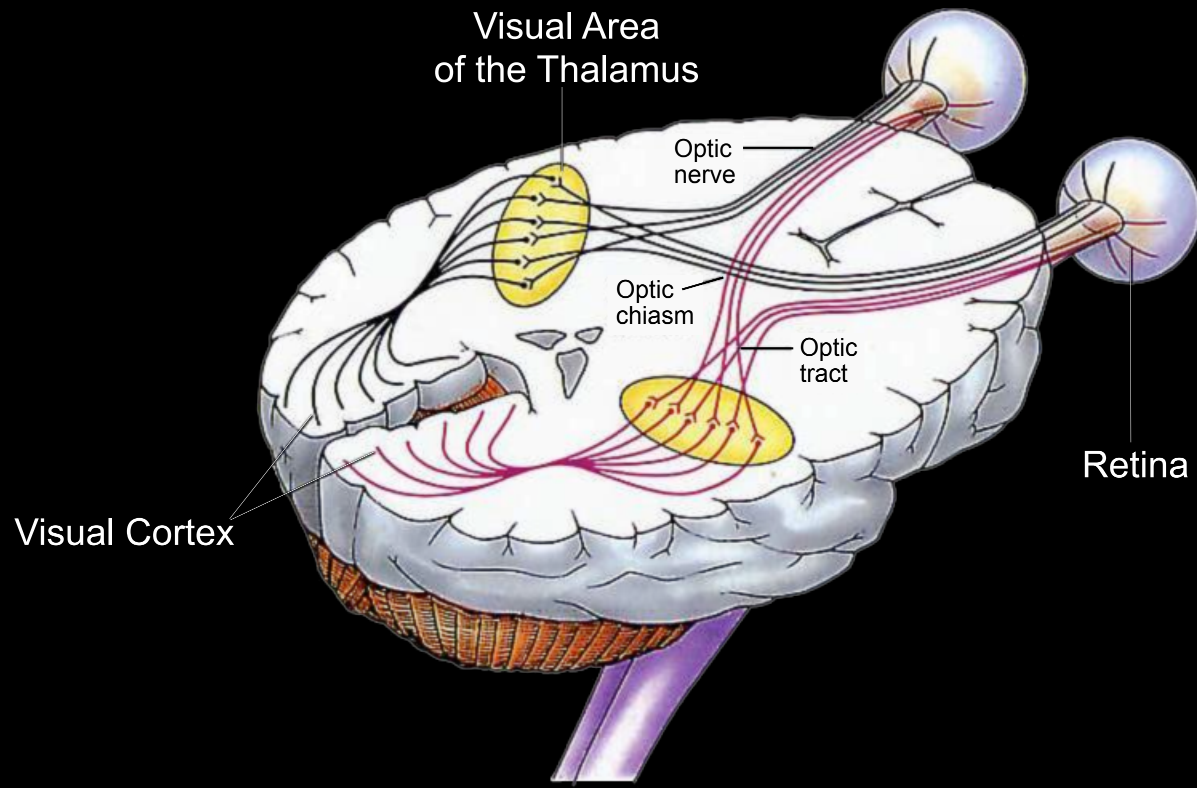
MATLAB Demo

```
%% Simulate Slice Profile using Bloch Simulation
x  = (-2:.01:2);              % in cm
f  = 0;                       % in Hz
dt = pulseduration/samples/1e3;
t  = (1:length(b1))*dt;  % in usec

% Bloch Simulation
[mx,my,mz] = bloch(b1(:),g(:),t(:),1,.2,f(:),x(:),0);

% Transverse Magnetization
mxy_bloch = mx+1i*my;

%% Simulate Slice Profile using Small Tip Approximation
samples_st = 4096;
f_st = linspace(-0.5/dt,0.5/dt,samples_st)/1e3;
x_st = -f_st/(gamma_2pi*gmax);

rfs_zp = zeros(1,samples_st);
rfs_zp(1:samples) = rfs;

mxy_st = fftshift(fftn(fftshift(rfs_zp)))/30;
```

http://www-mrsrl.stanford.edu/~brian/blochsim/

Image Contrast

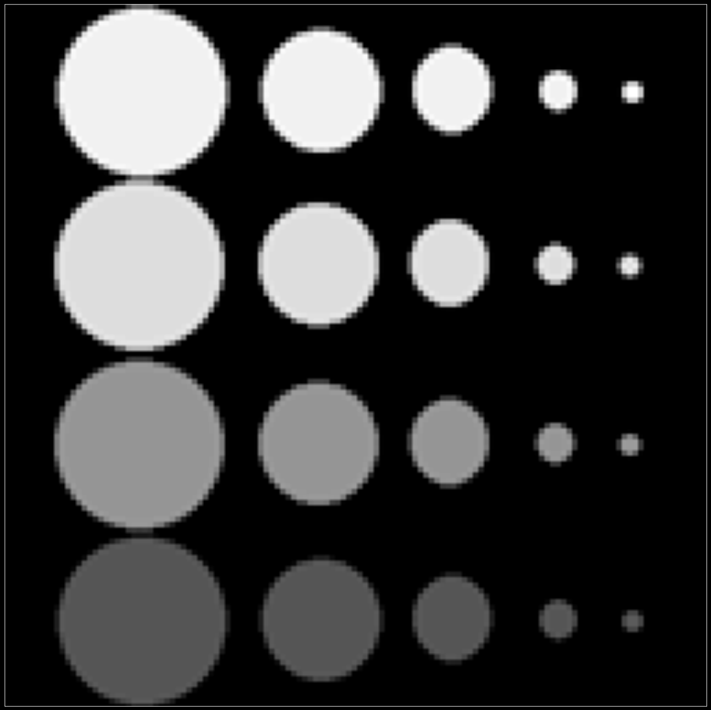
Why Image Contrast?



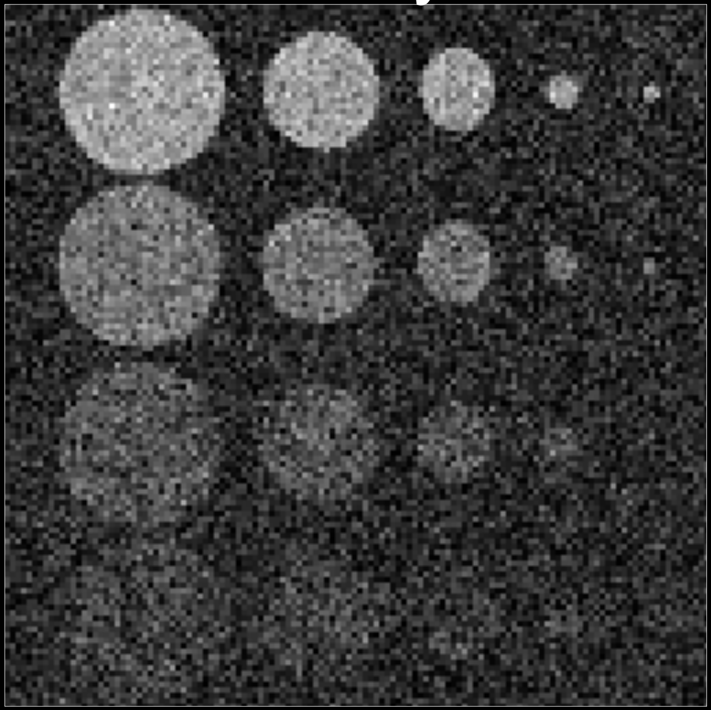
The human visual system is more sensitive to contrast than absolute luminance.

Signal to Noise Ratio (SNR)

Noise Free

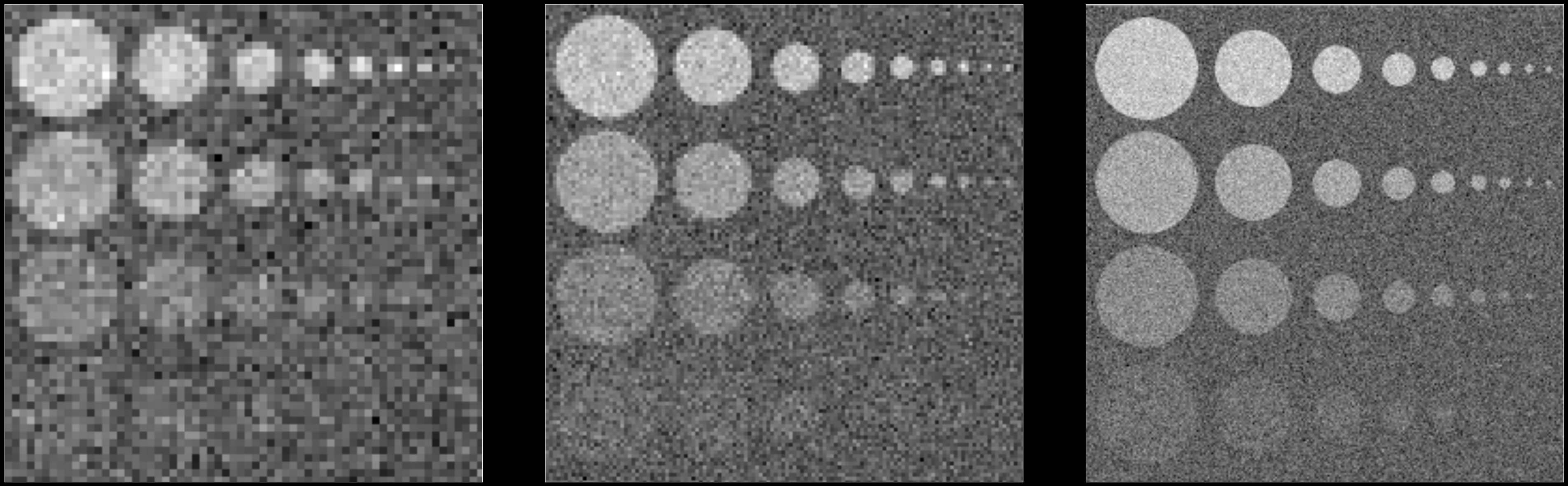


Noisy



SNR vs. Resolution

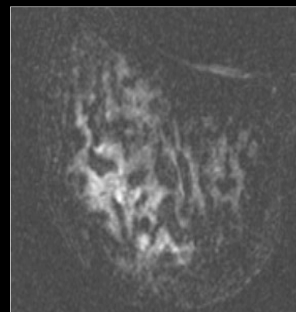
Low Resolution   Intermediate Resolution   High Resolution

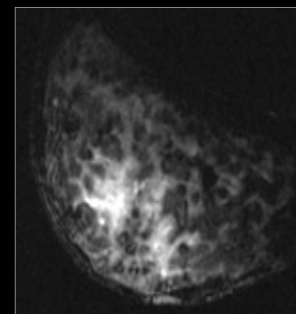


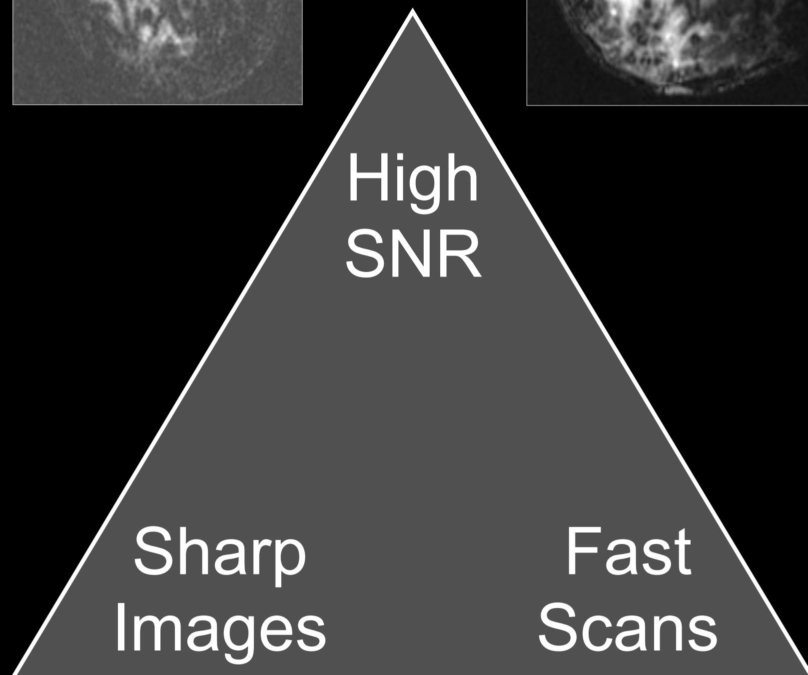
Small low-contrast objects are easier to see with higher resolution.

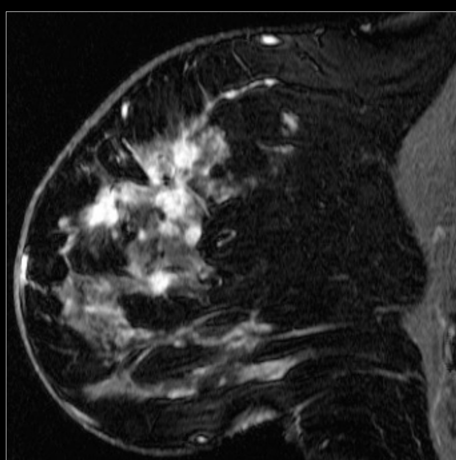
Image signal-to-noise is constant.

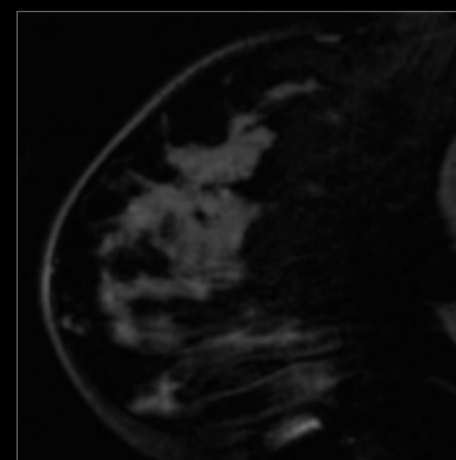
SNR vs Resolution vs Scan Time









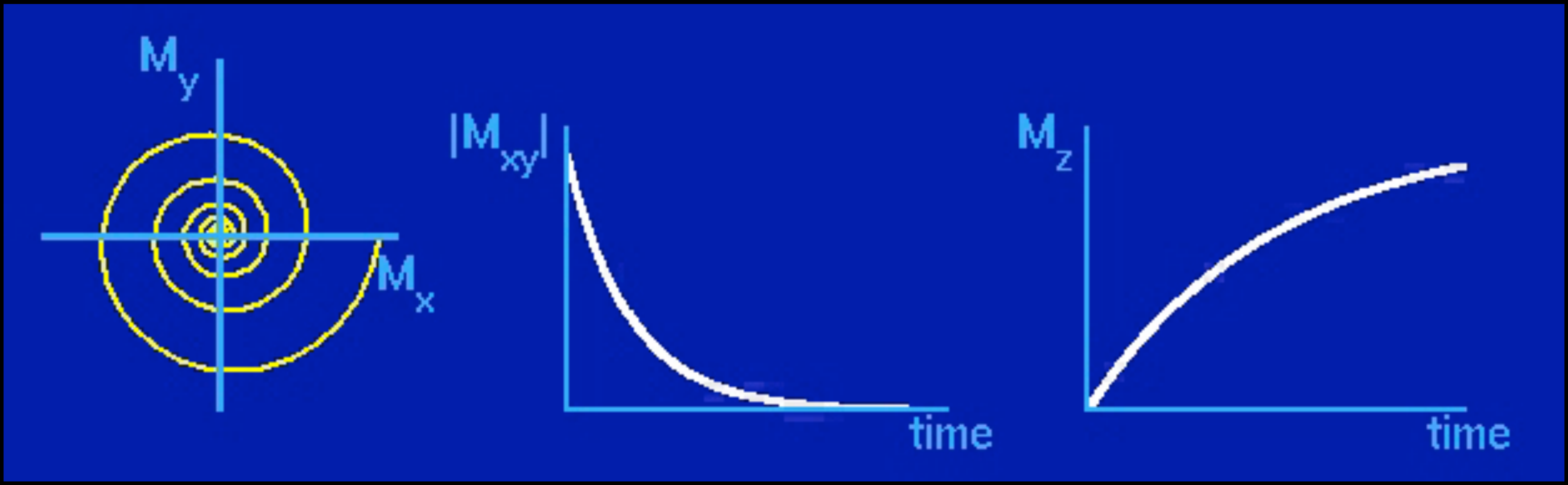


Coils, field strength, pulse sequence affect starting point!

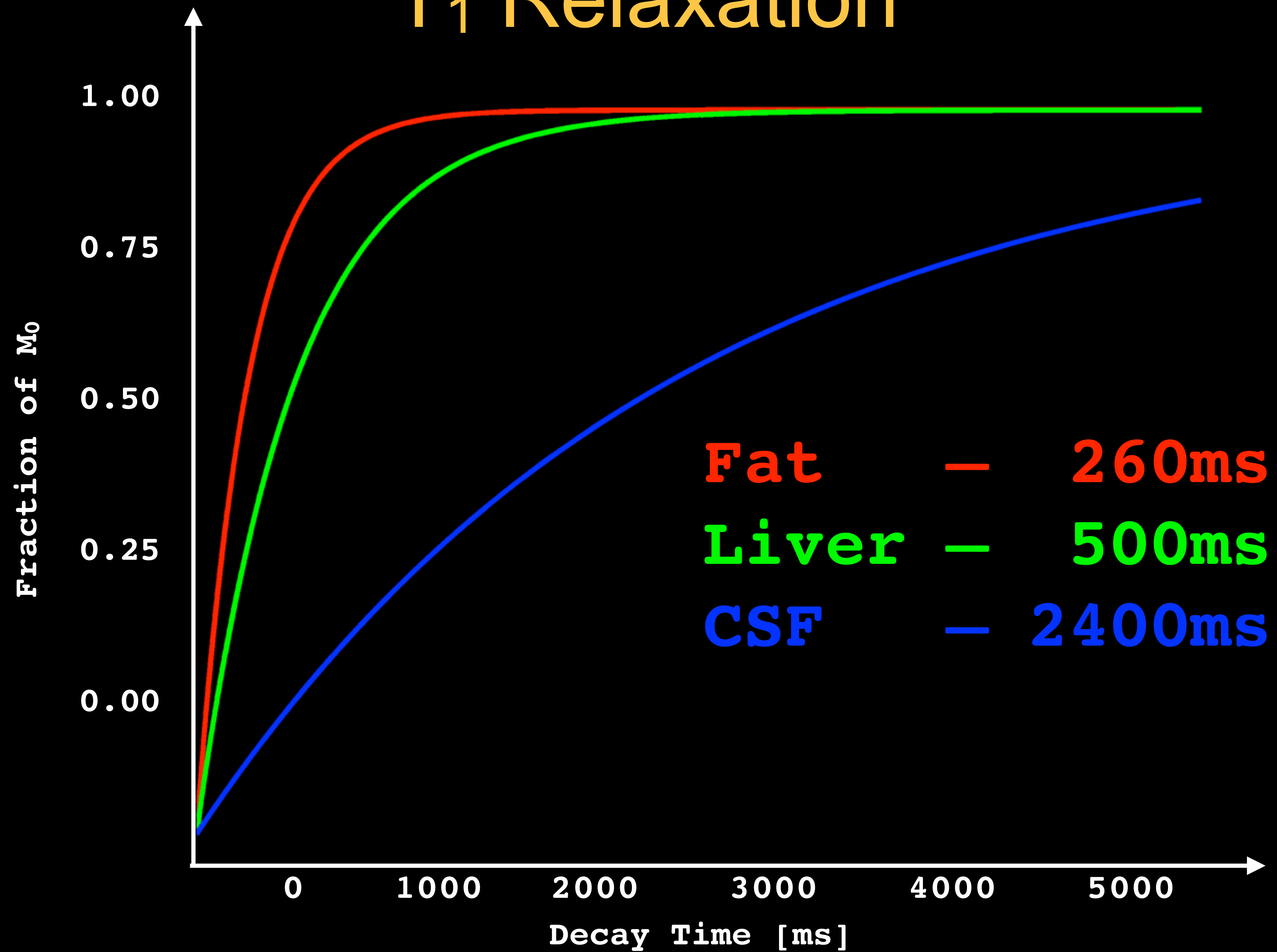
T_1 & T_2 Relaxation

Relaxation

- Magnetization returns exponentially to equilibrium:
  - Longitudinal recovery time constant is T1
  - Transverse decay time constant is T2
- Relaxation and precession are independent



T_1 Relaxation

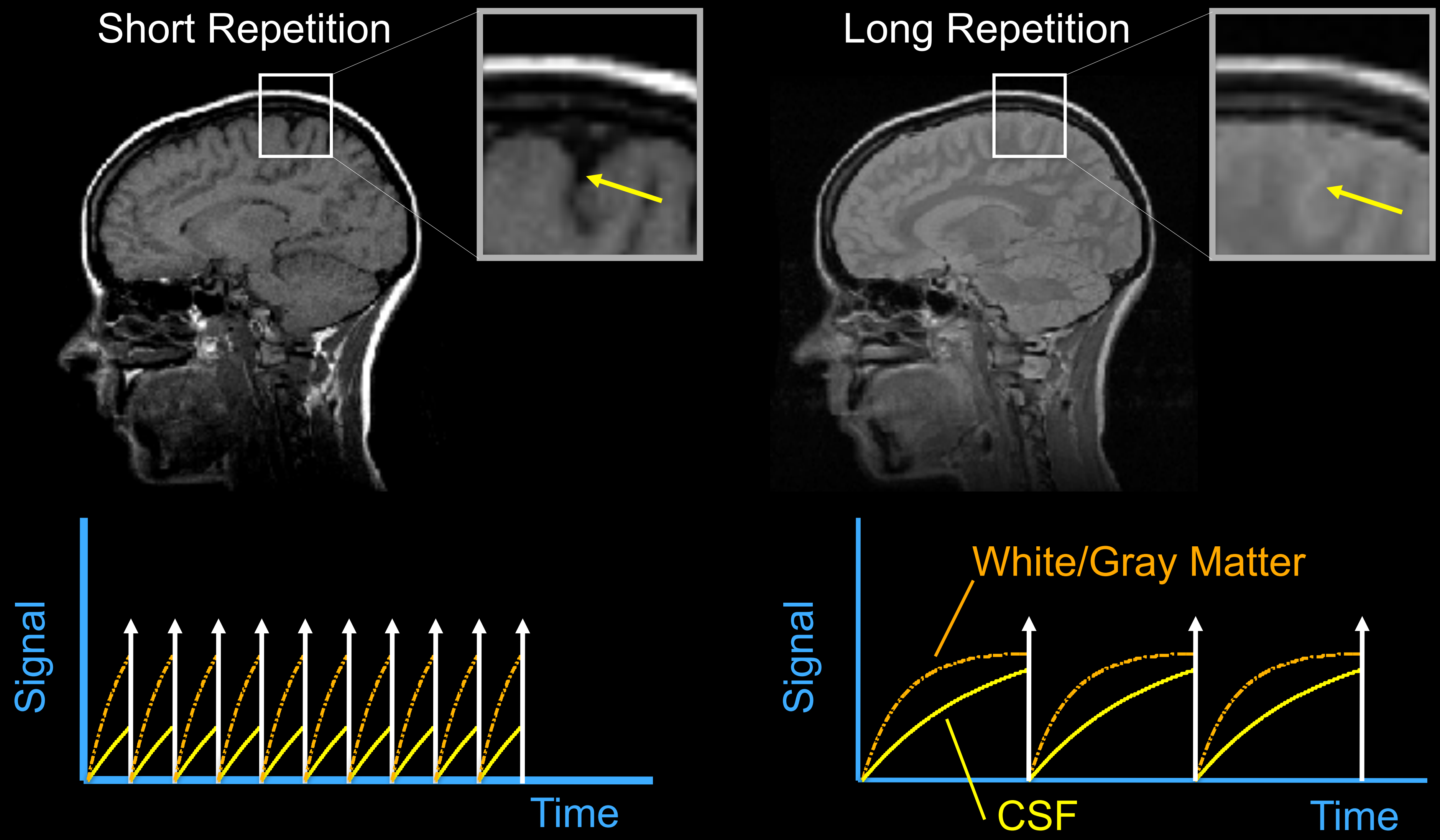


T_1 Relaxation

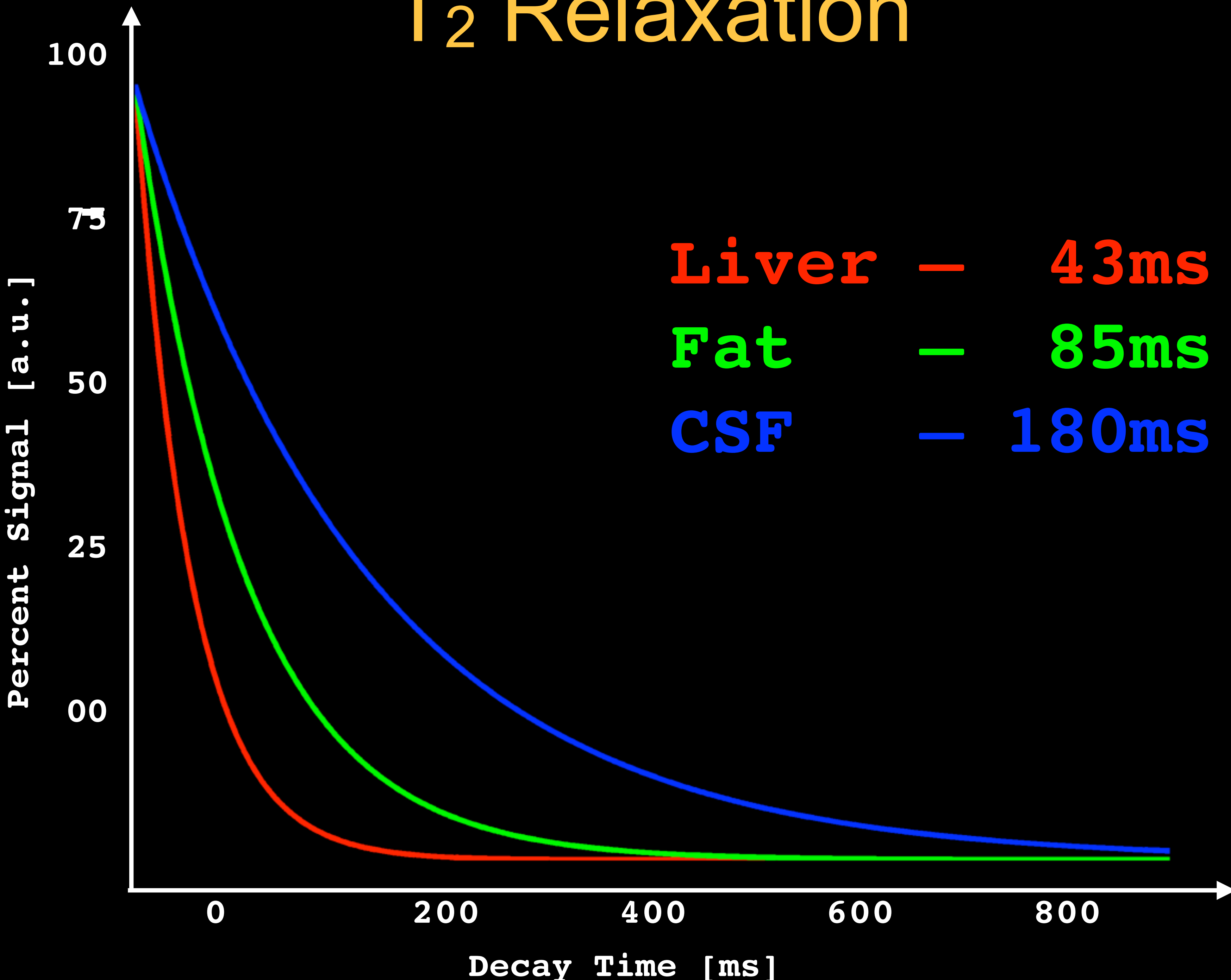
- Longitudinal or spin-lattice relaxation
  - Typically, (10s ms) < T1 < (100s ms)
- T1 is long for
  - Small molecules (water)
  - Large molecules (proteins)
- T1 is short for
  - Fats and intermediate-sized molecules
- T1 increases with increasing B0
- T1 decreases with contrast agents

Short T_1s are bright on T_1-weighted image

T1 Contrast


Short Repetition
Long Repetition
Signal
Time
Signal
White/Gray Matter
CSF
Time

T_2 Relaxation

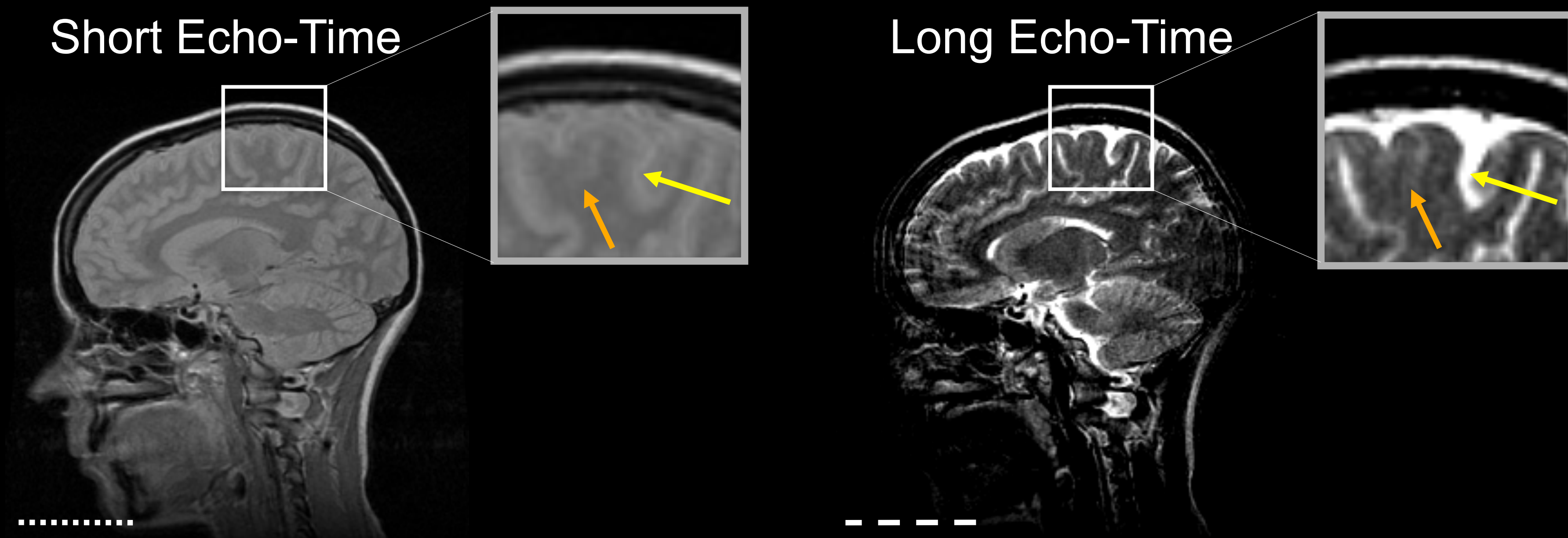


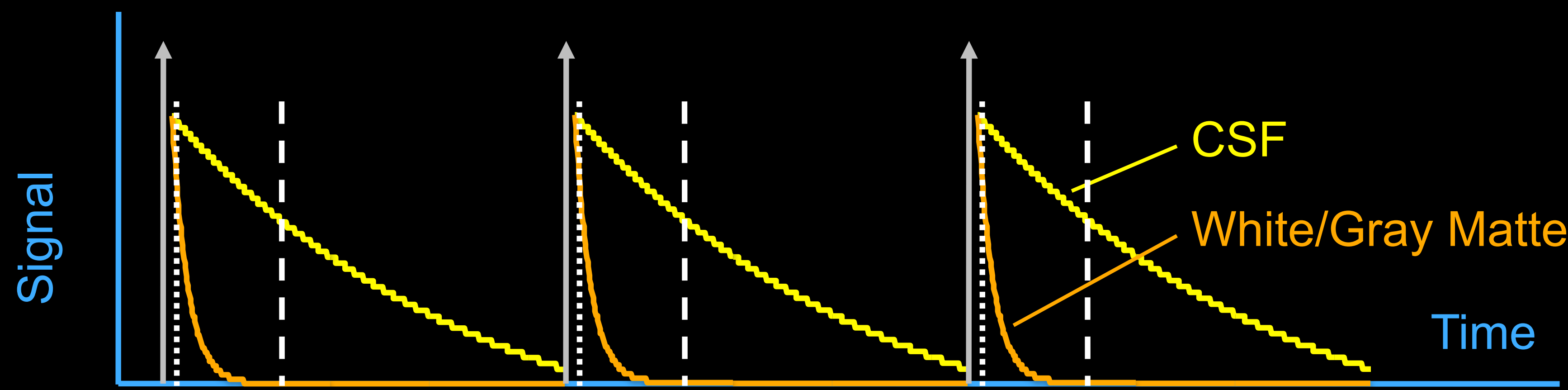
T_2 Relaxation

- Transverse or spin-spin relaxation
  - Molecular interaction causes spin dephasing
  - Typically, T2 < (10s ms)
- Increasing molecular size, decrease T2
  - Fat has a short T2
- Increasing molecular mobility, increases T2
  - Liquids (CSF, edema) have long T2s
- Increasing molecular interactions, decreases T2
  - Solids have short T2s
- T2 relatively independent of B0

Long T_2 is bright on T_2 weighted image

T2 Contrast


Short Echo-Time
Long Echo-Time


Signal
CSF
White/Gray Matter
Time

T_1 and T_2 Values @ 1.5T

Tissue	T_1 [ms]	T_2 [ms]
gray matter	925	100
white matter	790	92
muscle	875	47
fat	260	85
kidney	650	58
liver	500	43
CSF	2400	180

Each tissue has “unique” relaxation properties, which enables “soft tissue contrast”.

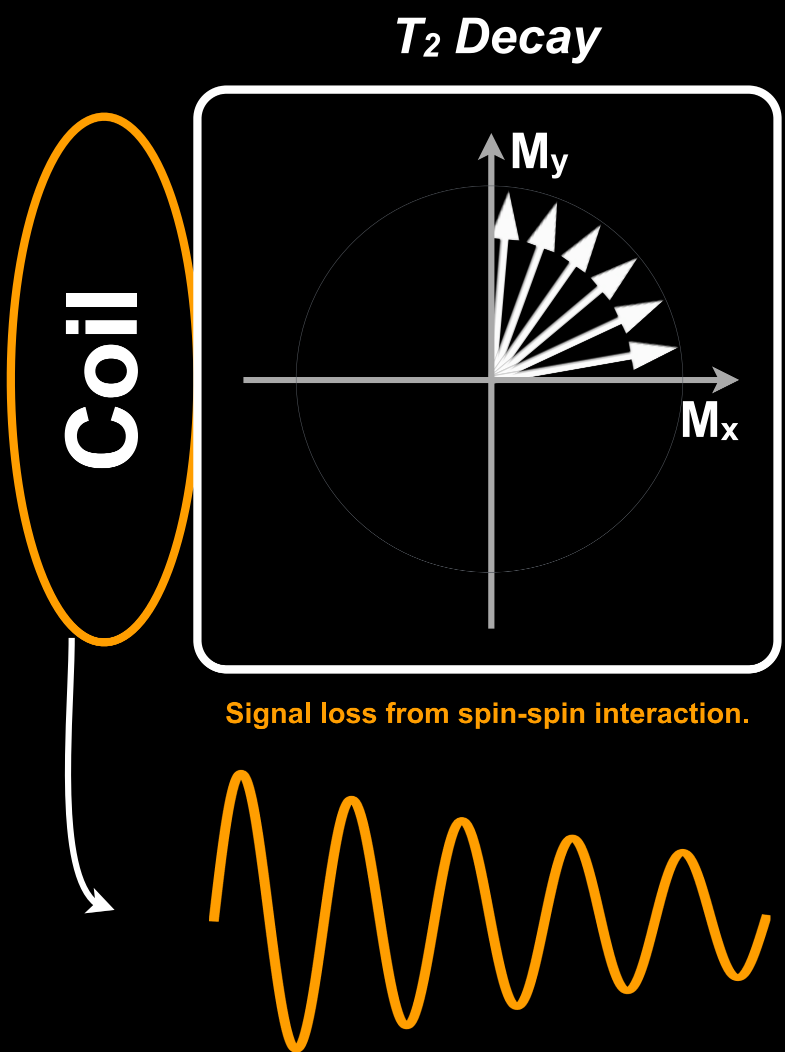
T_2^* Relaxation

T_2^* Relaxation

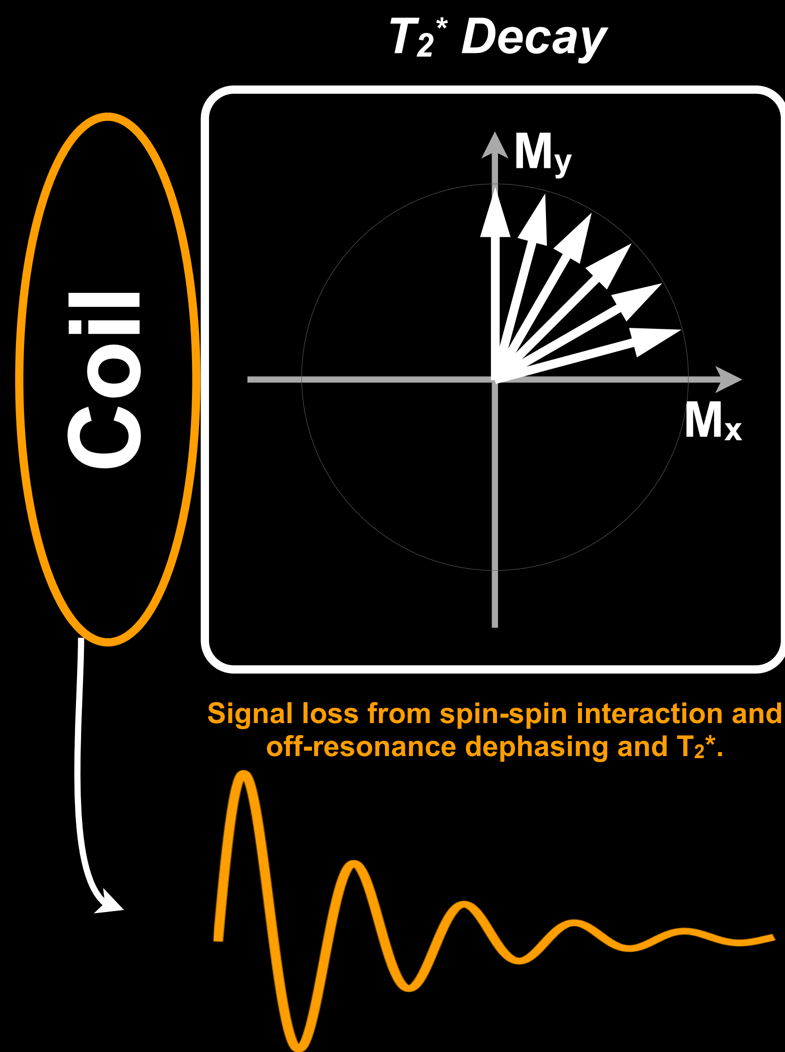
$$\frac{1}{T_2^*} = \frac{1}{T_2} + \gamma \Delta B_0$$

- T_2^* is “observed” transverse relaxation time constant
- T_2^* consists of *irreversible spin-spin (T_2) dephasing* and *reversible intravoxel spin de-phasing* due to off-resonance
- Sources of off-resonance:
  - B_0 inhomogeneity
  - susceptibility differences (e.g. air spaces)

T_2 versus T_2^*



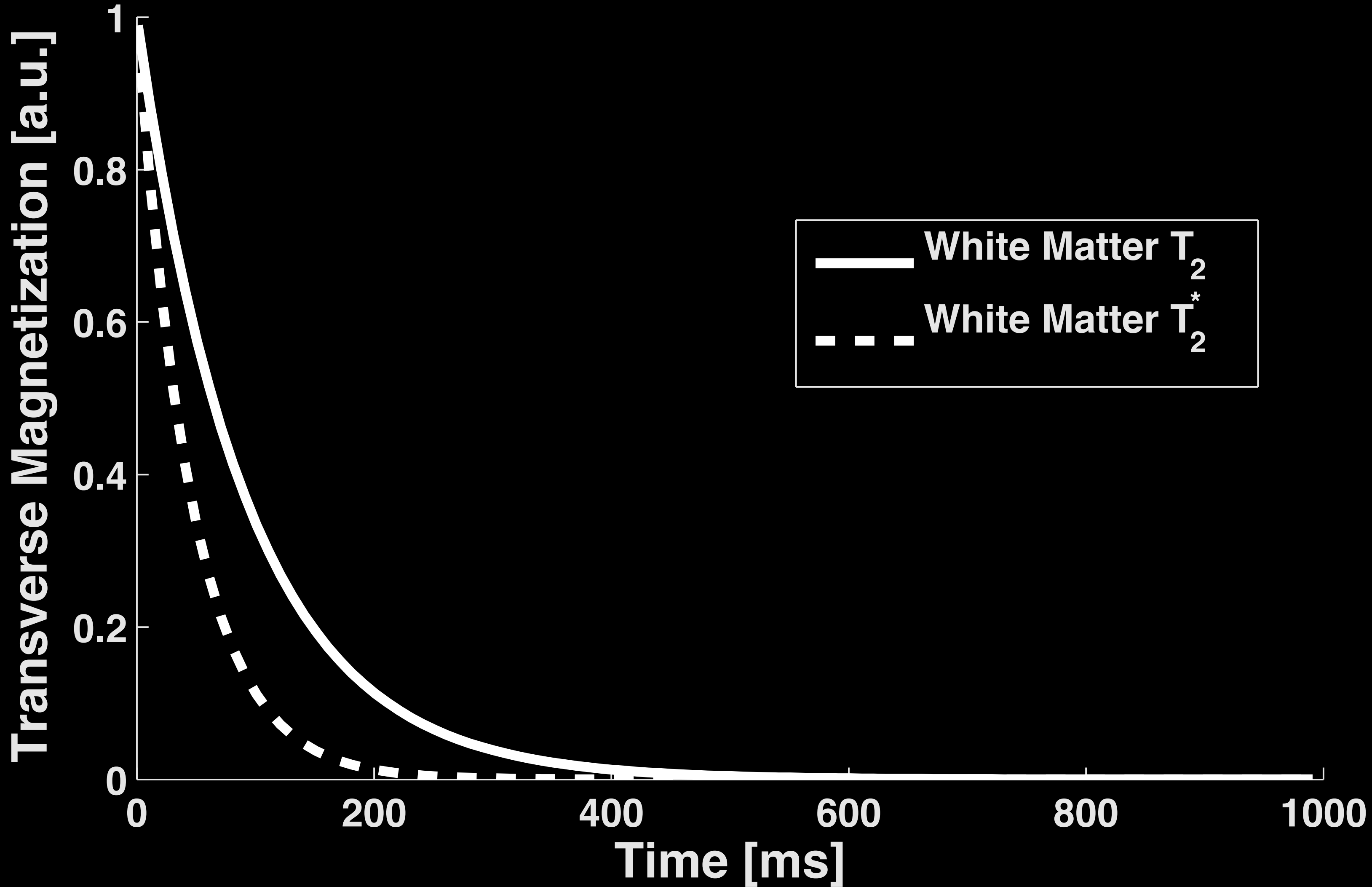
Signal loss from spin-spin interaction.



Signal loss from spin-spin interaction and off-resonance dephasing and T_2^*.

T_2^* is signal loss from spin dephasing and T_2

T2*<T2 (always!)



Transverse Magnetization [a.u.]
1
0.8
0.6
0.4
0.2
0
0
200
400
600
800
1000
Time [ms]
White Matter T_2
White Matter T_2^*

To the Board

Questions?

- Related reading materials
  - Nishimura - Chap 6 and 7

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