M219 Principles and Applications of MRI (Winter 2022)
Homework Assignment #1 (20 points)
Assigned: 1/10/2022, Due: 1/26/2022 at 5 pm by email

E-mail a PDF (entitled M219_HW01_[Last Name].pdf). Please only submit neat and clear solutions. If your assignments are hard to read, poorly commented, or sloppy points may be deducted. As appropriate, each solution should be obtained using Matlab; provide the code.

For all problems – clearly state the value of all constants and free variables that you use, show your work, provide units, and label your axes. This is not a group assignment. Please work individually.

Problem #1 (4 points) – B0 vs. B1 fields

For a 1ms ^{1}H RF hard pulse with an $\alpha=\pi/2$ flip angle at B_0 = 3T:

A. Find ω_1, the precession frequency for the B_1 field. (1 point)

B. Plot the M_x and M_y components for Part-A (above) from 0 to 1 ms (the RF pulse duration) in the rotating frame using MATLAB. Use 1,000 points for your simulation. (1 point)

C. Find ω_0, the precession frequency with respect to the B_0 field. How many cycles of precession take place during the duration of the RF pulse? How does it compare to the B_1 precession frequency? (1 point)

D. Plot M_x and M_y components from Part-C (above) in the laboratory frame using Matlab. Note that you'll need to change the limits of your graph and number of simulation points compared to part B. to see the precession behavior clearly. Why is this necessary? (1 point)

E. **Extra Credit:** How does the amplitude of this B_1 pulse compare to the amplitude of the B_0 field? Explain how B_1 can be effective at perturbing the spin system when B_0 is so different in magnitude. (1 point)

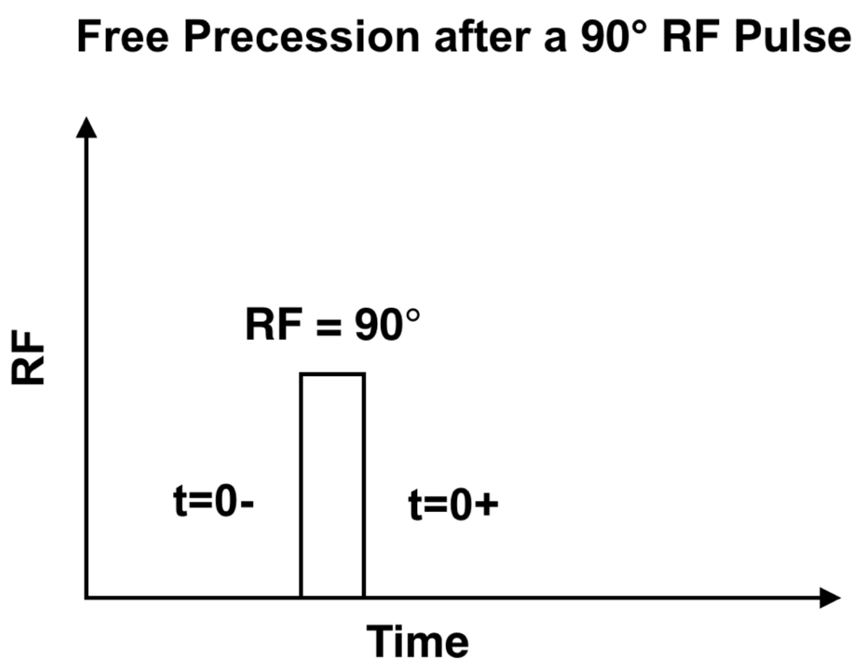
Problem #2 (4 points) – Free Precession in the Laboratory Frame Without Relaxation

A. Write an expression for each component $M_x(0_-)$, $M_y(0_-)$, and $M_z(0_-)$ immediately prior to a 90 degree RF pulse as shown in the included figure. (1 point)

B. Write an expression for each component $M_x(0_+)$, $M_y(0_+)$, and $M_z(0_+)$ immediately after a 90 degree RF pulse. Explain what RF phase you used. (1 point)

C. Write a general expression (include the RF phase!) as a function of time for each component $M_x(t)$, $M_y(t)$, and $M_z(t)$ beginning immediately after the 90° RF pulse. (1 point)

D. Plot all three components of your answer from (C) in MATLAB. Assume B_0 = 1.5T. Remember to make your simulation time-steps sufficiently small to observe the time varying magnetization. (1 point)



Problem #3 (4 points) – RF Pulses

Design the shortest possible inversion RF half sin-pulse (i.e. sin on [0, π]) for ^{31}P at 1.5T that does not exceed the RF amplifier's ability to output $B_{1,max}$ = 25μT . In doing so, fully specify the variables in the following equation:

$$B_1(t) = B_1^e(t)e^{-i\omega_{RF}t}$$

Plot the components of the resultant B_1 pulse amplitude in the laboratory and rotating frames as a function of time. (2 point)

Now design the shortest possible saturation RF half sin-pulse (i.e. sin on [0, π]) for ^{19}F at 3.0T that does not exceed the same RF amplifier's ability to output $B_{1,max}$ = 25μT. Plot the components of the resultant B_1 pulse amplitude in the laboratory frame as a function of time. (2 point)

Problem #4 (8 points) – Forced Precession in the Rotating Frame

The goal is to derive the solution for the individual magnetization components for Forced Precession in the Rotating Frame without Relaxation when the RF phase is non-zero. To do so, consider the following steps.

A. First, define $\vec{B}_{1,rot}(t)$ from the general form of the RF pulse in the laboratory frame: (1 point)

$$\vec{B}_1(t) = B_1^E(t)[\cos(\omega_{RF}t + \theta)\,\hat{\imath} - \sin(\omega_{RF}t + \theta)\,\hat{\jmath}]$$

B. Then, show the steps required to define the system of differential equations in the rotating frame in the presence of $B_0\hat{k}$ and $\vec{B}_{1,rot}$. You'll want to define $\vec{B}_{eff}$ first. (1 point)

C. Then, write the general solution for this system of differential equations. (1 point)

D. Next, write a specific solution when $B_1^e(t) = B_1 \sqcap \left(\frac{t-\tau_p/2}{\tau_p}\right)$. (1 point)

E. Finally, use homogeneous coordinate operators to simulate the action of this hard RF pulse for α=90° RF pulse (θ=π/4) with $B_{1,max}$ =1μT and B_0 =0.5 Gauss in the rotating frame. Plot the components of the bulk magnetization as a function of time. (2 points)

G. Add relaxation to your simulation in the rotating frame. Use T_1=1000 ms and T_2=100 ms. Plot the components of the bulk magnetization during the application of the same RF pulse. Overlay a plot when T_1=250 ms and T_2=25 ms. (2 points)