Fast Imaging, Advanced Image Reconstruction

M219 Principles and Applications of MRI

Holden H. Wu, Ph.D.

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UCLA

Department of Radiological Sciences

David Geffen School of Medicine at UCLA

Review: Basic Recon

The Fourier Transform

$$S(\vec{k}) = \int_{-\infty}^{+\infty} I\left(\vec{r}\right) e^{-i2\pi\vec{k}\cdot\vec{r}} d\vec{r}$$

MRI Signal Equation

$$S(\vec{k}) \xleftrightarrow{\mathcal{F}} I(\vec{r})$$

$$S(k_x) = \int_{-\infty}^{+\infty} I\left(x\right) e^{-i2\pi(k_x x)} dx$$ 1D

$$S(k_x, k_y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I\left(x, y\right) e^{-i2\pi(k_x x + k_y y)} dx dy$$ 2D

$$S(k_x, k_y, k_z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I\left(x, y, z\right) e^{-i2\pi(k_x x + k_y y + k_z z)} dx dy dz$$ 3D

Finite Sampling

$S(k)$ is measured at $k \in \mathcal{D}$

$$\mathcal{D} = \{n\Delta k, -N/2 \leq n \leq +N/2\}$$

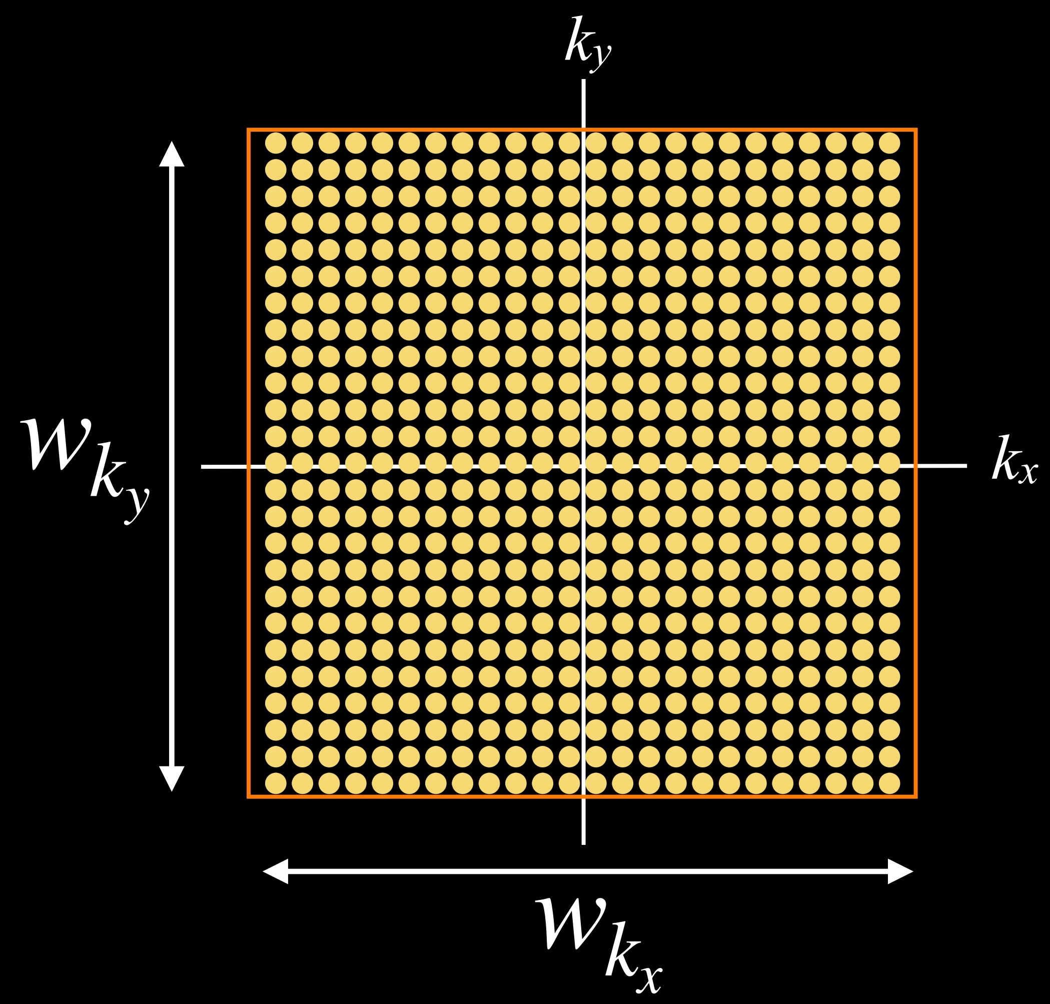
Fourier Step-size

Number of Sample Points

$$I(x) = \Delta k \sum_{n=-N/2}^{N/2-1} S[n] e^{i2\pi n \Delta k x}, \;\; |x| < \frac{1}{\Delta k}$$ Eqn. 6.20

This is the fundamental image reconstruction equation for MRI.

Sampling Considerations



$$\Delta k_x = \frac{1}{FOV_x} \qquad \Delta k_y = \frac{1}{FOV_y}$$

$$w_{k_x} = \frac{1}{\Delta x} \qquad w_{k_y} = \frac{1}{\Delta y}$$

Review Sampling Theorem

Review Lectures 9&10 on Spatial Localization

Noise Considerations

- Signal-to-Noise Ratio (SNR)
  - A fundamental measure of image quality
  - $$SNR \triangleq \frac{signal\ amplitude}{\sigma\ of\ noise}$$
  - $$SNR_{dB} = 20 \cdot log(SNR)$$

Nishimura Ch. 7.5

Noise Considerations

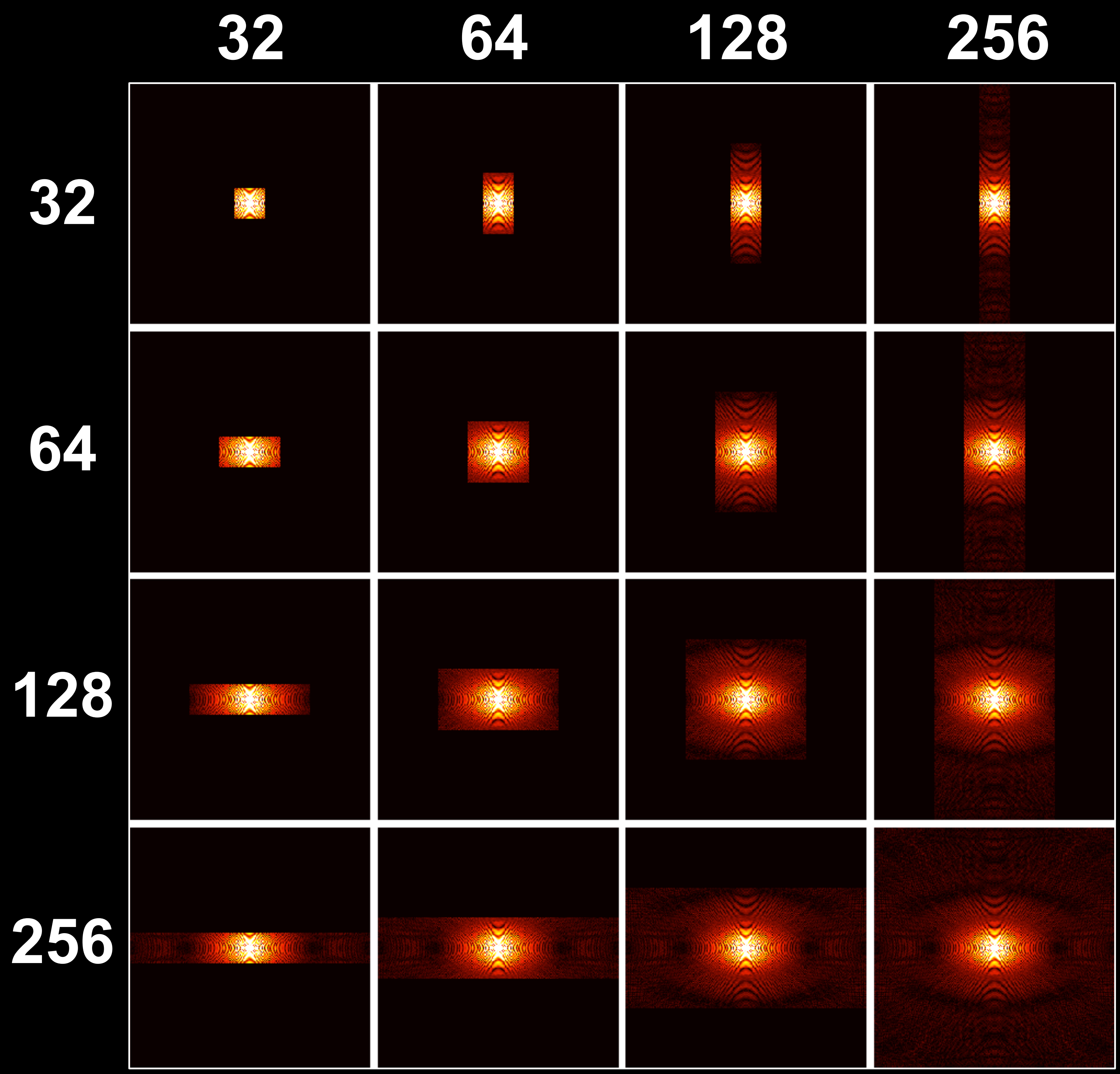
- Summary of Acquisition Time Effects
  - $SNR \propto \sqrt{N_{ave} \cdot T_{read}}$
  - $SNR \propto \sqrt{measurement\ time}$

- Effect of Spatial Resolution
  - $SNR \propto (\delta_x)(\delta_y)(\delta_z)$

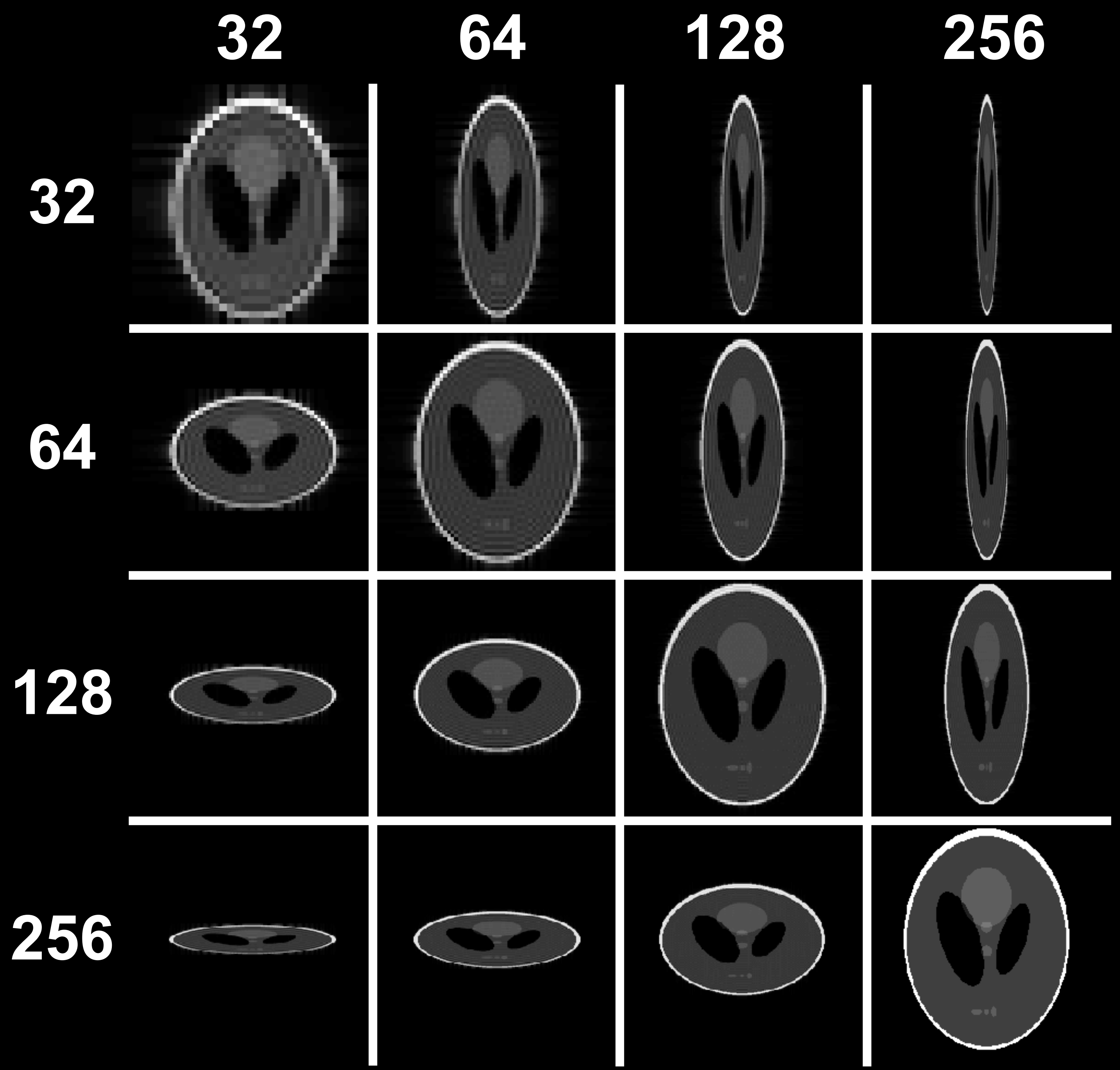
- Other factors
  - $SNR \propto f(\rho, T_1, T_2, \ldots)$

Nishimura Ch. 7.5

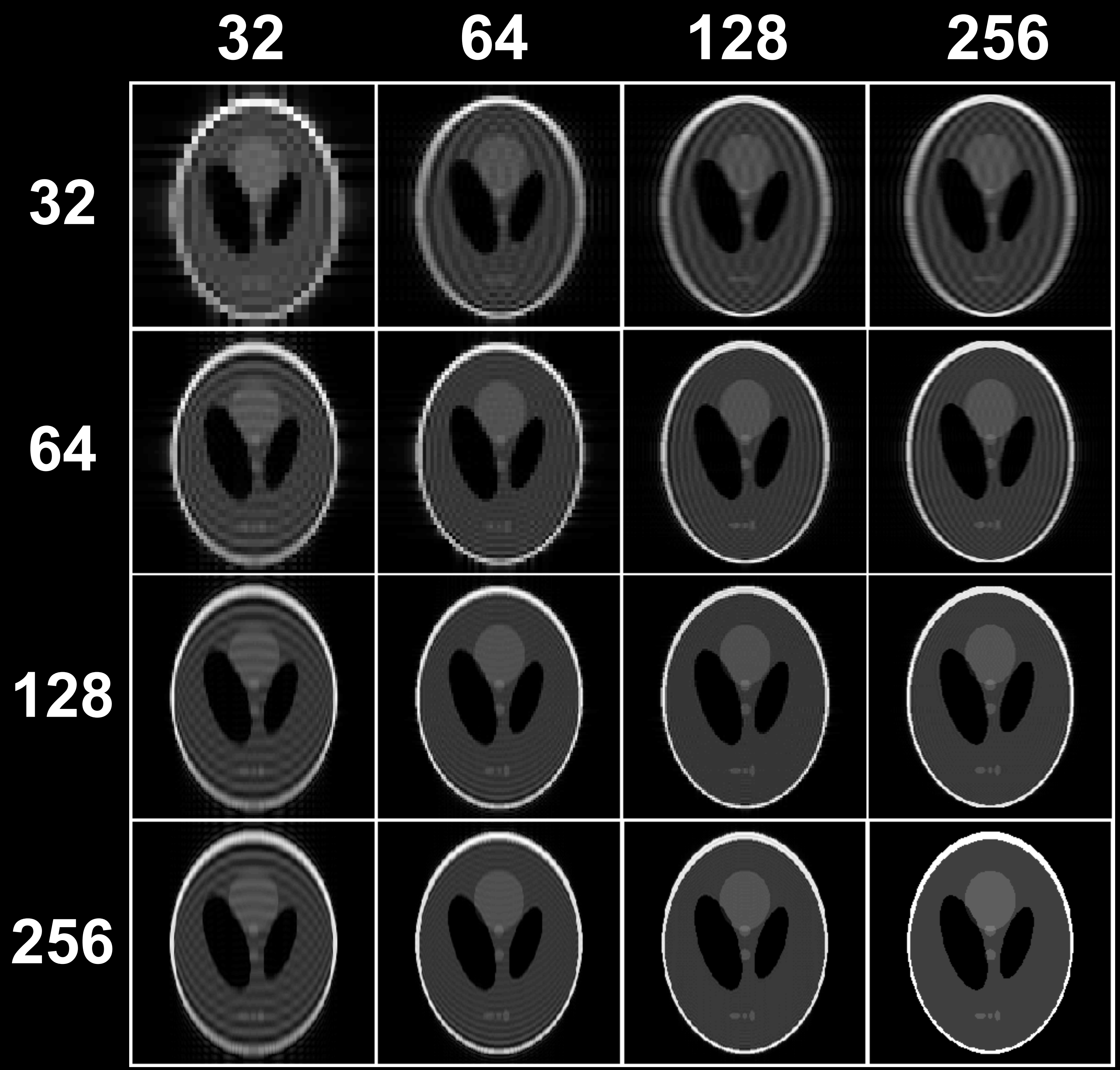
Gibb's Ringing



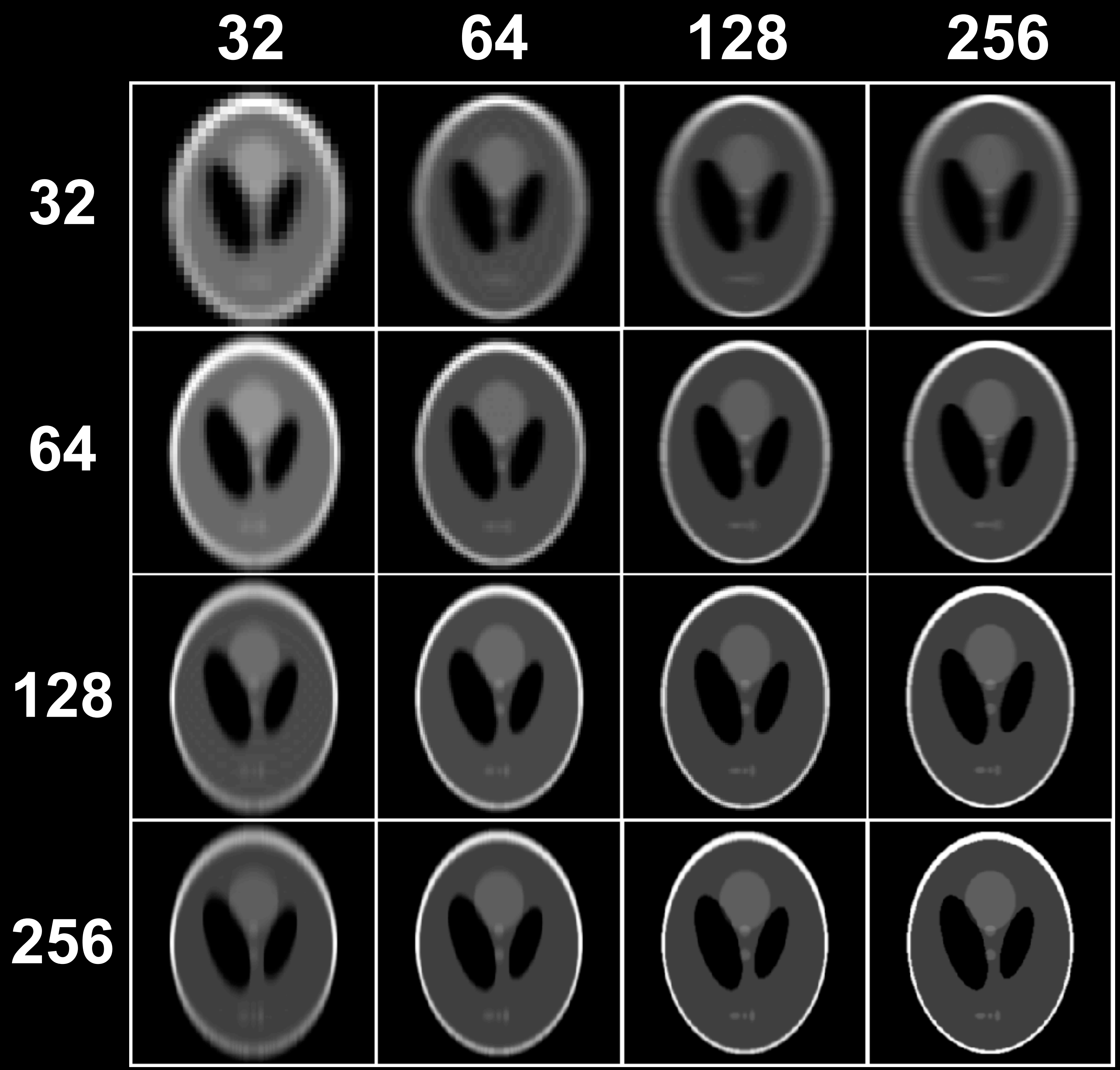
Gibb's Ringing



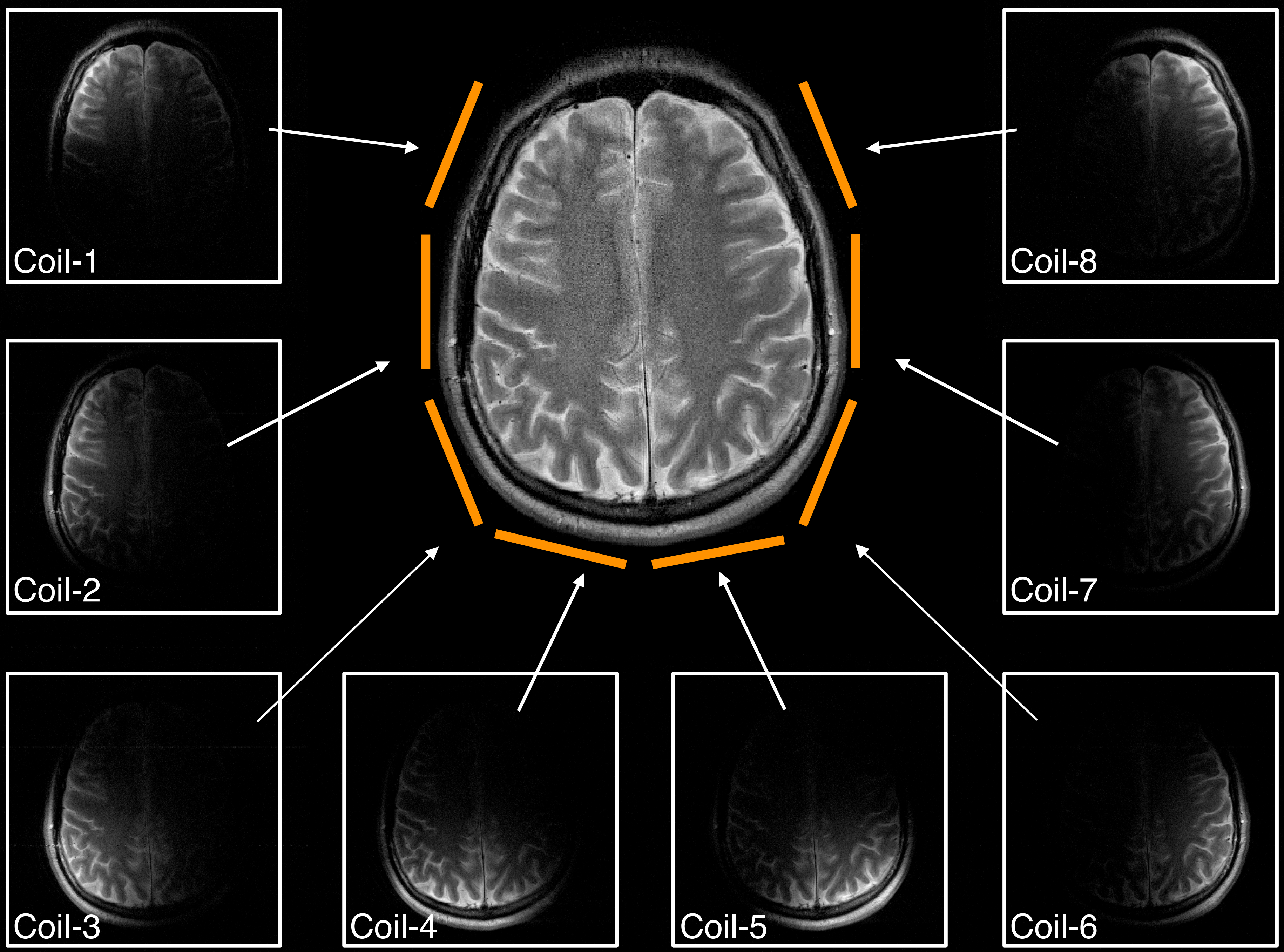
Zero-Pad



Hamming Window & Zero-Pad



Multi-Coil Reconstruction



Each coil element (channel) has a unique sensitivity profile – $\vec{B}_r\left(\vec{r}\right)$

Outline

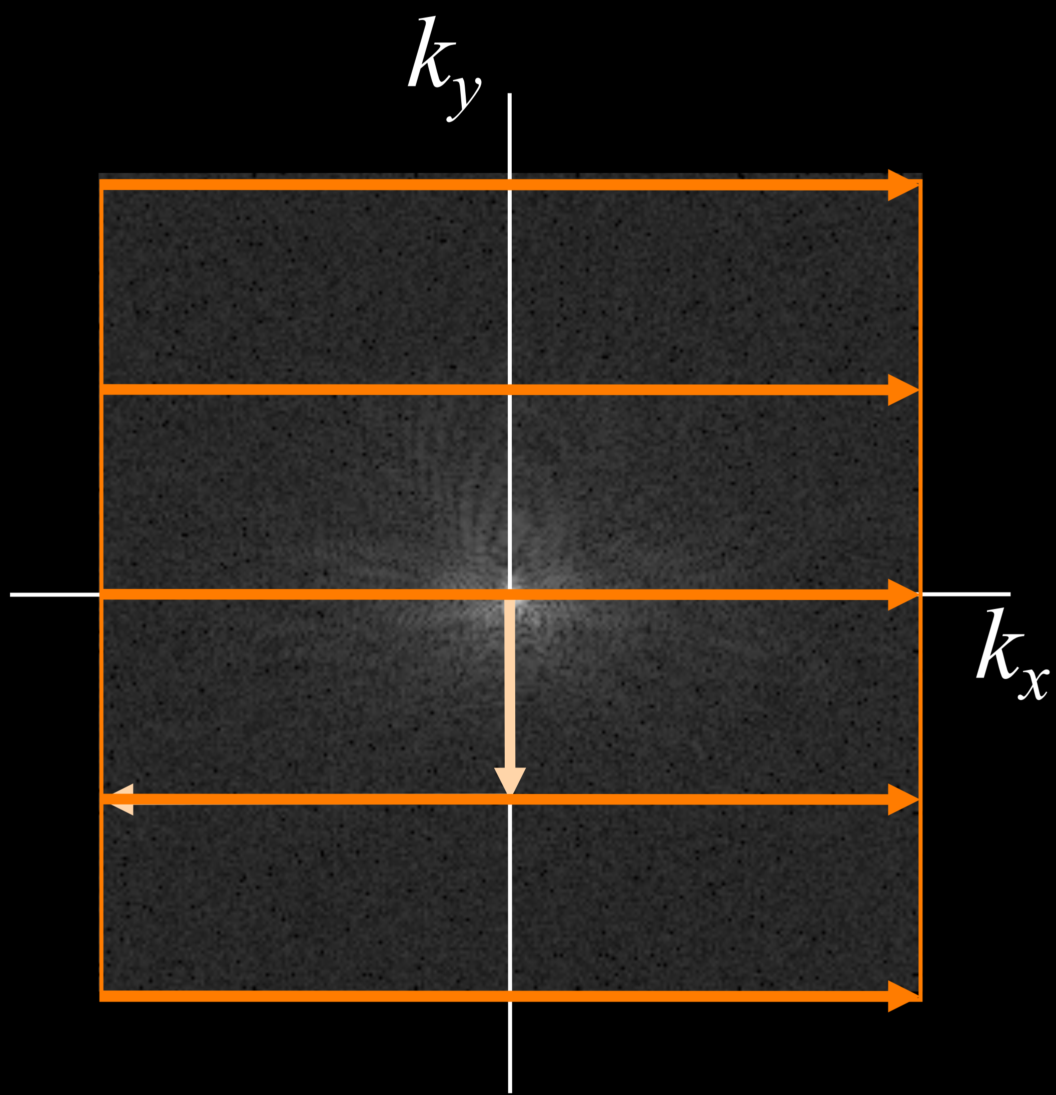
- Fast Imaging
  - Non-Cartesian MRI
  - Echo-planar imaging (EPI)
- Advanced MR Image Reconstruction
  - Parallel imaging
  - … *and more in upcoming lectures*

Overview

- Motivation
  - MRI is relatively slow; need to accelerate
- Strategies
  - Efficient pulse sequences
  - Fast k-space sampling trajectories
  - Data undersampling + advanced recon
- Many challenges and trade-offs
- Key drivers for MRI research

Fast Imaging

k-Space Sampling



set of $s(t)$ covers $m(k_x, k_y)$

Pulse Sequence Diagram

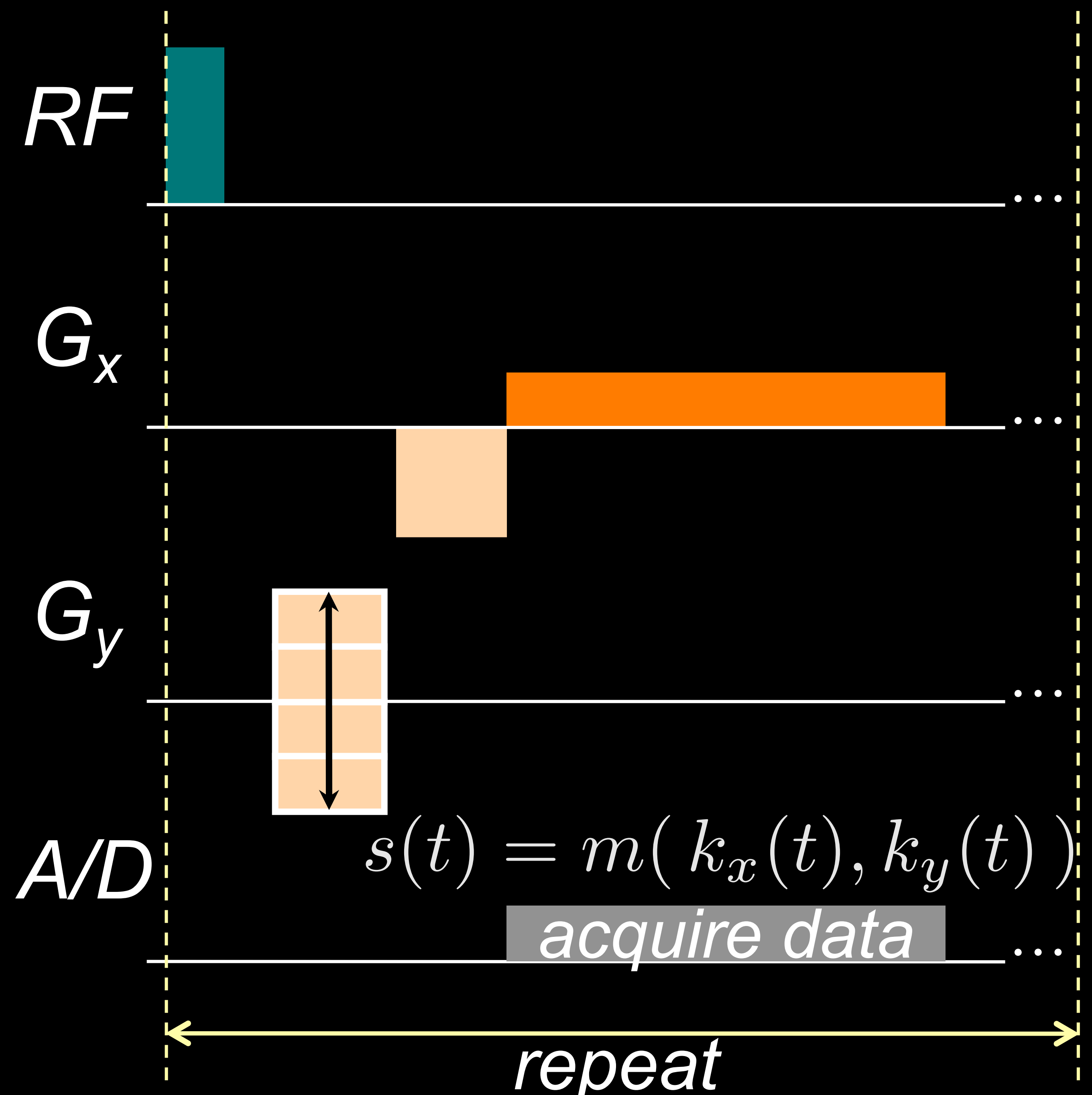
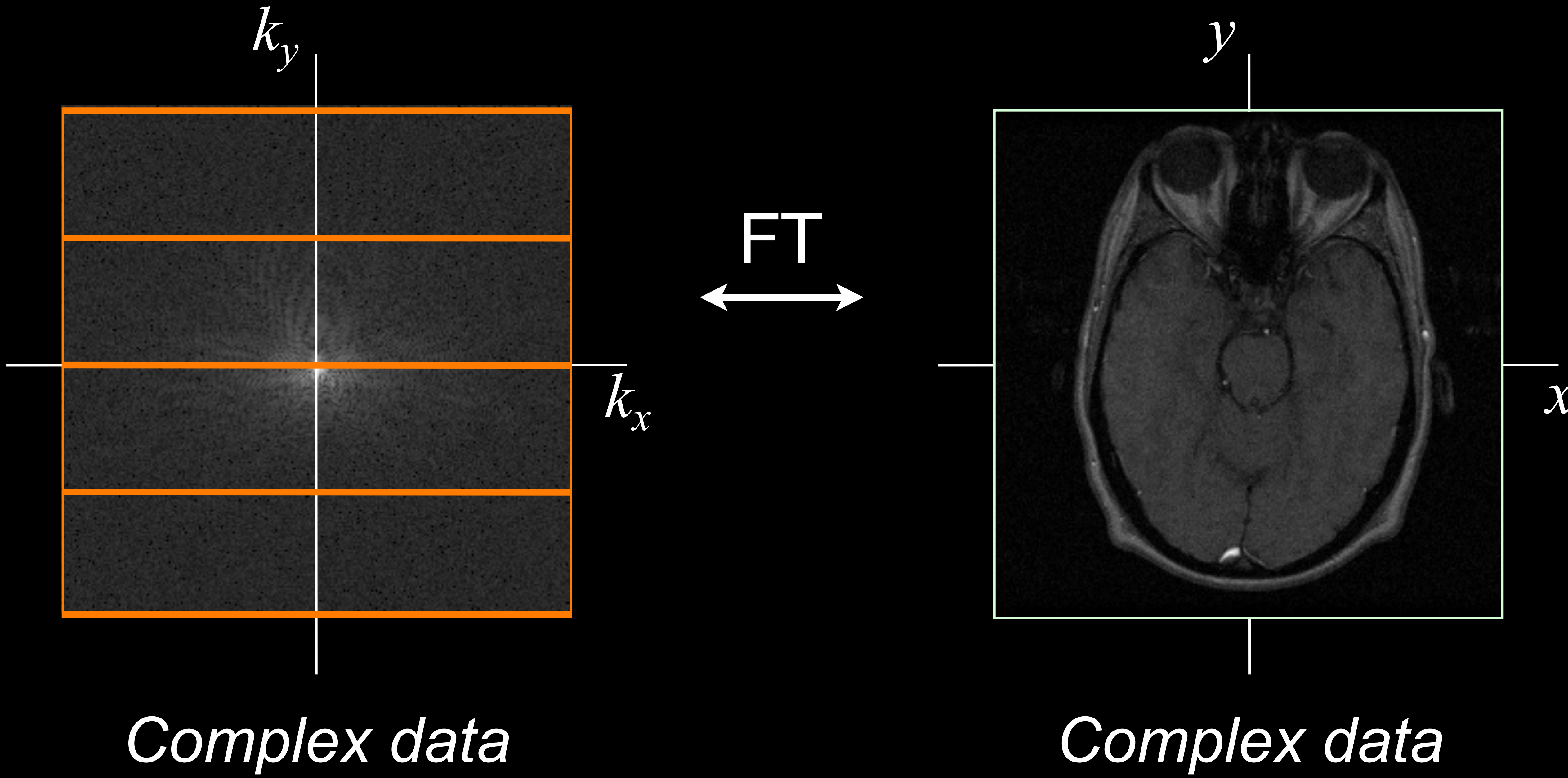
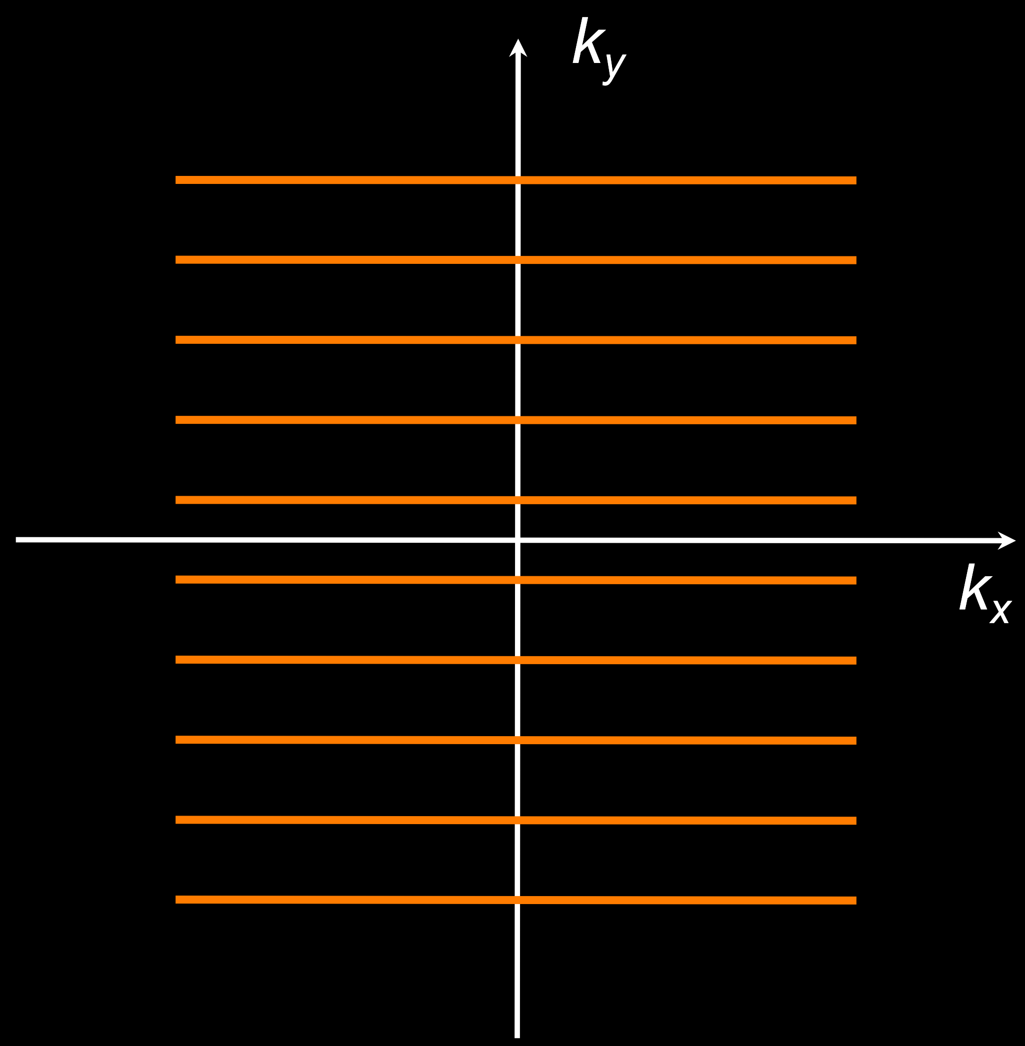


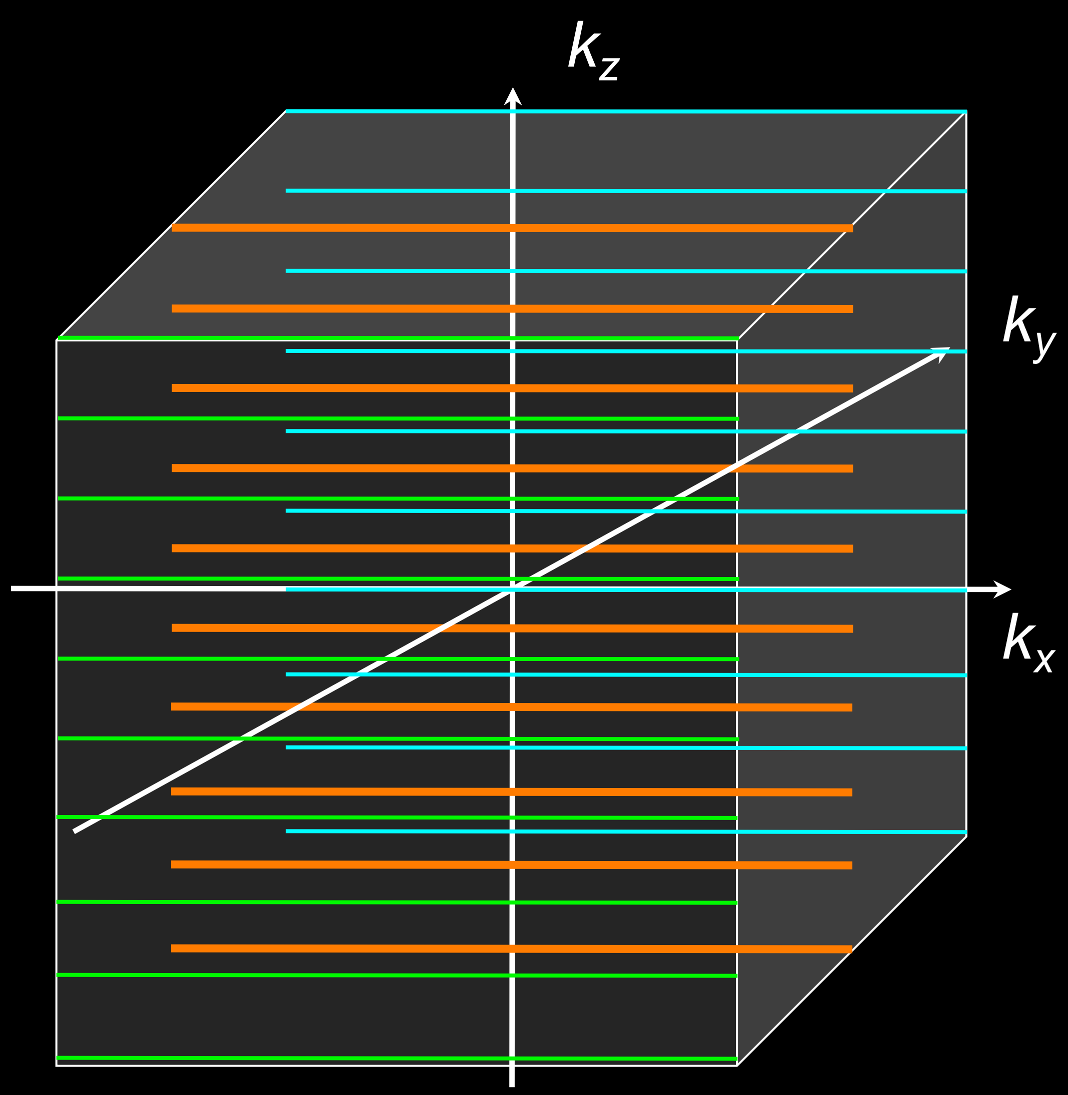
Image Reconstruction



Cartesian Sampling



Cartesian 2DFT



Cartesian 3DFT

MR Signal Equation

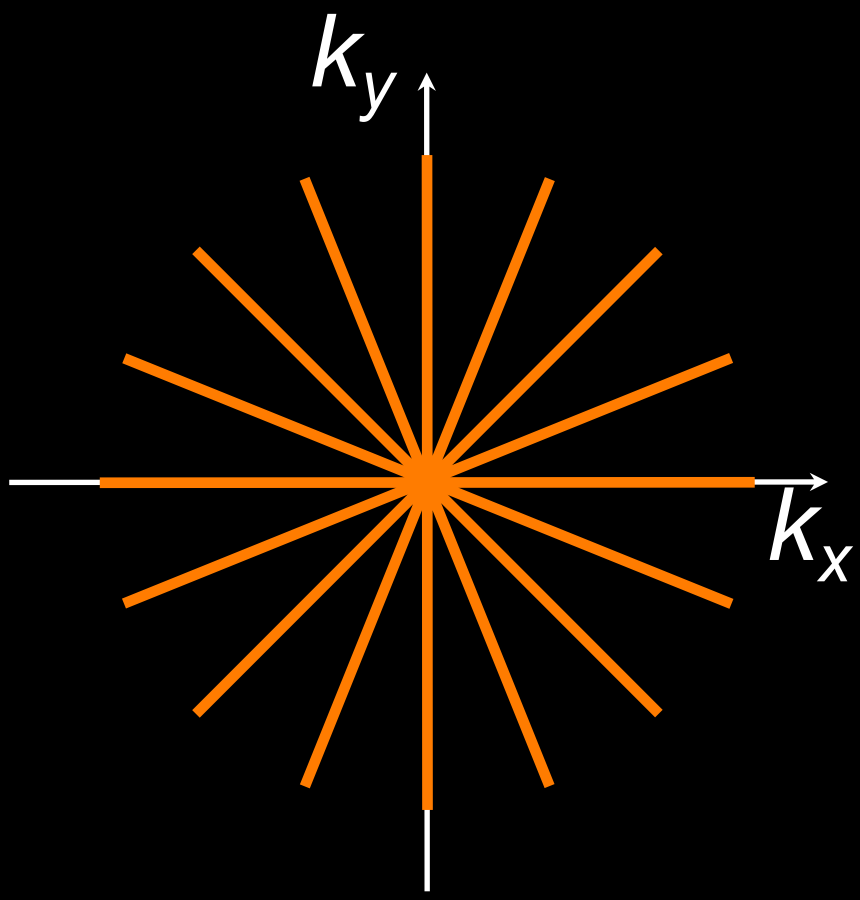
$$s(t) = \iint\limits_{X,Y} M(x,y) \cdot \exp(-i2\pi \cdot [k_x(t)\, x + k_y(t)\, y])\, \mathrm{d}x\, \mathrm{d}y$$

$$= m(\, k_x(t), k_y(t)\,) \qquad \boxed{k_x(t) = \frac{\gamma}{2\pi} G_x t,\; k_y(t) = \frac{\gamma}{2\pi} G_y t}$$

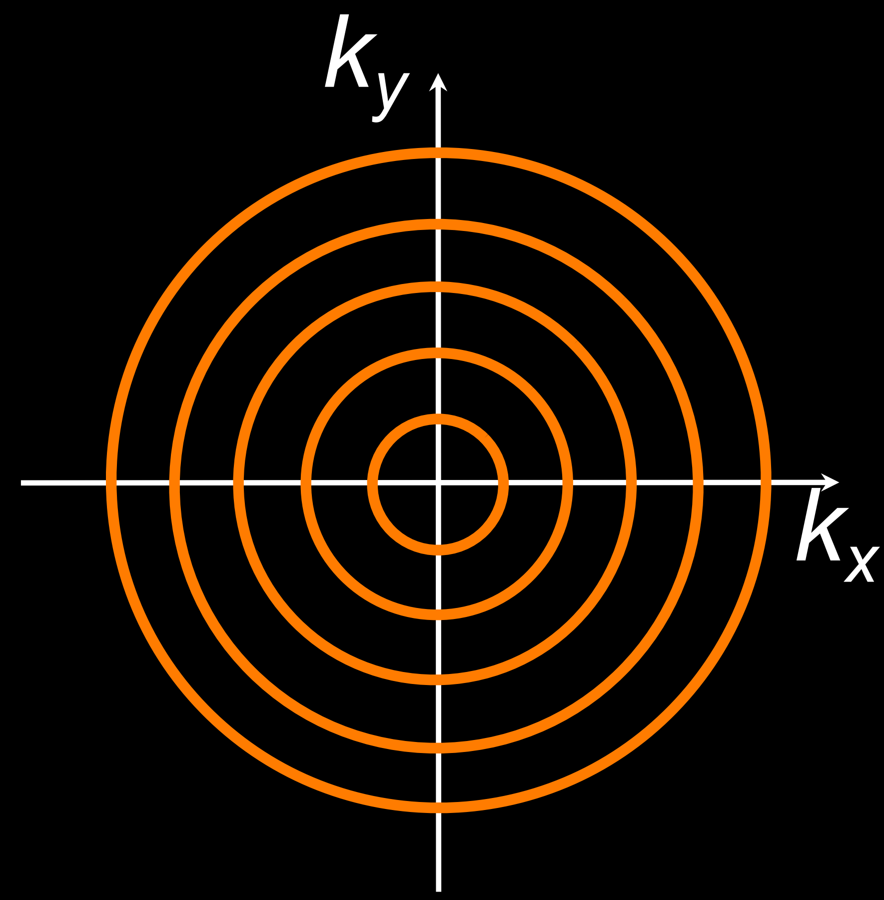
$$m = \mathcal{FT}(\, M(x,y)\,)$$

$$\boxed{k_x(t) = \frac{\gamma}{2\pi} \int_0^t G_x(\tau)\, \mathrm{d}\tau,\; k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau)\, \mathrm{d}\tau}$$

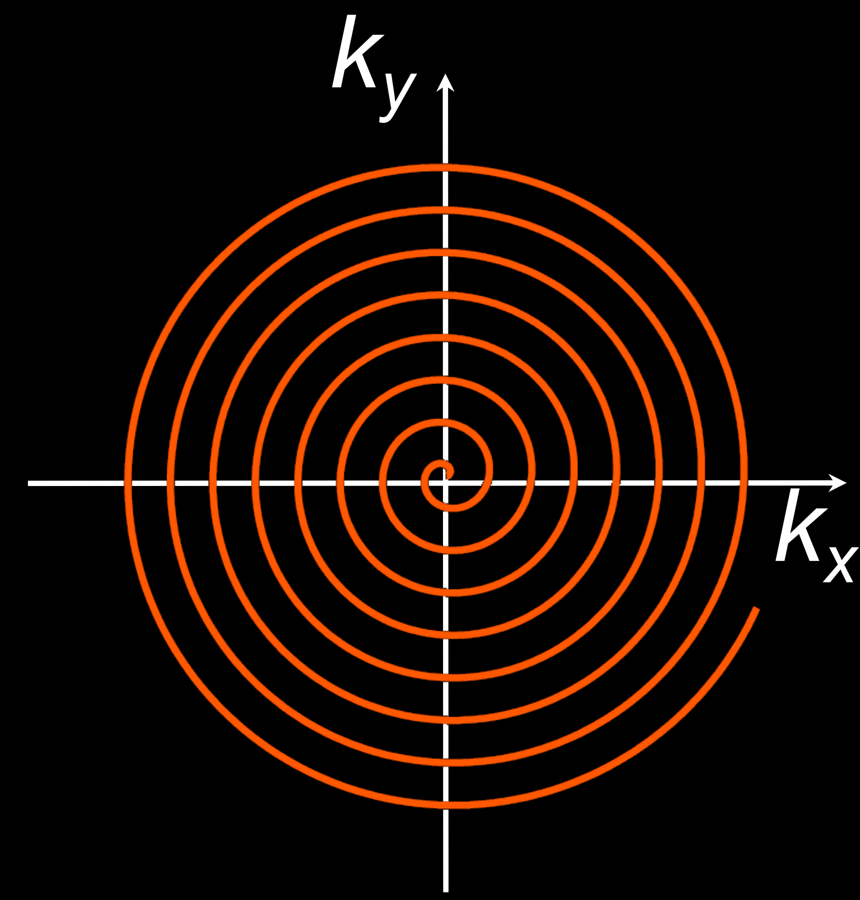
Non-Cartesian Sampling



2D Radial



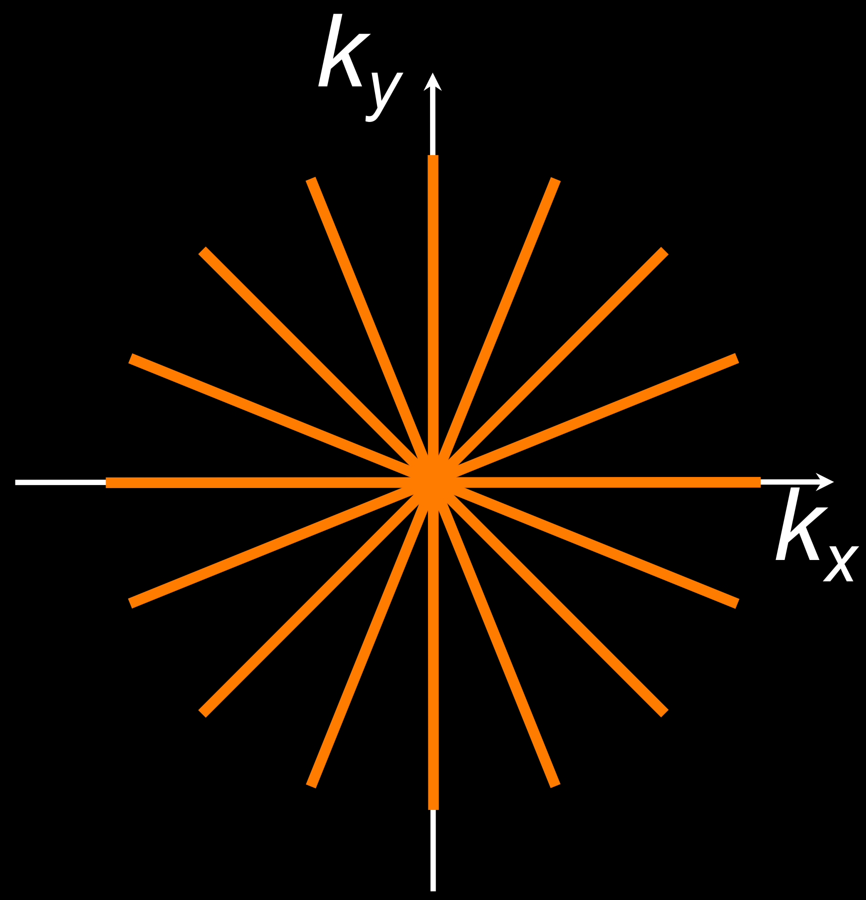
2D Concentric Rings



2D Spiral

and much more ...

Radial: Pros and Cons



Pros

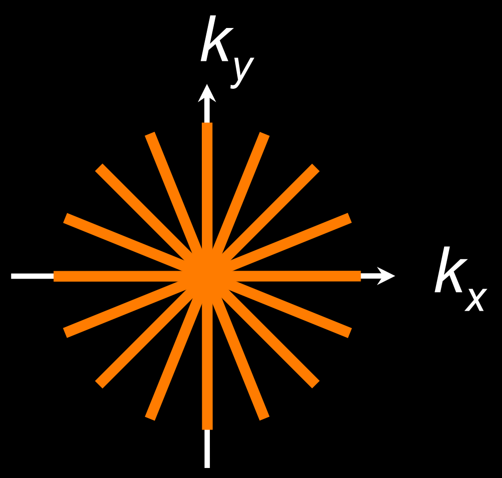
- Robust to motion (get DC every TR)
- Can tolerate a lot of undersampling
- Half-spoke PR has very short TE

Cons

- SNR penalty (non-uniform density)
- May have mixed contrast
- Sensitive to gradient delays
- Sensitive to off-resonance effects

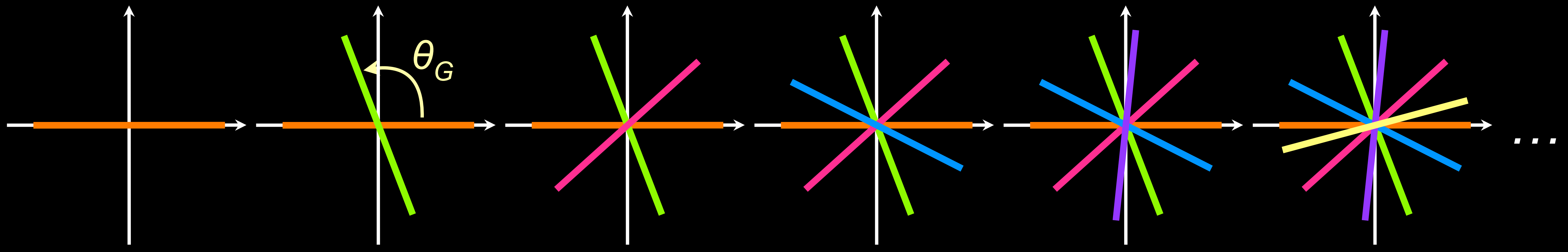
Radial: Real-time MRI

2D Radial MRI



- Robust to motion (oversample center of k-space)
- Can tolerate a lot of undersampling

Golden Angle Ordering



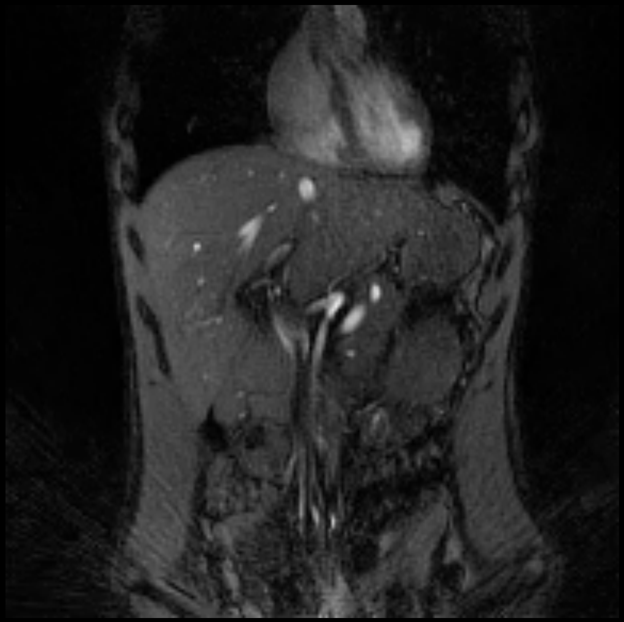
- Almost uniform sampling of *k-t* space
- Flexible choice of temporal frame location and width

Radial: Real-time MRI

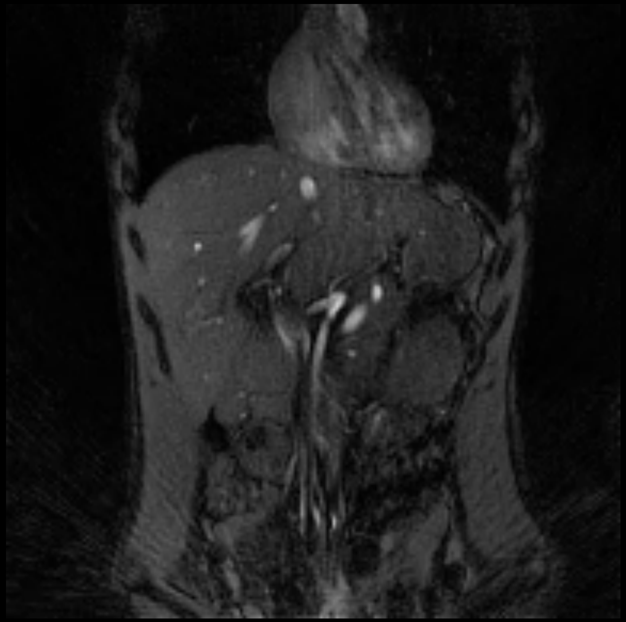
<u>Radial FLASH</u>
- golden-angle ordering
- 192 x 192 matrix
- TR = 3.1 ms
  (1 spoke per TR)
- 3.0 T

<u>Reconstruction</u>
- sliding window of 20 TRs
  (display at 16 frames/sec)
- parallel imaging (SPIRiT)
  (300 spokes for Nyquist)

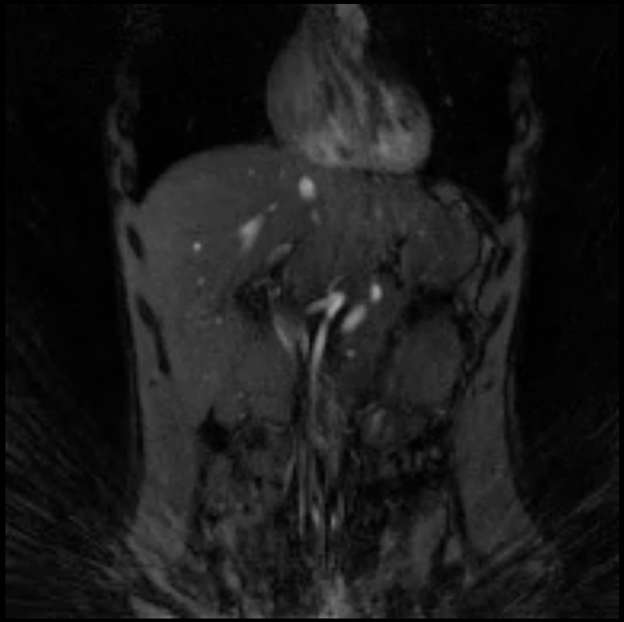
255 spokes/frame
(791 ms/frame)



144 spokes/frame
(446 ms/frame)



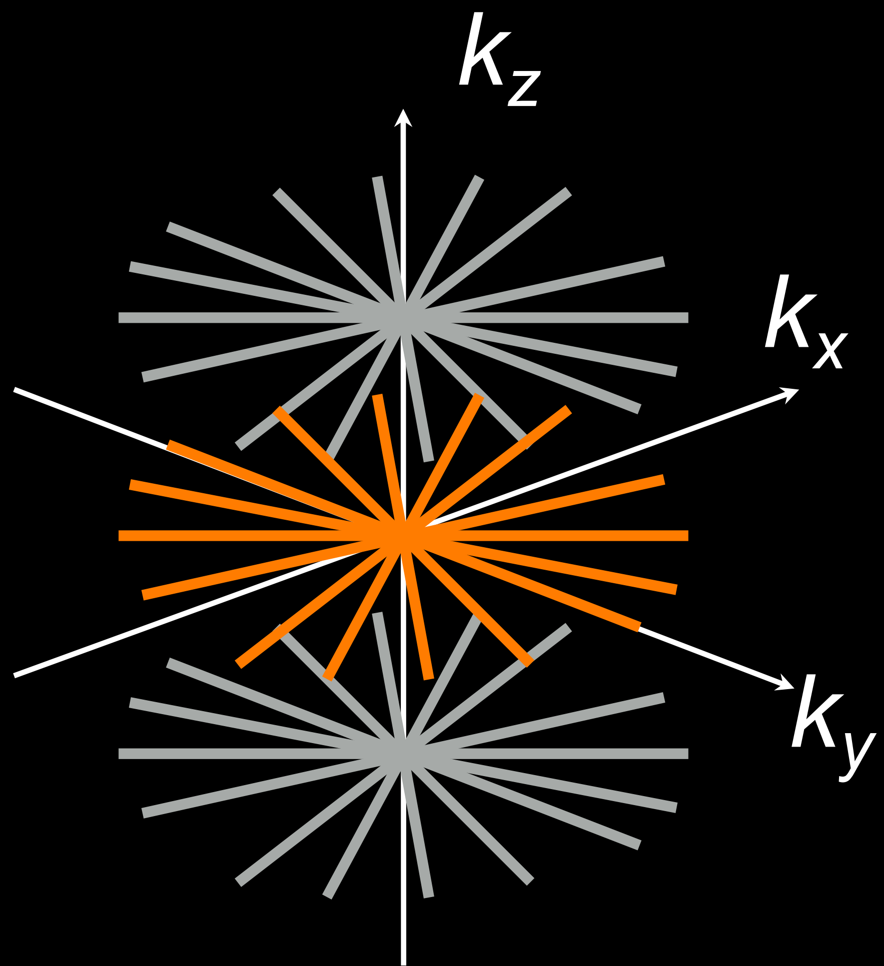
89 spokes/frame
(276 ms/frame)



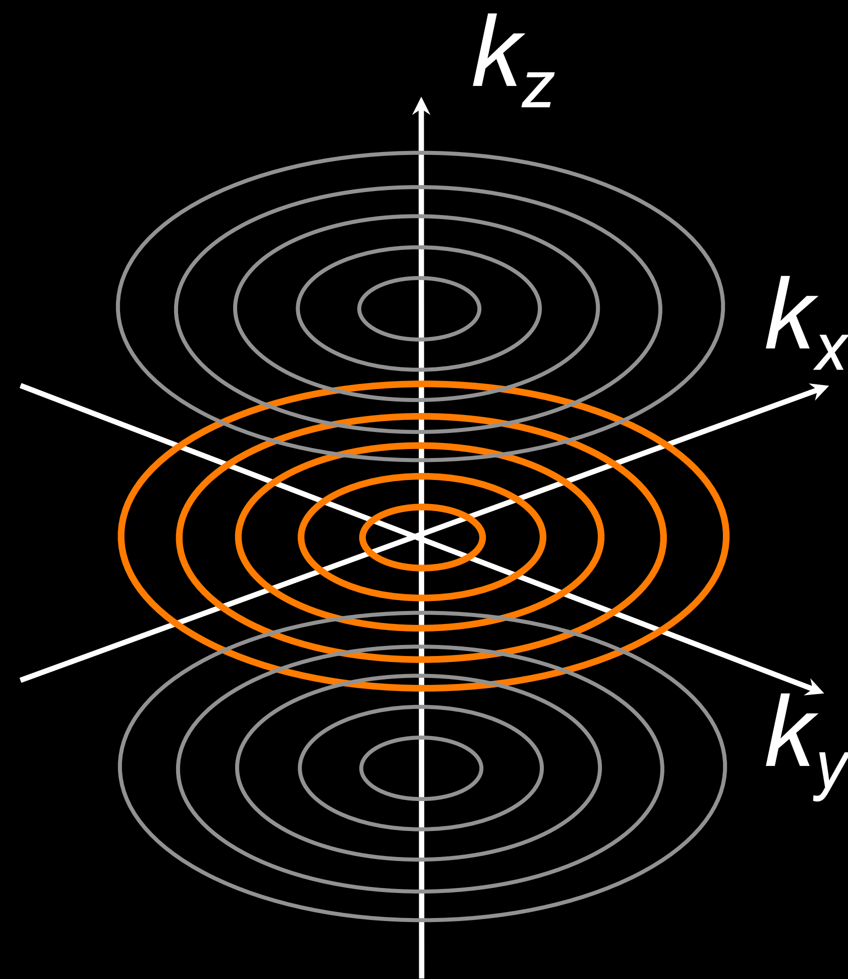
55 spokes/frame
(171 ms/frame)

courtesy of Samantha Mikaiel

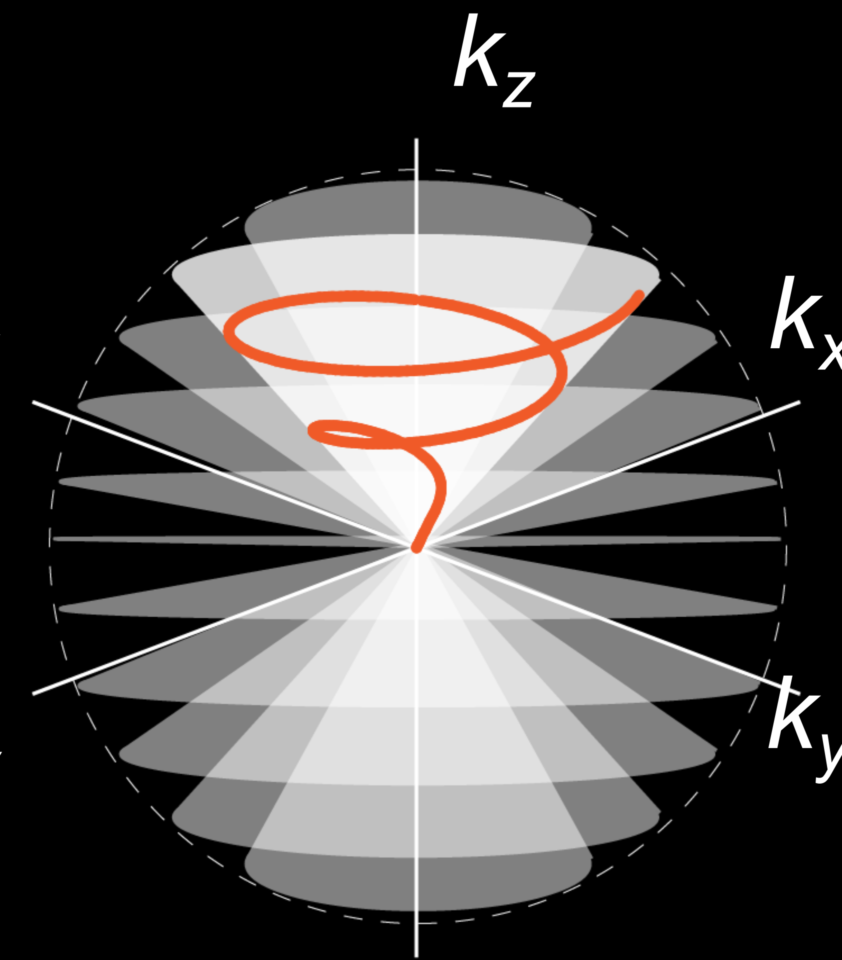
Non-Cartesian Sampling



3D Stack of Stars



3D Stack of Rings

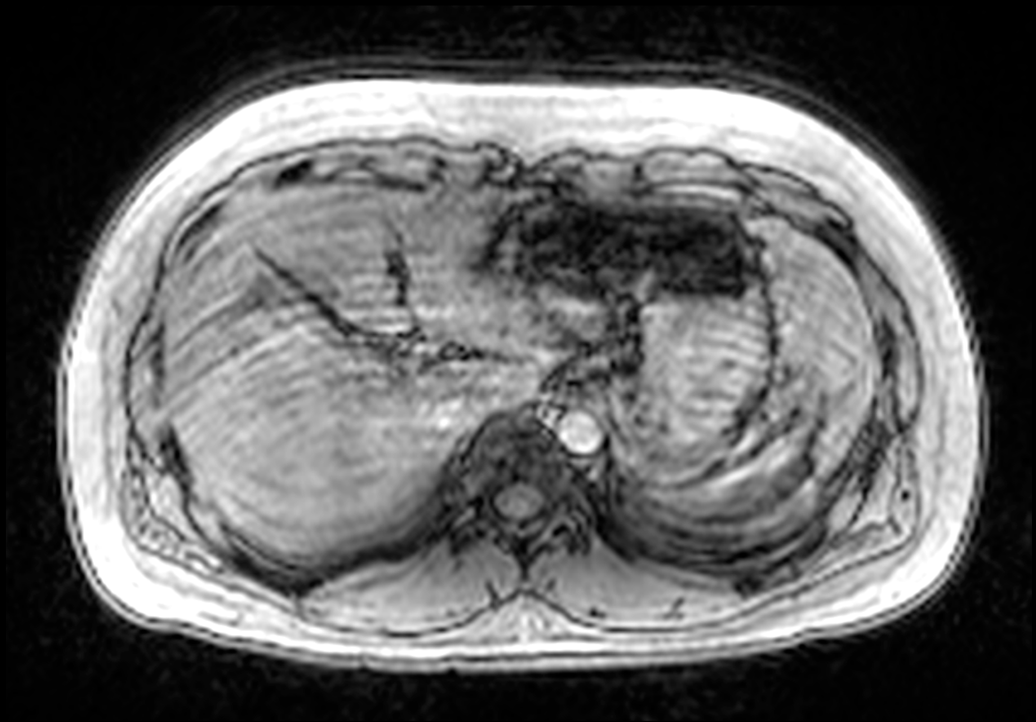


3D Cones

and much more ...

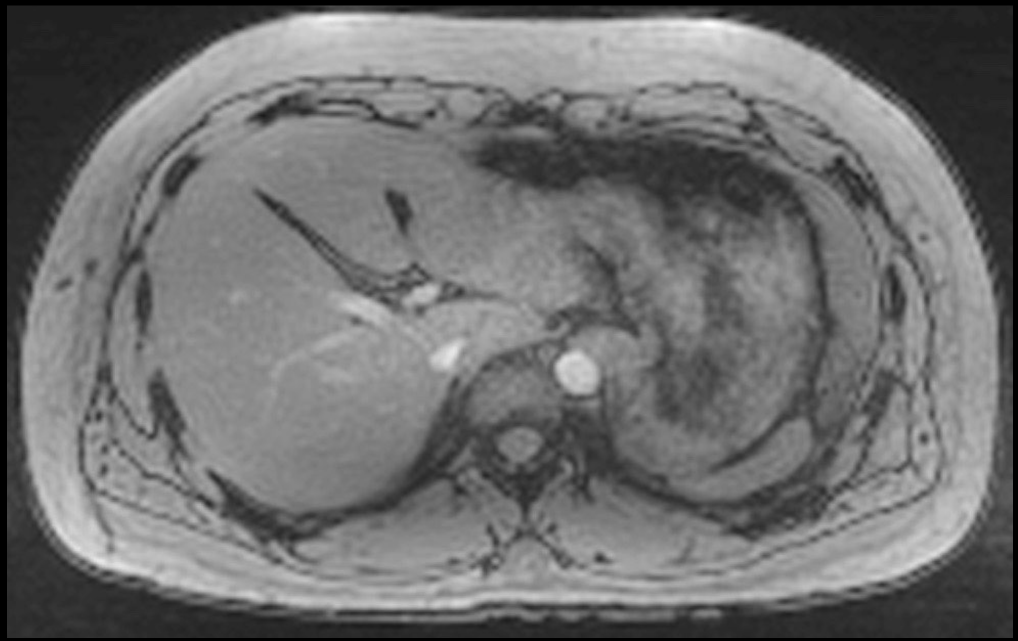
3D Stack-of-Radial: Liver MRI

3D Cartesian MRI

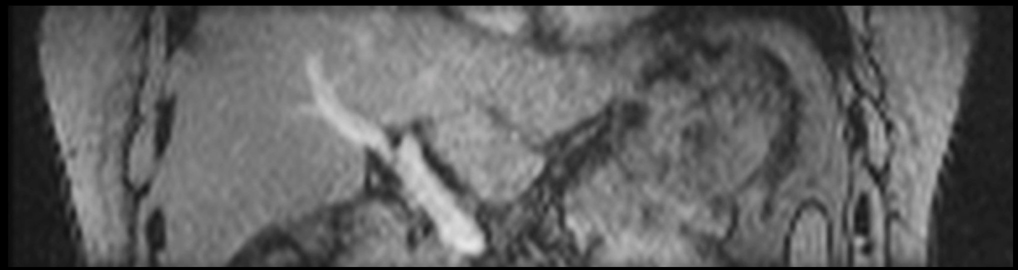


Insufficient breath-holding

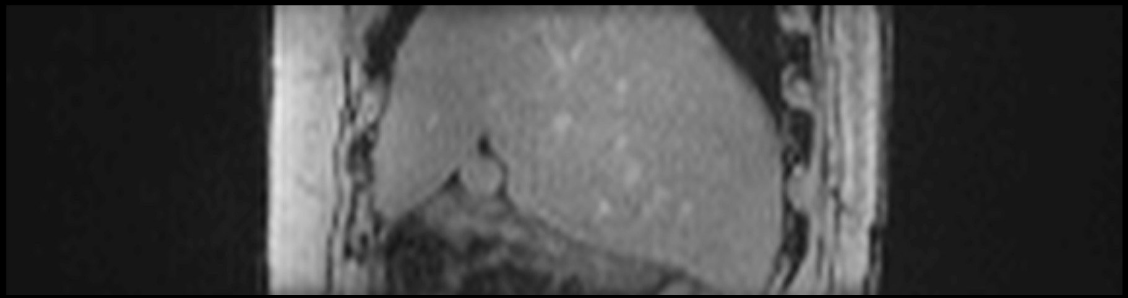
Free-breathing 3D Stack-of-Radial MRI



Axial



Coronal



Sagittal

courtesy of Tess Armstrong

3D Radial: Coronary MRA

Contrast-Enhanced MRA at 3.0T

ECG-gated, fat-saturated, inversion-recovery prepared spoiled gradient echo sequence

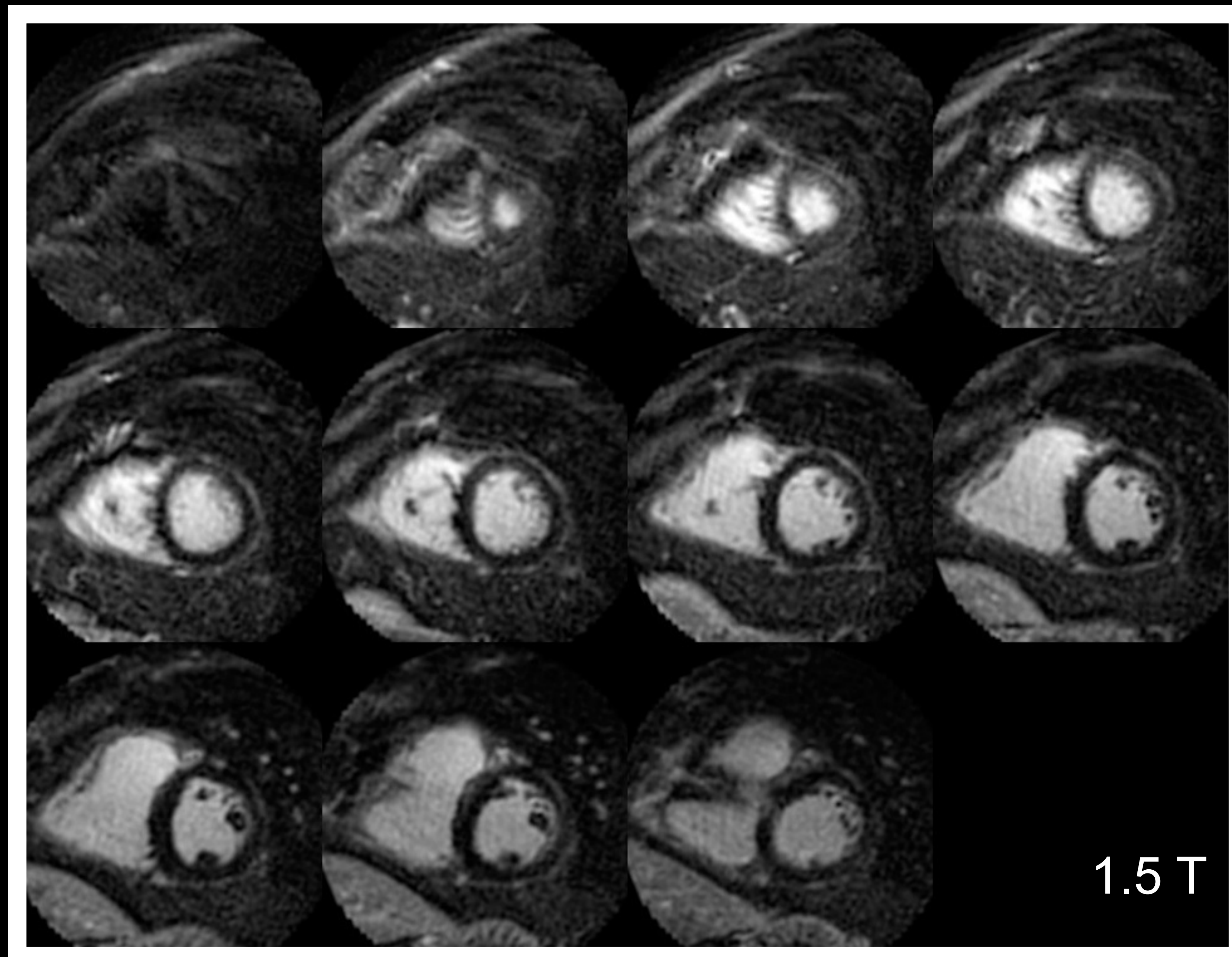
$(1.0\ \text{mm})^3$ spatial resolution, 1D self navigation, CG-SENSE recon, 5.4 min scan time

courtesy of Debiao Li and J Pang (Cedars-Sinai)

3D Stack-of-Spirals: LGE MRI

<u>3D Spiral IR-GRE</u>
- 6-interleaf VD spiral
- 7.5-ms readout
- 90 x 90 x 11 matrix
- outer volume suppr
- water-only RF exc
- TR = 15.48 ms
- 8-HB BH scan

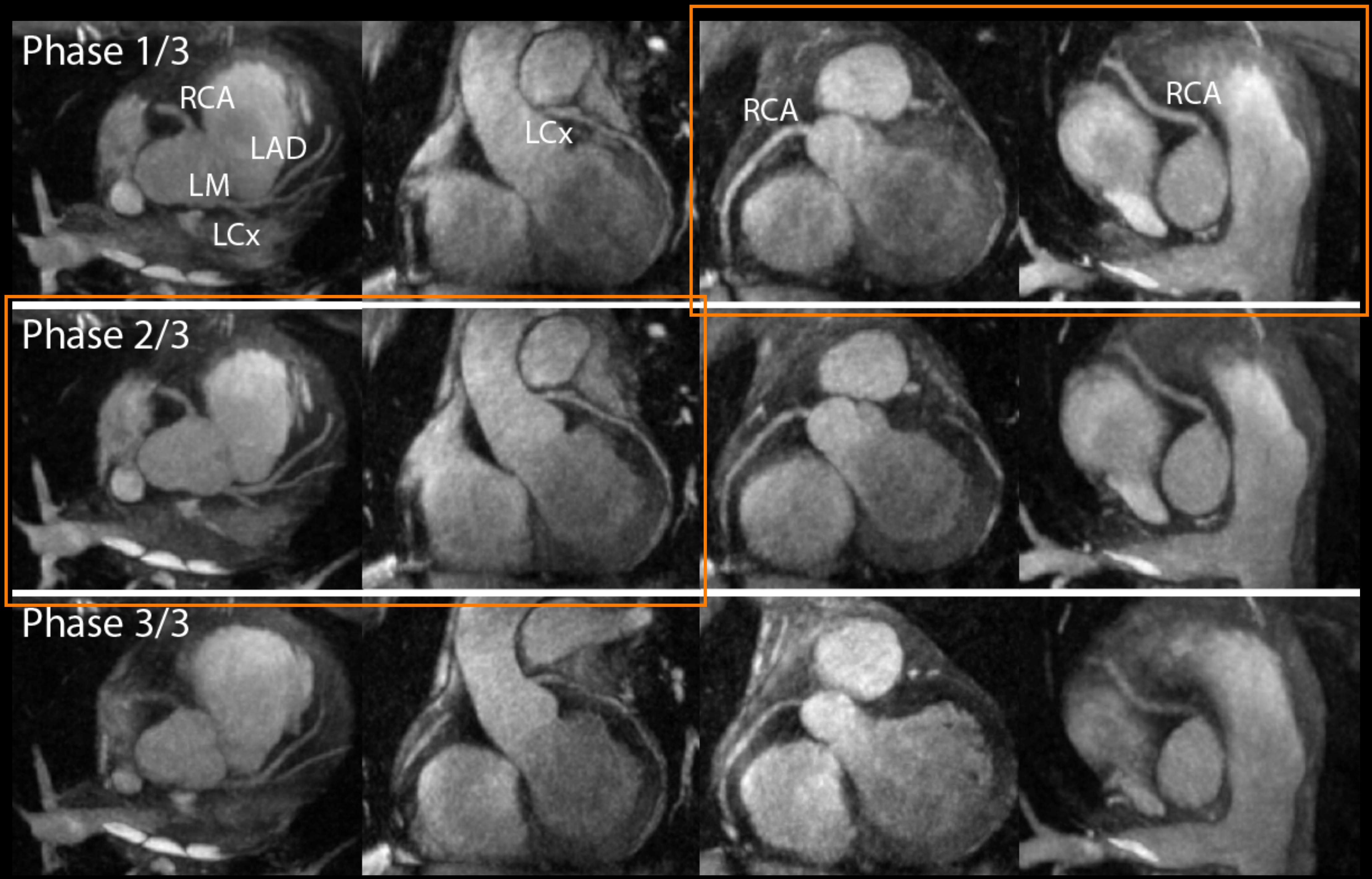
<u>Reconstruction</u>
- SPIRiT (R = 2)
- ~5-sec recon



courtesy of Joelle Barral & Juan Santos (HeartVista)

3D Cones: Coronary MRA

Multi-Phase Thin-Slab MIP Reformats



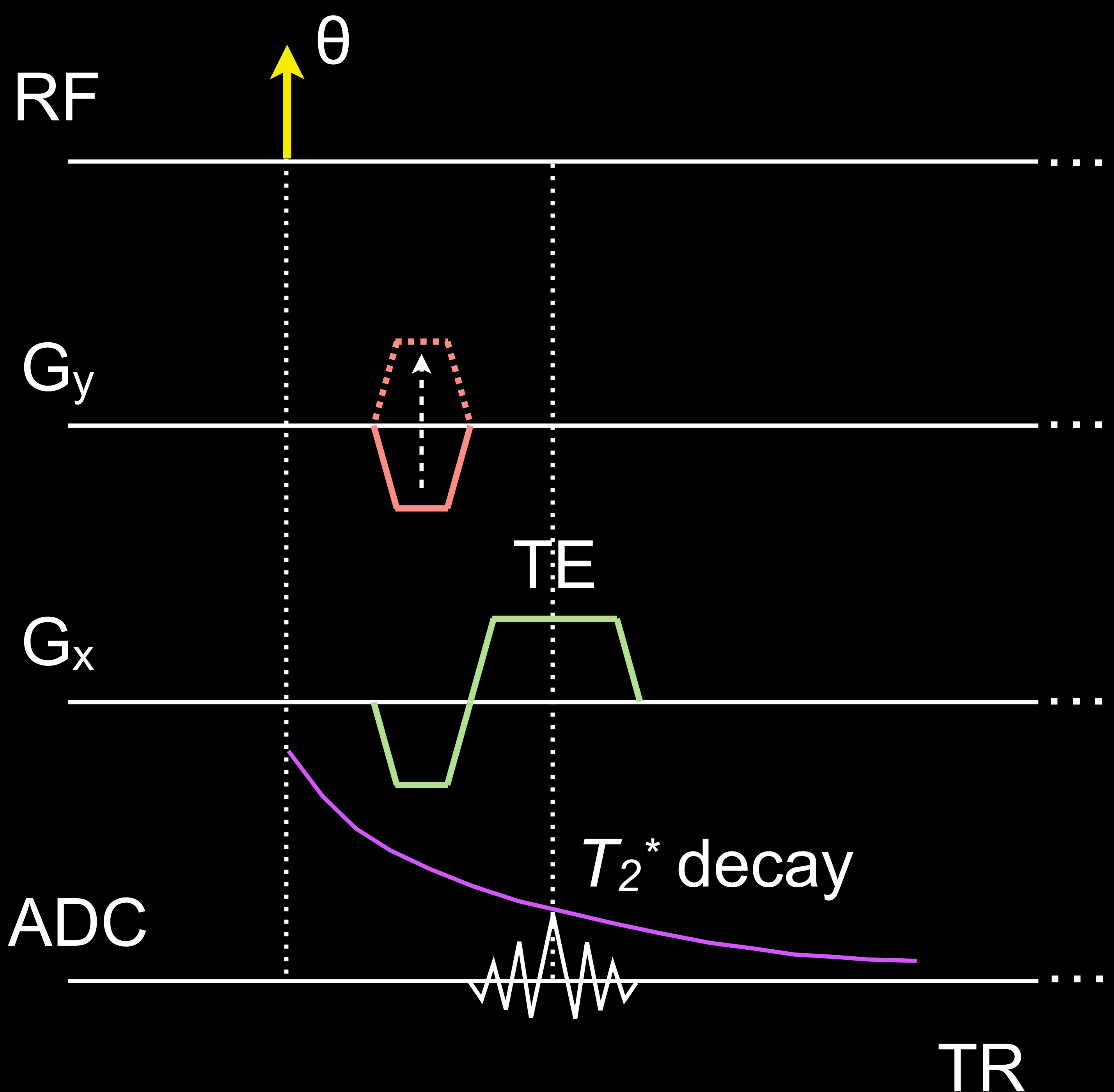
Wu HH et al., MRM 2013; 69: 1083-1093

Echo-Planar Imaging

- Echo-Planar Imaging (EPI)[1]
- Ultra-fast imaging (<100 ms/frame)
- Imperfections and artifacts
- Ongoing topic of rapid MRI research

[1] *Mansfield P, J Phys C: Solid State Phys 1977*

Gradient Echo



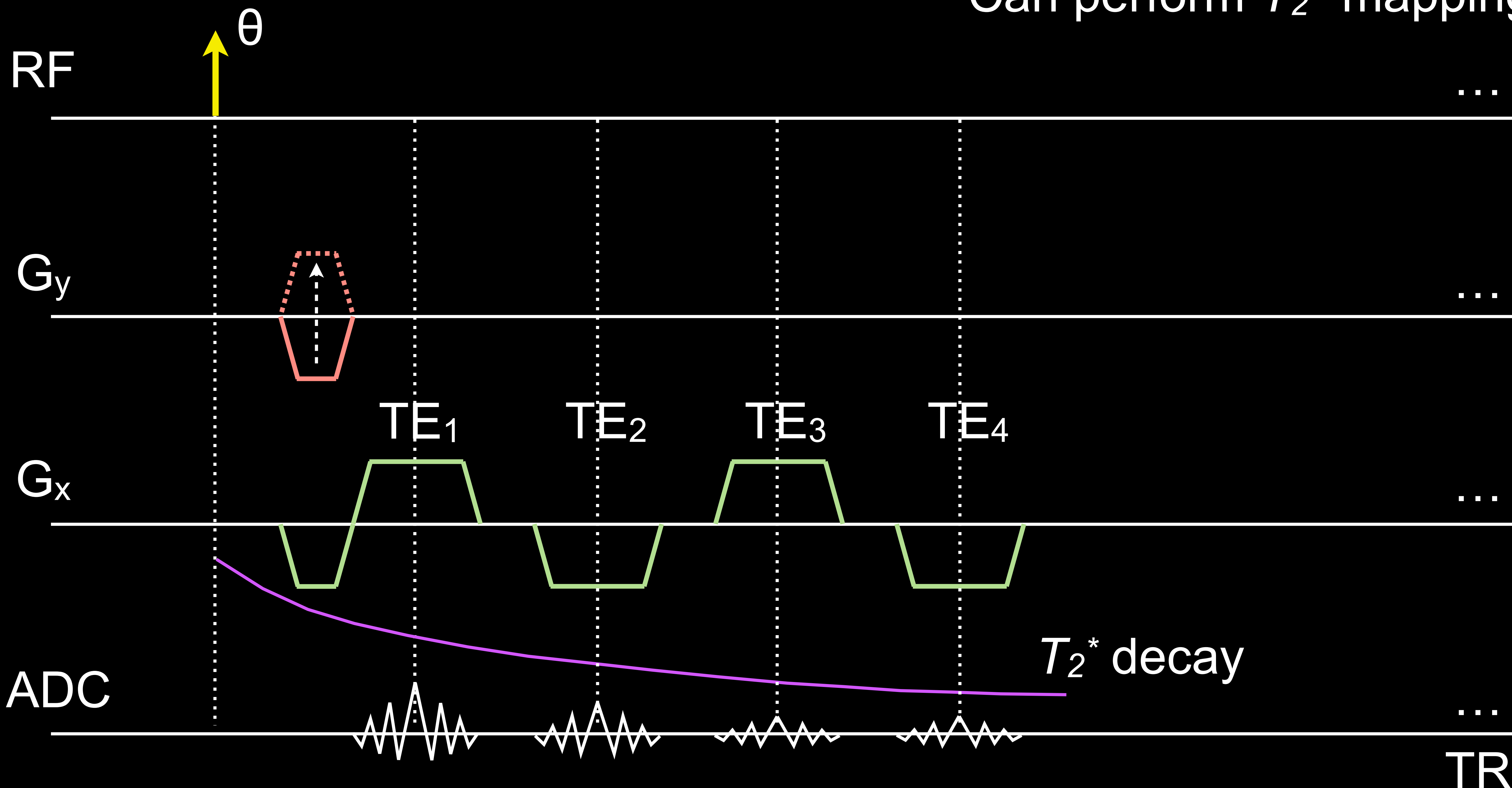
- Utilization of transverse magnetization
  - With T_s = 8 μs and N_x = 128, T_{acq} = 1.024 ms
  - <2% of T_2^* in brain at 3 T![1]
- Scan time
  - $T_{GRE} = N_{pe} \times TR$
  - TR = 10 ms, N_{pe} = 256: T_{GRE} = 2.56 sec

[1]*Peters, et al., Proc ISMRM 2006*

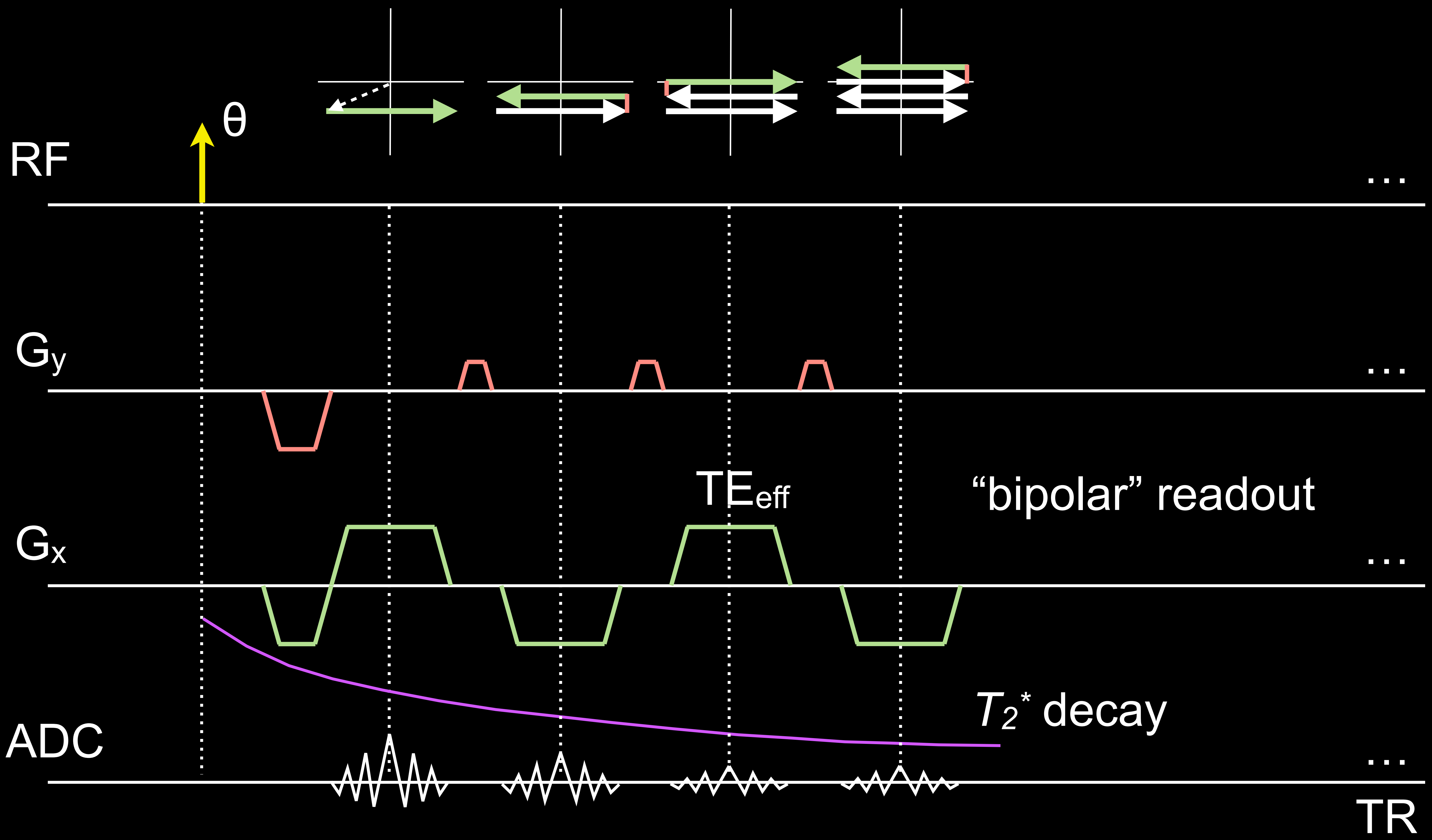
Multi-echo Gradient Echo

ΔTE can be non-uniform

Can perform T_2* mapping



Gradient-Echo EPI



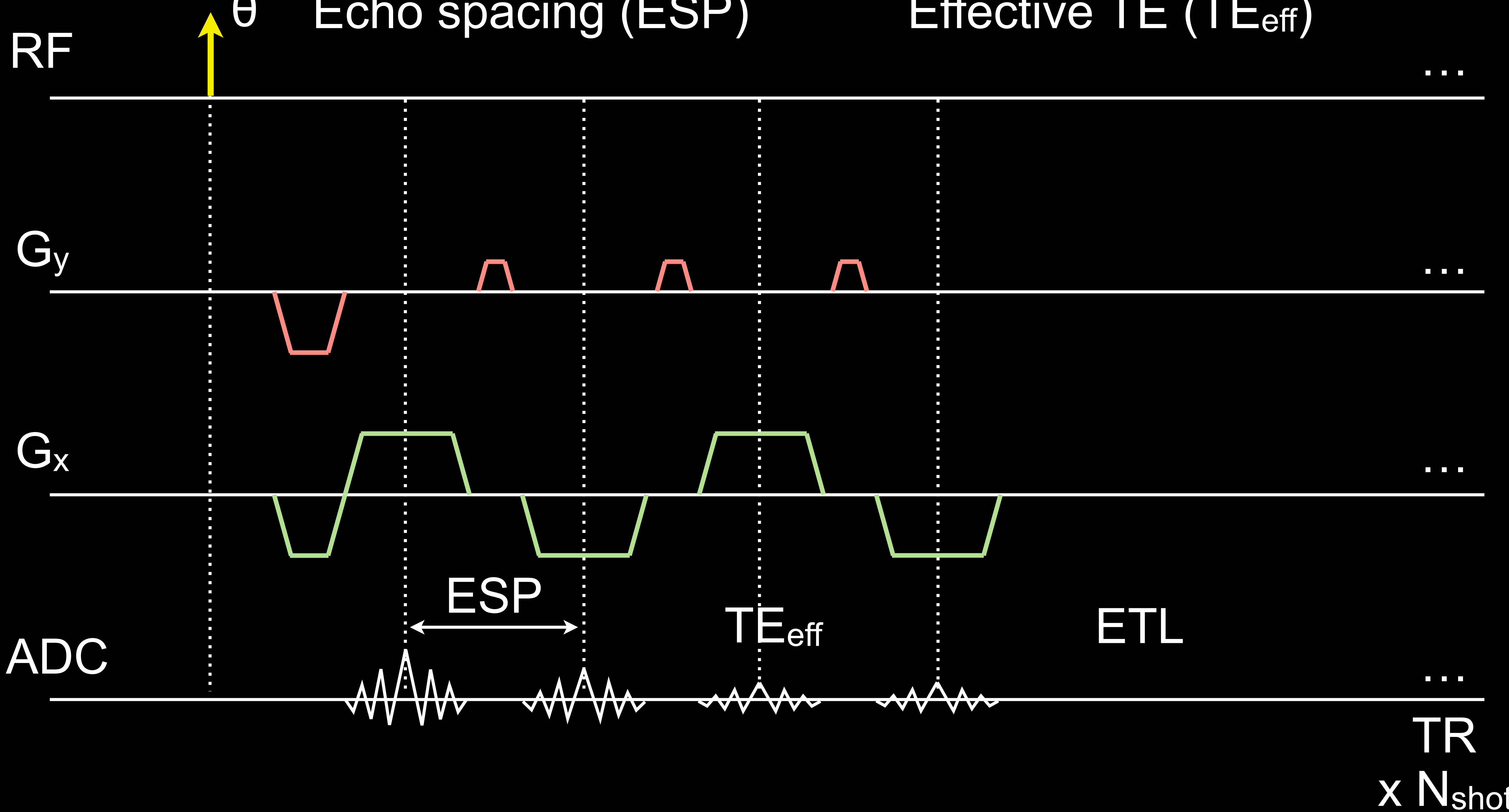
EPI Sequence Parameters

Echo train length (ETL)

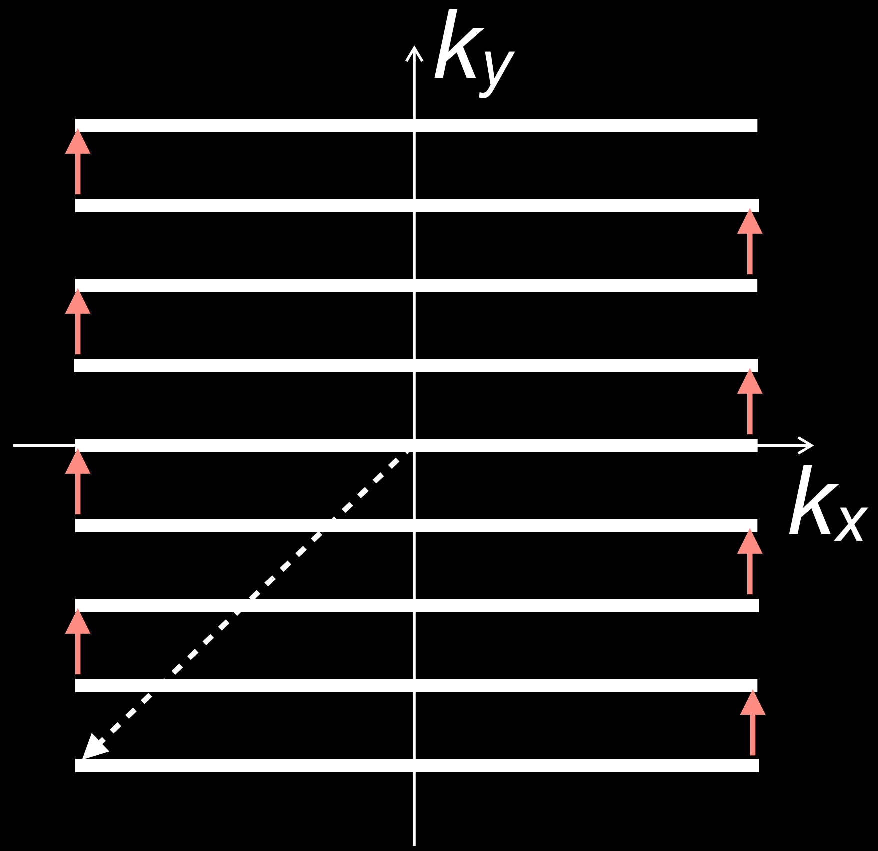
Echo spacing (ESP)

Number of shots (N_{shot})

Effective TE (TE_{eff})



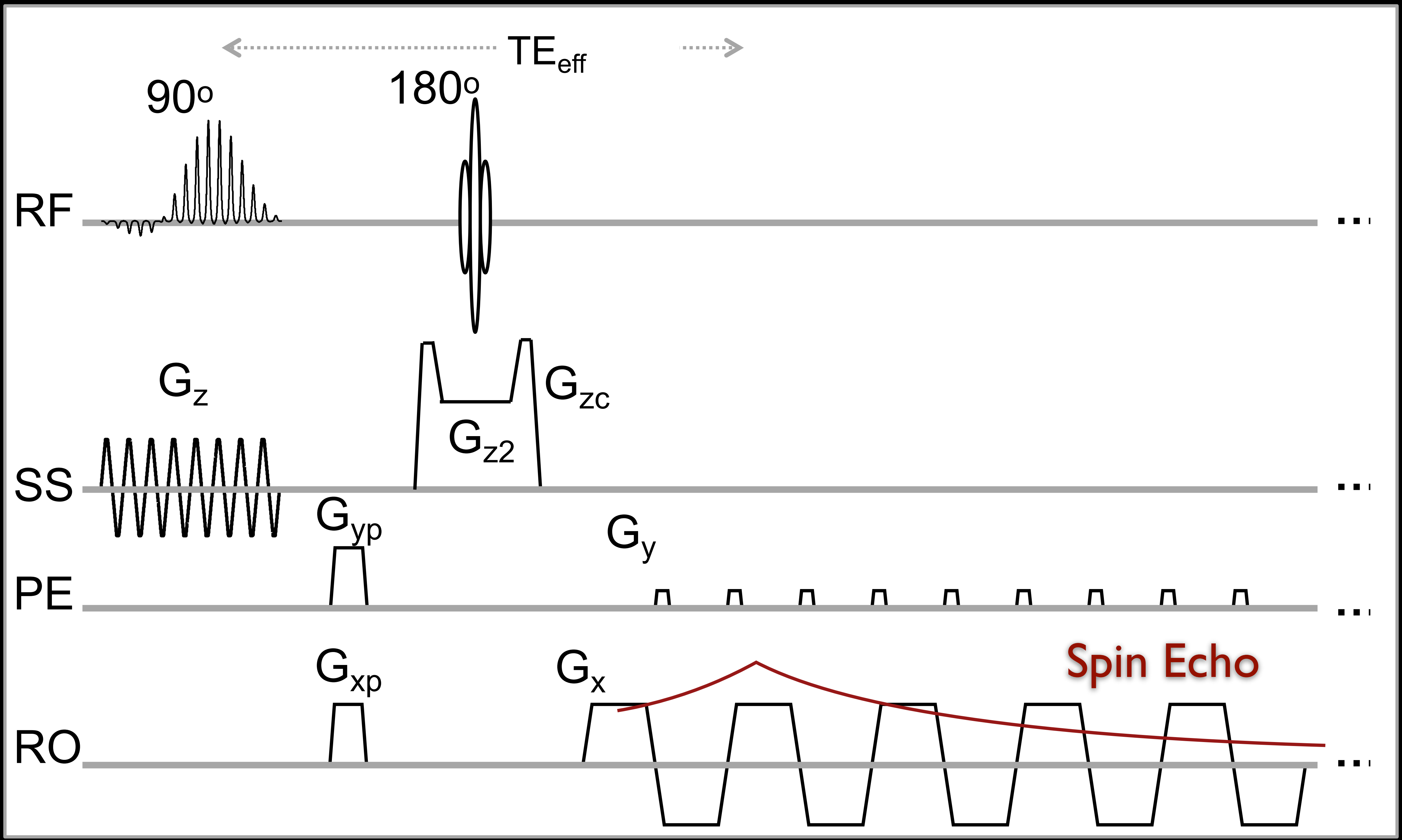
EPI k-Space Sampling



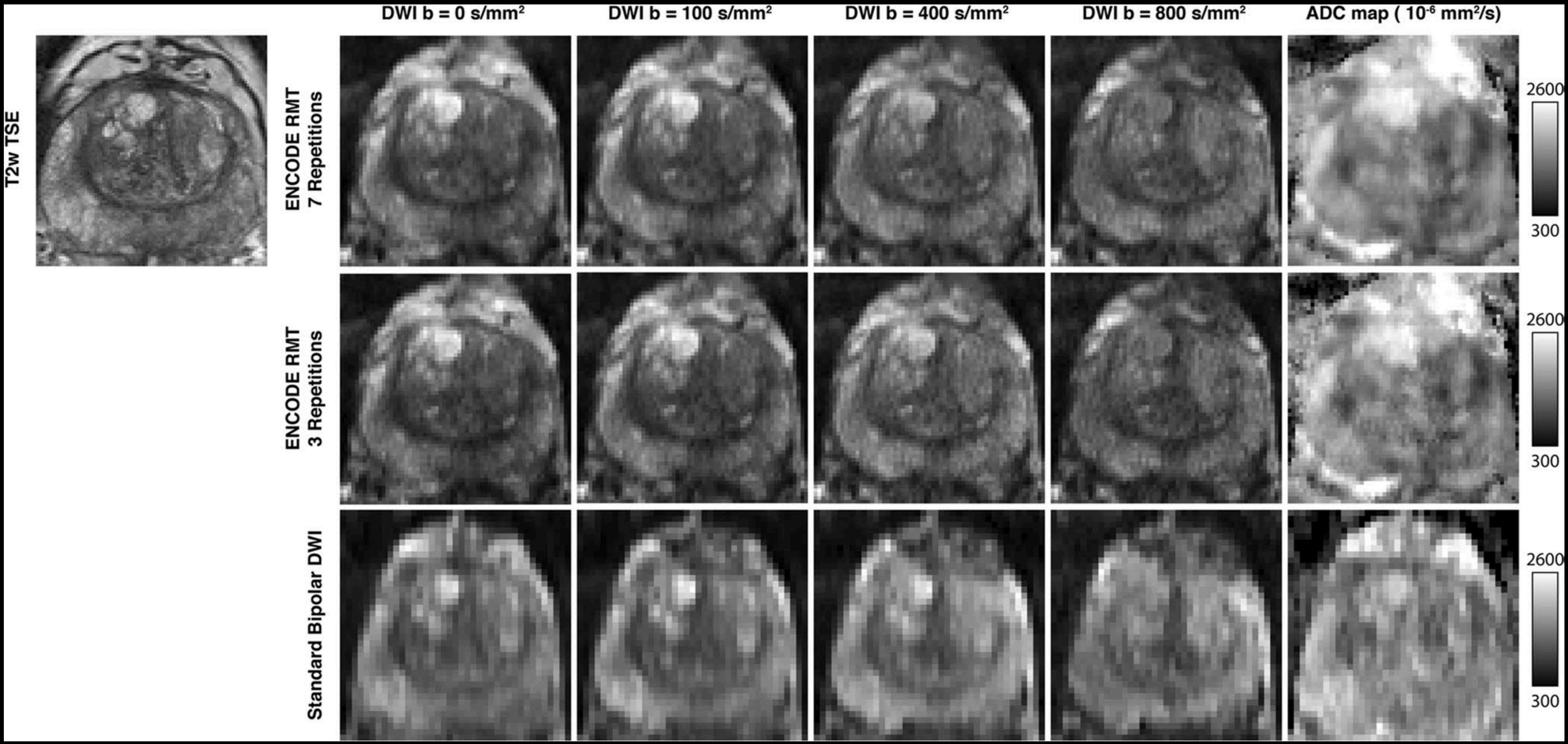
- ETL can be 4-64 or higher
  - Limited by T_2* decay, off-resonance effects
  - aka “EPI factor”

- ESP typically ~1 ms
  - Must accommodate RF, gradients, ADC
  - Short ESP facilitates high ETL

Spin-Echo (SE) EPI



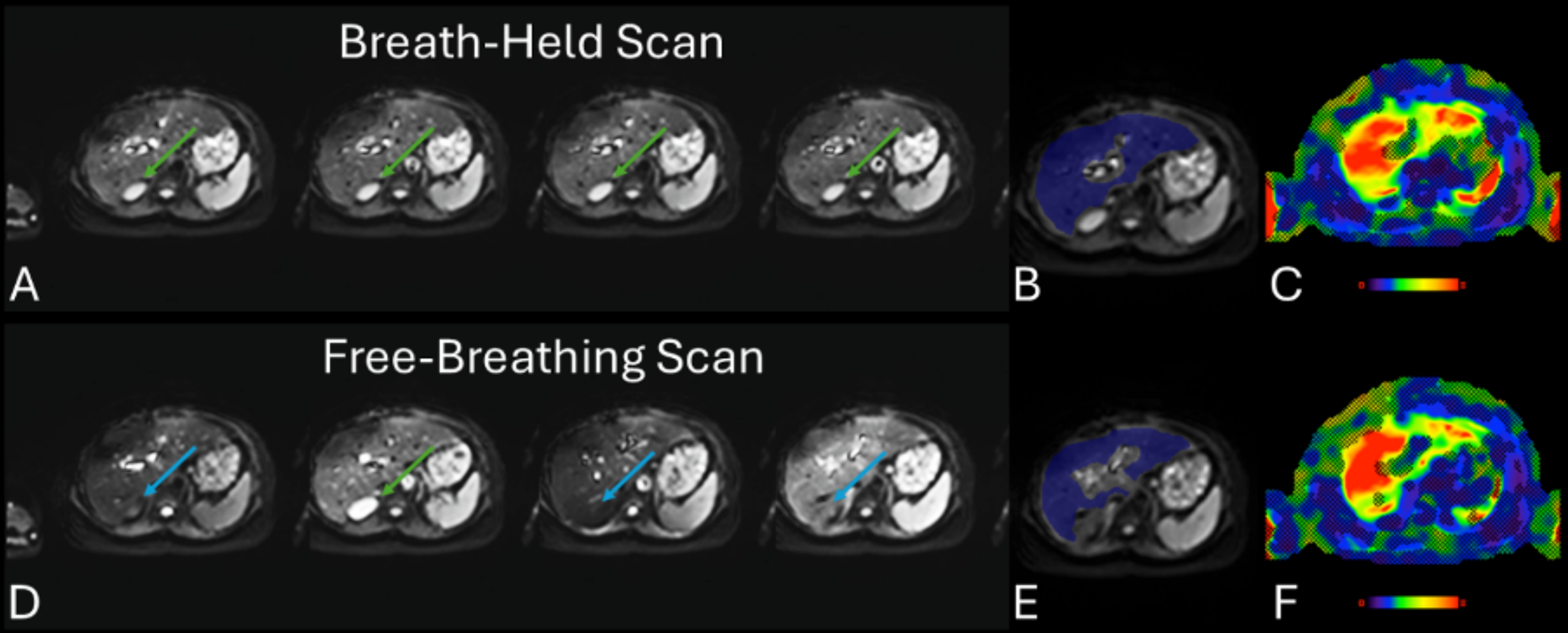
SE-EPI: Prostate Diffusion MRI



ENCODE RMT: TR=4800 ms, 16 slices, 3 averages, parallel imaging R=2, scan time=2m 30s

Zhang Z, et al., MAGMA 2024

SE-EPI: Liver MR Elastography



SE-EPI MRE: TR=1000 ms, 4 slices, 4 wave encodings, parallel imaging R=2, scan time=11s

Delgado T, et al., ISMRM 2025, in press

Fast Imaging Challenges

- Trajectory and gradient design
- Hardware performance
- Gradient fidelity
- Off-resonance effects
- Reconstruction: non-uniform FT

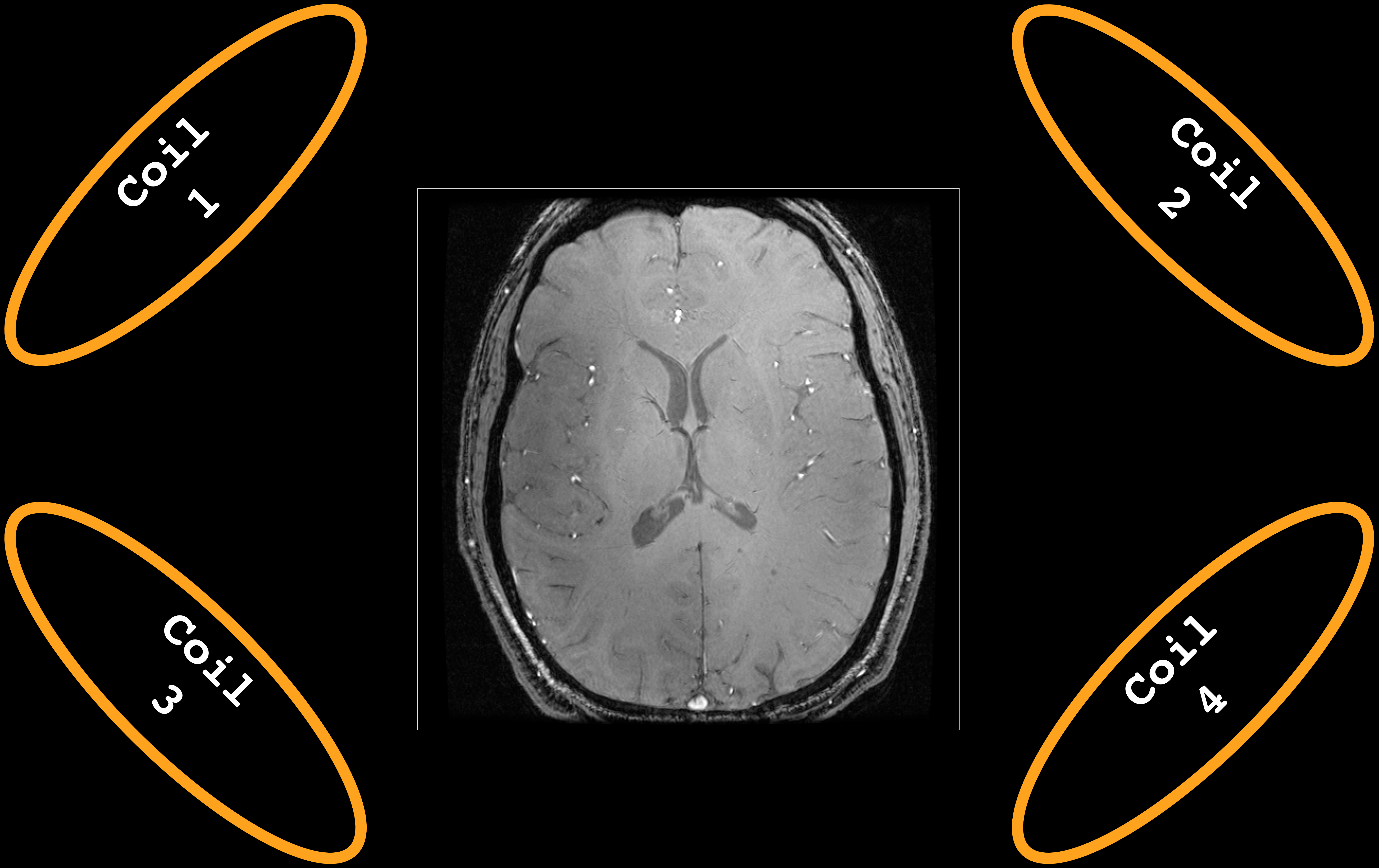
- Methods have been developed!

Fast Imaging Trajectories

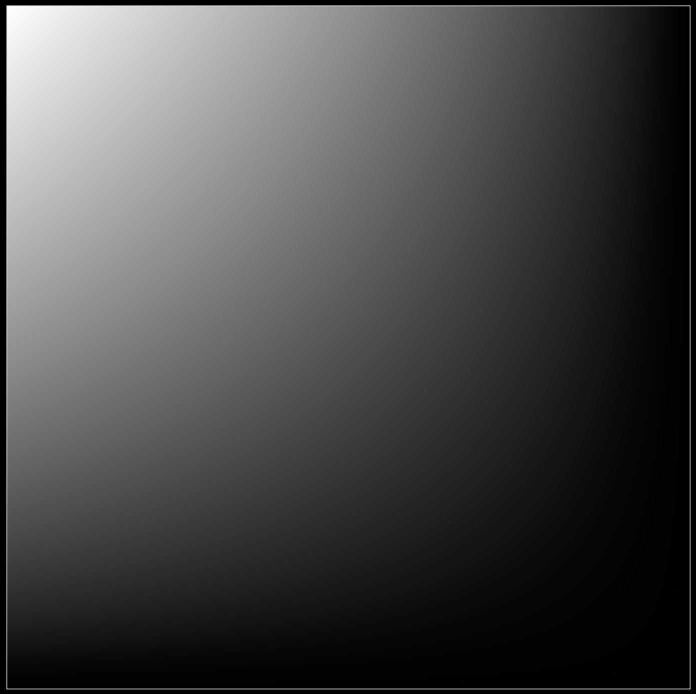
- Benefits
  - Reduced scan time
  - Robustness to motion and flow
  - Short echo time
- Applications
  - Dynamic MRI
  - Real-time MRI
  - Cardiovascular MRI
  - Short-TE MRI
- Challenges
  - Hardware performance
  - Gradient fidelity
  - Off-resonance effects
  - Design and implementation
- Challenges addressed
- On-going research
- Use judiciously!

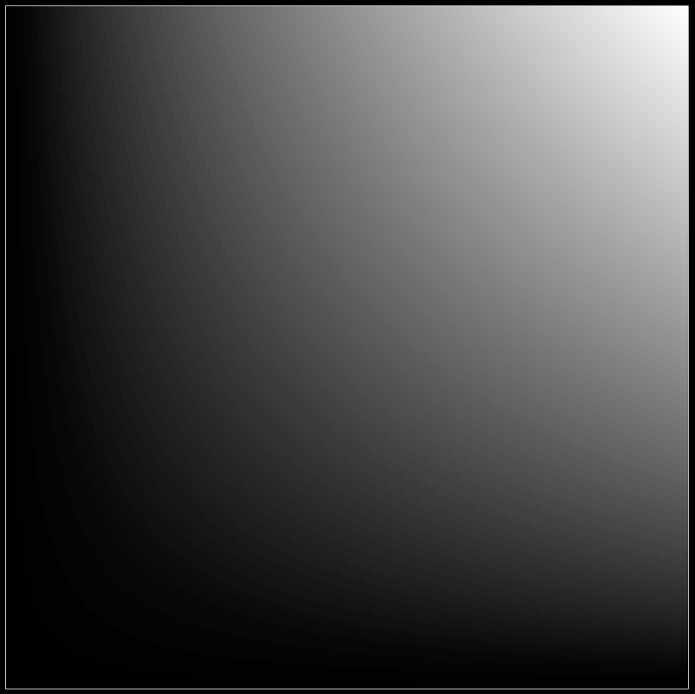
Parallel Imaging

Multi-coil Arrays

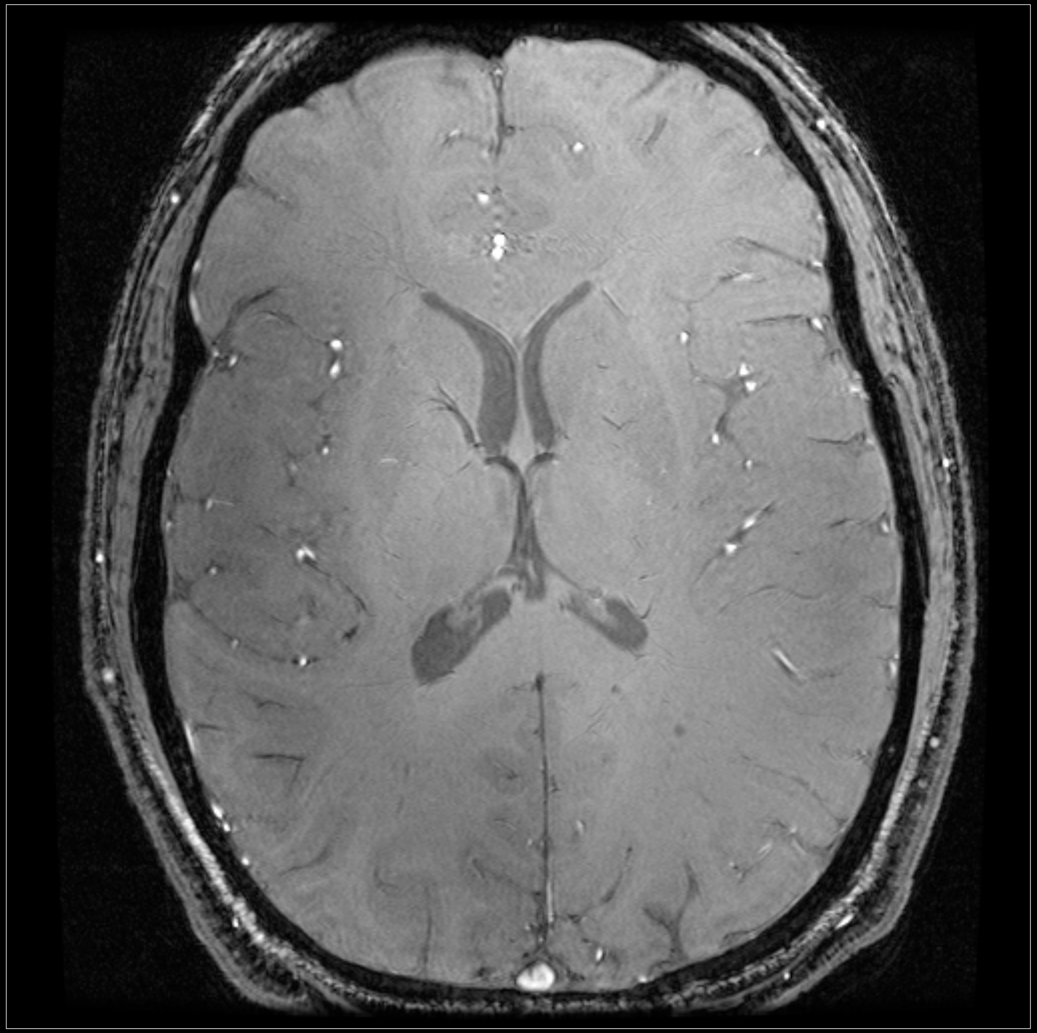


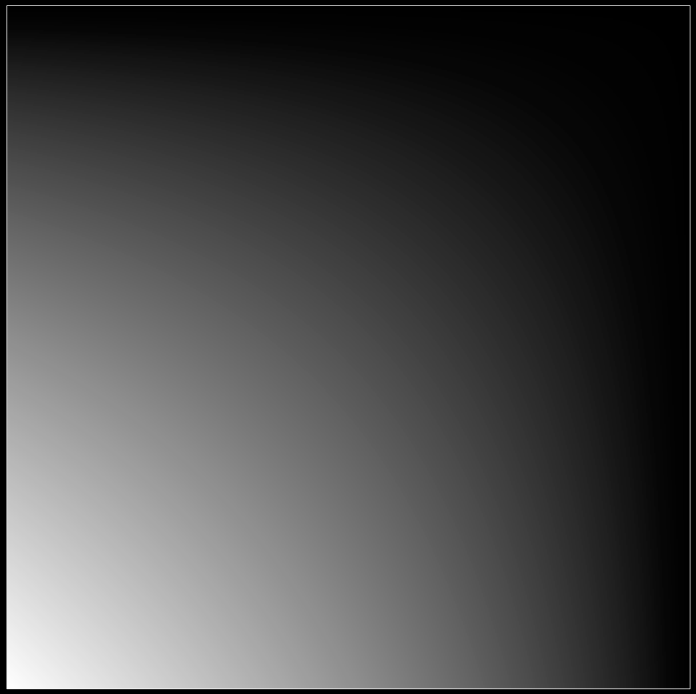
Multi-coil Sensitivity

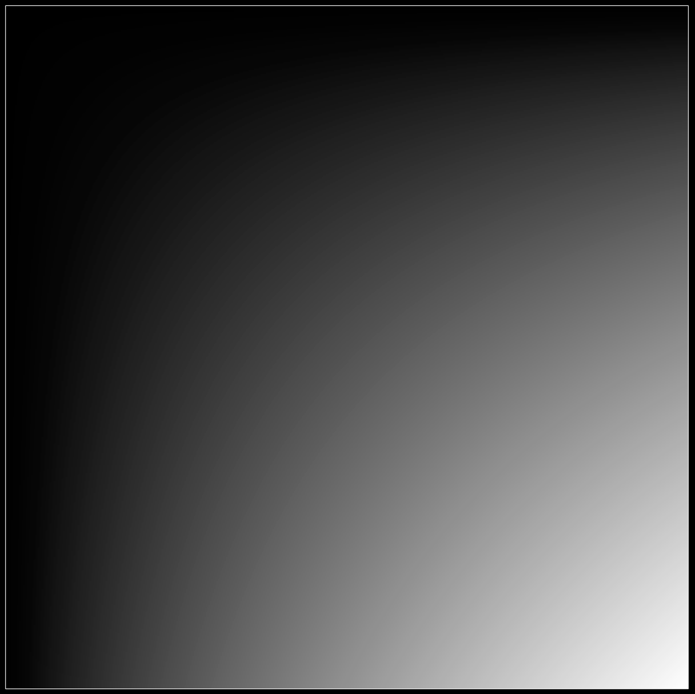




$$\left\| \vec{B}\left(\vec{r}\right) \right\|$$

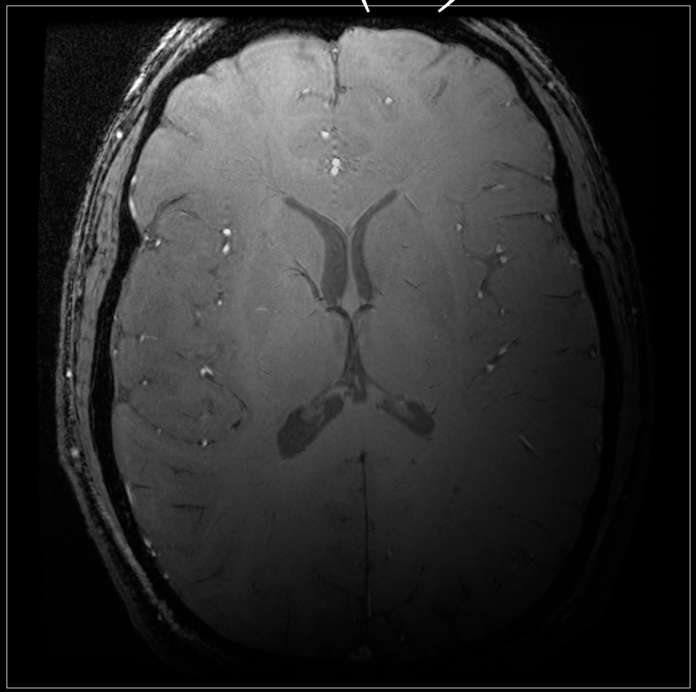






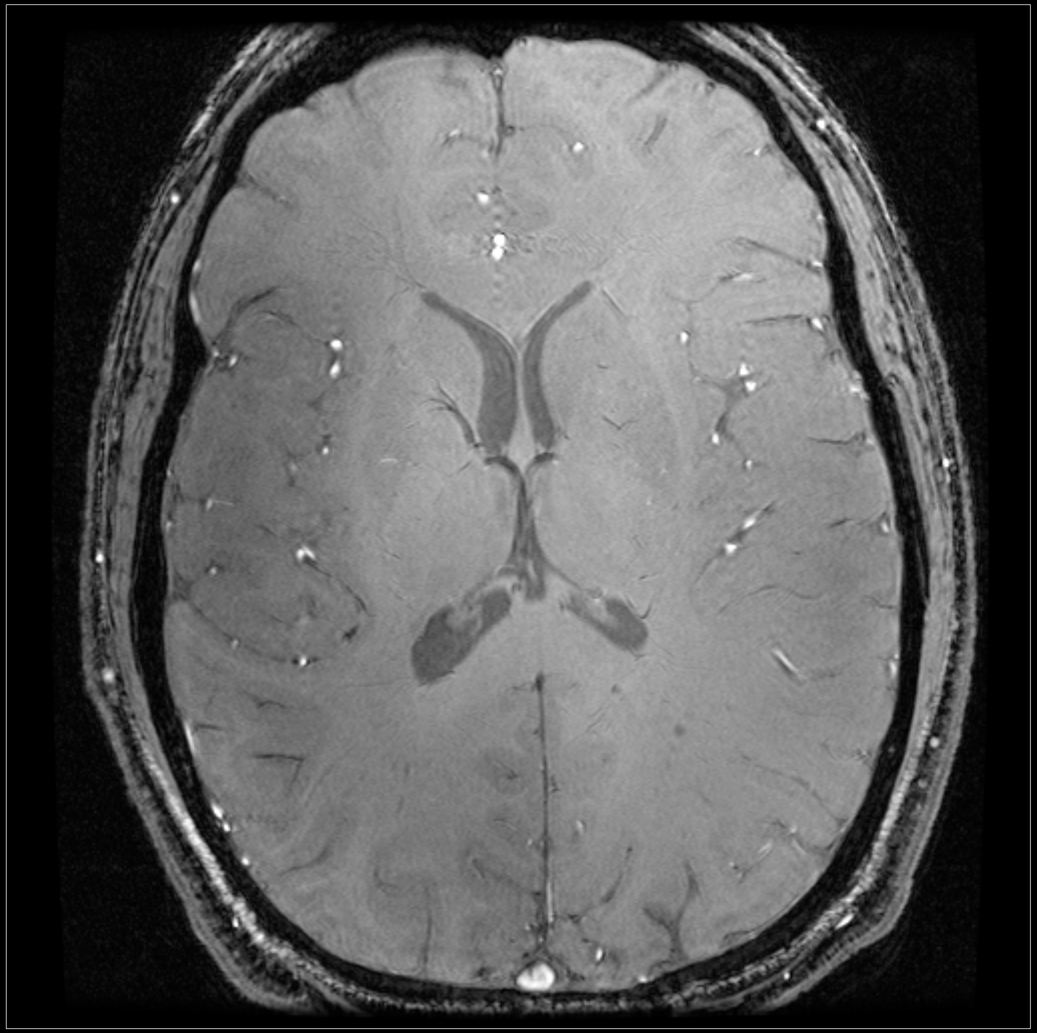
Multi-coil Images

$m_1(x)$

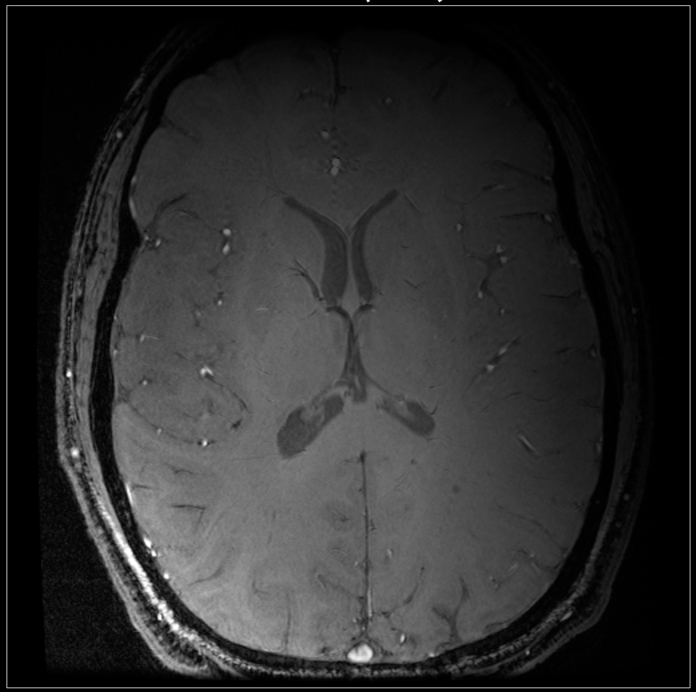


$m_2(x)$

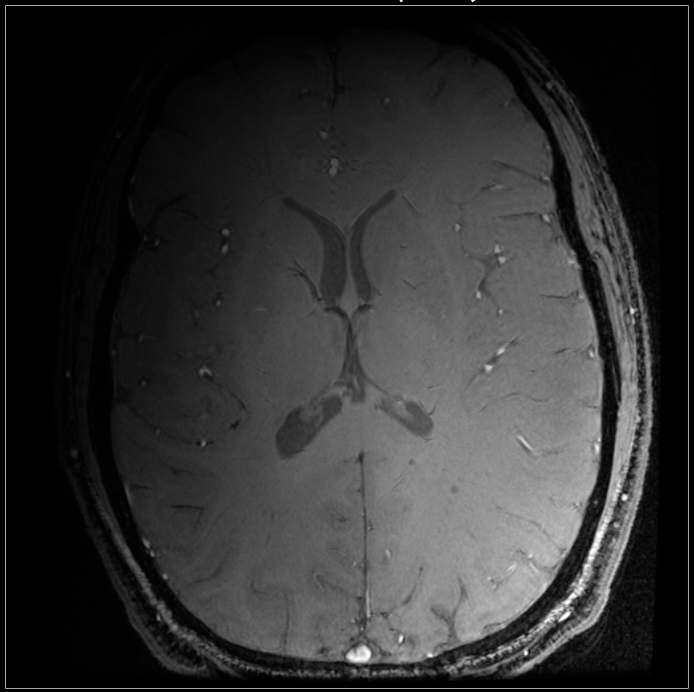
$m_s(x)$



$m_3(x)$



$m_4(x)$



Multi-coil Reconstruction

- Each coil has a complete image of whole FOV and an amplitude and phase sensitivity

$$C_l(\vec{x}) \qquad l = 1,2,\ldots L$$

- Coils are coupled, so noise is correlated

$$E[n_i n_j] = \Psi$$

- Received data from coil l:

$$m_l(\vec{x}) = C_l(\vec{x})m(\vec{x}) + n_l(\vec{x})$$

- Given $m_l(x)$, how do we reconstruct $m(x)$?

Multi-coil Reconstruction

For a particular voxel x

$$\begin{pmatrix} m_1(\vec{x}) \\ m_2(\vec{x}) \\ \cdot \\ \cdot \\ \cdot \\ m_L(\vec{x}) \end{pmatrix} = \begin{pmatrix} C_1(\vec{x}) \\ C_2(\vec{x}) \\ \cdot \\ \cdot \\ \cdot \\ C_L(\vec{x}) \end{pmatrix} m(\vec{x}) + \begin{pmatrix} n_1(\vec{x}) \\ n_2(\vec{x}) \\ \cdot \\ \cdot \\ \cdot \\ n_L(\vec{x}) \end{pmatrix}$$

OR

$$\underbrace{m_s(\vec{x})}_{L \times 1} = \underbrace{C}_{L \times 1} m(\vec{x}) + \underbrace{n}_{L \times 1}$$

Minimum Variance Estimate

$$\hat{m}(\vec{x}) = \underbrace{(C^*\Psi^{-1}C)^{-1}}_{1 \text{ x } 1}\underbrace{C^*\Psi^{-1}}_{1 \text{ x } L}\underbrace{m_s(\vec{x})}_{L \text{ x } 1}$$

Covariance (variance)

$$COV(\hat{m}(\vec{x})) = C^*\Psi^{-1}C$$

What if Ψ is $\sigma^2 I$?

$$\hat{m}(\vec{x}) = \underbrace{(C^*C)^{-1}}_{\text{Intensity Correction}}\underbrace{C^*}_{\text{Phase Correction}}m_s(\vec{x})$$

Approximate Solution

- Often we don't know $C_l(x)$, but we have

$$m_l(\vec{x}) = C_l(\vec{x})m(\vec{x})$$

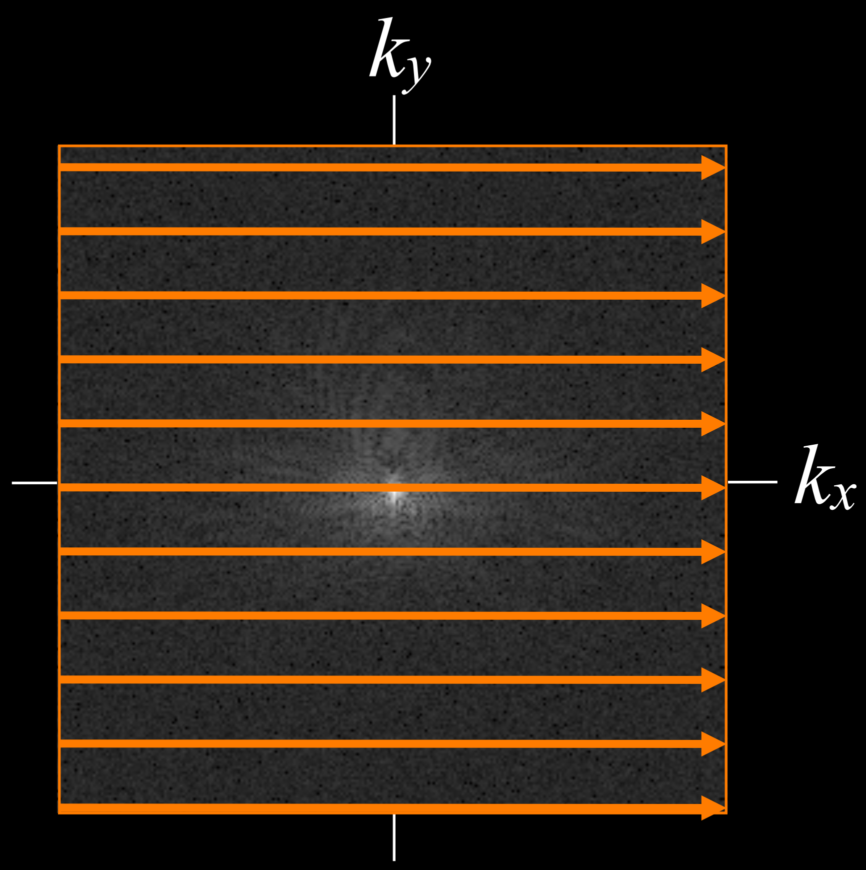
- Approximate solution:

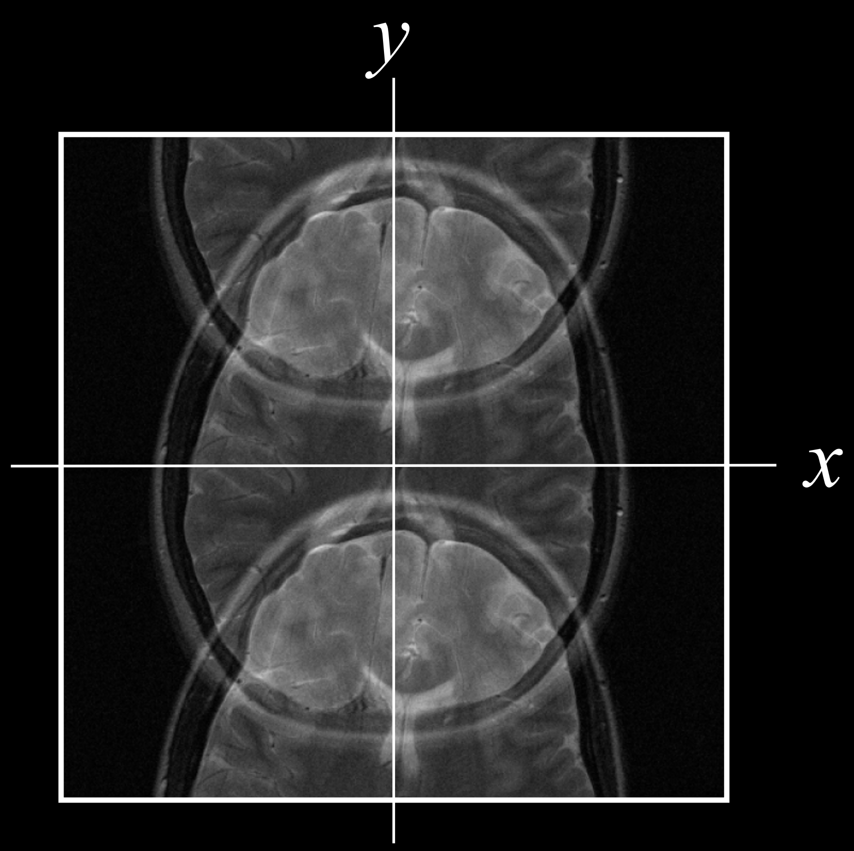
$$\hat{m}_{SS}(\vec{x}) = \sqrt{\sum_l m_l^*(\vec{x})m_l(\vec{x})}$$

- For SNR > 20, within 10% of optimal solution

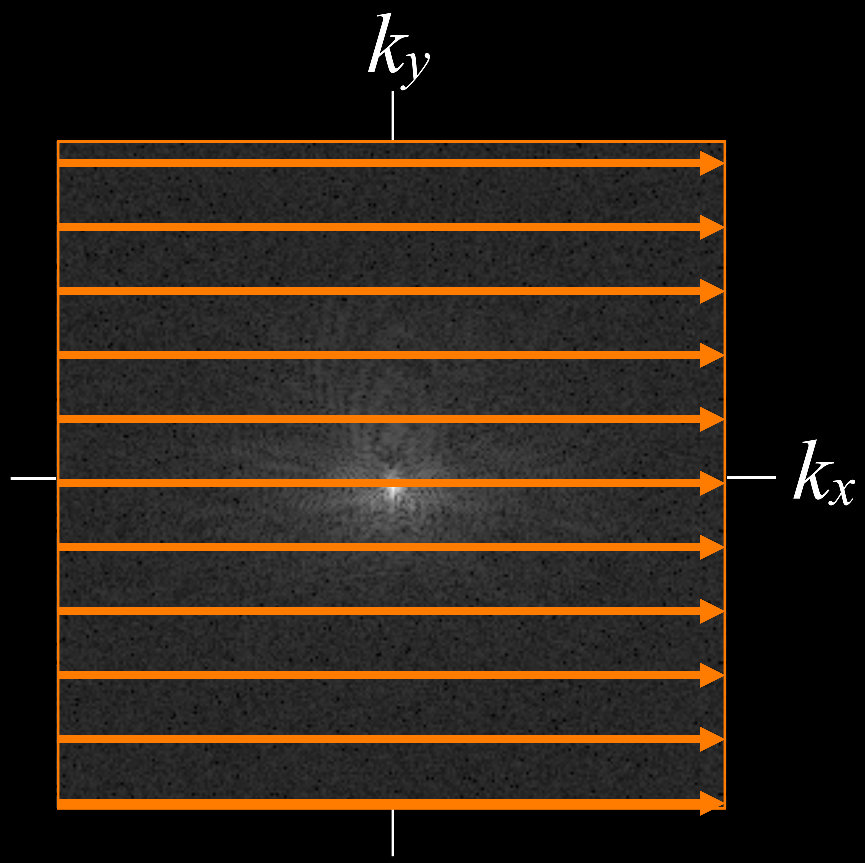
PB Roemer et al. MRM 1990

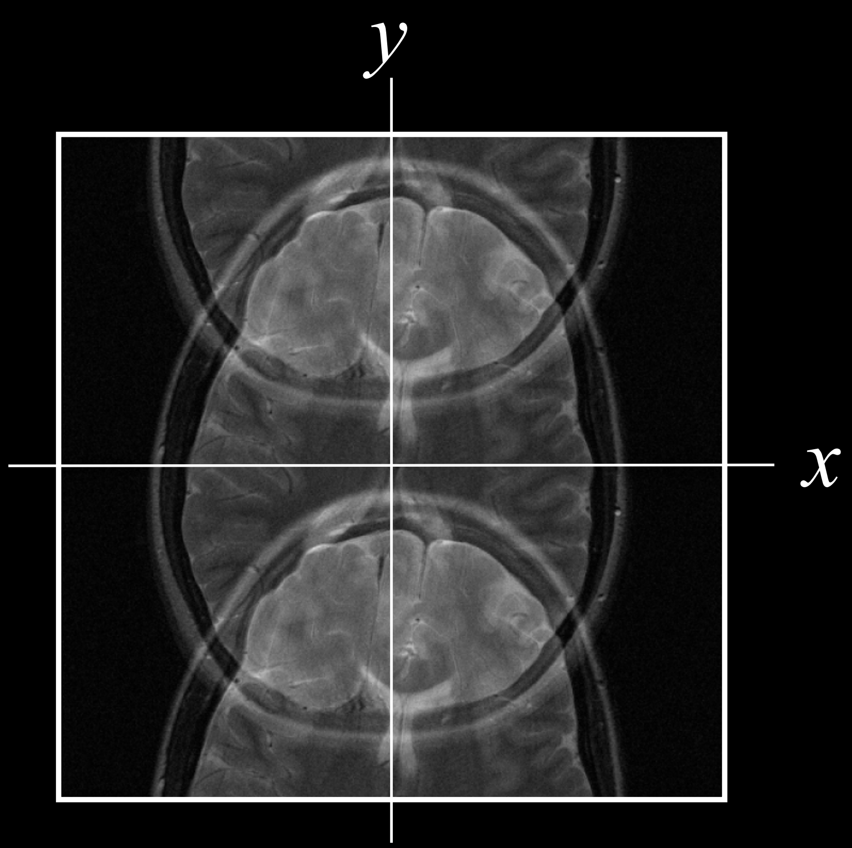
Accelerate Imaging with Array Coils





Accelerate Imaging with Array Coils





- Parallel Imaging
  - Coil elements provide some localization
  - Undersample in k-space, producing aliasing
  - Sort out in reconstruction

Parallel Imaging

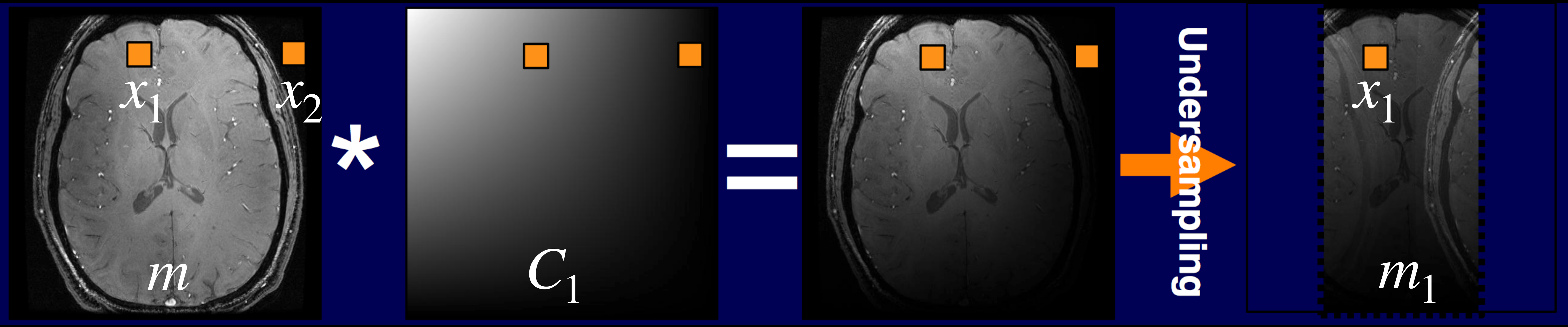
- Many approaches:
  - Image domain - SENSE
  - k-space domain - SMASH, GRAPPA
  - Hybrid - ARC

- We will introduce one:
  - SENSE: optimal if you know coil sensitivities

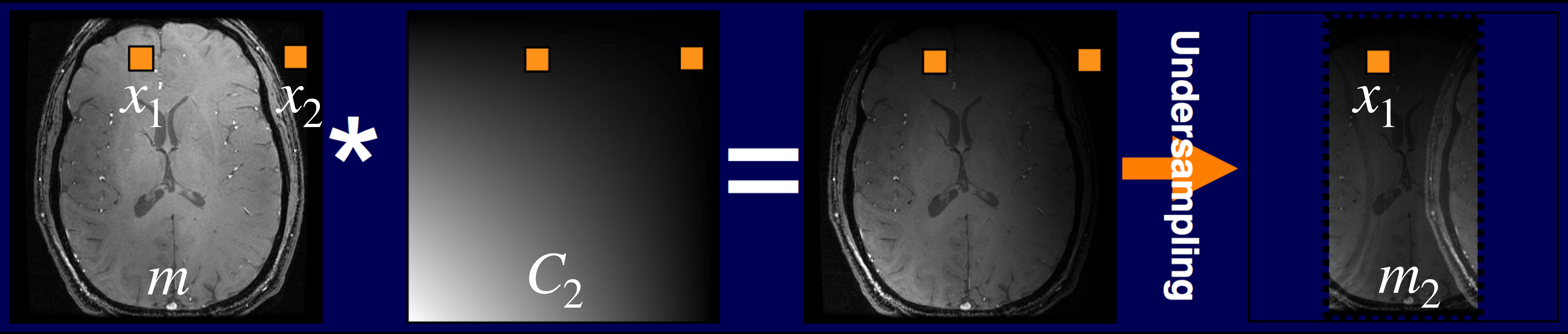
Pruessmann et al. MRM 1999
https://pubmed.ncbi.nlm.nih.gov/10542355/

Cartesian SENSE

$$m_1(\vec{x_1}) = C_1(\vec{x_1})m(\vec{x_1}) + C_1(\vec{x_2})m(\vec{x_2})$$



$$m_2(\vec{x_1}) = C_2(\vec{x_1})m(\vec{x_1}) + C_2(\vec{x_2})m(\vec{x_2})$$



$$\underbrace{\begin{pmatrix} m_1(\vec{x_1}) \\ m_2(\vec{x_1}) \\ \cdot \\ \cdot \\ \cdot \\ m_L(\vec{x_1}) \end{pmatrix}}_{\text{Aliased Images}} = \underbrace{\begin{pmatrix} C_1(\vec{x_1}) & C_1(\vec{x_2}) \\ C_2(\vec{x_1}) & C_2(\vec{x_2}) \\ \cdot & \\ \cdot & \\ \cdot & \\ C_L(\vec{x_1}) & C_L(\vec{x_2}) \end{pmatrix}}_{\text{Sensitivity at Source Voxels}} \underbrace{\begin{pmatrix} m(\vec{x_1}) \\ m(\vec{x_2}) \end{pmatrix}}_{\text{Source Voxels}} + \begin{pmatrix} n_1(\vec{x_1}) \\ n_2(\vec{x_1}) \\ \cdot \\ \cdot \\ \cdot \\ n_L(\vec{x_1}) \end{pmatrix}$$

OR

$$\underbrace{m_s}_{\text{L x 1}} = \underbrace{C}_{\text{L x 2}} \overset{\text{2 x 1}}{m} + \underbrace{n}_{\text{L x 1}}$$

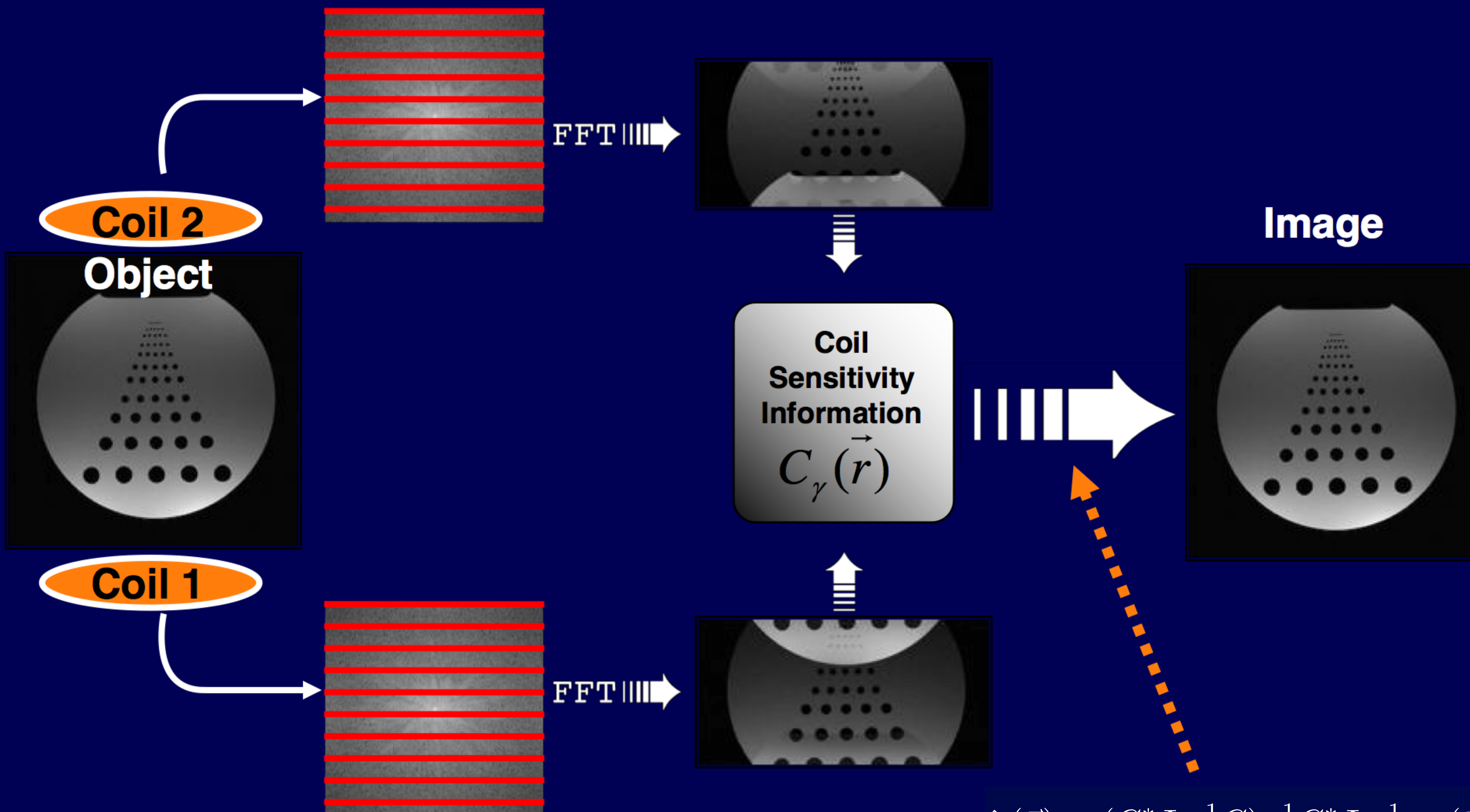
$$\hat{m}(\vec{x}) = \underbrace{(C^*\Psi^{-1}C)}_{2 \times 2}{}^{-1}\underbrace{C^*\Psi^{-1}}_{2 \times L}\underbrace{m_s(\vec{x})}_{L \times 1}$$

L aliased reconstruction resolves 2 image pixels

For an N x N image, we solve (N/2 x N) 2 x 2 inverse systems

For an acceleration factor R, we solve (N/R x N) R x R inverse systems

SENSE Reconstruction



$$\hat{m}(\vec{x}) = (C^*\Psi^{-1}C)^{-1}C^*\Psi^{-1}m_s(\vec{x})$$

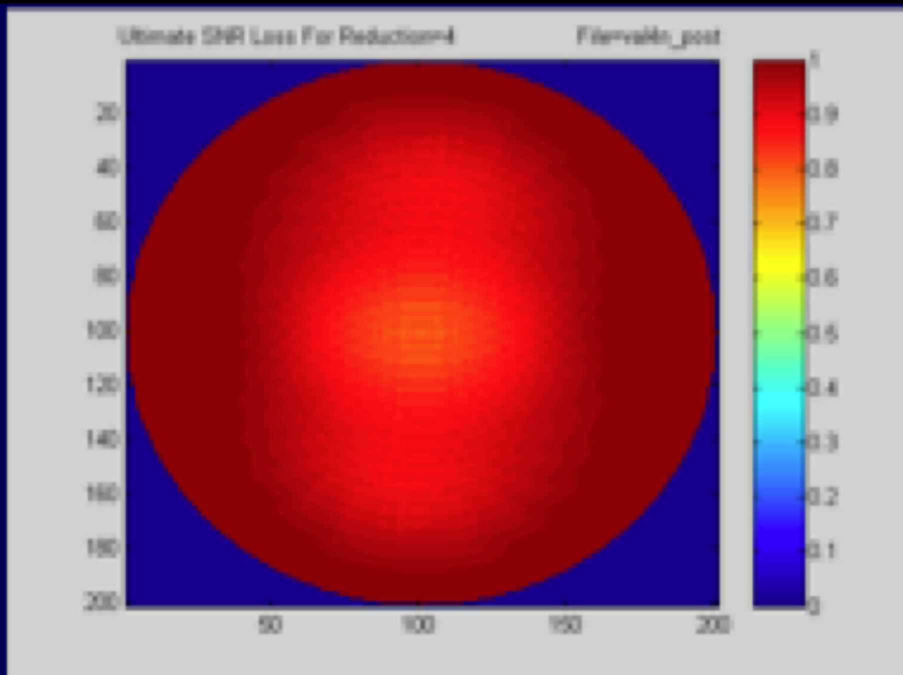
Unwrap fold over in image space

SNR Cost

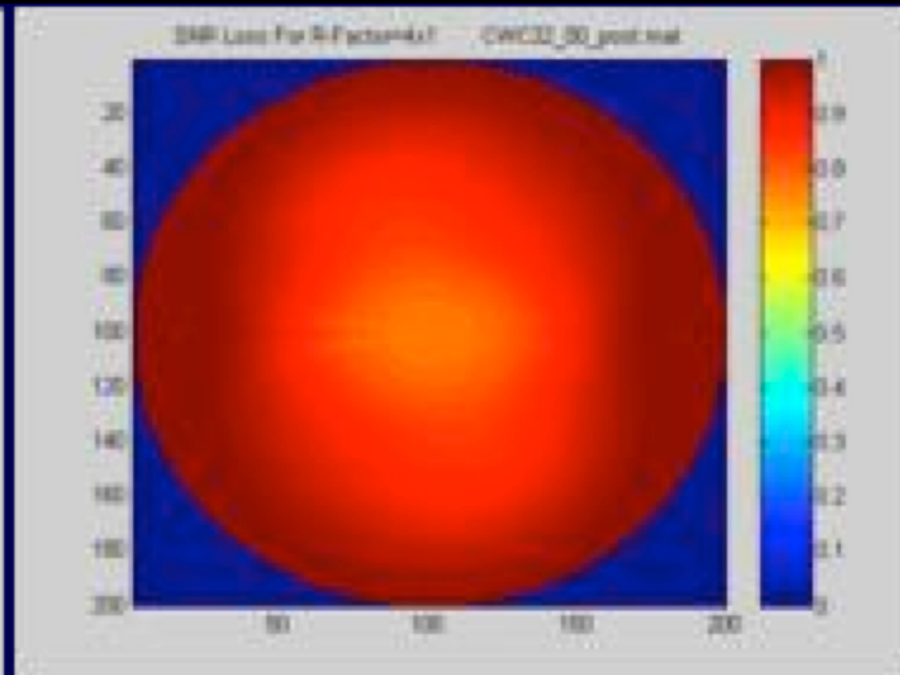
- How large can R be?
- Two SNR loss mechanisms
  - Reduced scan time
  - Condition of the SENSE decomposition
- SNR Loss

$$SNR_{SENSE} = \frac{SNR}{\underbrace{g}_{\text{Geometry Factor}}\underbrace{\sqrt{R}}_{\text{Reduced Scan Time}}}$$

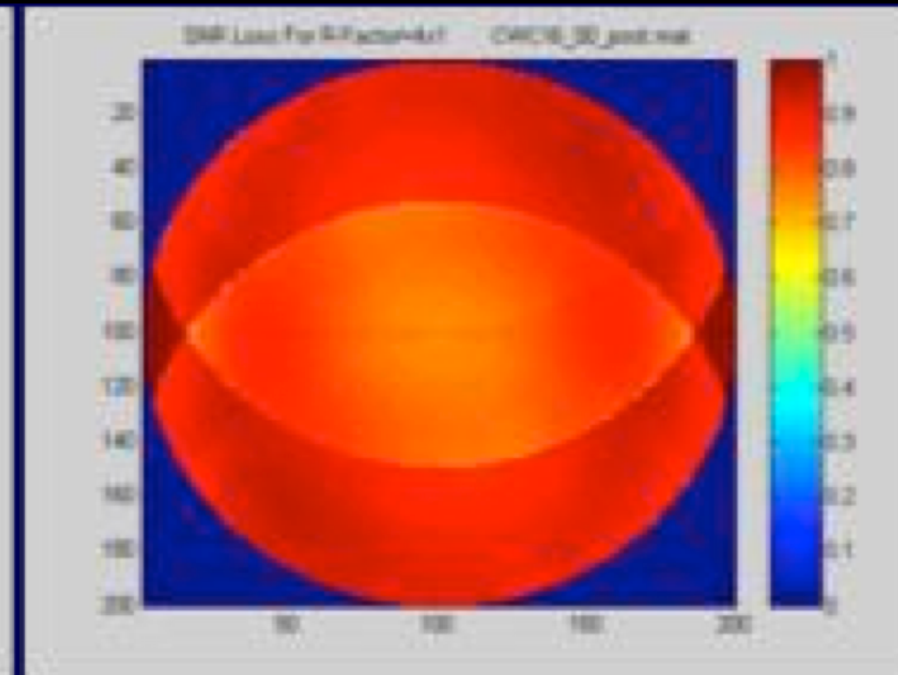
1/g-factor Map for R=4



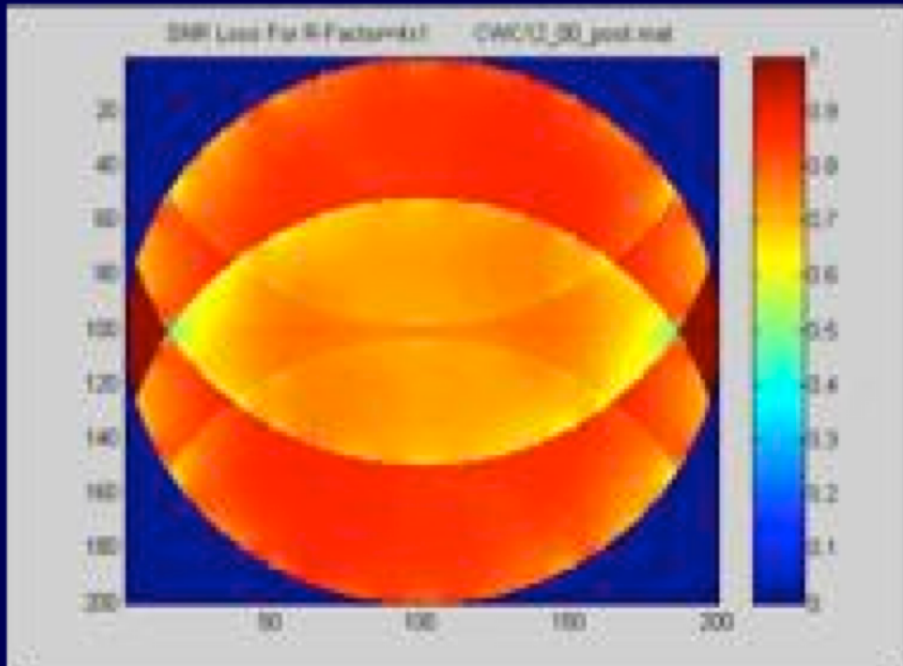
∞ elements



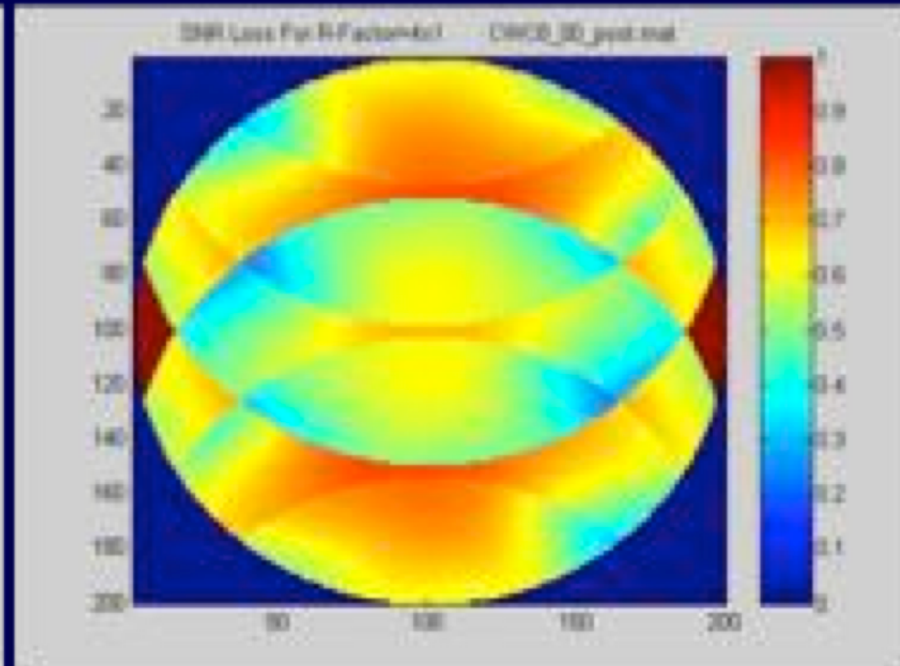
32 elements



16 elements



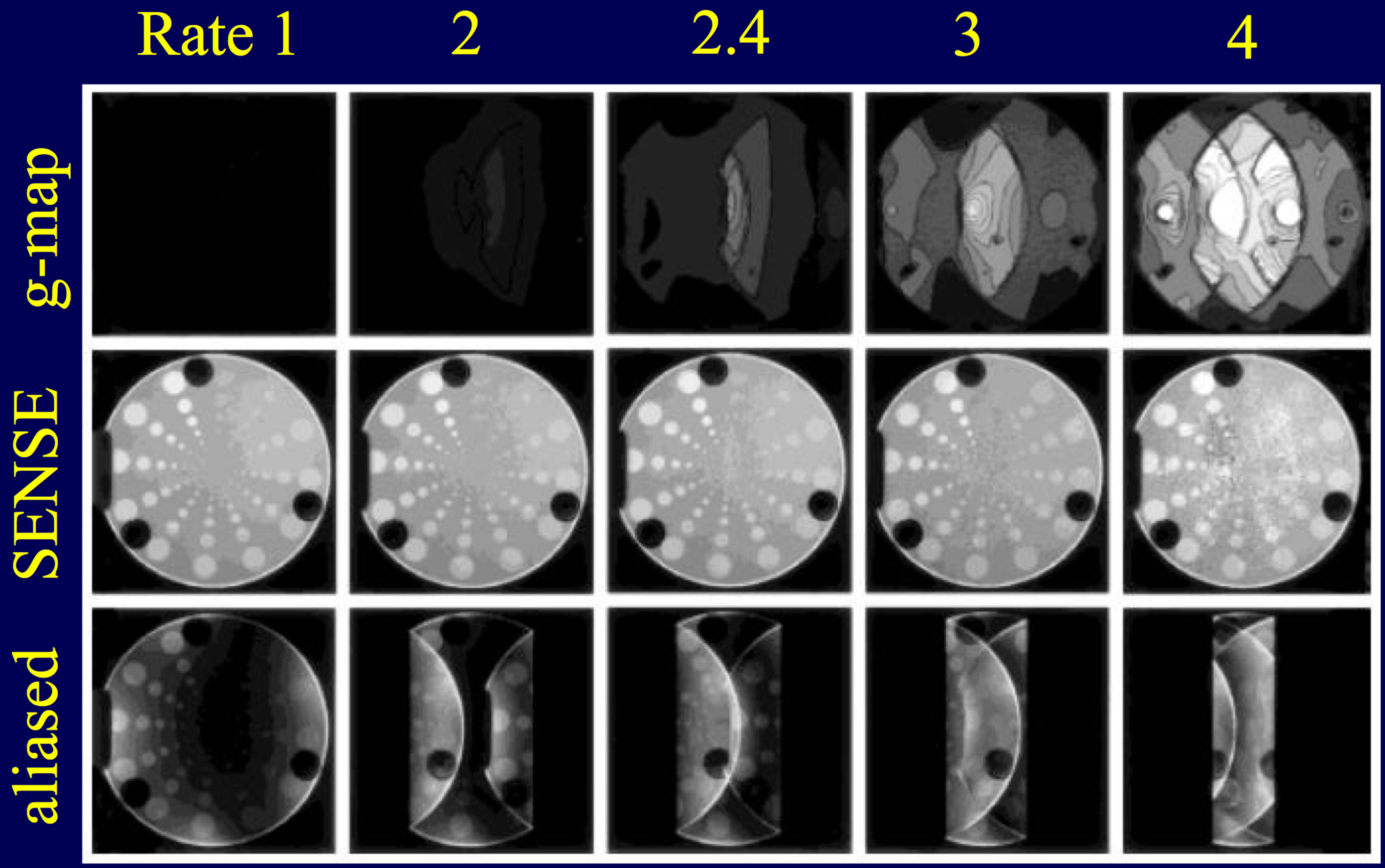
12 elements



8 elements

Relative SNR Scale

g-factor and its Impact on Images



Parallel Imaging

- Utilizes coil sensitivities to increase the speed of MRI (typical R=2-4)

- Can be combined with other techniques
  - Efficient pulse sequences
  - non-Cartesian trajectories
  - Constrained reconstruction

Parallel Imaging

- Cases for parallel imaging
  - Higher patient throughput
  - Real-time imaging/Interventional imaging
  - Motion suppression

- Cases against parallel imaging
  - Low SNR applications

Thanks!

- Interested in more?
- M219 upcoming lectures
- M229 in Spring
  - Fast imaging sequences
  - Fast sampling trajectories
  - Parallel imaging
  - Constrained reconstruction
  - Deep learning-based methods

Thanks!

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Holden H. Wu, Ph.D.

HoldenWu@mednet.ucla.edu

http://mrrl.ucla.edu/wulab