magnetic dipole

①

$$* \quad \vec{\tau} = \vec{\mu} \times \vec{B} \qquad \qquad \vec{S} = \vec{r} \times \vec{p}$$

$$\vec{\mu} = \gamma \vec{S}$$

$\vec{\mu}$: magnetic moment
$\vec{S}$: spin angular moment

$$\vec{\mu} = \gamma \cdot \vec{S}$$

$$\frac{d\vec{S}}{dt} = \frac{1}{\gamma} \cdot \frac{d\vec{\mu}}{dt} \longleftrightarrow \frac{d\vec{S}}{dt} = \vec{\mu} \times \vec{B}$$

$$\frac{1}{\gamma} \cdot \frac{d\vec{\mu}}{dt} = \vec{\mu} \times \vec{B}$$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \cdot \vec{B}$$

$$\vec{S} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times m\vec{v}$$

$$\frac{d\vec{S}}{dt} = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times m\frac{d\vec{v}}{dt}$$

$$= \vec{r} \times m\vec{a}$$
$$= \vec{r} \times \vec{F}$$
$$= \vec{\tau}$$

$$\vec{M} = \sum_{n=1}^{N_s} \vec{\mu}_n$$

$$\frac{d\vec{M}}{dt} = \sum_{n=1}^{N_s} \frac{\vec{\mu}_n}{dt} = \sum_{n=1}^{N_s} \left(\vec{\mu}_n \times \gamma \cdot \vec{B} \right)$$

$$= \vec{M} \times \gamma \vec{B}$$

Equation of motion

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

let $\vec{B} = B_0 \cdot \hat{k}$

$$\frac{d\vec{M}}{dt} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$\frac{dM_x}{dt} = M_y \cdot \gamma B_0 \quad , \quad \frac{M_y}{dt} = -M_x \cdot \gamma B_0 \quad , \quad \frac{dM_z}{dt} = 0$$

$$\frac{dM_x^2}{dt^2} = \frac{dM_y}{dt} \cdot \gamma B_0 \quad , \quad \frac{dM_y^2}{dt^2} = -\frac{dM_x}{dt} \cdot \gamma B_0$$

$$= -(\gamma B_0)^2 \cdot M_x \qquad\qquad = -(\gamma B_0)^2 M_y$$

Assume, $M_x(t) = A \cos(\gamma B_0 t) + B \sin(\gamma B_0 t)$

$$\vec{M}^0 = \begin{bmatrix} M_x^0 \\ M_y^0 \\ M_z^0 \end{bmatrix}$$

$M_x(t=0) = A = M_x^0$

$$\frac{dM_x}{dt} = -A\gamma B_0 \sin(\gamma B_0 t) + B\gamma B_0 \cos(\gamma B_0 t)$$

$$= M_y \cdot \gamma B_0$$

$M_y(t=0) = B = M_y^0$

Finally,

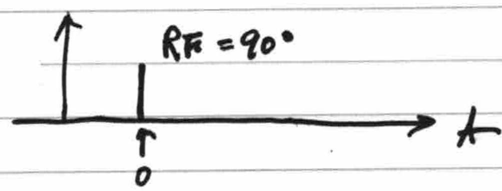
$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$
$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$
$$M_z(t) = M_z^0$$

or,

$$\begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\gamma B_0 t) & \sin(\gamma B_0 t) & 0 \\ -\sin(\gamma B_0 t) & \cos(\gamma B_0 t) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_z(\gamma B_0 t)} \begin{bmatrix} M_x^0 \\ M_y^0 \\ M_z^0 \end{bmatrix}$$

$$\omega = \gamma B_0$$
$$\vec{\omega} = \gamma \vec{B_0} = \gamma B_0 \hat{k}$$

Ex1>



$$\vec{M}(0_-) = \begin{bmatrix} 0 \\ 0 \\ M_z^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix}$$

$$\vec{M}(0_+) = \begin{bmatrix} M_0 \\ 0 \\ 0 \end{bmatrix}$$ ← Immediately after a 90° RF pulse

$$\vec{M}(t) = R_z(\gamma B_0 t) \begin{bmatrix} M_0 \\ 0 \\ 0 \end{bmatrix}$$ ← $\vec{M}(t)$ after the 90° RF pulse

$$\left.\begin{aligned} M_x(t) &= M_0 \cos(\gamma B_0 t) \\ M_y(t) &= -M_0 \sin(\gamma B_0 t) \end{aligned}\right] \leftarrow \text{free precession}$$
$$M_z(t) = 0$$