– Signal equation $s(t)$

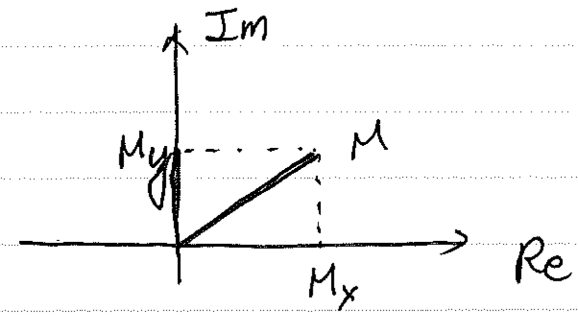
→ M_z does not provide any signal in MRI

→ The only thing we can detect in MRI is time-varying magnetization in the transverse plane. (M_x & M_y)

→ M_{xy} ⇒ FID $s_r(t)$

in a simplified notation

$$M = M_x + i M_y \quad ; \text{ complex rep of } M_{xy}$$



length $|M|$

phase $\angle M$

$$\frac{dM}{dt} = \frac{dM_x}{dt} + i\frac{dM}{dt}$$

$$= -\left(i\gamma B_0 + \frac{1}{T_2}\right) M \qquad (\text{see eq. } 5.8 - 5.12)$$

$$= -\left(i\omega_0 + \frac{1}{T_2}\right) M$$

rotation on the complex plane

decay

②

$\Rightarrow$ solution ; $M(t) = M_0 \underbrace{e^{-i\omega_0 t}}_{\text{precession}} \underbrace{e^{-t/T_2}}_{T_2 \text{ relaxation}}$

With $\vec{G}(t)$, spatial & time dimensions need to be considered

$$M(\vec{r}, t) = M_x(x, y, z, t) + i M_y(x, y, z, t)$$

$$T_2(\vec{r})$$

$$\vec{B}(\vec{r}, t) = [B_0 + \Delta B(\vec{r}, t)] \hat{k}$$

$\Rightarrow$ solution ;

$$M(\vec{r}, t) = M(\vec{r}, 0) e^{-i\omega_0 t} e^{-t/T_2(\vec{r})} \underbrace{e^{-i\int_0^t \Delta\omega(\vec{r}, t)dt}}_{\text{time dependent phase factor}}$$

③

$$M(\vec{r},t) = \overbrace{M(\vec{r},0)\, e^{-i\omega_0 t} e^{-t/T_2(\vec{r})}}^{M_a} \cdot e^{-i\int_0^t \Delta\omega(\vec{r},\tau)\,d\tau}$$

$$\Delta\omega(\vec{r},t) = \gamma \Delta B(\vec{r},t)$$

$$\vec{B}(\vec{r},t) = \left(B_0 + \Delta B(\vec{r},t) \right) \hat{k}$$

* $\Delta\omega(\vec{r},t)$

1) linear gradient

$$\Delta\omega(\vec{r},t) = \gamma\, \vec{G} \cdot \vec{r} = \gamma \left(G_x \cdot x + G_y \cdot y + G_z \cdot z \right)$$

$$M(\vec{r},t) = M_a\, e^{-i 2\pi \frac{\gamma}{2\pi} \vec{G} \cdot \vec{r} \cdot t}$$

2) time-varying

$$\Delta\omega(\vec{r},t) = \gamma\, \vec{G}(t) \cdot \vec{r}$$

$$M(\vec{r},t) = M_a \cdot \underbrace{e^{-i 2\pi \frac{\gamma}{2\pi} \int_0^t \vec{G}(\tau)\vec{r}\, d\tau}}_{\text{spatially varying phase due to } \vec{G}(\vec{r},t)}$$