Imaging Principles

M219 - Principles and Applications of MRI

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1/29/2025

Course Overview

- 2025 course schedule
  - https://mrrl.ucla.edu/pages/m219_2025
- Assignments
  - Homework #1 due today by 5pm

- TA office hours, Mon 4-6pm
- Office hours, Fri 10-11am

Combined B_0 and Gradient Fields

- Gradients contribute to the net B-field, but only along the z-direction

$$(\overrightarrow{G} \cdot \vec{r})\ \hat{k} = (G_x \cdot x + G_y \cdot y + G_z \cdot z)\ \hat{k}$$

$$\overrightarrow{B}(\vec{r}, t) = (B_0 + \overrightarrow{G}(t) \cdot \vec{r})\ \hat{k}$$

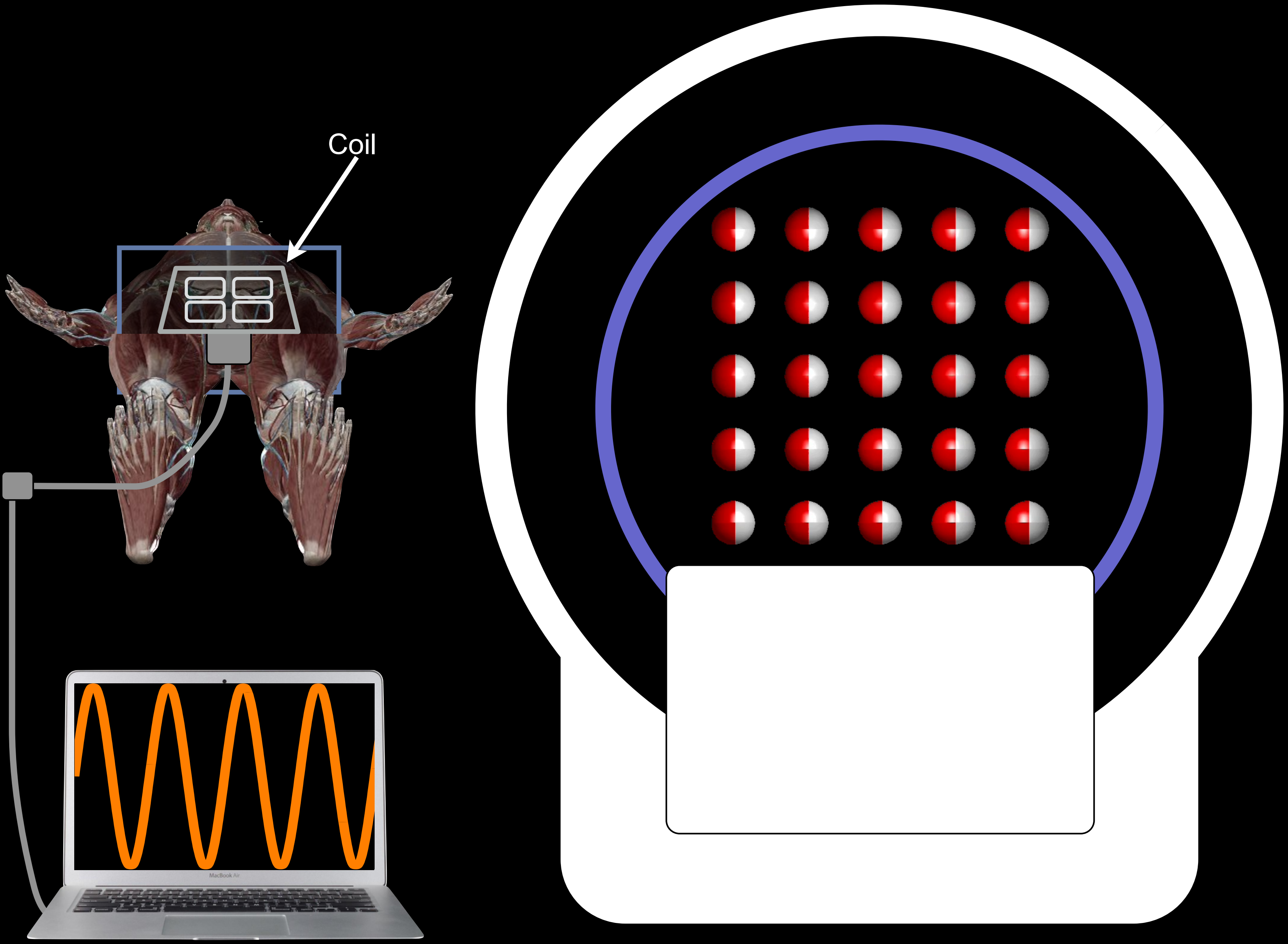
- Each gradient coil can be activated independently and simultaneously

B-Field Assumptions in MRI

- **B_0-field is:**
  - Perfectly uniform over space.
    - “B_0 homogeneity”
  - Perfectly stable with time.
- **B_1-field is:**
  - Perfectly uniform over space.
    - “B_1 homogeneity”
  - Temporally modulated exactly as specified.
- **Gradient Fields are:**
  - Perfectly linear over space.
    - “Gradient linearity”
  - Temporally modulated exactly as specified

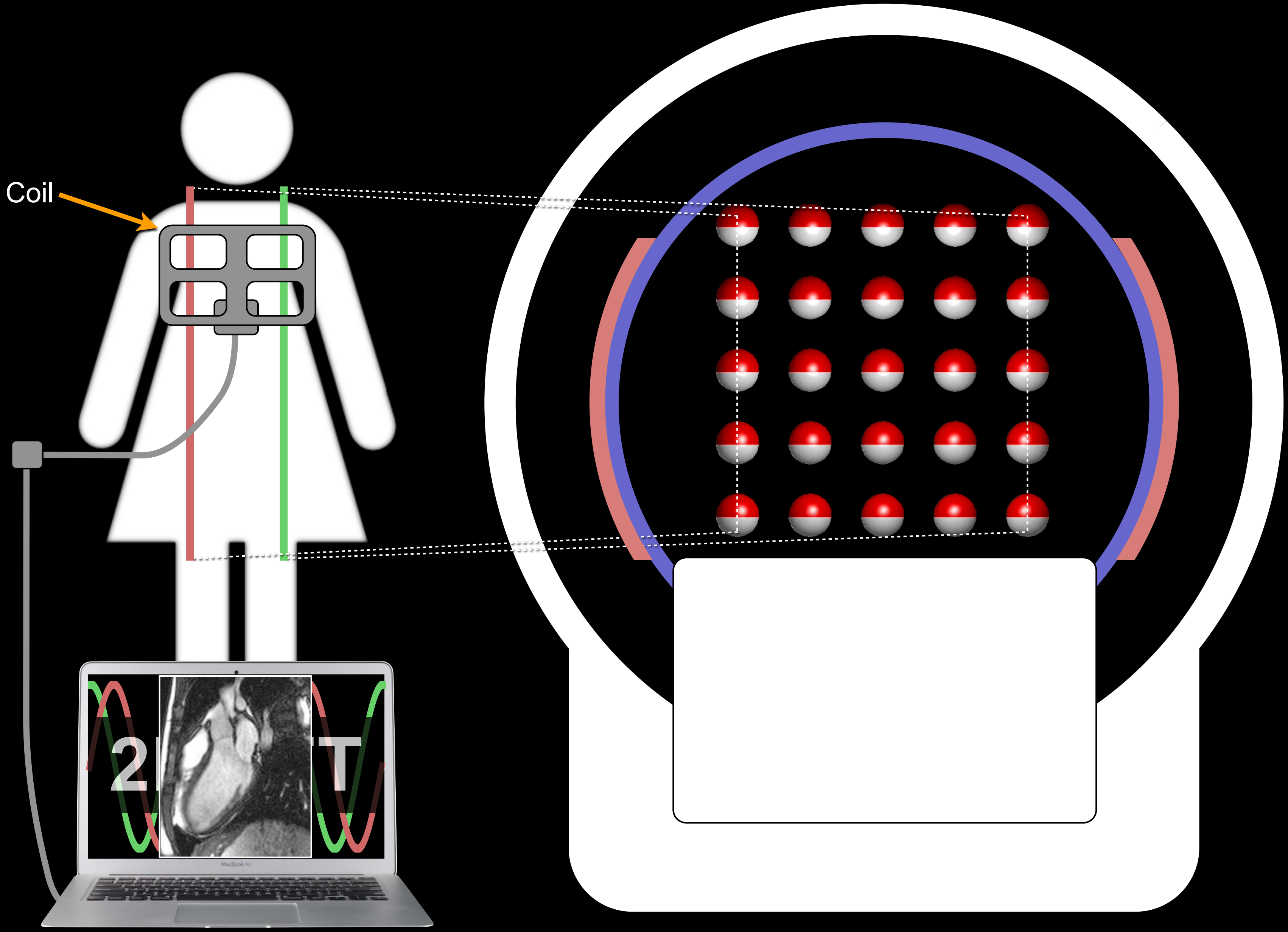
How do we measure M_{xy}?

Faraday’s Law of Induction



Precessing spins *induce* a current in a nearby coil.

Faraday's Law of Induction



The trick is to encode spatial information and image contrast in the echo.

Basic Detection Principles

We get here

$$S(t) = \int_{\mathrm{object}} M_{xy}(r,0)e^{-i\gamma\Delta B(r)t}dr$$

From Here

$$V(t) = -\frac{\partial\Phi(t)}{\partial t} = -\frac{\partial}{\partial t}\int_{object}\vec{B}(\vec{r})\cdot\vec{M}(\vec{r},t)\,d\vec{r}$$

with 25 pages of Math!

Basic Detection Principles

$$S(t) = \int_{\text{object}} M_{xy}(\boldsymbol{r},0)e^{-i\gamma\Delta B(\boldsymbol{r})t}d\boldsymbol{r}$$

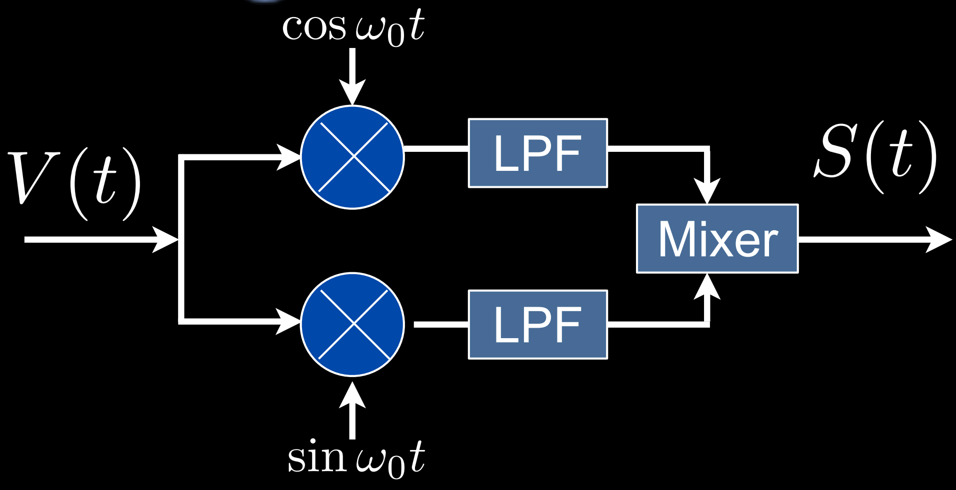
Observations

Detected signal is the vector sum of all transverse magnetizations in the "rotating frame" within the imaging volume.

The Larmor frequency precession (Lab frame rotation) is necessary for detection, although only the baseband signal matters for imaging

Signals in MRI

$V(t)$



$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

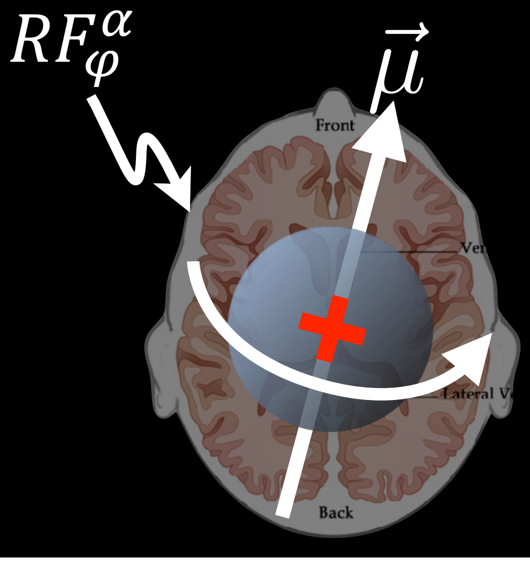
$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

$S(\vec{k})$

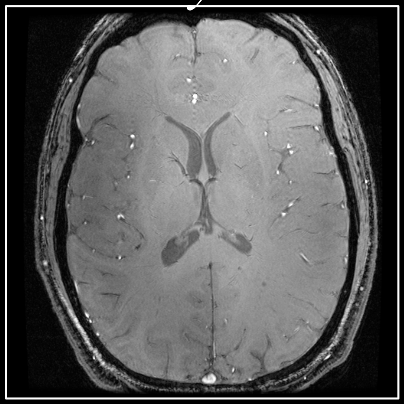
We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?

$M_{xy}(\vec{r}, t)$

RF_φ^α $\vec{\mu}$

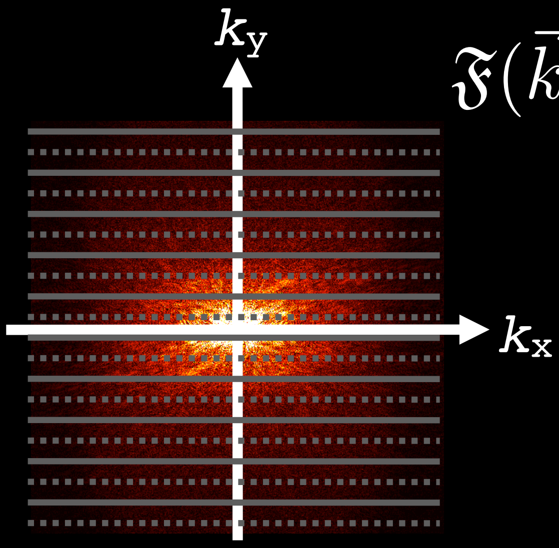


$M_{xy}(\vec{r})$

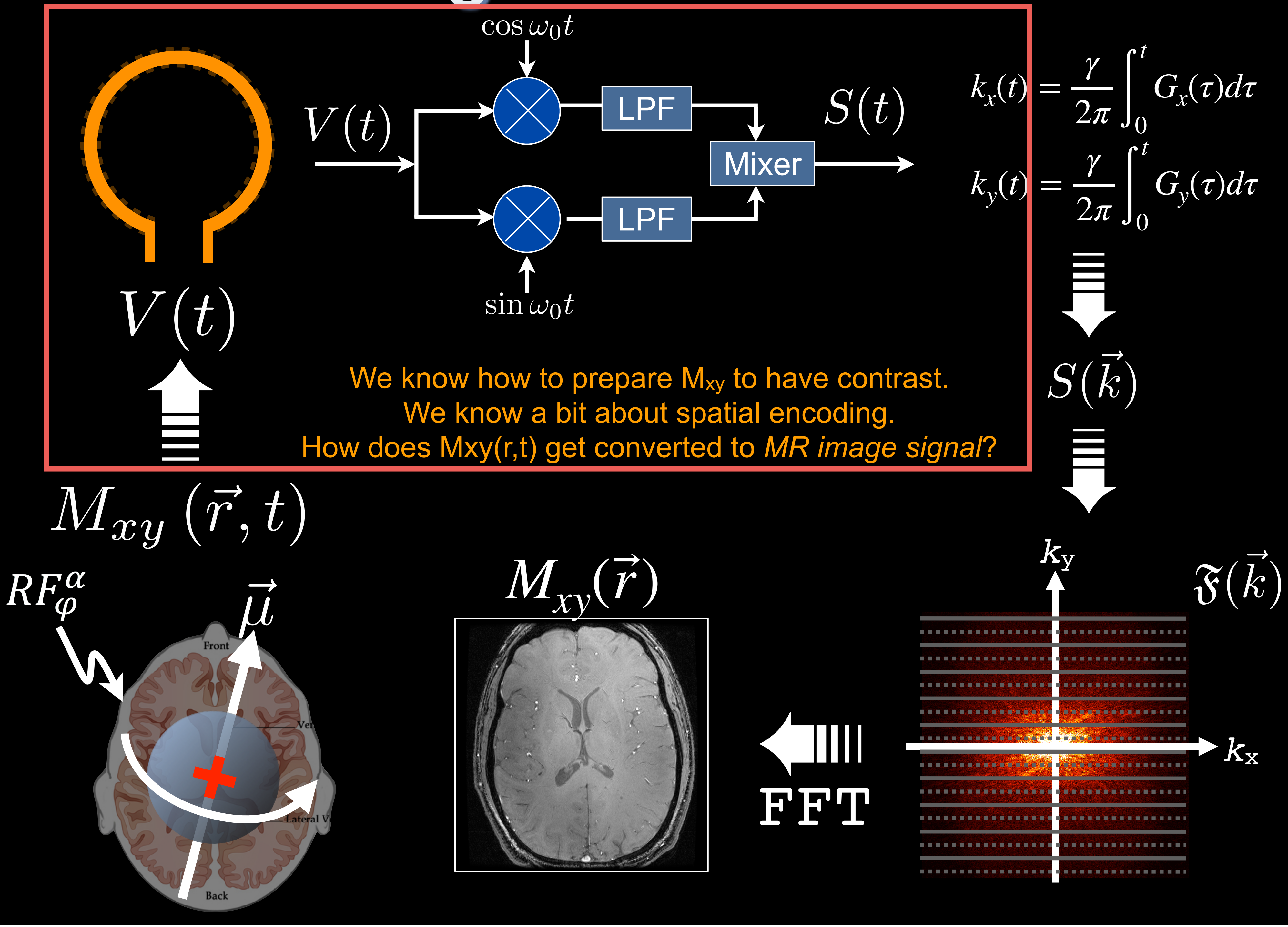


FFT

$\mathfrak{F}(\vec{k})$



Signals in MRI



$V(t)$

$V(t)$ LPF Mixer LPF $S(t)$

$\cos\omega_0 t$

$\sin\omega_0 t$

$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

$S(\vec{k})$

We know how to prepare M_{xy} to have contrast.
We know a bit about spatial encoding.
How does Mxy(r,t) get converted to *MR image signal*?

$M_{xy}\left(\vec{r}, t\right)$

RF_{φ}^{α}

$\vec{\mu}$

$M_{xy}(\vec{r})$

FFT

$\mathfrak{F}(\vec{k})$

k_y

k_x

To the Board

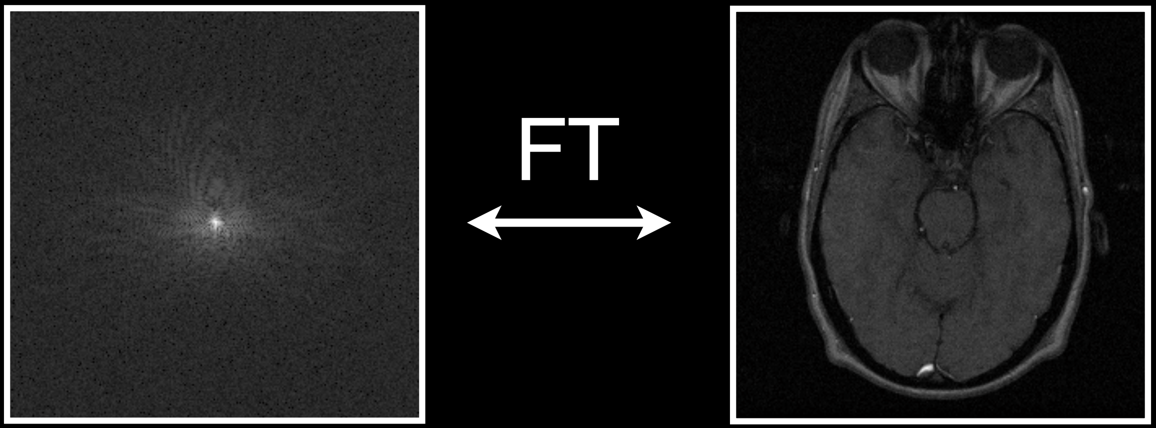
MR Signal Equation

$$s(t) = \int_x \int_y M(x,y) e^{-i2\pi(k_x(t)\cdot x + k_y(t)\cdot y)} dx\,dy$$

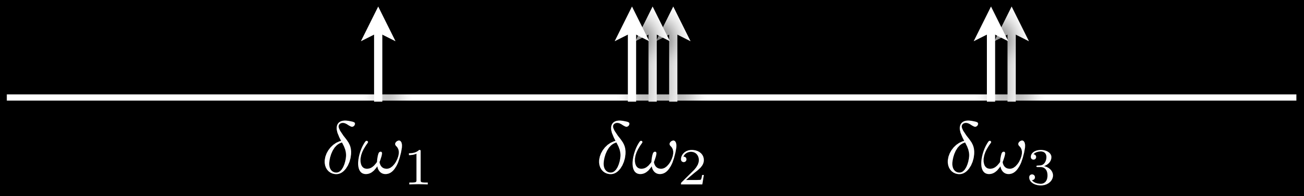
$$k_x(t) = \frac{\gamma}{2\pi}\int_0^t G_x(\tau)d\tau \quad k_y(t) = \frac{\gamma}{2\pi}\int_0^t G_y(\tau)d\tau$$

$$s(t) = m(k_x(t), k_y(t))$$

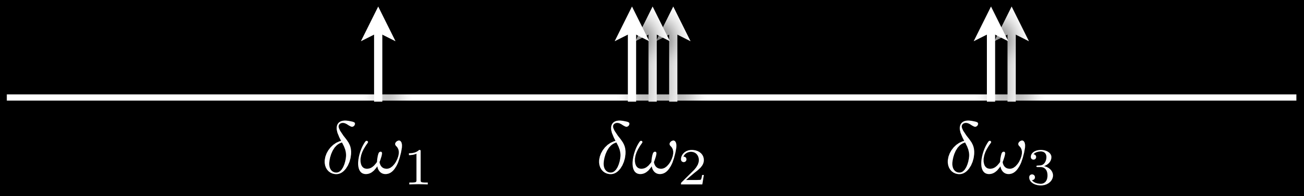
$$m = \mathcal{FT}(\,M(x,y)\,)$$



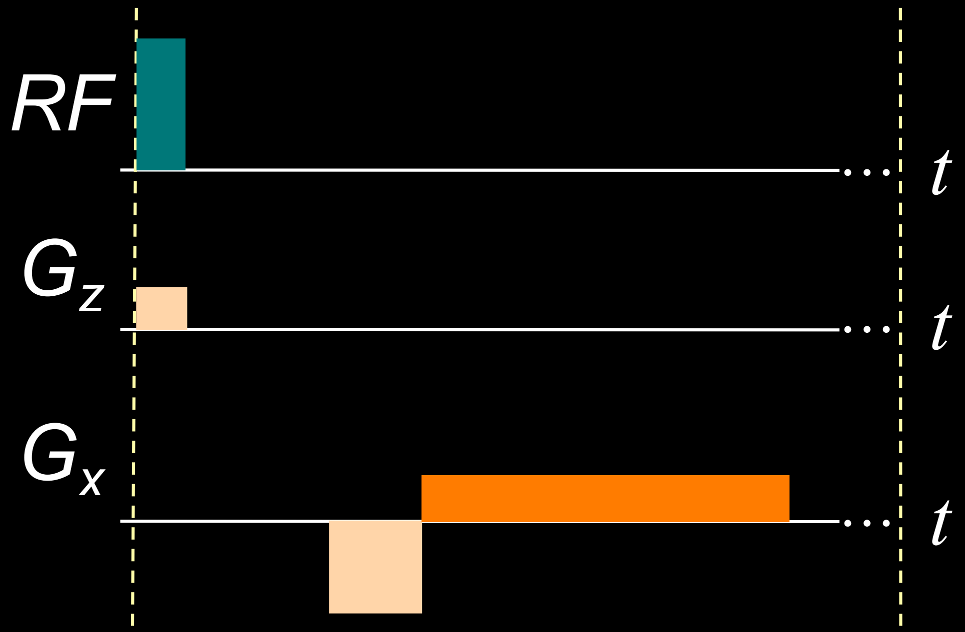
1D Imaging



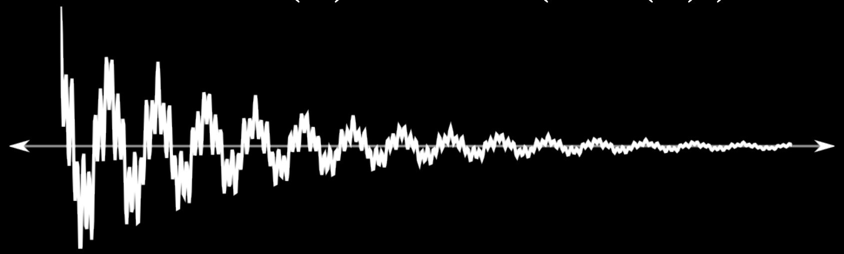
1D Imaging



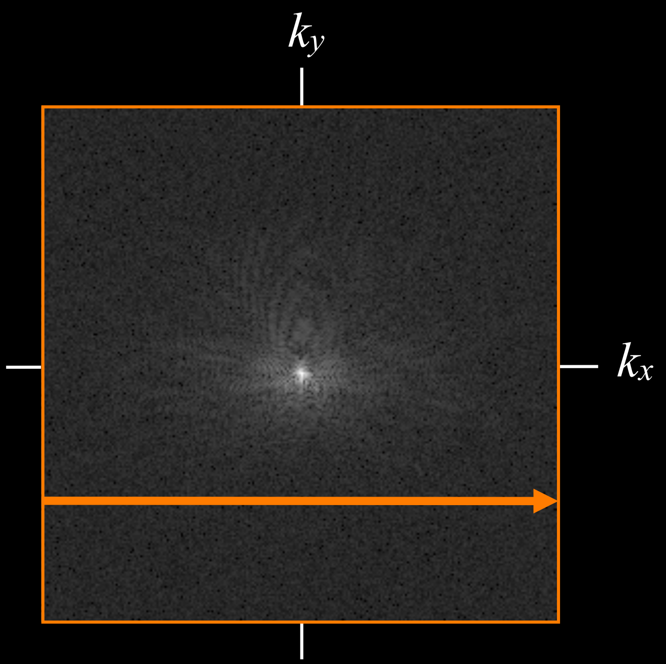
Pulse Sequence Diagram



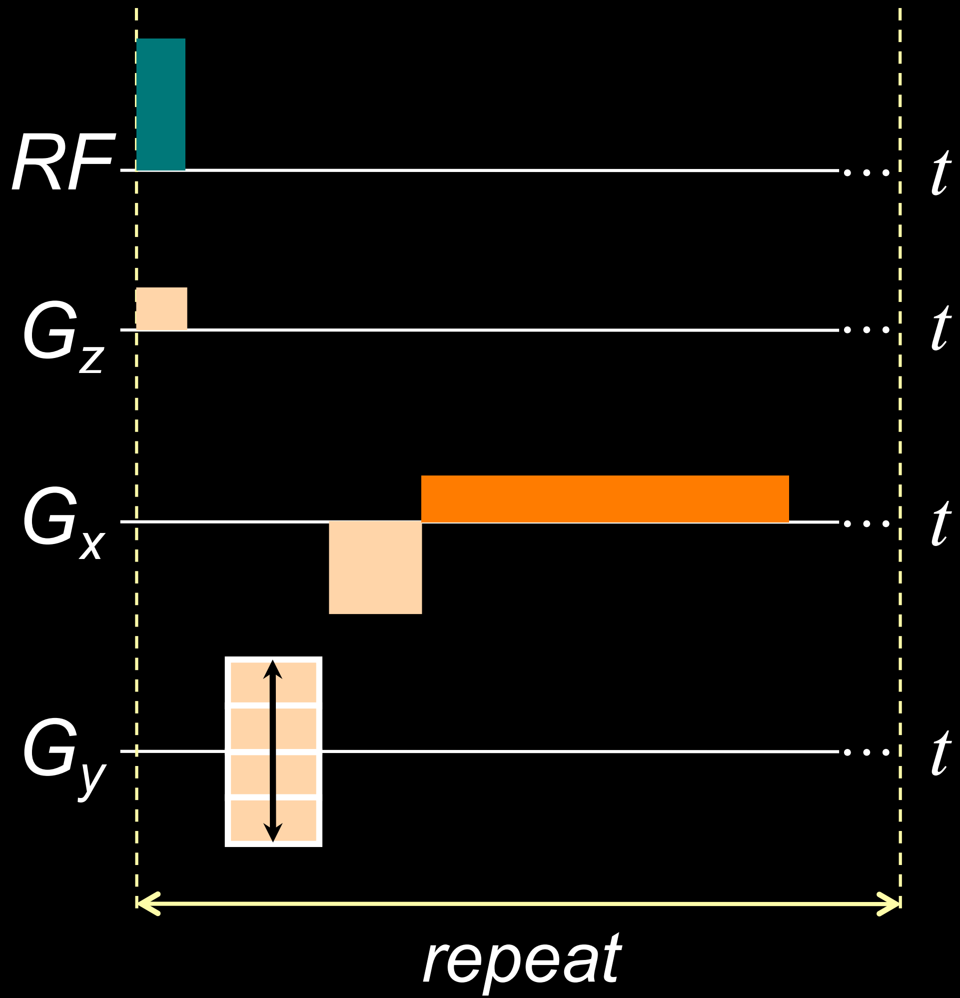
$$s(t) = m(k_x(t))$$



2D Imaging

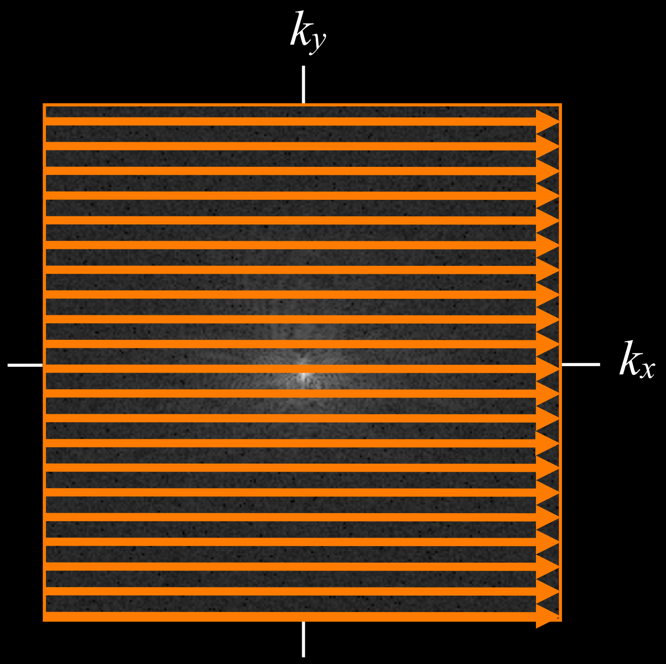


$$s(t) = m(\,k_x(t), k_y(t)\,)$$



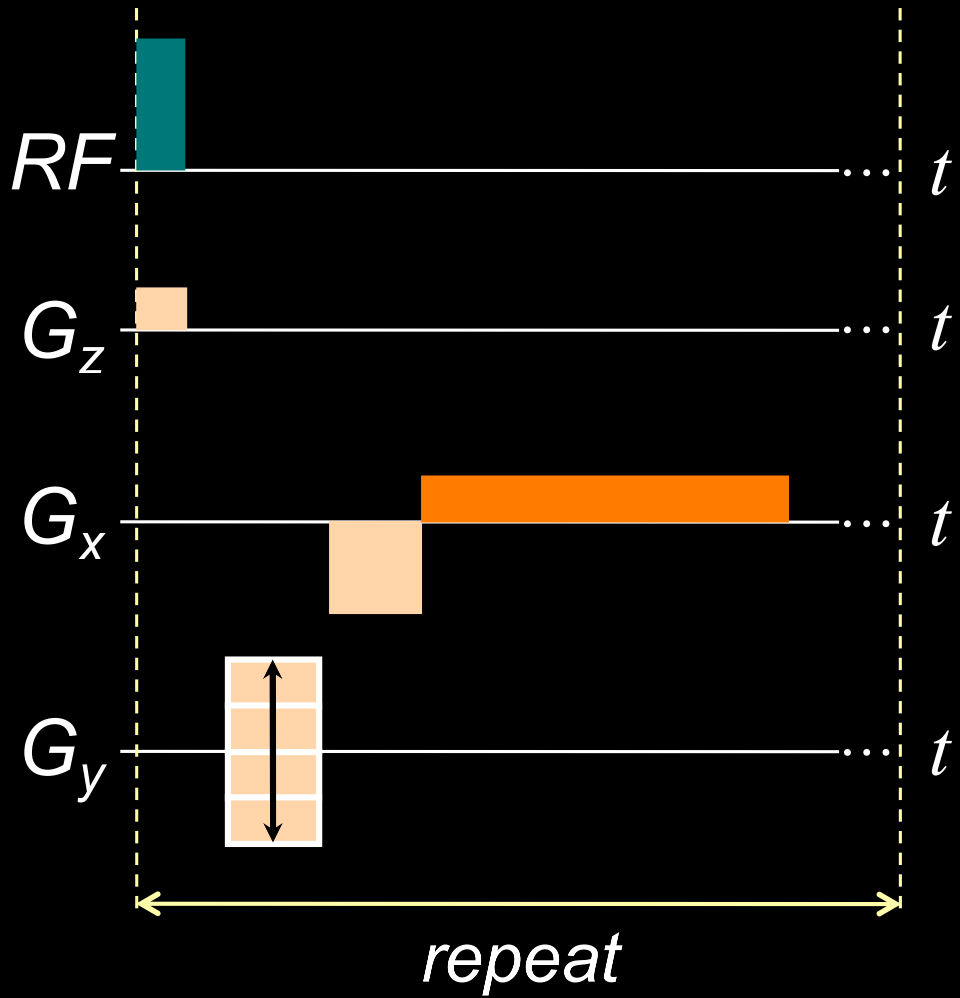
Pulse Sequence Diagram

2D Imaging

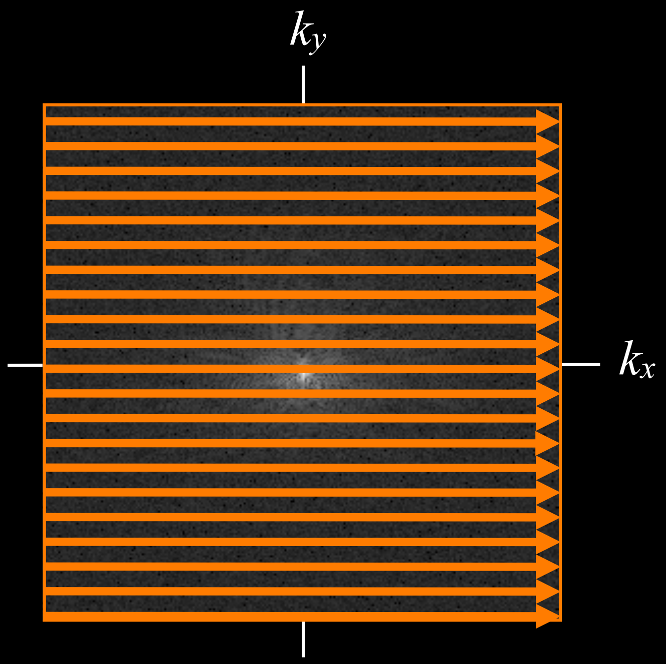


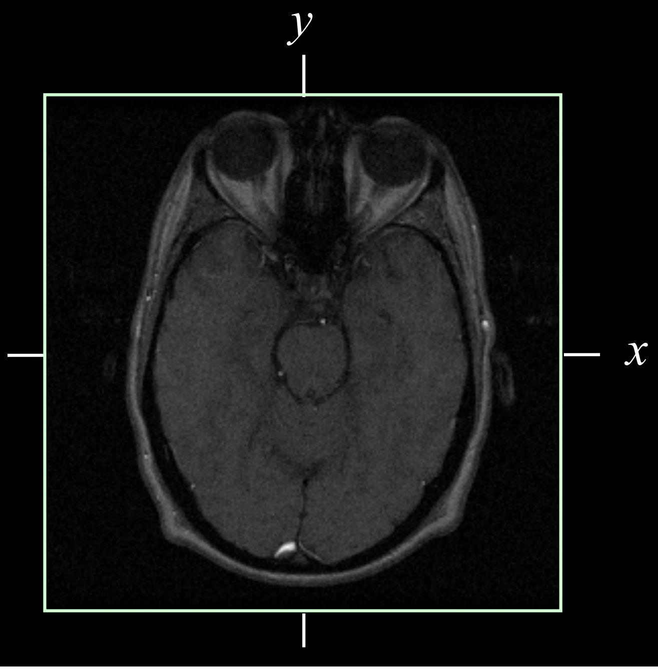
$$s(t) = m(\,k_x(t), k_y(t)\,)$$

Pulse Sequence Diagram

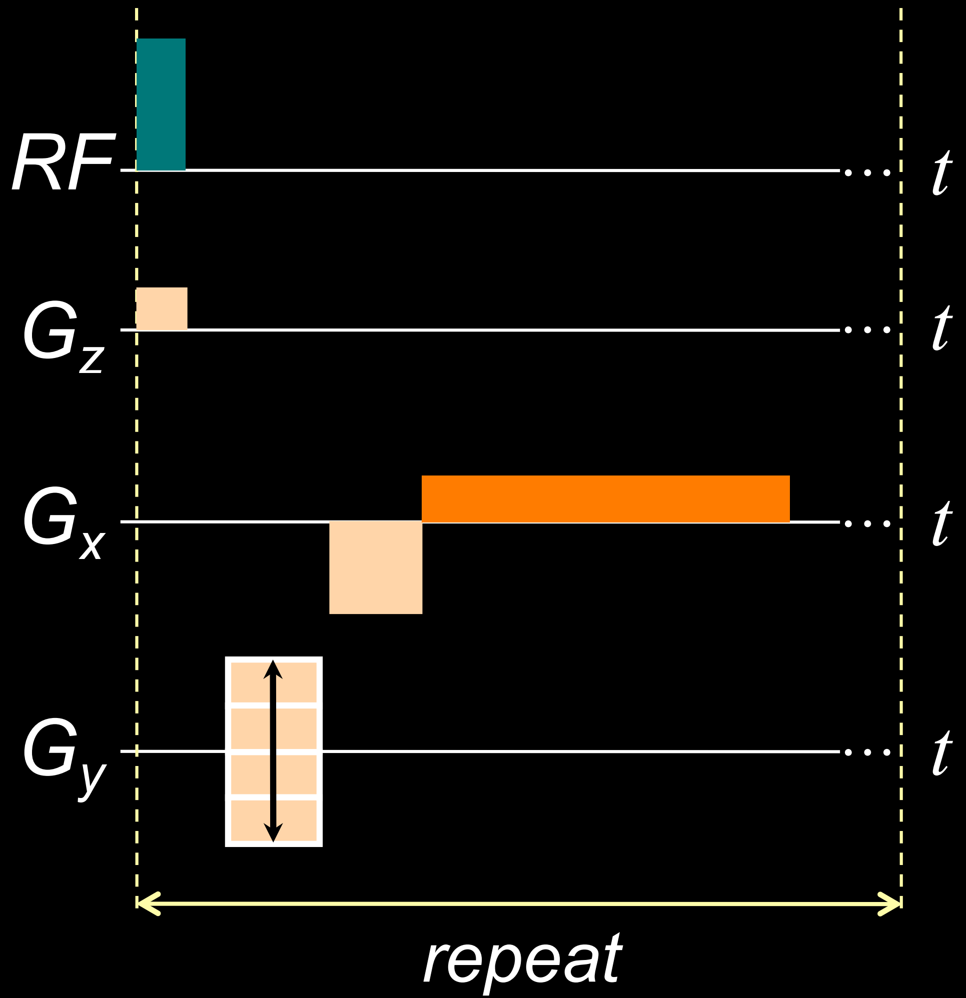


2D Imaging



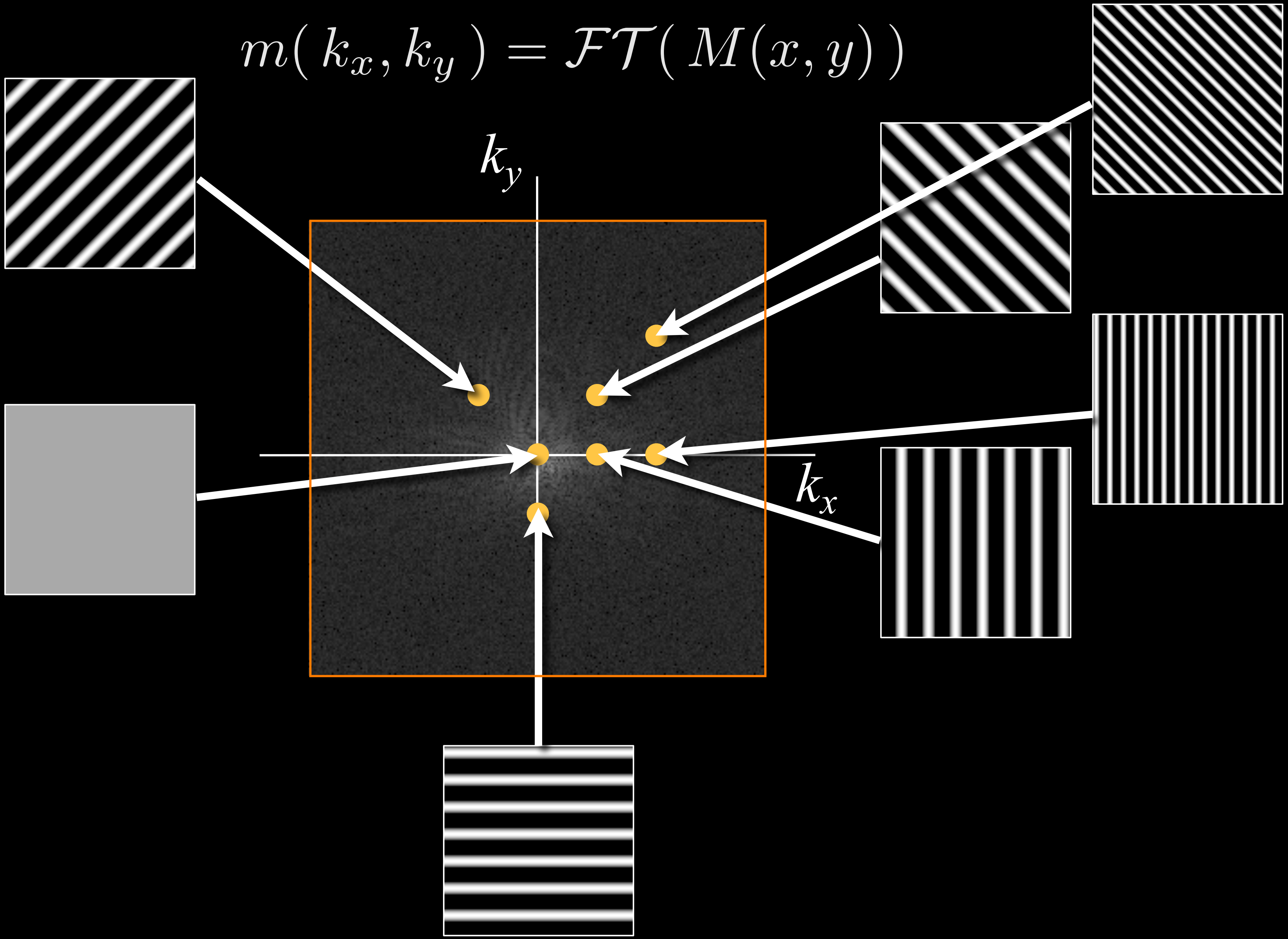


Pulse Sequence Diagram



2D k-Space: MRI Data

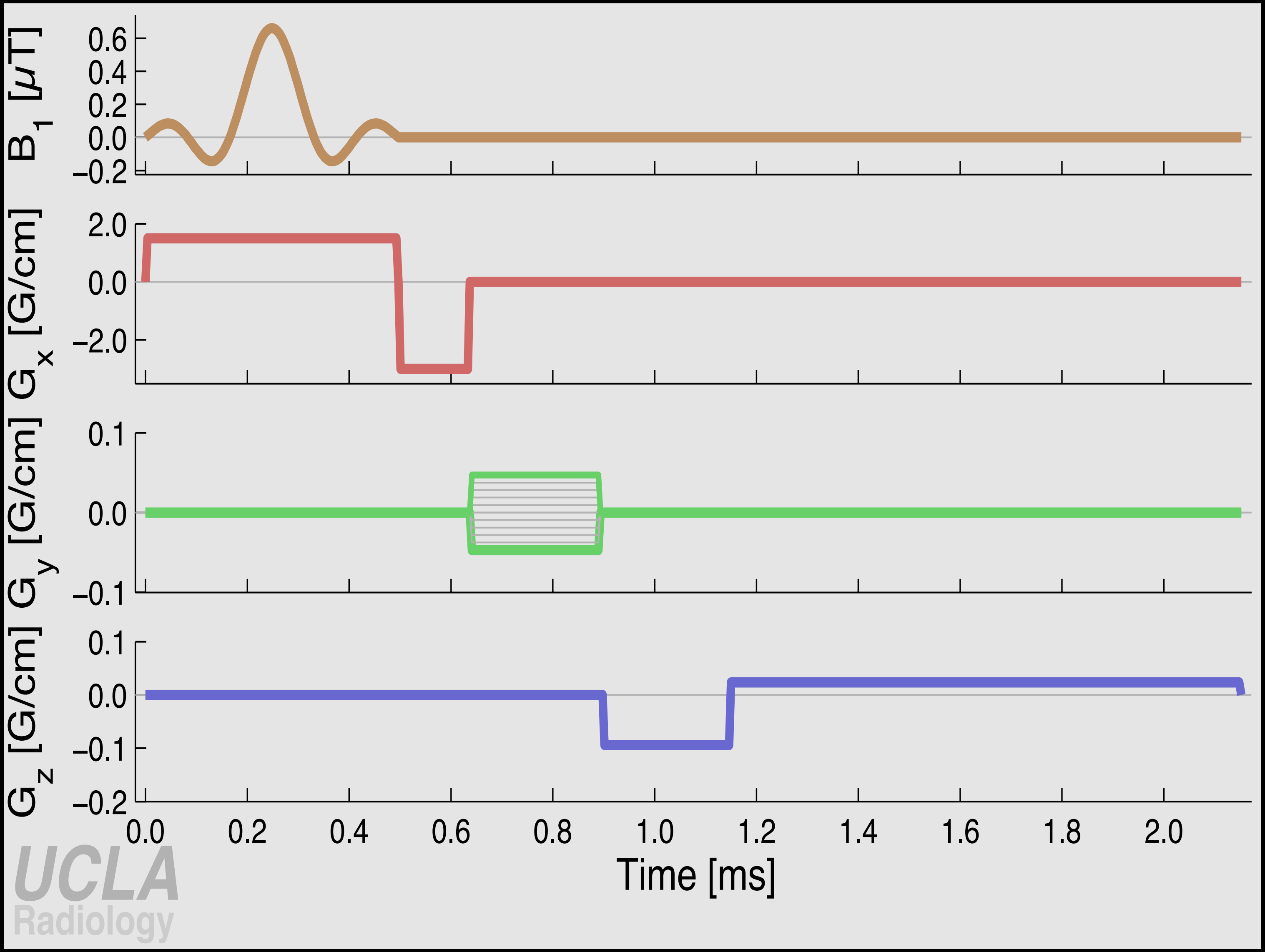
$$m(k_x, k_y) = \mathcal{FT}(M(x,y))$$



Spatial Encoding

- **Three key steps:**
  - **Slice selection**
    - You have to pick slice!
  - **Phase Encoding**
    - You have to encode 1 of 2 dimensions within the slice.
  - **Frequency Encoding (aka *readout*)**
    - You have to encode the other dimension within the slice.

3 Steps for Spatial Localization



Slice Selection

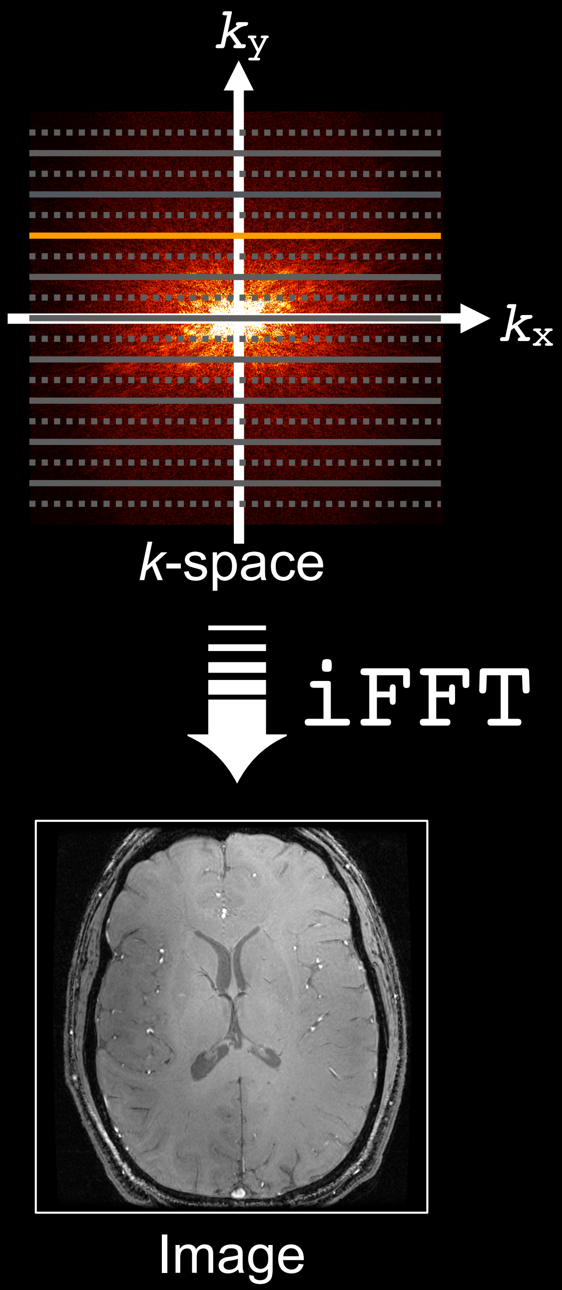
Phase Encoding

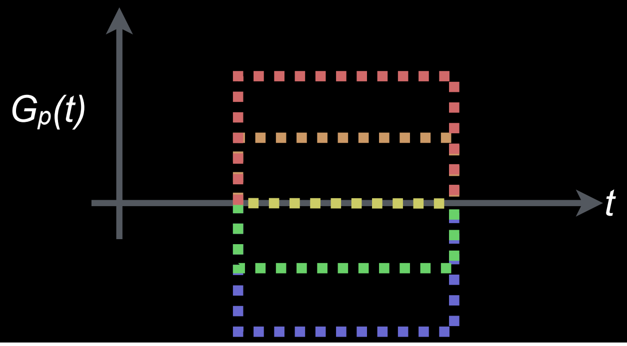
Frequency Encoding

Pulse Sequence Diagram - Timing diagram of the RF and gradient events that comprise an MRI pulse sequence.

Phase Encoding

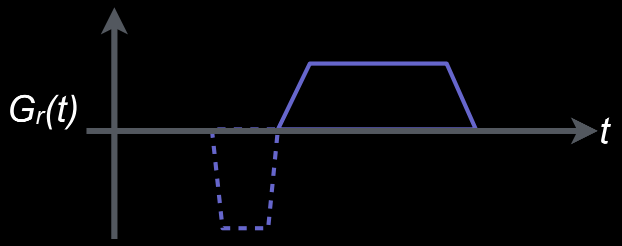
- **Consists of:**
  - **Phase encoding gradient**
    - **Magnitude changes with each TR**
    - **Can be played with other gradients**
      - **Crushers, Slice-selection rephaser, readout dephasing**
- **Used with Cartesian imaging**
- **After excitation, before readout**
- **Adds linear spatial variation of phase**
- **Phase encode in**
  - **one direction for 2D imaging**
  - **two directions for 3D imaging**
- **Only one PE step per echo**



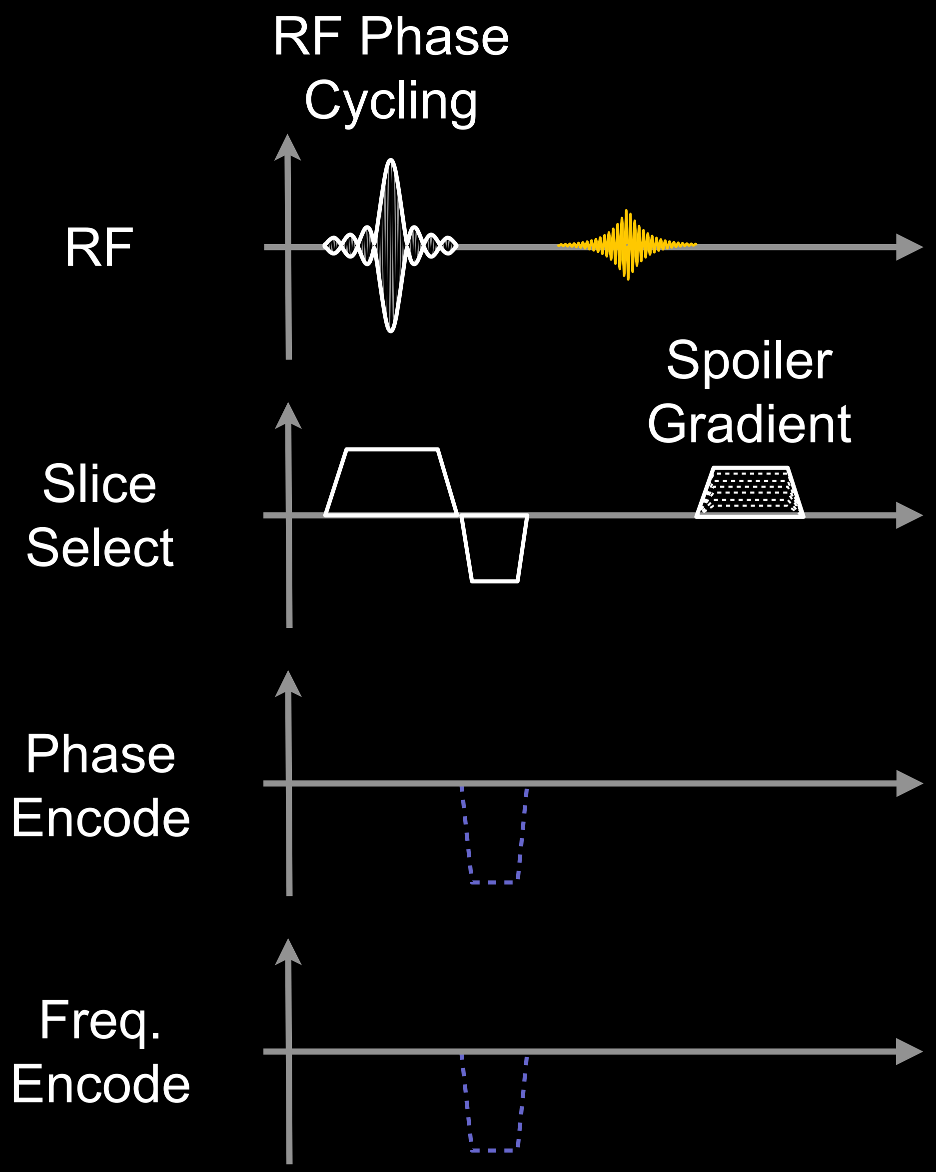


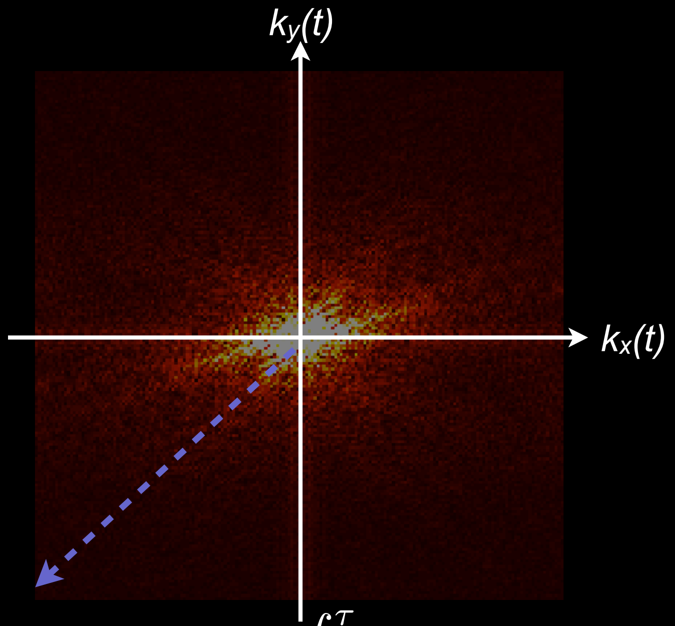
Frequency Encoding

- **Consists of:**
  - **Frequency encoding gradient**
    - **Constant magnitude for Cartesian imaging**
  - **No simultaneous**
    - **RF (B_1)**
    - **Other gradients**
      - **phase encoding, slice encoding, crushers**
  - **Readout pre-phasing gradient**
    - **Prepares spin phase so peak echo amplitude occurs at middle of readout (TE)**
    - **AKA “readout de-phasing gradient”**
- **Adds linear spatial variation of frequency**
- **Helps form an *echo***

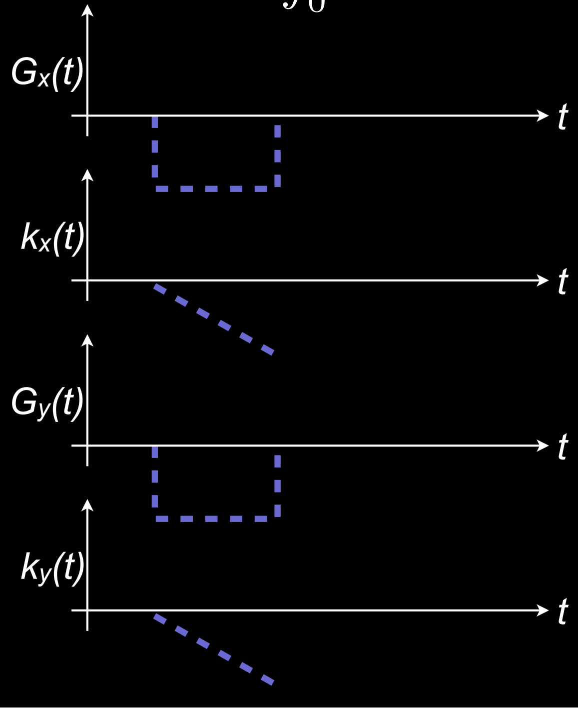


Where am I in *k*-space?



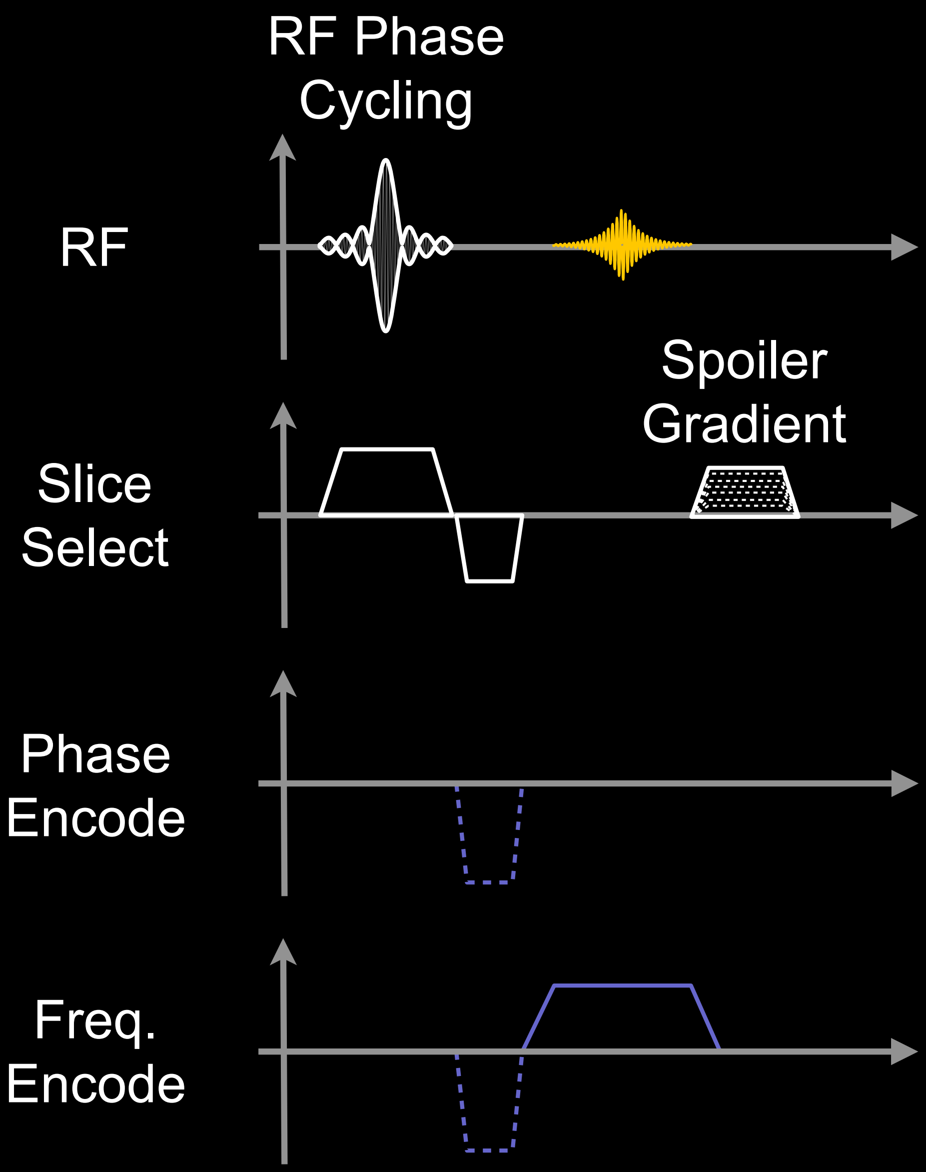


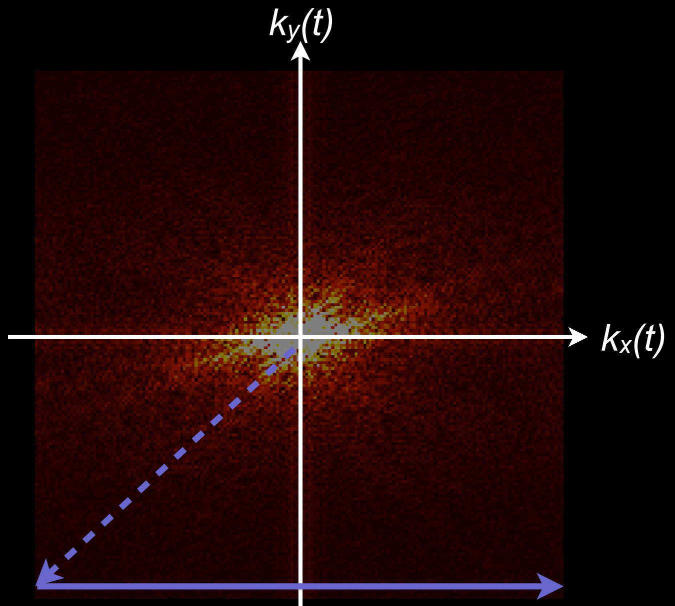
$$\vec{k}\,(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}\,(t)\,d\tau$$



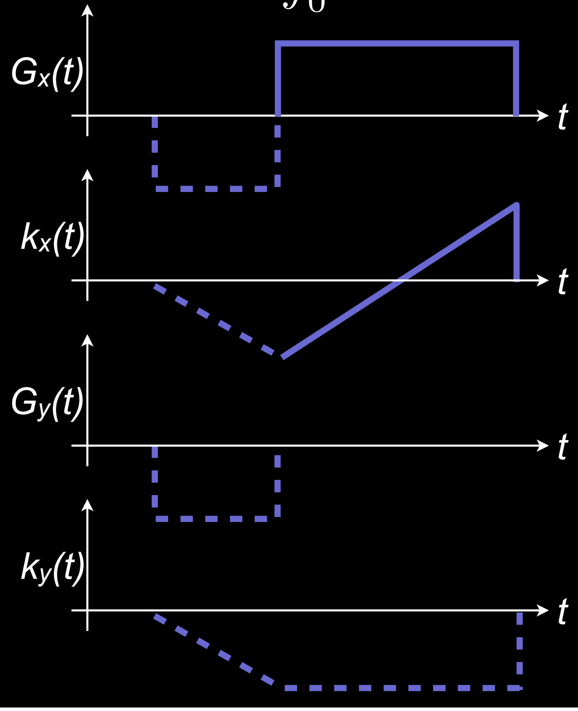
One phase encoded echo is acquired per TR.

Where am I in *k*-space?





$$\vec{k}(t) = \frac{\gamma}{2\pi}\int_0^{\tau} \vec{G}(t)\,d\tau$$



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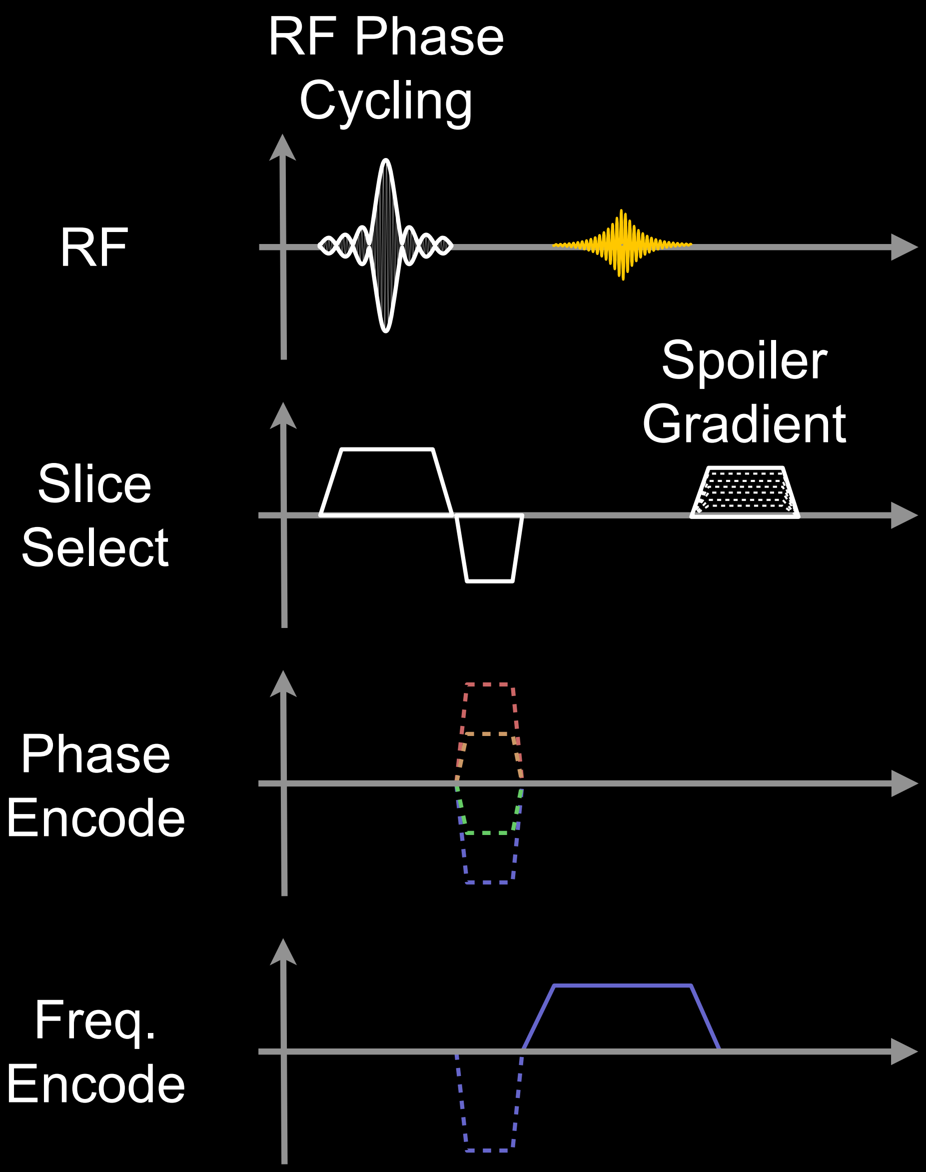

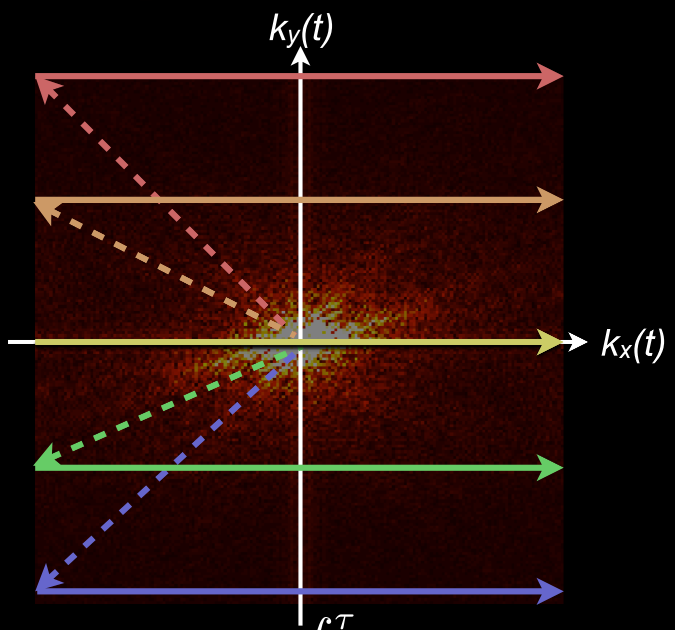

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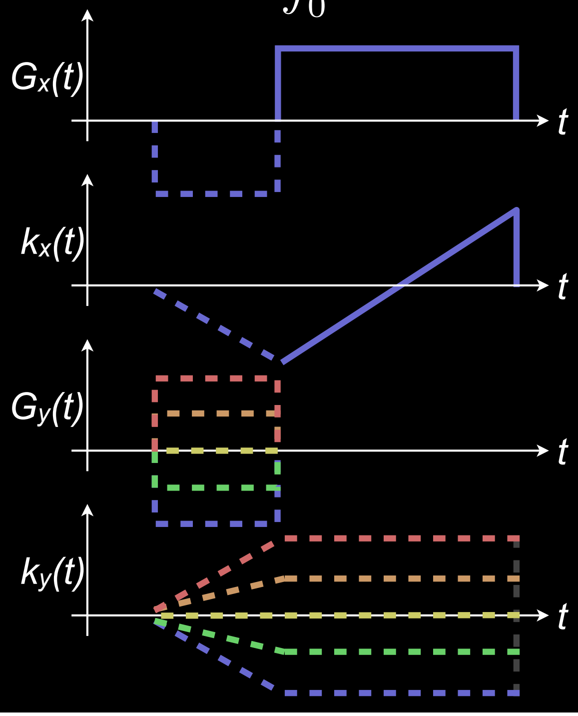

One phase encoded echo is acquired per TR.

Questions?

- Related reading materials
  - Nishimura - Chap 5

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