Image Reconstruction

Compressed Sensing MRI

M229 Advanced Topics in MRI

Kyung Sung, Ph.D.

2018.05.29

Class Business

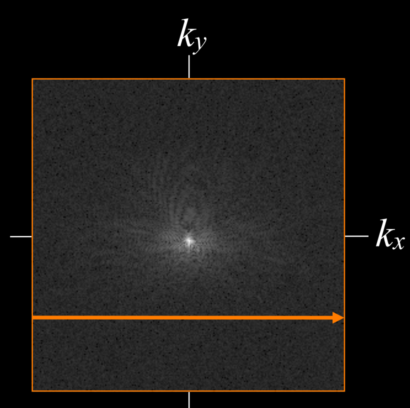
- Final project abstract due on 6/8 Friday
- Final project presentation on 6/7 (9-12pm) and 6/8 (3-6pm)
- Guest Lecturers:
  - Machine Learning in Neurovascular Imaging by Dr. Fabien Scalzo (5/31)
  - Peng Hu (6/5)

Today's Topics

- Motivation
- Background
  - Reconstruction Domain
  - Compressibility or Sparsity
  - Incoherent Measurement
  - Reconstruction
- CS-MRI Examples
- Current Research

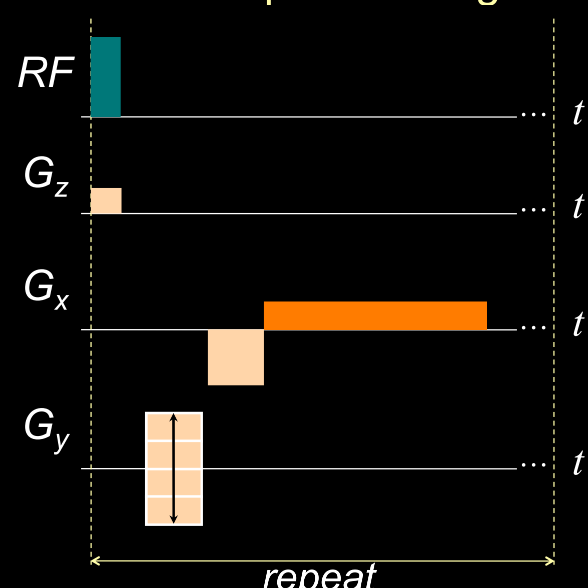
2D Imaging

Frequency-space (k-space)



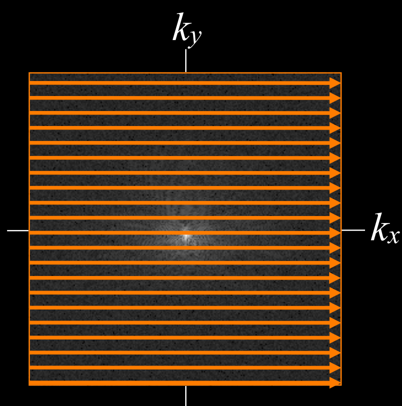
$$s(t) = m(\, k_x(t), k_y(t)\,)$$

Pulse Sequence Diagram



2D Image Reconstruction

Frequency-space (k-space)

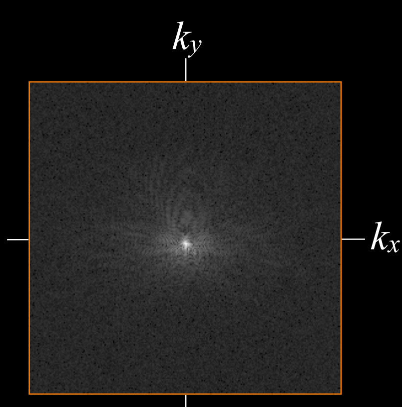


Time for one line = 10ms
lines = 256

Total imaging time?
~2.5 sec

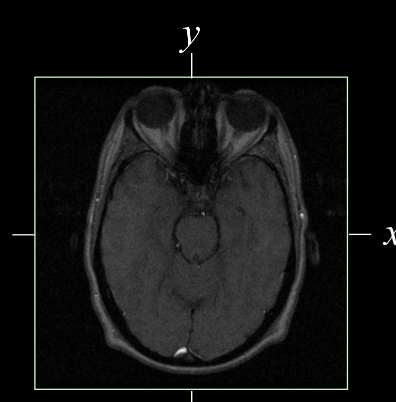
2D Image Reconstruction

Frequency-space (k-space)



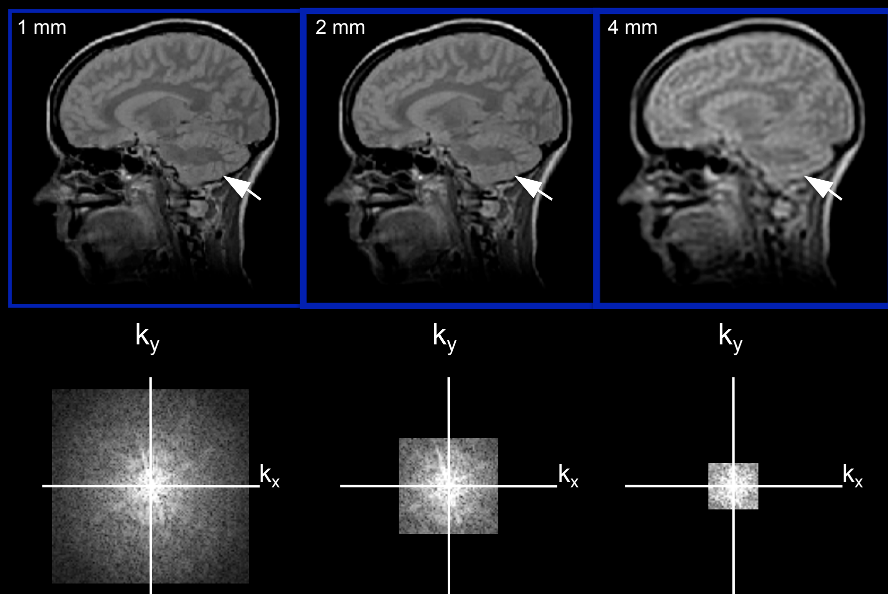
Fourier Transform

Image space

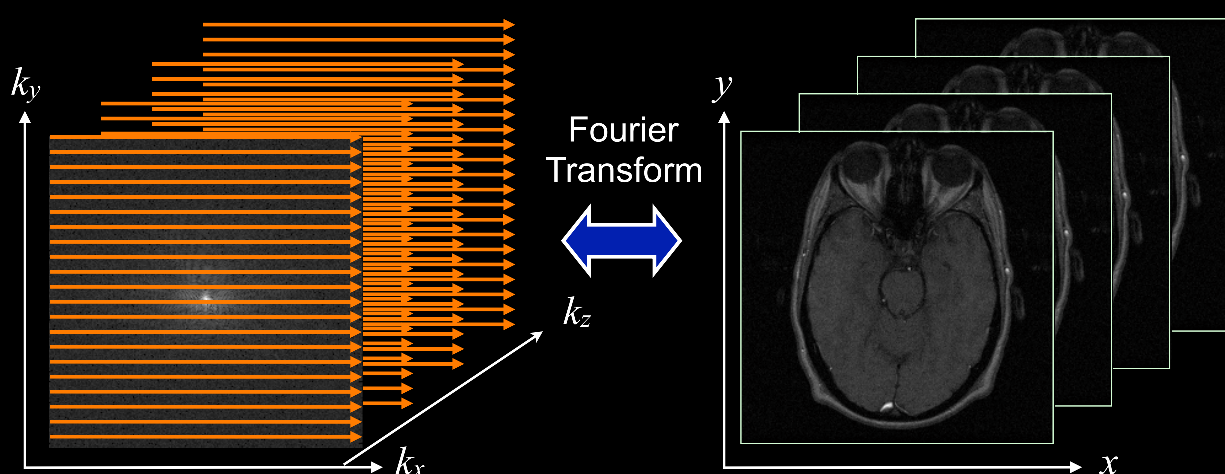


Spatial Resolution

Image resolution increases as higher spatial frequencies are acquired

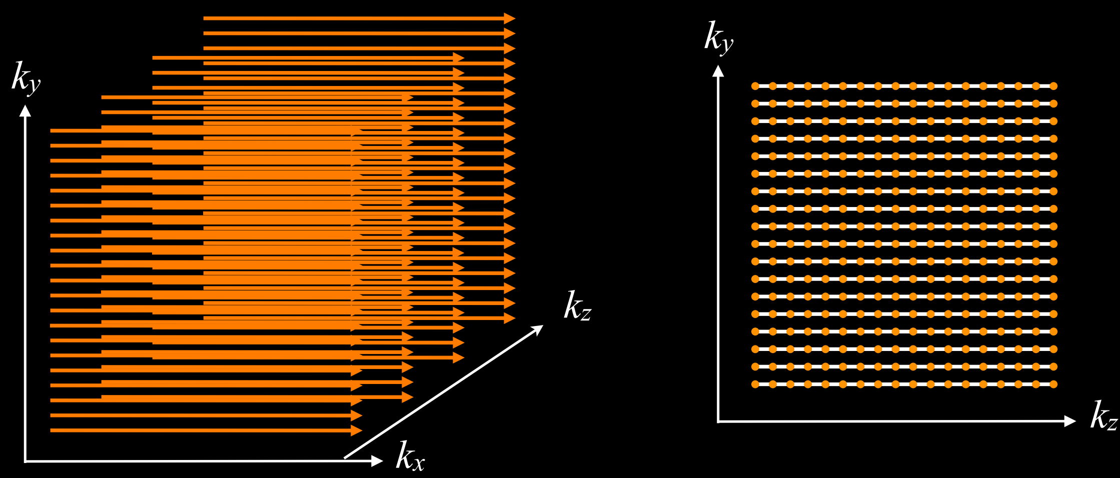


3D Imaging

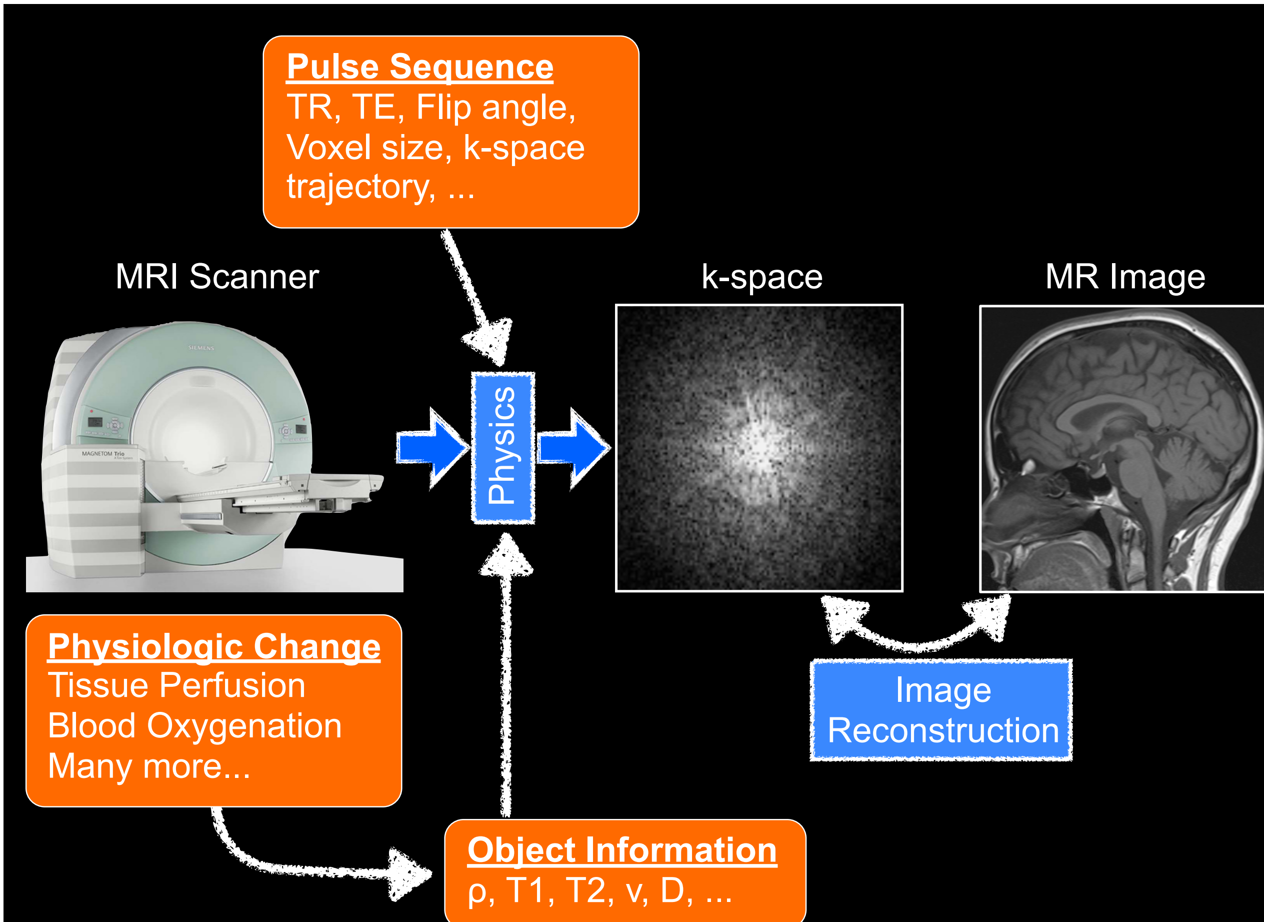


k-space Sampling

Total imaging time? ~20 min



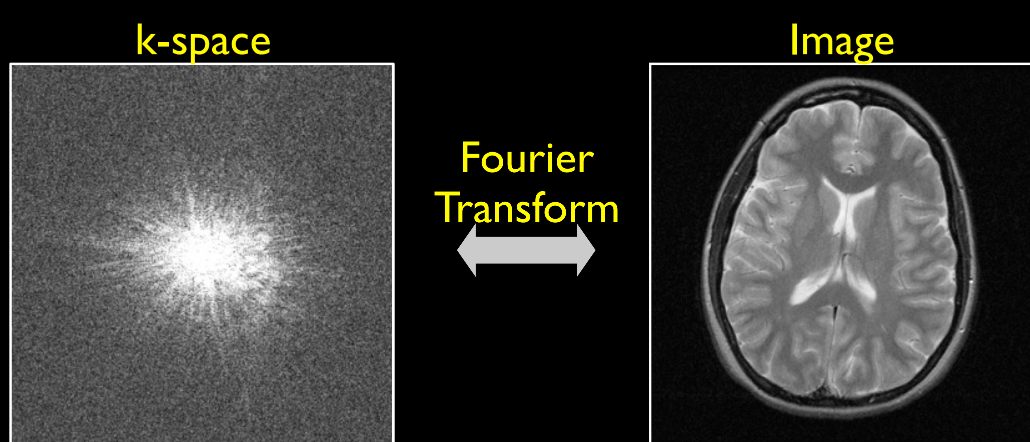
Time for one line (or one dot) = 10ms
lines = 256 x 256



Rapid MR Imaging

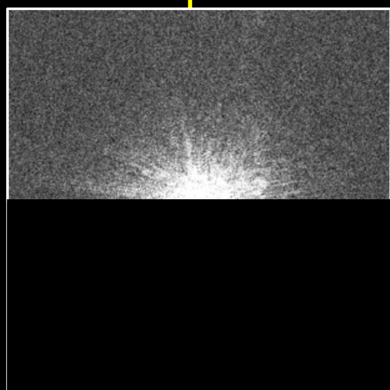
- Improving speed of MRI is great for many MRI applications because it can:
  - increase overall throughputs
  - reduce imaging costs per patient
  - reduce motion artifacts
  - improve temporal resolution for dynamic imaging
  - many more...
- Reducing acquired k-space data is one common way but creates aliasing artifacts

k-space Sampling



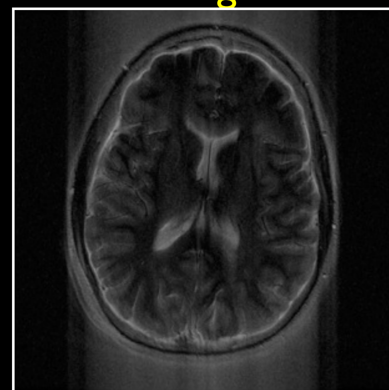
k-space Sampling

k-space



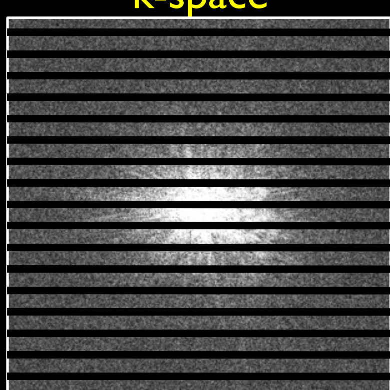
Fourier Transform

Image



k-space Sampling

k-space



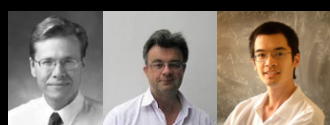
Fourier Transform

Image

Can we estimate missing k-space data?
YES, we can!

Fast MRI Techniques

- Many reconstruction methods minimize aliasing artifacts by exploiting *information redundancy* (or *prior knowledge*)
  - Parallel imaging
  - Compressed sensing



Donoho, IEEE TIT, 2006
Candes et al., Inverse Problems, 2007

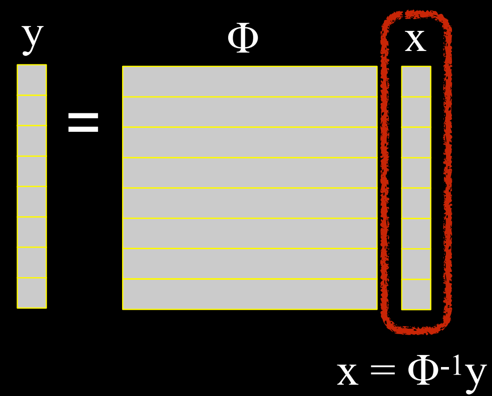
What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

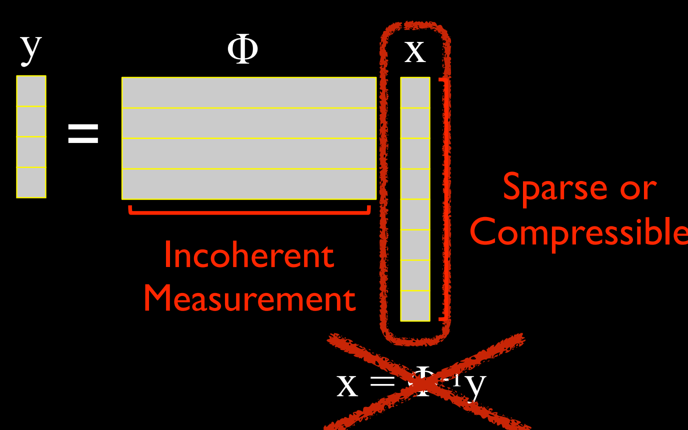
8 Equations
8 Unknowns



What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

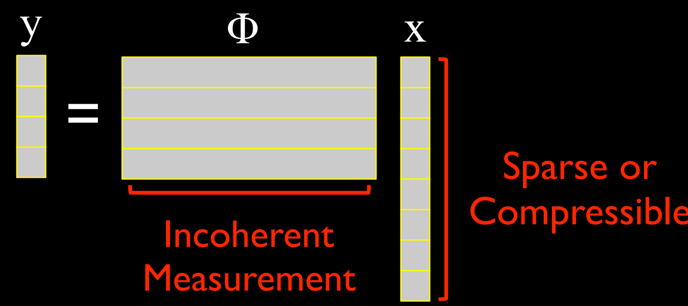
4 Equations
8 Unknowns



What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

4 Equations
8 Unknowns



We still can find 8 unknowns!

Math Background

L0-norm ($|x|_0$): a number of non-zero coefficients

L1-norm ($|x|_1$): a sum of absolute values of coefficients

L2-norm ($|x|_2$): a sum of squared values of coefficients

$$x = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \qquad x = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad x = \begin{pmatrix} 1 \\ 1 \\ -2 \\ 3 \end{pmatrix}$$

Simple Example

$$
\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}}
$$

Simple Example

$$
\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}} \quad \begin{matrix} \\ 0 \\ 0 \\ \\ \\ \\ \end{matrix}
$$

$x_2+x_3 = 0$

Simple Example

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x_1 \\ 0 \\ 0 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}} \quad \begin{matrix} 0 \\ \\ \\ 0 \\ 1 \\ \\ \end{matrix}$$

$$x_4 + x_5 = 1$$

$$x_1 + x_2 + x_5 = 1 \quad (x_2 \to 0)$$

Simple Example

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ x_6 \end{pmatrix}} \quad \begin{matrix} \\ \\ \\ \\ \\ 3 \end{matrix}$$

$$x_5 + x_6 = 4 \quad (x_5 \to 1)$$

Simple Example

2) This should be "smart"

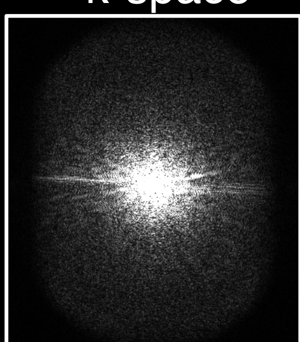
1) This should be "sparse"

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 3 \end{pmatrix}}$$

3) Reconstruction should be "feasible"

Compressed Sensing MRI

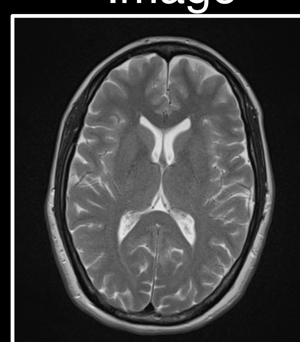
k-space

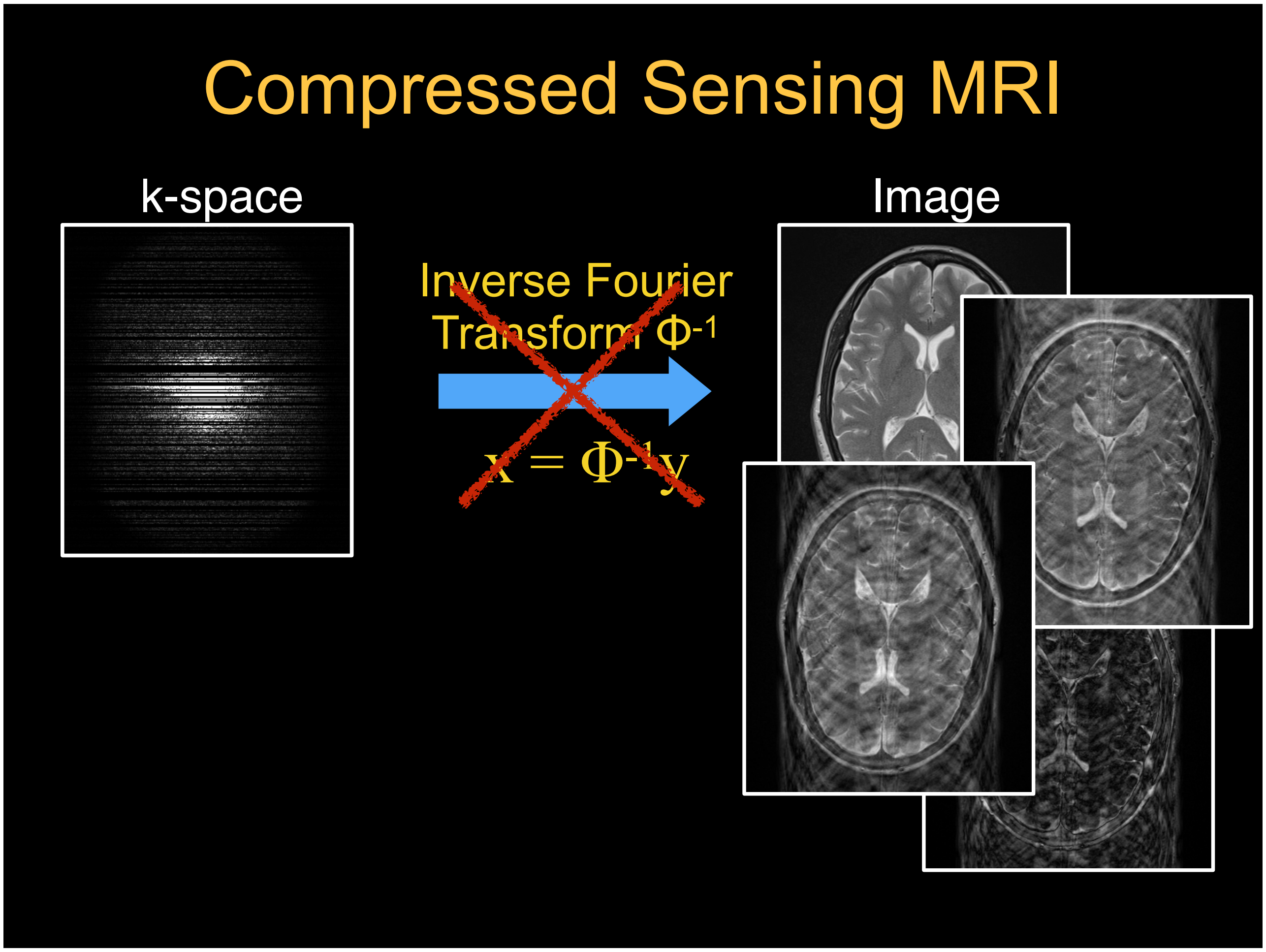


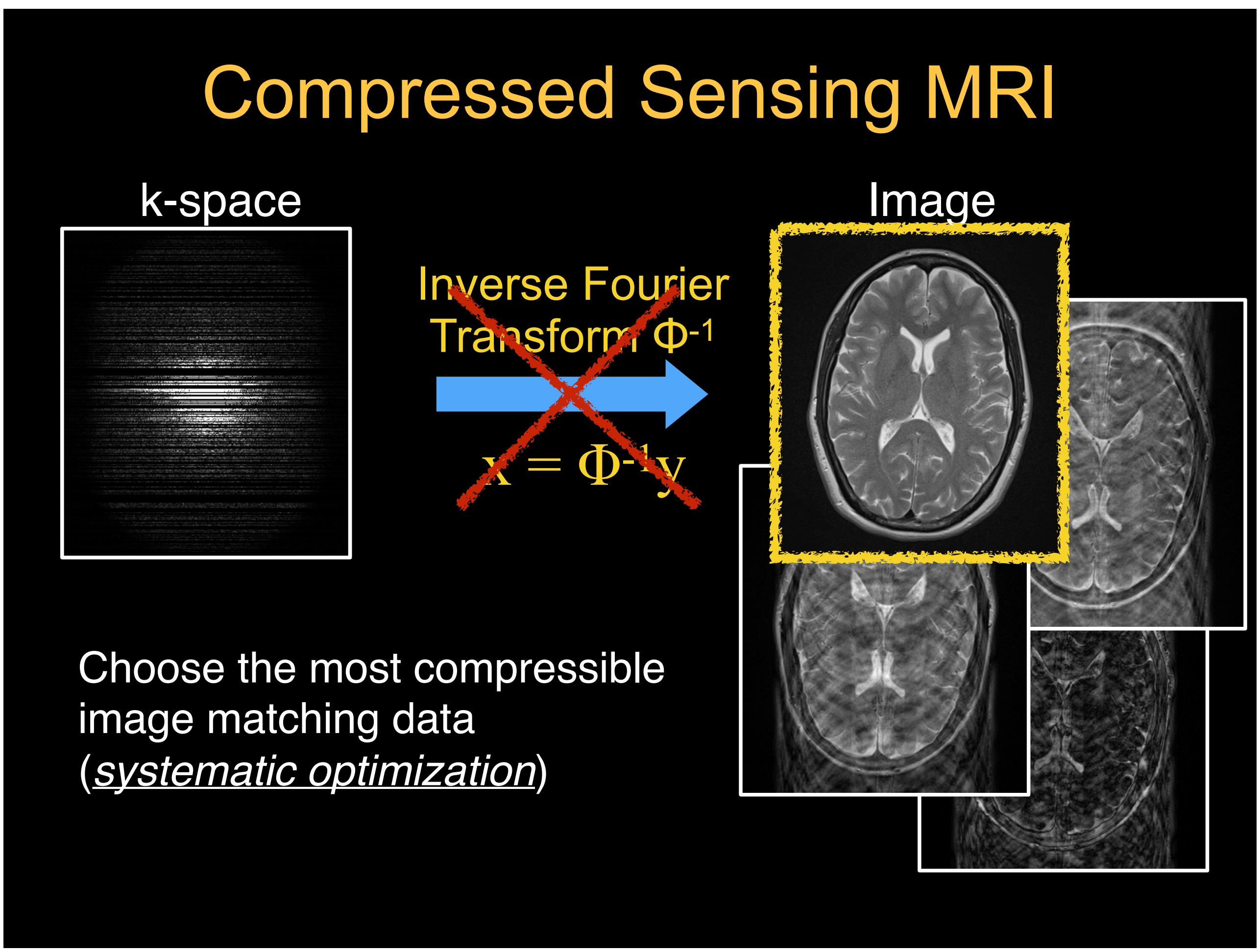
Inverse Fourier Transform Φ^{-1}

$$x = \Phi^{-1} y$$

Image




Compressed Sensing MRI
k-space
Image
Inverse Fourier
Transform Φ-1
x = Φ-1y


Compressed Sensing MRI
k-space
Image
Inverse Fourier
Transform Φ-1
x = Φ-1y
Choose the most compressible
image matching data
(systematic optimization)

Systematic Optimization

- Assuming *sparsity* and *incoherence* are provided, an image can be recovered with highly undersampled data by:

minimize $|\Psi x|_1$, subject to $y = \Phi x$

Sparse Transform (e.g., Wavelet Transform) — Randomly Undersampled Fourier Transform

Systematic Optimization

- Assuming *sparsity* and *incoherence* are provided, an image can be recovered with highly undersampled data by:

minimize $|\Psi x|_1$, subject to $y = \Phi x$

Sparse Transform (e.g., Wavelet Transform) — Randomly Undersampled Fourier Transform

- We can relax the minimization by using regularization,

minimize $F(x): |y - \Phi x|_2^2 + \lambda |\Psi x|_1$

Regularization Parameter (λ)

Three Tenets of CS

minimize $F(x): |y - \Phi x|_2^2 + R(x)$

Data Consistency — Compressibility Constraint

- Three key elements of Compressed Sensing:

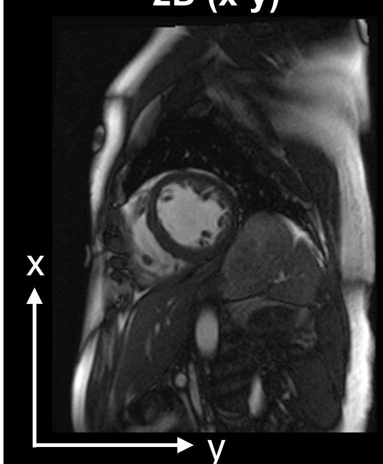
Compressibility

Incoherence

Nonlinear Reconstruction

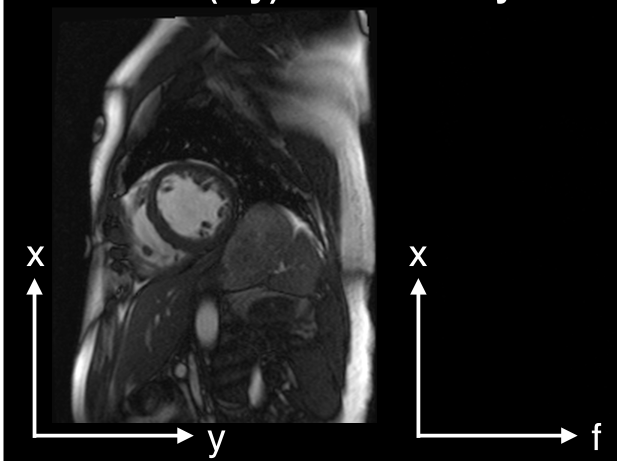
Defining Reconstruction Domain

2D (x-y)



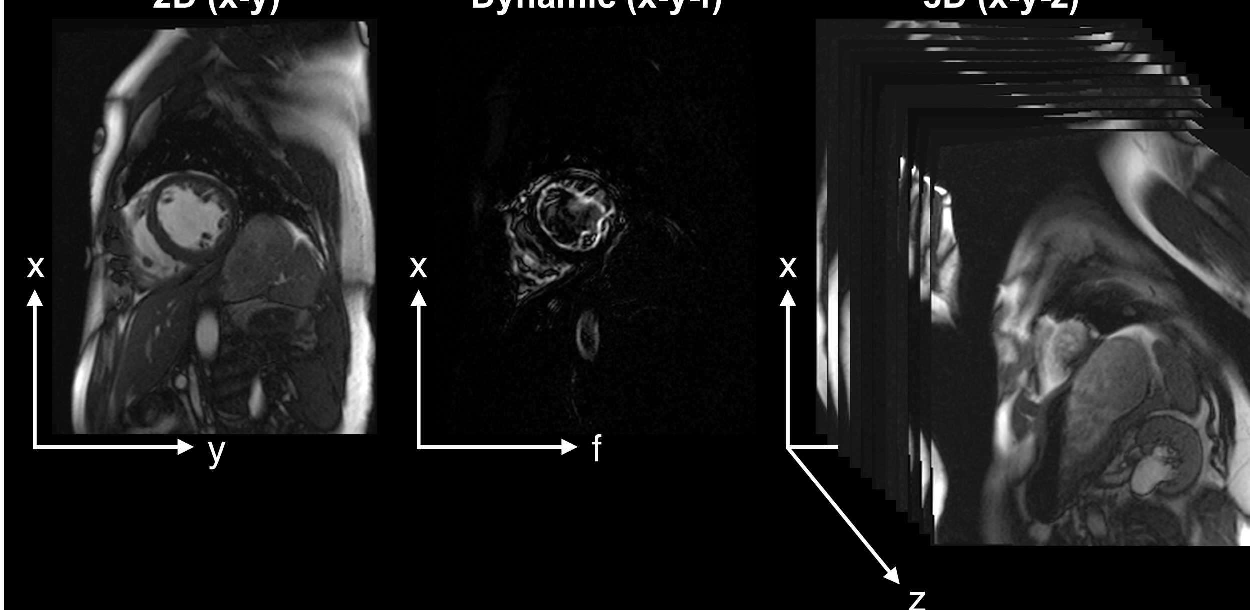
Defining Reconstruction Domain

2D (x-y) **Dynamic (x-y-f)**



Defining Reconstruction Domain

2D (x-y) **Dynamic (x-y-f)** **3D (x-y-z)**



Single Coil vs. Coil Combined

Compressibility Constraint

minimize $F(x): |y - \Phi x|^2_2 + R(x)$

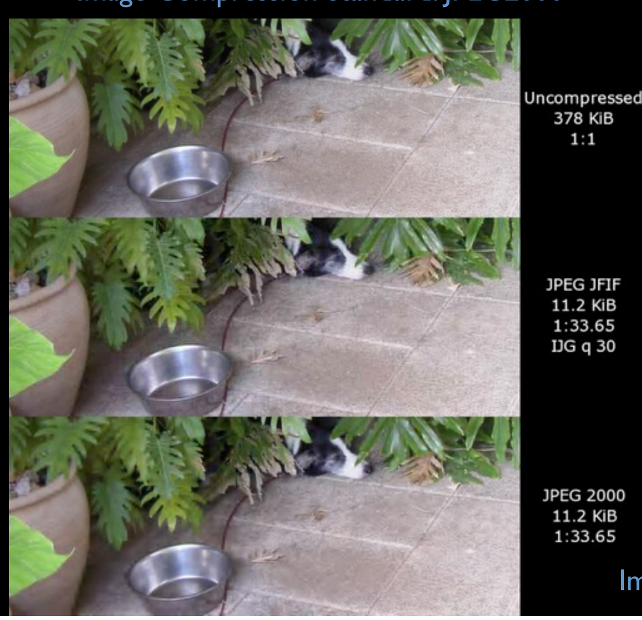
Compressibility Constraint

- $R(x) = \lambda|x|_1$ (*Identity Transform*)
- $R(x) = \lambda|\Psi x|_1$ (*Wavelet Transform*)
- $R(x) = \lambda H(x)$ (*Total Variation*)
- $R(x) = \lambda|x|_*$ (*Rank or Nuclear Norm*)
- Many more…

Wavelet Transform

- Natural images are compressible using wavelet transforms

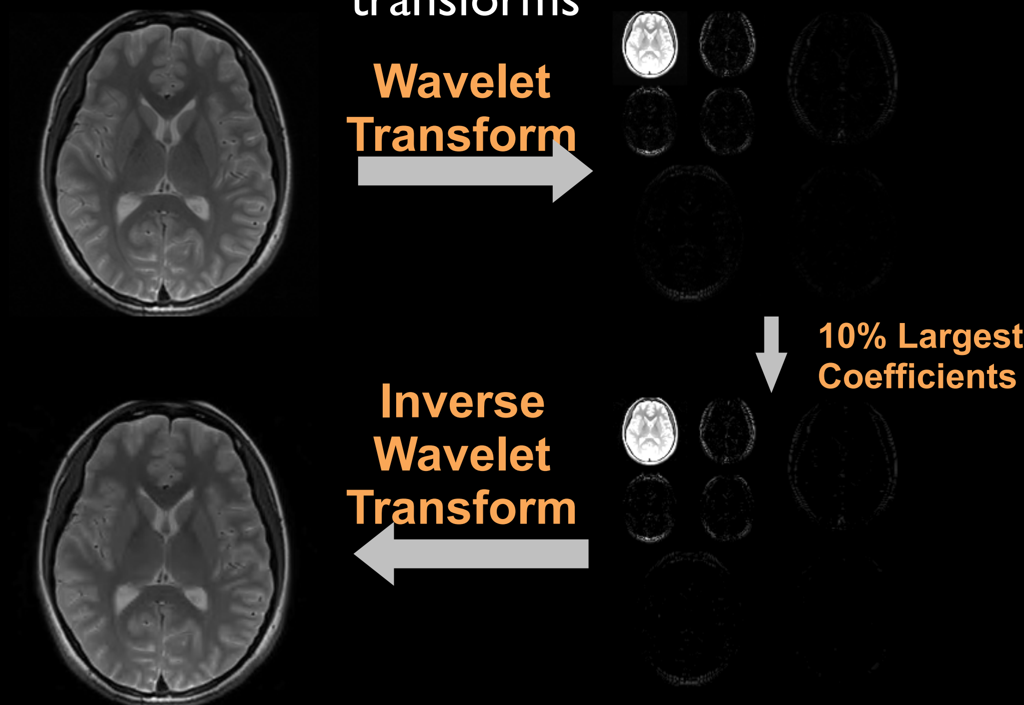
Image Compression Standard: JPEG2000



Images from Wikipedia

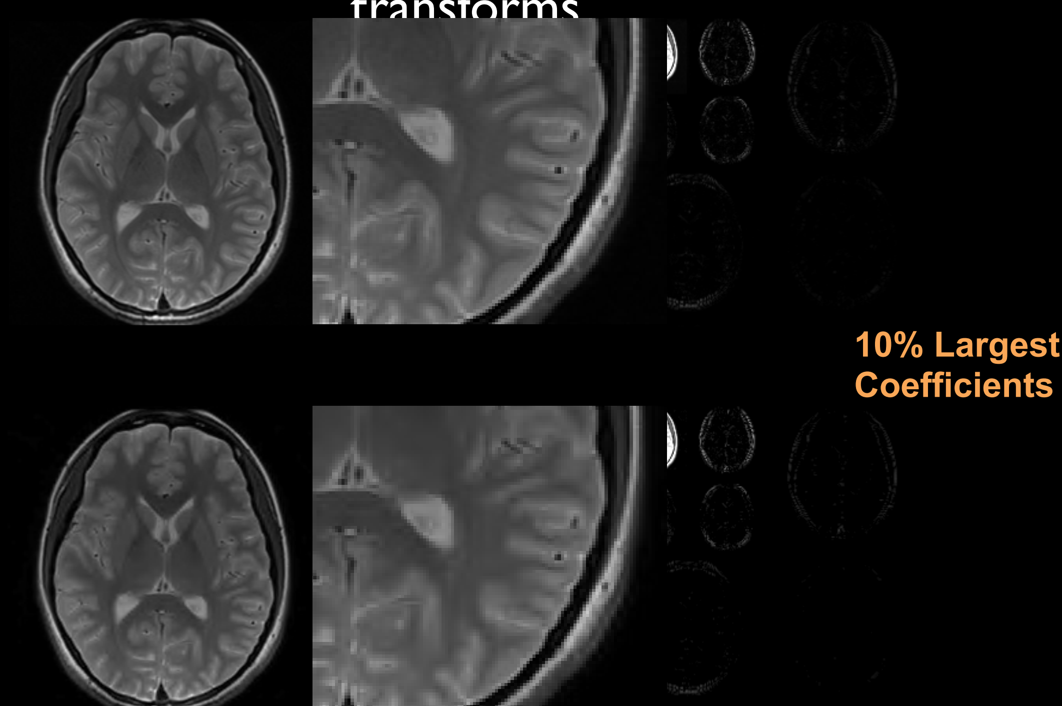
Wavelet Transform

MR images are mostly compressible using wavelet transforms



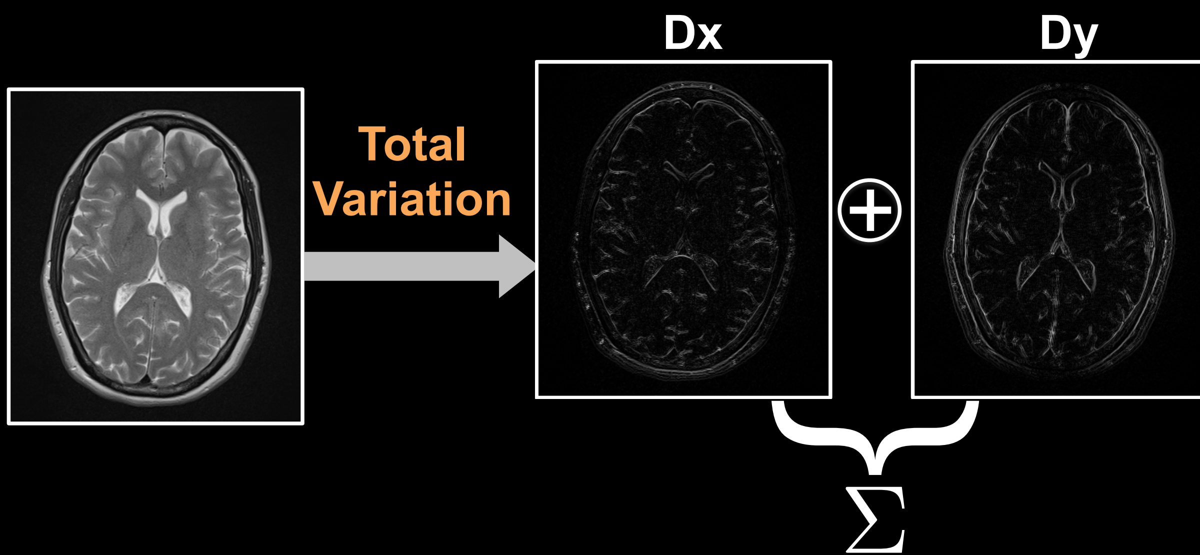
Wavelet Transform

MR images are mostly compressible using wavelet transforms



Total Variation

$$H(x) = \sum_{i,j} \sqrt{\underbrace{|x_{i+1,j} - x_{i,j}|^2}_{Dx} + \underbrace{|x_{i,j+1} - x_{i,j}|^2}_{Dy}}$$



Total Variation

Limitations / Considerations

- Define reconstruction domain and exploit information redundancy (or prior knowledge)
  - More apparent when MRI is repeated on a same object (e.g., repeating with different time points, flip angles, TEs, etc)
- Be aware of underlying assumptions of each constraint
  - Wavelet / TV denoising
- Consistent compressibility is desirable to easily anticipate reconstruction quality

Limitations / Considerations

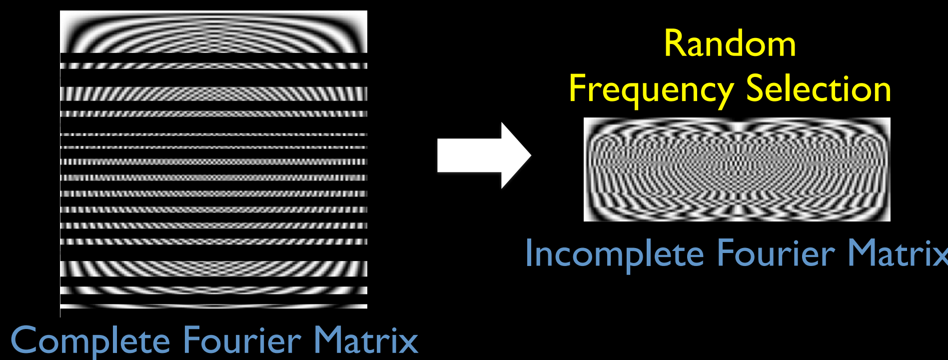
- High vs. low computational complexities
  - Wavelet transform
  - Total Variation
  - Nuclear norm
- Multiple compressibility constraints vs. single constraint
  - Reconstruction quality
  - Reconstruction stability

Incoherent Measurements

minimize $F(x): |y - \Phi x|_2^2 + R(x)$

Incoherent Measurement

- Incoherent measurement provides “perfect” reconstruction
  - Random projection bases are incoherent when the number of measurement is greater than 3S (sparsity)



Incoherent Measurements - Random Frequency Selection

- How do we randomly select frequencies?
  - Uniform / variable density random undersampling
  - Golden angle radial undersampling
  - Variable density spiral undersampling
  - Many more…

CS Reconstruction

- Assuming *sparsity* and *incoherence* are provided, an image can be recovered with highly undersampled data by:

$$\text{minimize } |\Psi x|_1, \text{ subject to } \quad y = \Phi x$$

Sparse Transform (e.g., Wavelet Transform) — Randomly Undersampled Fourier Transform

- We can relax the minimization by using regularization,

$$\text{minimize } F(x): |y - \Phi x|_2^2 + \lambda|\Psi x|_1$$

Regularization Parameter (λ)

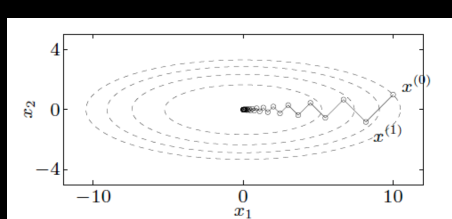
- When λ carefully chosen, unconstrained minimization becomes identical to original minimization

Solving L1 Minimization

- How can we solve this?

$$\text{Minimize}\{ f(x) = |y - \Phi x|_2^2 + \lambda|\Psi x|_1 \}$$

- Review of convex optimization:



General descent method.

given a starting point $x \in \mathbf{dom}\, f$.
repeat
1. Determine a descent direction Δx.
2. *Line search.* Choose a step size $t > 0$.
3. *Update.* $x := x + t\Delta x$.

until stopping criterion is satisfied.

– A choice for search direction (Δx) can be different (e.g. gradient decent method, Newton's method, etc)

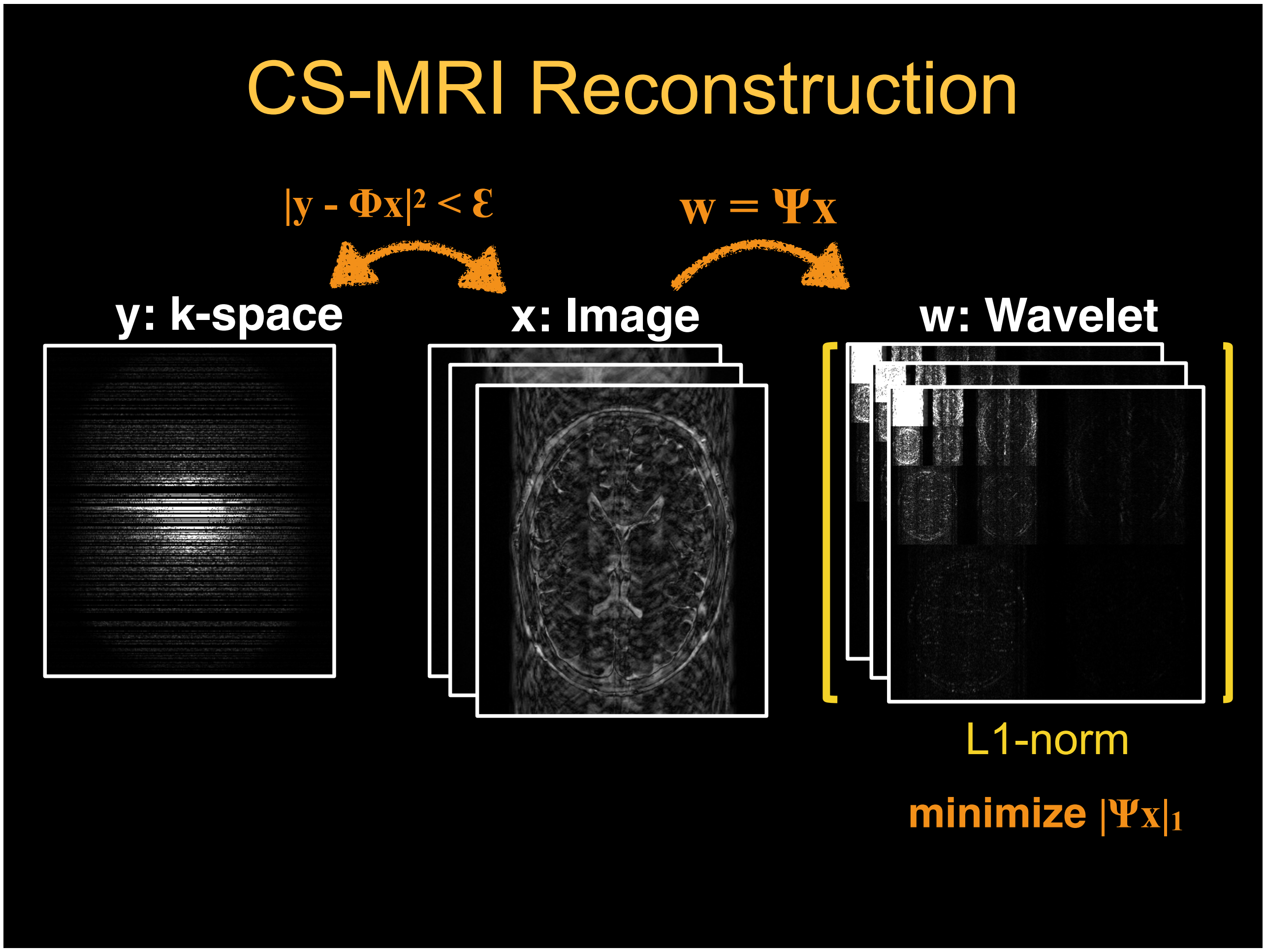
CS-MRI Reconstruction

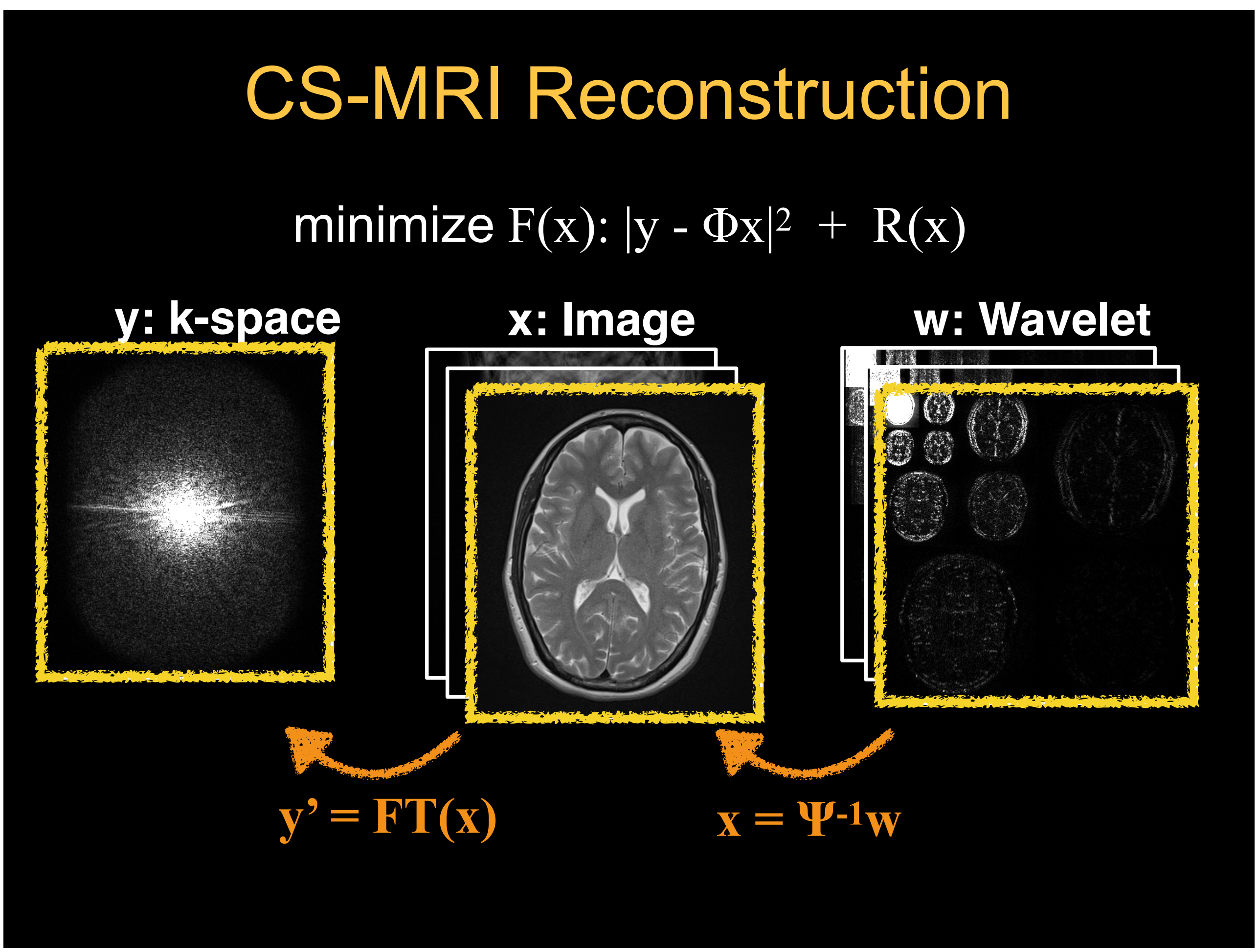
minimize $F(x): |y - \Phi x|_2^2 + R(x)$

- Minimizing F(x) is non-trivial since R(x) is not differentiable
  - Linear programming is challenging due to high computational complexity
- Simple gradient-based algorithms have been developed:
  - Re-weighted L1 / FOCUSS
  - IST / IHT / AMP / FISTA
  - Split Bregman / ADMM

I.F. Gorodnitsky, et al., J. Electroencephalog. Clinical Neurophysiol. 1995 Daubechies I, et al. Commun. Pure Appl. Math. 2004
Elad M, et al. in Proc. SPIE 2007
T. Goldstein, S. Osher, SIAM J. Imaging Sci. 2009

To the board ...


CS-MRI Reconstruction
$|y - \Phi x|^2 < \varepsilon$
$w = \Psi x$
y: k-space
x: Image
w: Wavelet
L1-norm
minimize $|\Psi x|_1$


CS-MRI Reconstruction
minimize $F(x): |y - \Phi x|^2 + R(x)$
y: k-space
x: Image
w: Wavelet
$y' = FT(x)$
$x = \Psi^{-1} w$

Summary So Far…

minimize $F(x): |y - \Phi x|_2^2 + R(x)$

Data Consistency — Compressibility Constraint

Reconstruction Domain
Compressibility Constraint
Incoherent Measurement
Reconstruction

Cardiac Function

- *Reconstruction Domain*:
  x (dynamic 2D MRI in x-f space)
- *Compressibility Constraint*:
  $|x|_1$: sparsity in x-f
- *Incoherent Measurement*: variable density random undersampling

minimize $F(x): |y - \Phi x|_2^2 + \lambda |x|_1$

- *Reconstruction*: non-linear CG L1 / FOCUSS

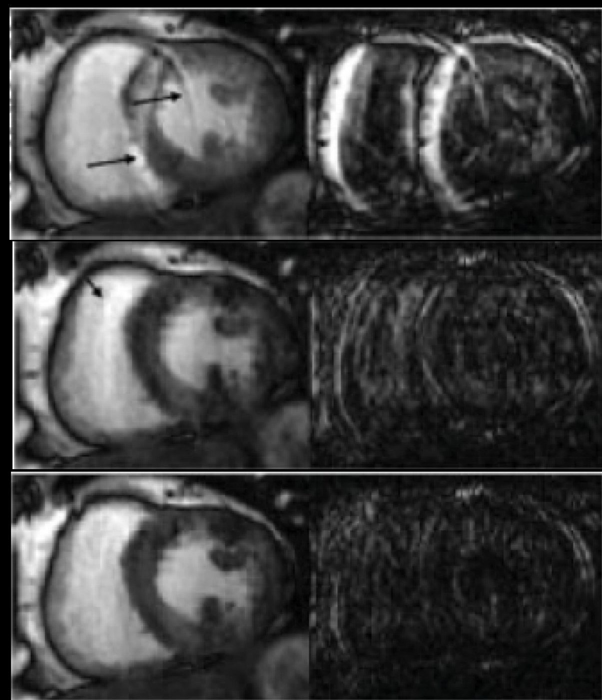
M. Lustig, et al., ISMRM 2006
H. Jung, et al., Physics in Medicine and Biology 2007
H. Jung, et al., MRM 2009

Cardiac Function (k-t FOCUSS)

k-t BLAST

k-t FOCUSS

k-t FOCUSS with ME/MC



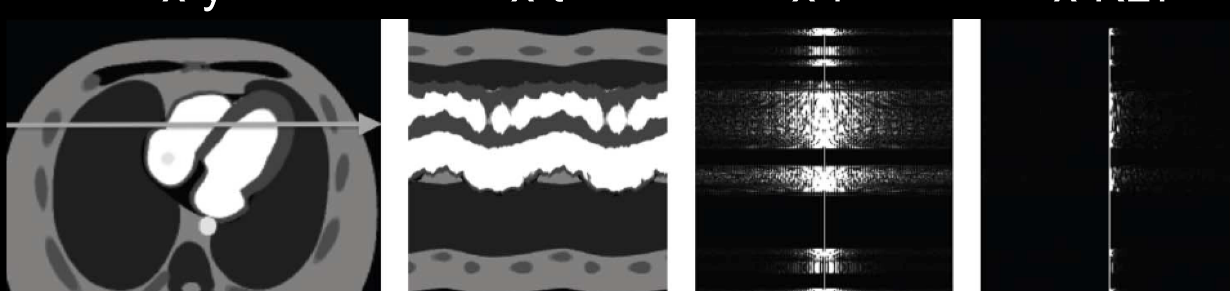
H. Jung, et al., MRM 2009

Cardiac Function (k-t SLR)

- ***Compressibility Constraint:***

$$|x|_* = \sum_i (\Sigma_{i,i}) \qquad x = U\Sigma V^*$$

x-y x-t x-f x-KLT



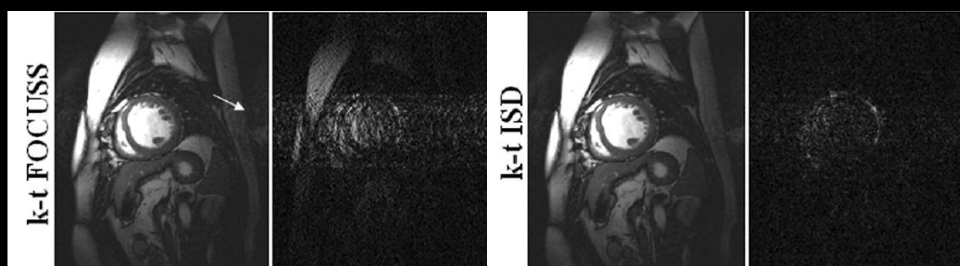
S.G. Lingala, et al., IEEE TMI 2011

Cardiac Function (k-t ISD)

- *Compressibility Constraint*:
  W: Diagonal weighting matrix (known support in x-f)
- *Incoherent Measurement*: variable density random undersampling

minimize $F(x)$: $|y - \Phi x|_2^2 + \lambda|Wx|_1$

- *Reconstruction*: FOCUSS



D. Liang, et al., MRM 2012

Phase Contrast

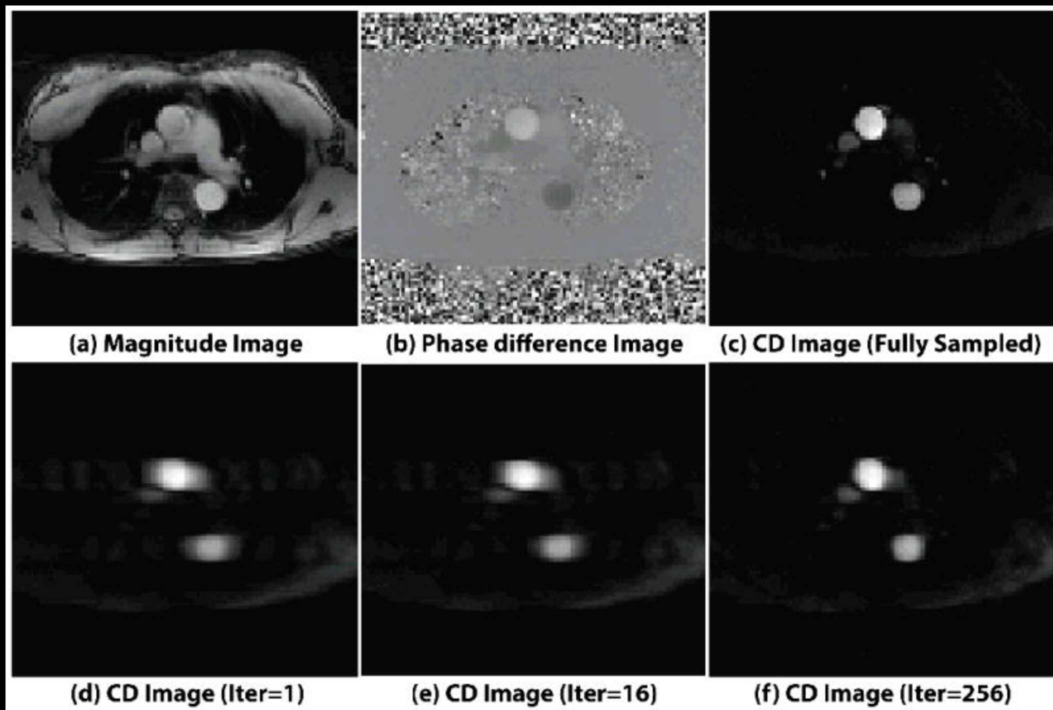
- *Reconstruction Domain*:
  x_1 (flow-compensated)
  x_2 (flow-encoded)
- *Compressibility Constraint*:
  $H(x_i)$: Total Variation
  $|x_1 - x_2|_1$: Complex Difference
- *Incoherent Measurement*: uniform random undersampling

minimize $F(x_1)$: $|y - \Phi x_1|_2^2 + \lambda_1 H(x_1) + \lambda_2 |x_1 - x_2|_1$
minimize $F(x_2)$: $|y - \Phi x_2|_2^2 + \lambda_1 H(x_2) + \lambda_2 |x_1 - x_2|_1$

- *Reconstruction*: Split Bregman

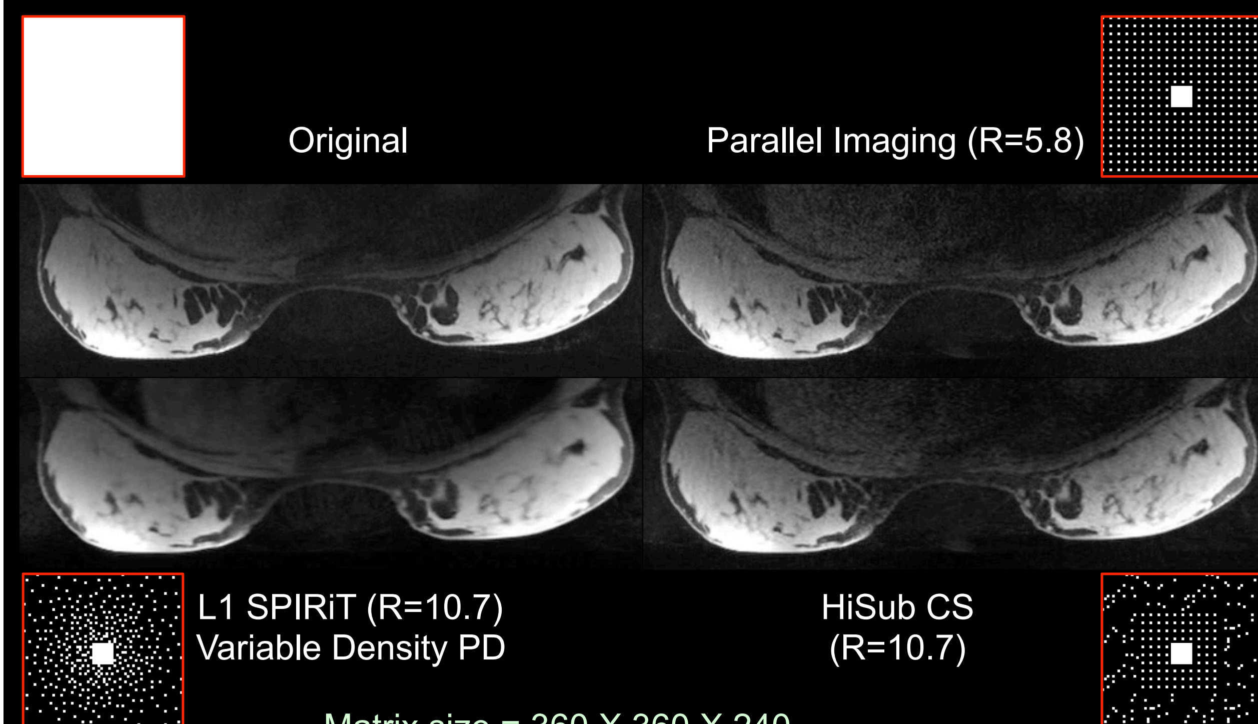
Y Kwak. et al., MRM 2012

Phase Contrast (Complex Difference)



Y Kwak. et al., MRM 2012

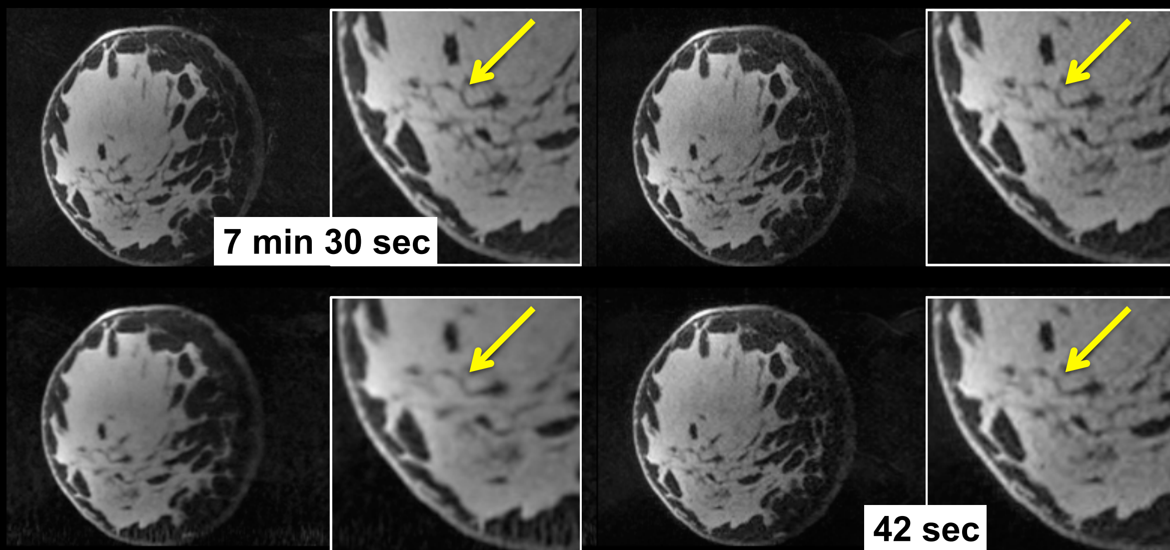
High-Frequency Subband CS



Matrix size = 360 X 360 X 240
Spatial resolution = 0.9 X 0.9 X 0.6 mm

High-Frequency Subband CS

Original | **Parallel Imaging (R=5.8)**

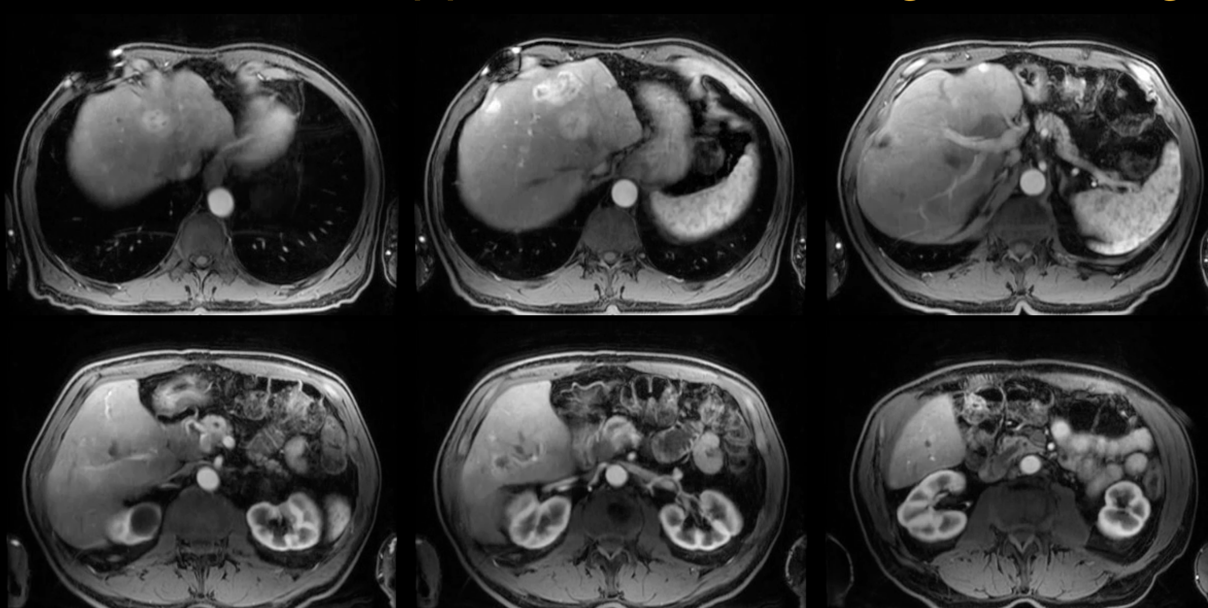


L1 SPIRiT (R=10.7)
Variable Density PD

HiSub CS (R=10.7)

K. Sung, et al. MRM 2013

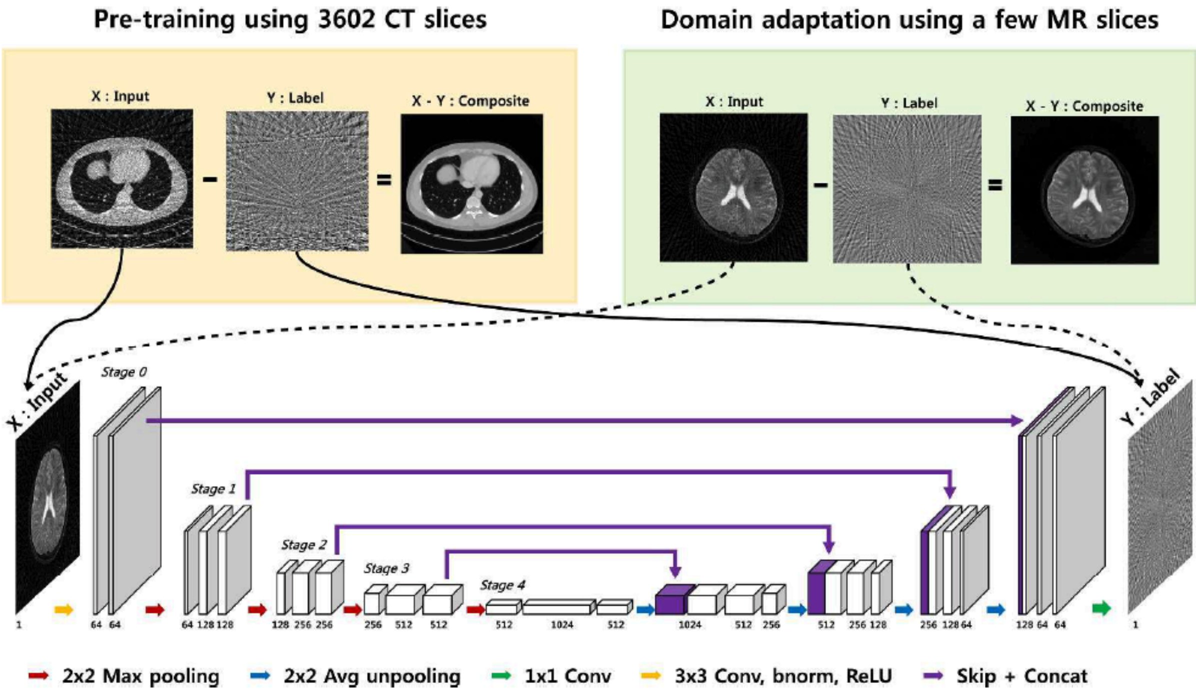
Liver DCE Imaging - LCAMP: Location Constrained Approximate Message Passing



Matrix size = 260 X 202 X 60
Temporal res = 4 sec and # temporal phases = 8
32 channel torso coil
12x acceleration

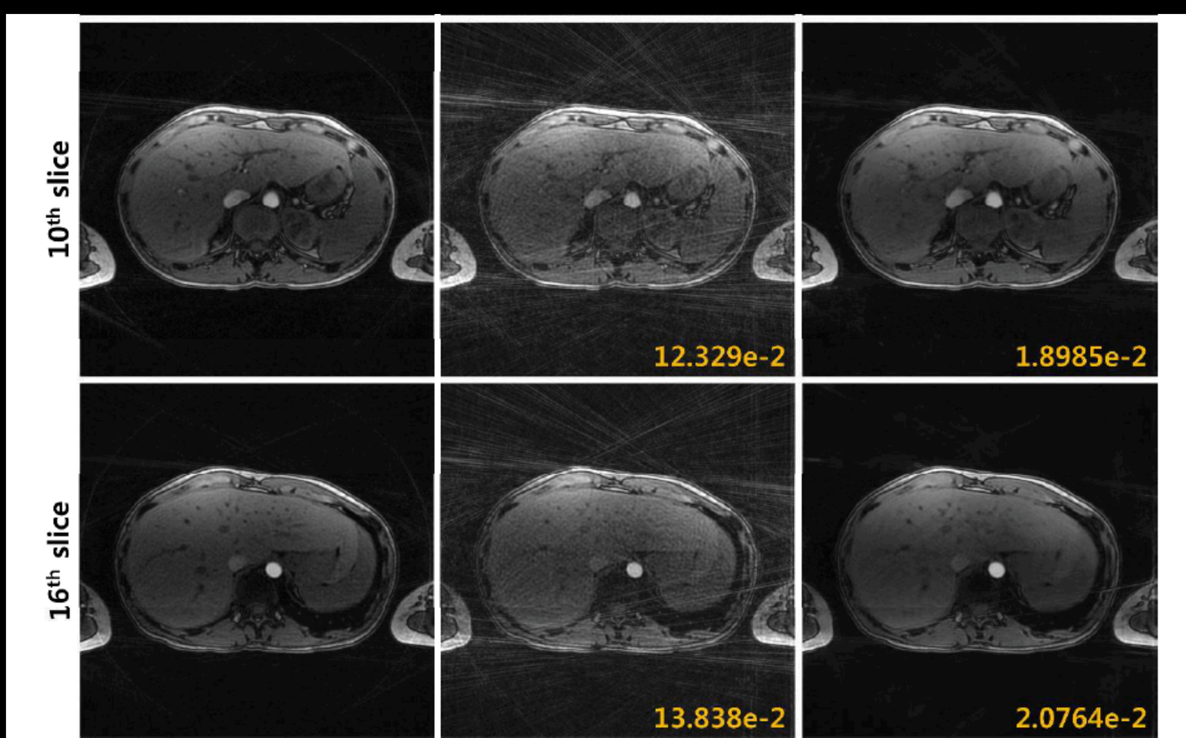
K. Sung, et al. MRM 2013

Deep Learning with Domain Adaptation



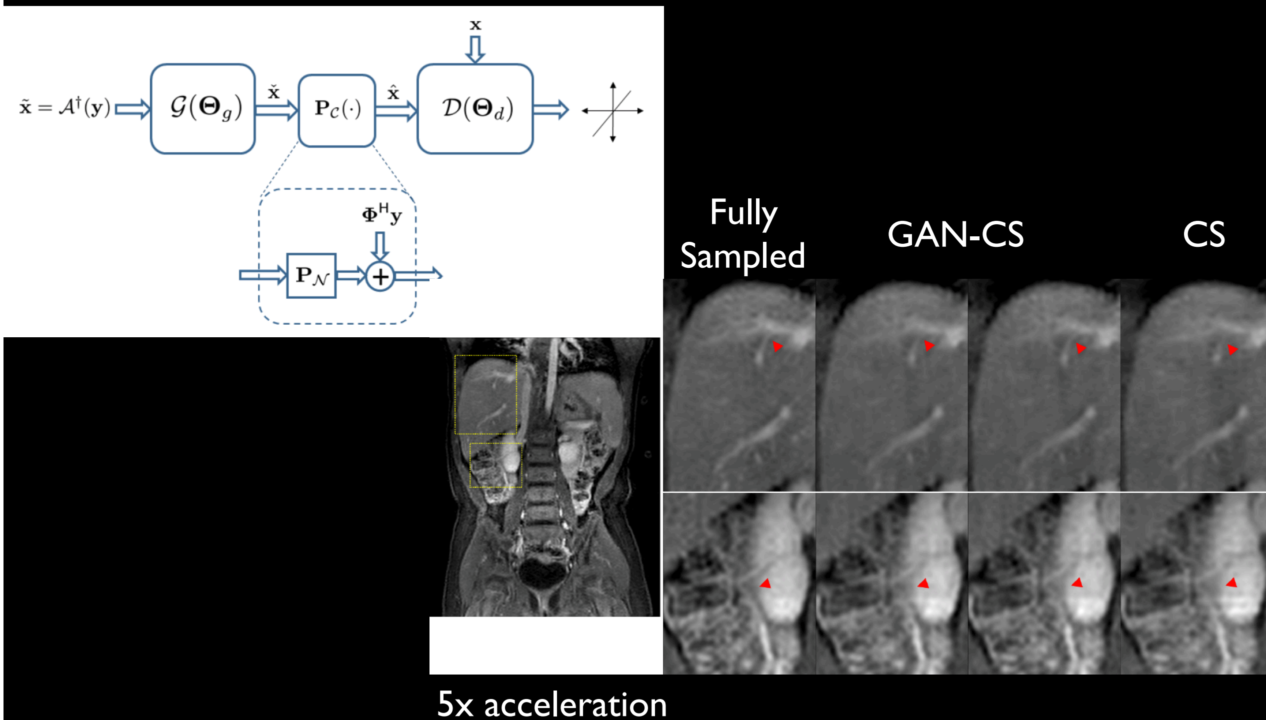
YS. Han, et al. MRM 2018

Deep Learning with Domain Adaptation



YS. Han, et al. MRM 2018

Deep Generative Adversarial Neural Networks for Compressive Sensing (GANCS) MRI



M. Mardani, et al. IEEE TMI 2018

State-of-the-Art CS/DL-MRI

- Reducing possible reconstruction failure
  - Improve sparse transformations
  - Develop k-space undersampling schemes
  - Develop and evaluate CS/DL reconstruction algorithms
- Integrating CS/DL with parallel imaging
  - Develop compatible undersampling patterns
  - Develop reconstruction methods

State-of-the-Art CS/DL-MRI

- Methods to evaluate CS/DL reconstructed images
  - RMSE / SSIM / Mutual Information
- Reducing CS/DL reconstruction time
  - Reduce computational complexity
  - Parallelize reconstruction problems
- Developing stable reconstruction algorithms
  - Minimize / avoid the number of regularization parameters

Summary

- CS-MRI has a lot of potential but is not a magic box!
- Always remember key components of CS:

Reconstruction Domain

Compressibility (or Sparsity)

Incoherent Measurement

Reconstruction

Thanks!

Kyung Sung, PhD

ksung@mednet.ucla.edu

http://kyungs.bol.ucla.edu