Accelerated MRI Techniques: Basics of Parallel Imaging and Compressed Sensing

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MRI…

- MRI has low signal levels
  - Polarization is PPM
    - Overcome with higher fields
  - Improve detection
    - High quality coil arrays
    - Mostly body noise limited today
- MRI is slow...
  - Slow to encode
    - Compare to digital camera!
  - Slow repetition times
    - Relaxation time constants are long
  - Need contrast agents
  - Need faster gradients (1990s)
    - Gradients are near optimal today

Gradient Encoding

$$S\left(\vec{k}\right) = \int_{r} M_{xy}\left(\vec{r}\right) e^{-i2\pi \vec{k}\bullet\vec{r}}\, d\vec{r}$$

- One-to-one correspondence between k-space location and MRI signal
- Speed of MRI is dependent on speed of travel in k-space
- K-space location is controlled by gradients
- One MRI signal sample at a time!
  - larger volume coverage -> longer scan time

Wait a minute…

- Can we increase the speed we travel in k-space using higher gradients and faster switching?

- Yes, you can, but…
  - Peripheral nerve stimulation
  - Gradient amplifier power considerations
  - SNR considerations

Peripheral Nerve Stimulation

- Switching of gradients -> time-varying magnetic field -> electrical current -> nerve stimulation -> tingling sensation
- PNS is not dangerous, but can be disturbing
- FDA limits PNS in MRI systems -> limits in switching speed of gradients
- Common Max slew rate: 200mT/m/ms

Gradient Amplifier

- Gradient amplifiers feed large electrical currents into the gradient coil
- $G_{max} \propto$ Current [I, amps]
- Slewrate $\propto$ Voltage [V, volts]
- Power=IV
- R-fold acceleration requires
  - R-fold increase in Gmax
  - R^2-fold increase in slewrate
  - Power=IV $\propto R^3$!!!

SNR Loss

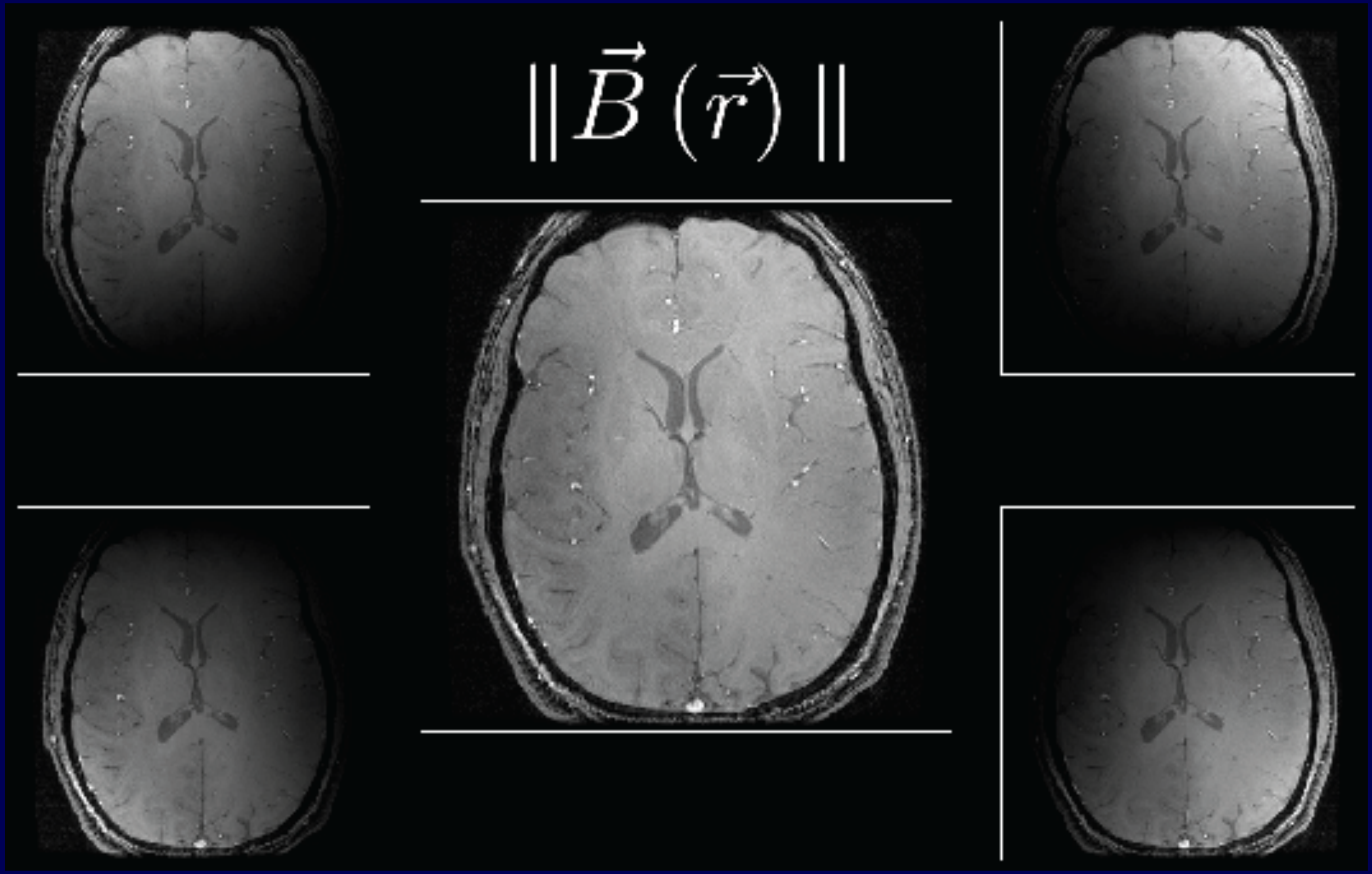
- Larger sampling bandwidth -> Larger anti-aliasing filter BW -> allowing more noise power into MRI signal -> decreasing SNR!
- Common Max Sampling Rate: 500KHz (2us period)

Alternative Technique to Speed up MRI

- Reduce k-space samples

Parallel Imaging!

Why MRI using Coil Arrays

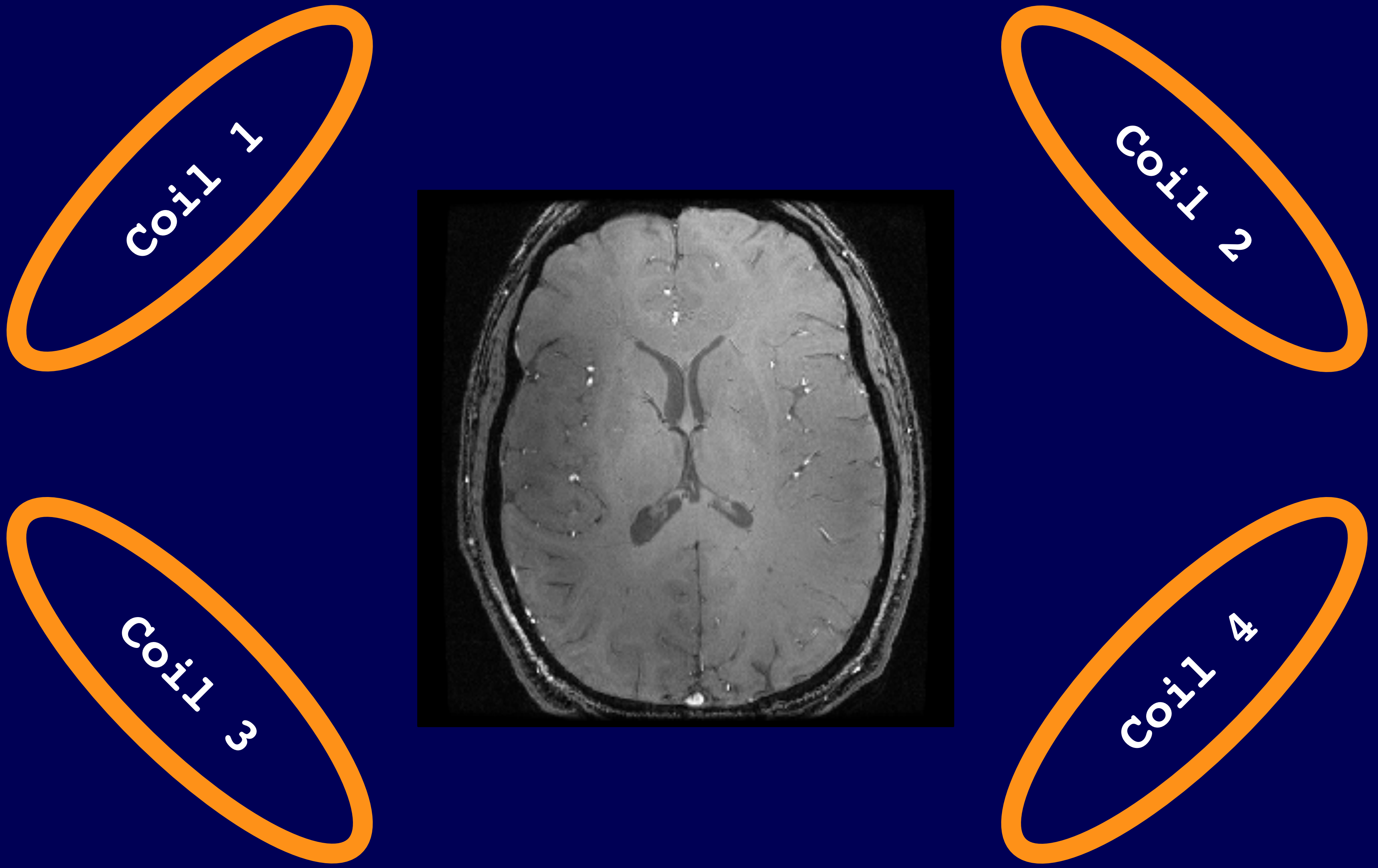


- Increased SNR

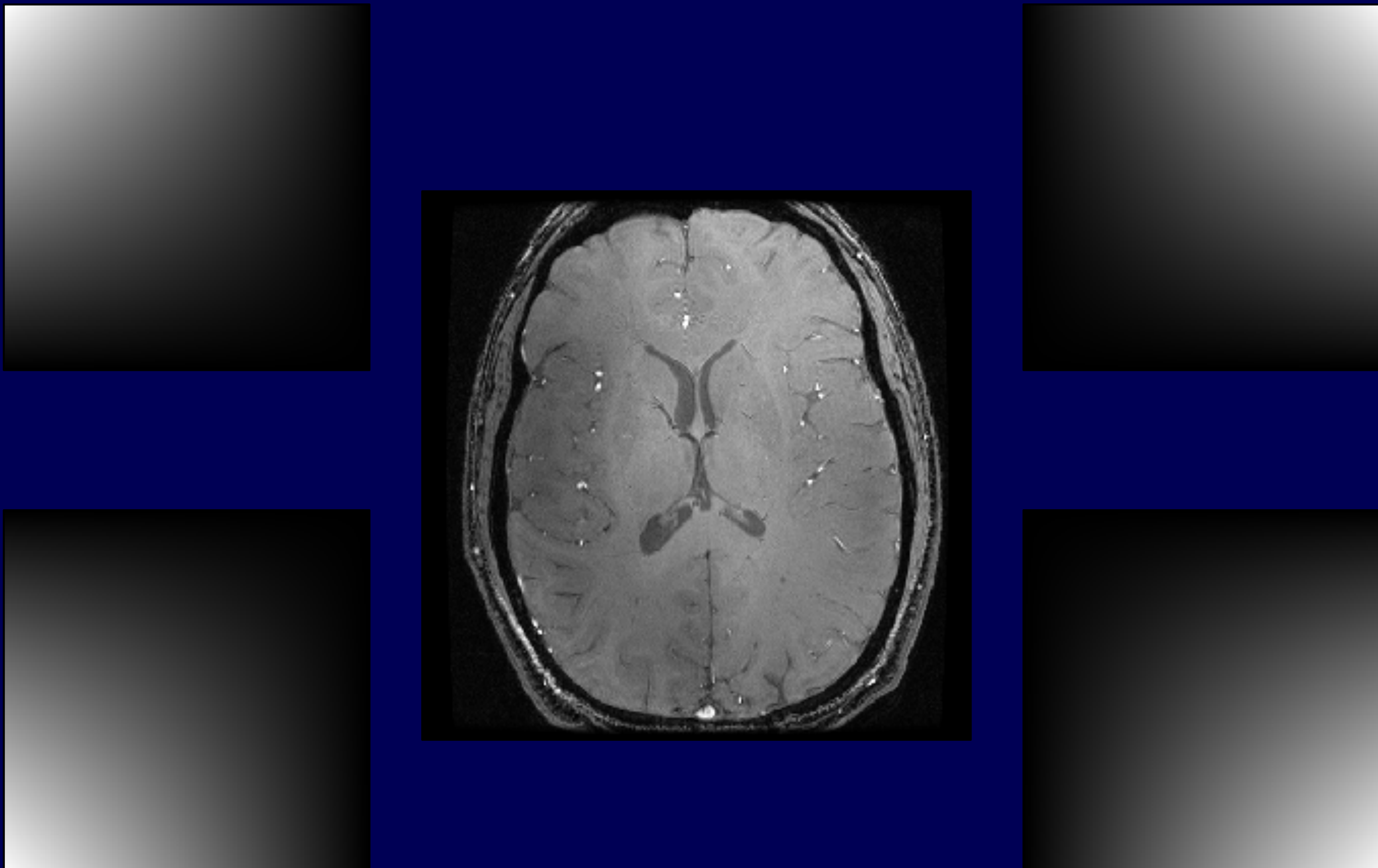
Sources of Noise in MRI

- Human Body
  - Noise from human body is most significant at high field
- Electronics
  - Coils, Pre-Amps, amplifiers, filters, A/D
- Interference
  - Less of an issue

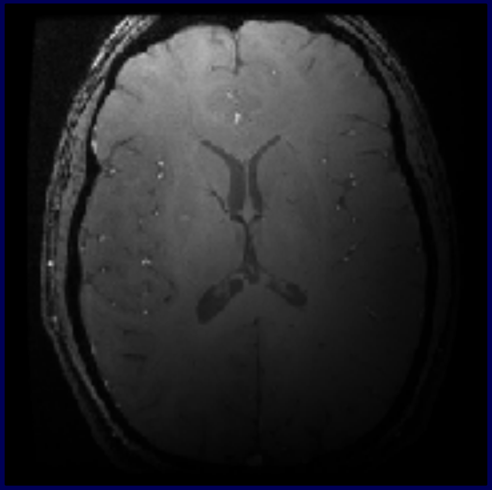
Multi-coil Reconstruction

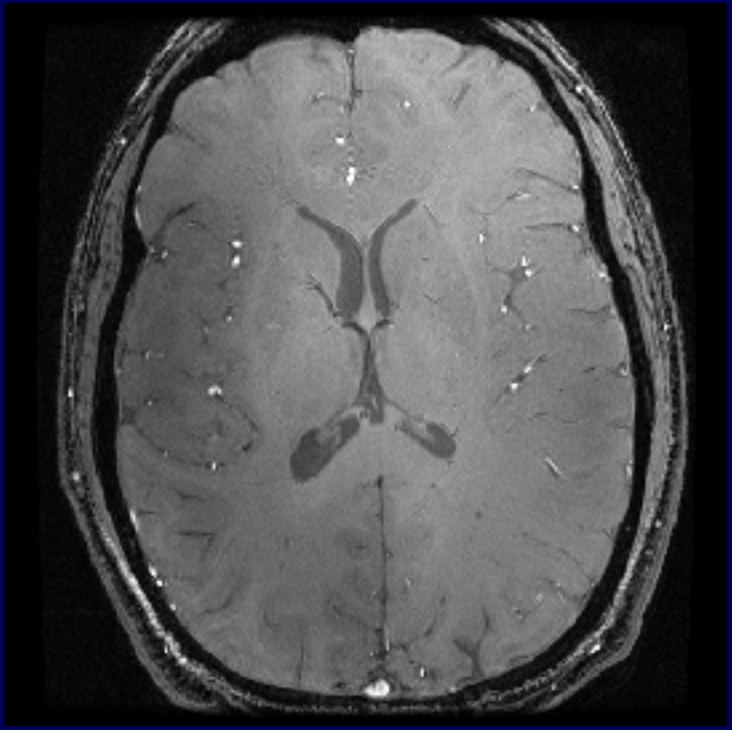


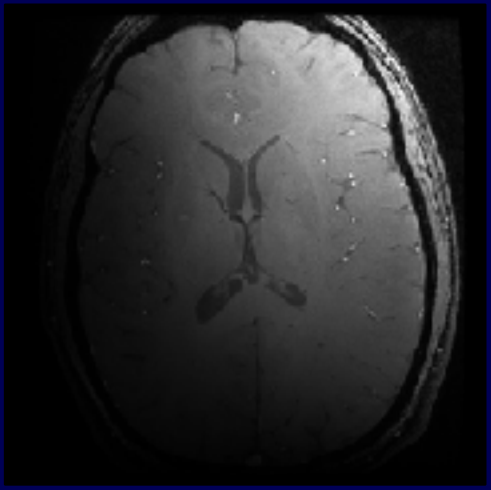
Multi-coil Reconstruction

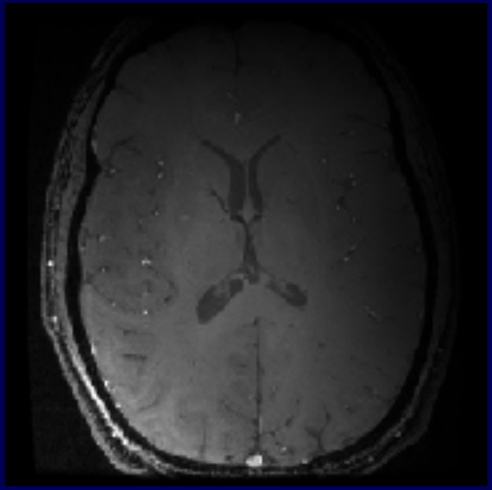


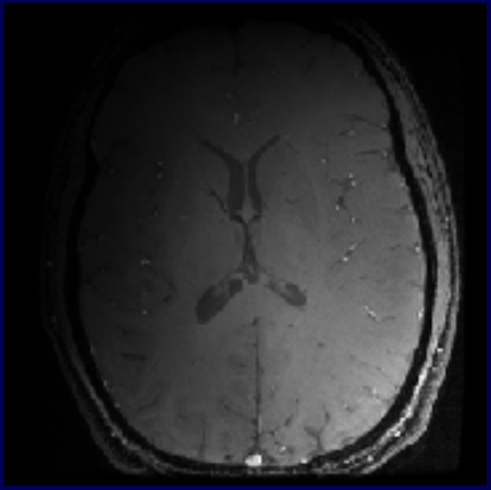
Multi-coil Reconstruction







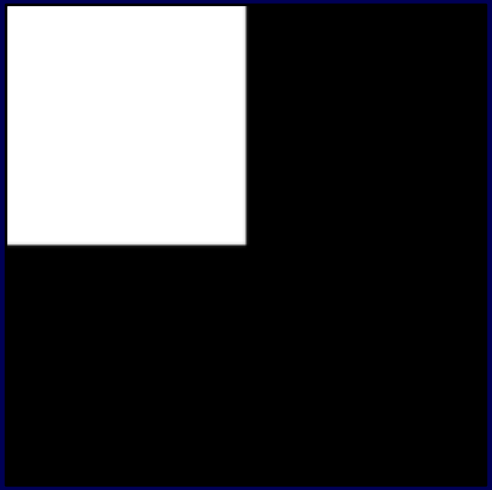
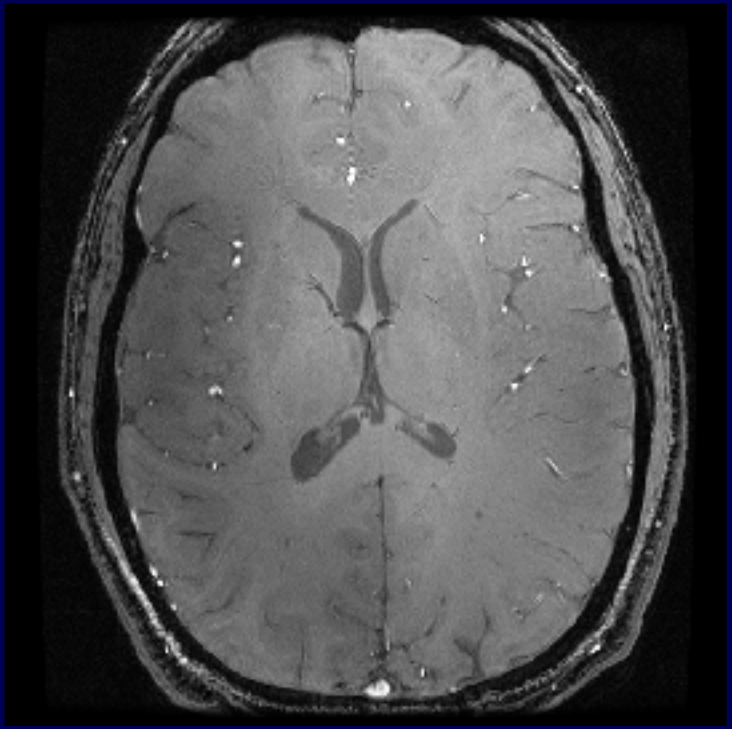
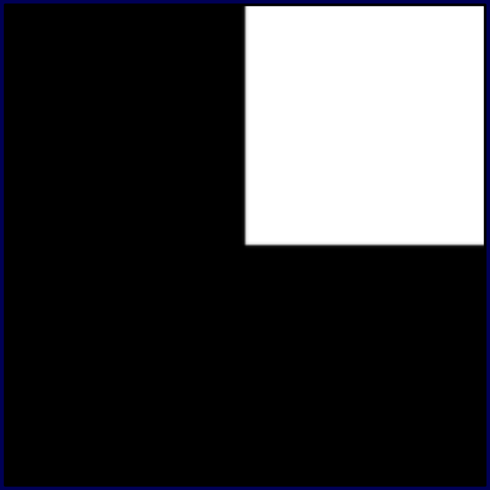
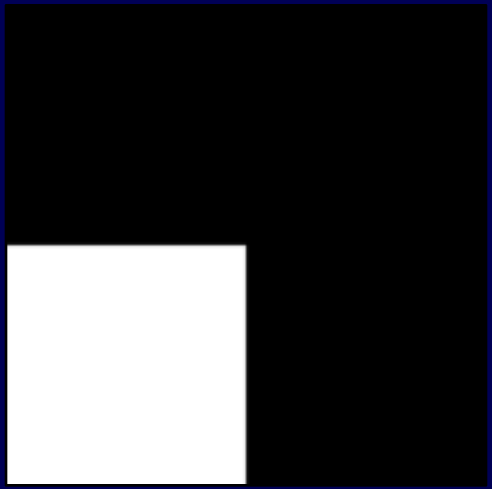
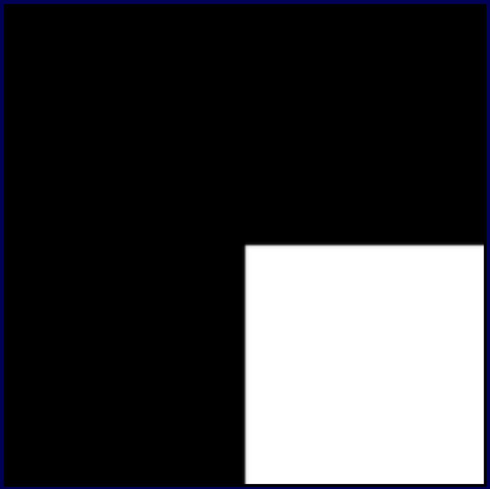




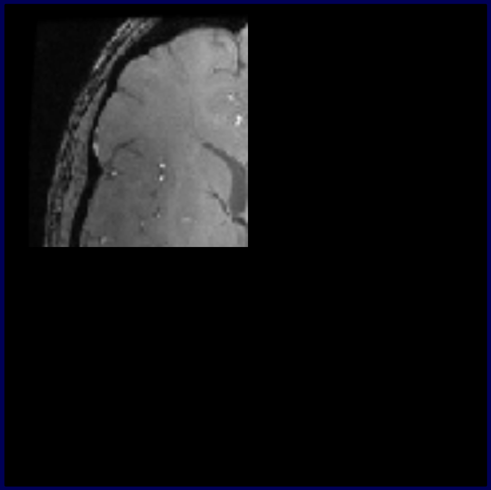
$$I = \sqrt{\left(\sum_{i=1}^{N_{\mathrm{coil}}} I^2_{\mathrm{Coil}_i} \right)}$$

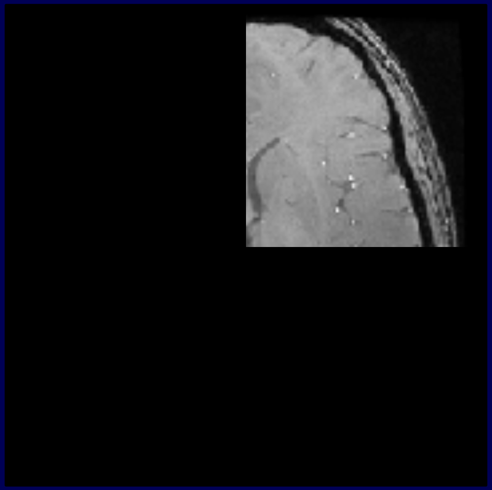
Recommended Reading: “The NMR Phase Array”, Roemer et al, MRM 1990

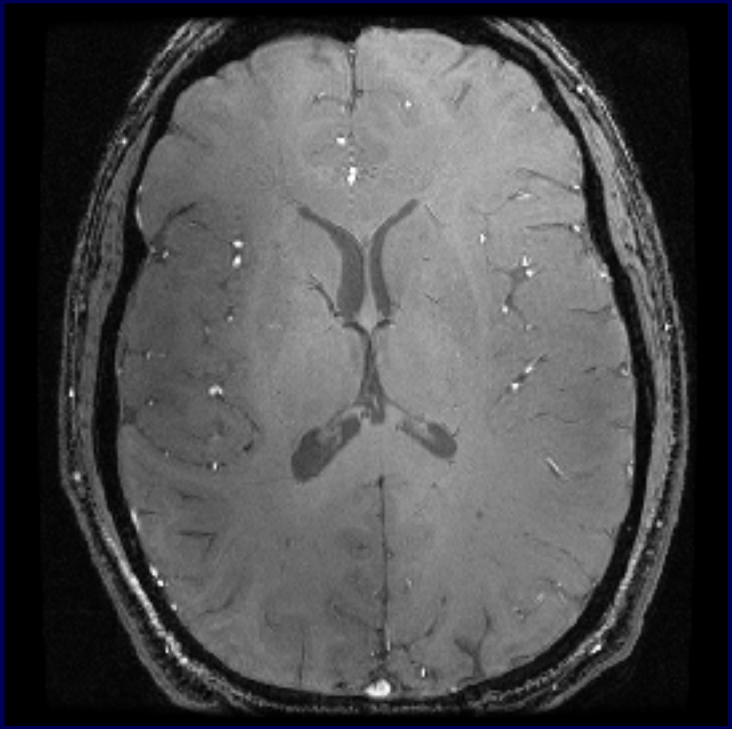
Ideal Coil Sensitivity

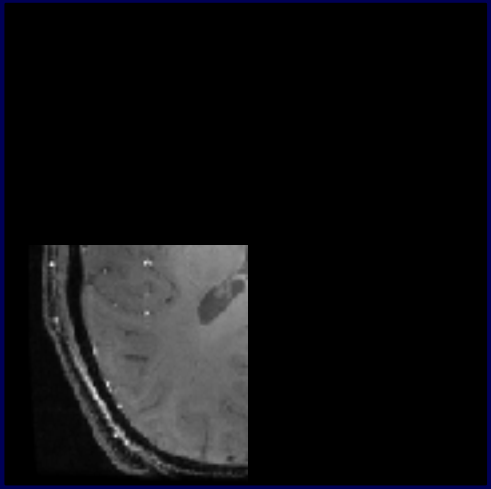






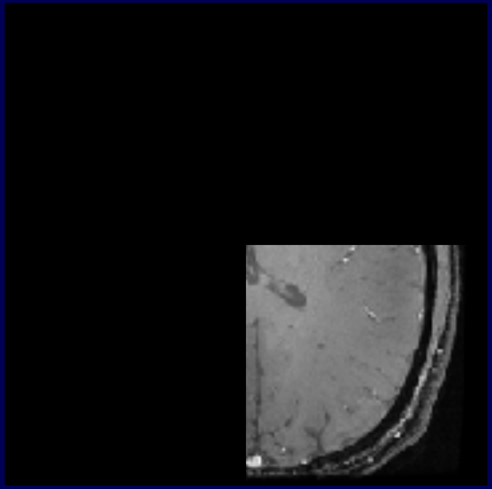
In the ideal world...











Signal Equation with Coils

$$S(\vec{k}) = \int_r M_{xy}(\vec{r}) e^{-i2\pi \vec{k} \bullet \vec{r}} d\vec{r}$$

Coil Sensitivity Modulation

$$S_\gamma(\vec{k}) = \int_r C_\gamma(\vec{r}) M_{xy}(\vec{r}) e^{-i2\pi \vec{k} \bullet \vec{r}} d\vec{r}$$

Coil Sensitivity

MR Signal Equation – Discrete Form

$$S_\gamma(\vec{k}) = \int_r C_\gamma(\vec{r}) M_{xy}(\vec{r}) e^{-i2\pi \vec{k}\cdot\vec{r}} d\vec{r}$$

1D Simplification

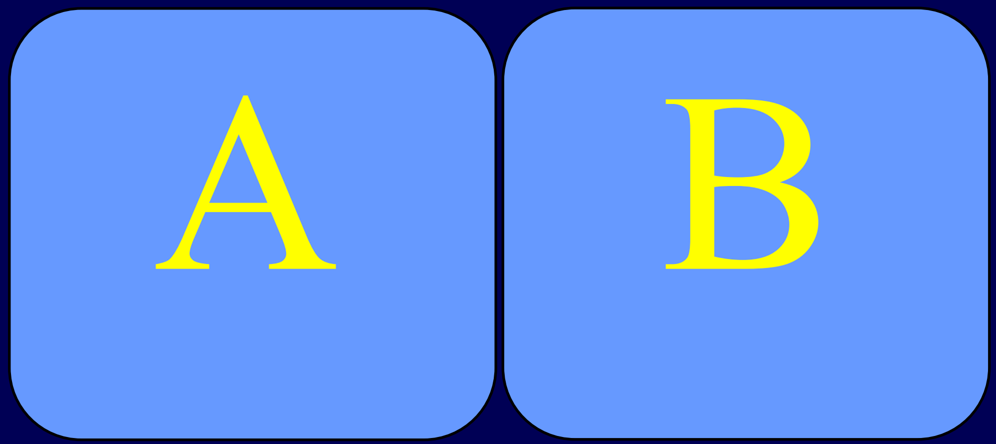
$$S_\gamma(k_y) = \int_y C_\gamma(y) M(y) e^{-i2\pi k_y y} dy$$

Discrete Form

$$S_\gamma(v) = \sum_{0 \le u \le N-1} C_\gamma(u) M(u) e^{-i2\pi \frac{uv}{N}}, 0 \le v \le N-1$$

2-Voxel Case

$$S(v) = \sum_{0<u<N-1} M(u) e^{-i2\pi \frac{uv}{N}}, 0 \le v \le N-1$$



$$S(0) = A + B \quad S(1) = A - B$$

$$S(0) = A + B \quad S(1) = A - B$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} S(0) \\ S(1) \end{pmatrix}$$

$$= \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} S(0) \\ S(1) \end{pmatrix}$$

4 Voxel Case

$$S(v) = \sum_{0<u<N-1} M(u) e^{-i2\pi\frac{uv}{N}}, 0 \le v \le N-1$$

A	B	C	D

$$S(0) = A + B + C + D \qquad S(1) = A - iB - C + iD$$

$$S(2) = A - B + C - D \qquad S(3) = A + iB - C - iD$$

Inverse Problem

$$S(0) = A + B + C + D \qquad S(1) = A - iB - C + iD$$

$$S(2) = A - B + C - D \qquad S(3) = A + iB - C - iD$$

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}^{-1} \begin{pmatrix} S(0) \\ S(1) \\ S(2) \\ S(3) \end{pmatrix}$$

Orthonormal Fourier Encoding Matrix!

4 Voxels with Coils

$$S_\gamma(v) = \sum_{0 \le u \le N-1} C_\gamma(u) M(u) e^{-i2\pi\frac{uv}{N}}, 0 \le v \le N-1$$

A B C D

Coil 1 Coil 2

$$S_1(0) = AC_1(0) + BC_1(1) + CC_1(2) + DC_1(3)$$

$$S_1(1) = AC_1(0) - iBC_1(1) - CC_1(2) + iDC_1(3)$$

$$S_1(2) = AC_1(0) - BC_1(1) + CC_1(2) - DC_1(3)$$

$$S_1(3) = AC_1(0) + iBC_1(1) - CC_1(2) - iDC_1(3)$$

$$S_2(0) = AC_2(0) + BC_2(1) + CC_2(2) + DC_2(3)$$

$$S_2(1) = AC_2(0) - iBC_2(1) - CC_2(2) + iDC_2(3)$$

$$S_2(2) = AC_2(0) - BC_2(1) + CC_2(2) - DC_2(3)$$

$$S_2(3) = AC_2(0) + iBC_2(1) - CC_2(2) - iDC_2(3)$$

Over-determined!

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} C_1(0) & C_1(1) & C_1(2) & C_1(3) \\ C_1(0) & -iC_1(1) & -C_1(2) & iC_1(3) \\ C_1(0) & -C_1(1) & C_1(2) & -C_1(3) \\ C_1(0) & iC_1(1) & -C_1(2) & -iC_1(3) \\ C_2(0) & C_2(1) & C_2(2) & C_2(3) \\ C_2(0) & -iC_2(1) & -C_2(2) & iC_2(3) \\ C_2(0) & -C_2(1) & C_2(2) & -C_2(3) \\ C_2(0) & iC_2(1) & -C_2(2) & -iC_2(3) \end{pmatrix}^{-1} \begin{pmatrix} S_1(0) \\ S_1(1) \\ S_1(2) \\ S_1(3) \\ S_2(0) \\ S_2(1) \\ S_2(2) \\ S_2(3) \end{pmatrix}$$

Under-Sampling!

$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} C_1(0) & C_1(1) & C_1(2) & C_1(3) \\ C_1(0) & -iC_1(1) & -C_1(2) & iC_1(3) \\ C_1(0) & -C_1(1) & C_1(2) & -C_1(3) \\ C_1(0) & iC_1(1) & -C_1(2) & -iC_1(3) \\ C_2(0) & C_2(1) & C_2(2) & C_2(3) \\ C_2(0) & -iC_2(1) & -C_2(2) & iC_2(3) \\ C_2(0) & -C_2(1) & C_2(2) & -C_2(3) \\ C_2(0) & iC_2(1) & -C_2(2) & -iC_2(3) \end{pmatrix}^{-1} \begin{pmatrix} S_1(0) \\ S_1(1) \\ S_1(2) \\ S_1(3) \\ S_2(0) \\ S_2(1) \\ S_2(2) \\ S_2(3) \end{pmatrix}$$

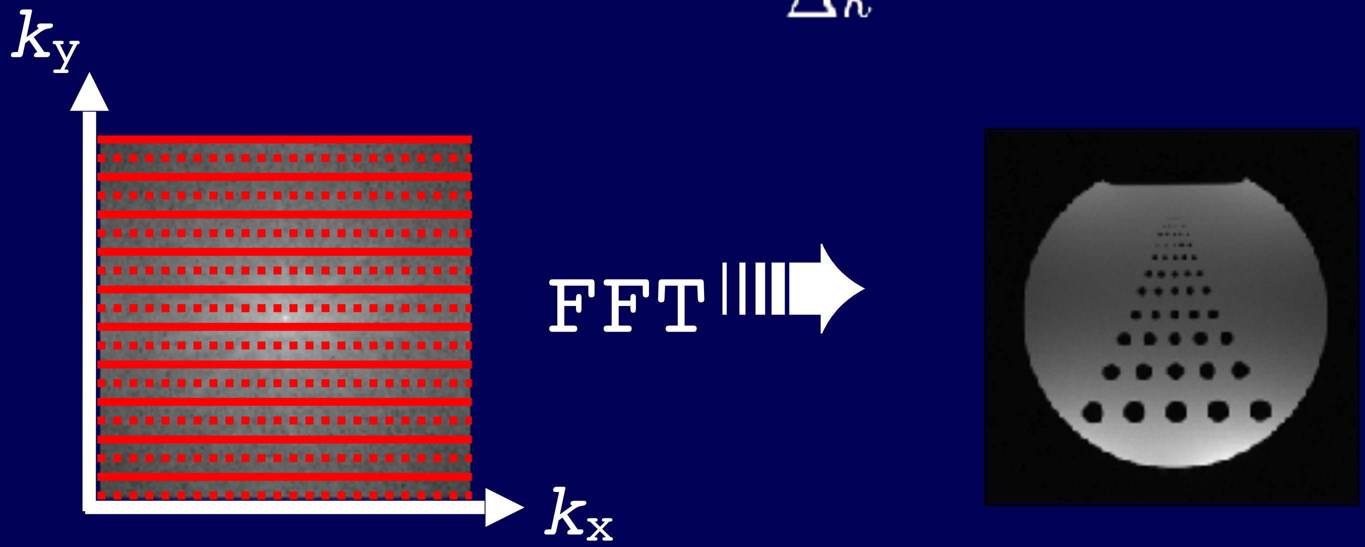
2X Acceleration

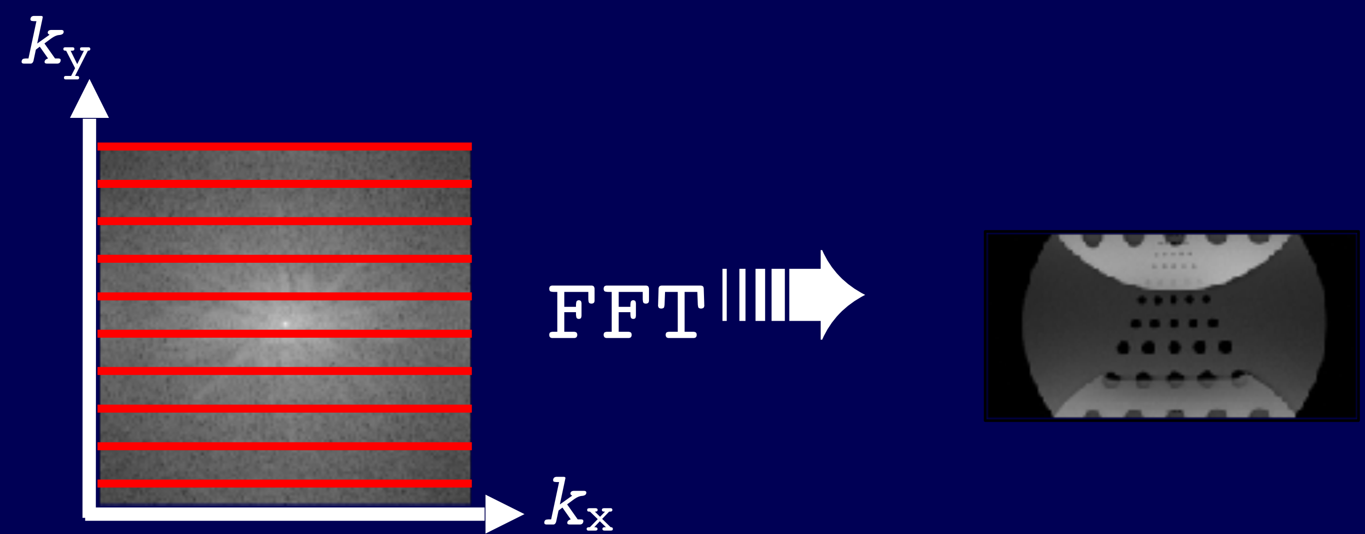
$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} C_1(0) & C_1(1) & C_1(2) & C_1(3) \\ C_1(0) & -C_1(1) & C_1(2) & -C_1(3) \\ C_2(0) & C_2(1) & C_2(2) & C_2(3) \\ C_2(0) & -C_2(1) & C_2(2) & -C_2(3) \end{pmatrix}^{-1} \begin{pmatrix} S_1(0) \\ S_1(2) \\ S_2(0) \\ S_2(2) \end{pmatrix}$$

Sensitivity Encoding Matrix!

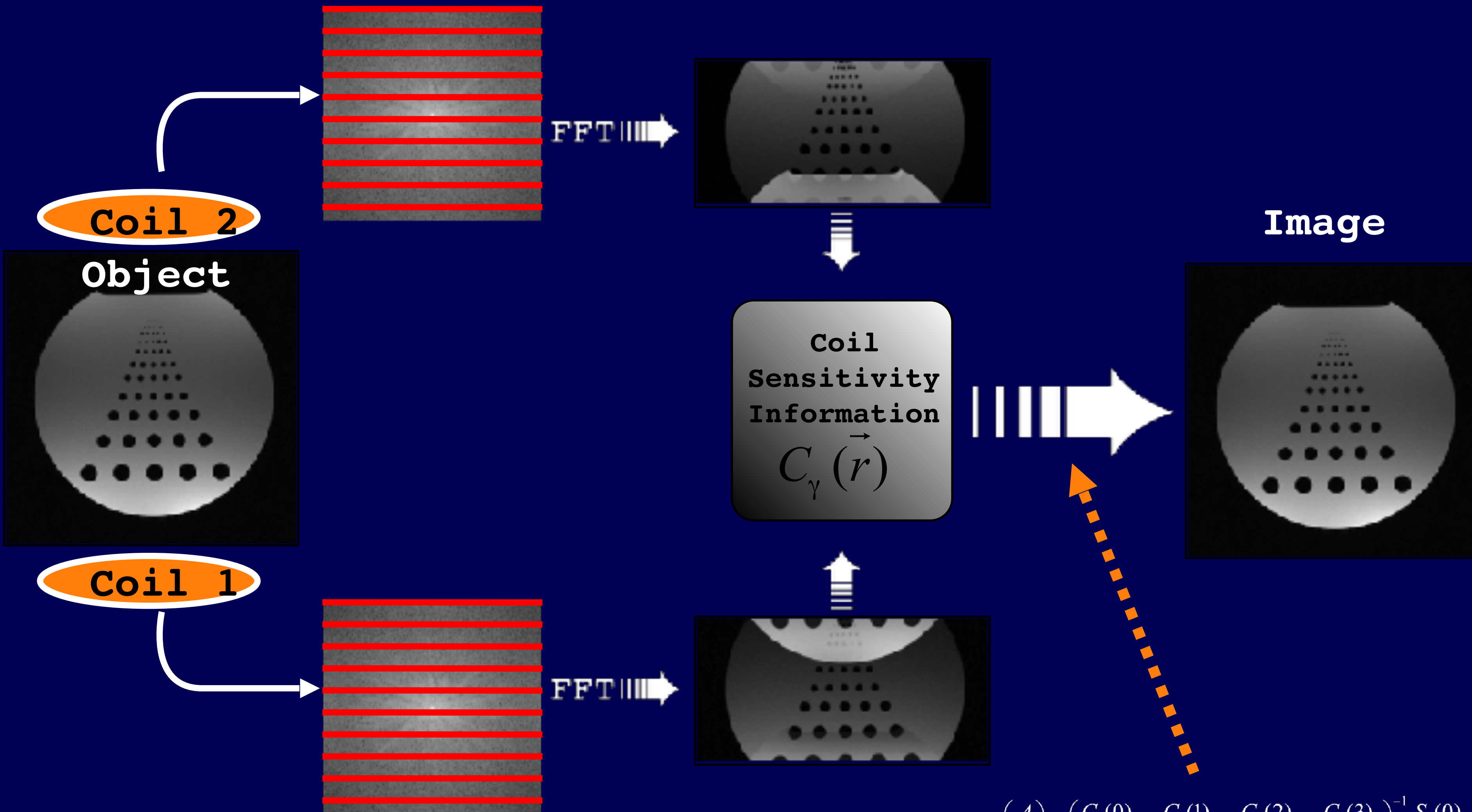
k-space Under-sampling

$$FOV = \frac{1}{\Delta k}$$





SENSE



Unwrap fold over in image space

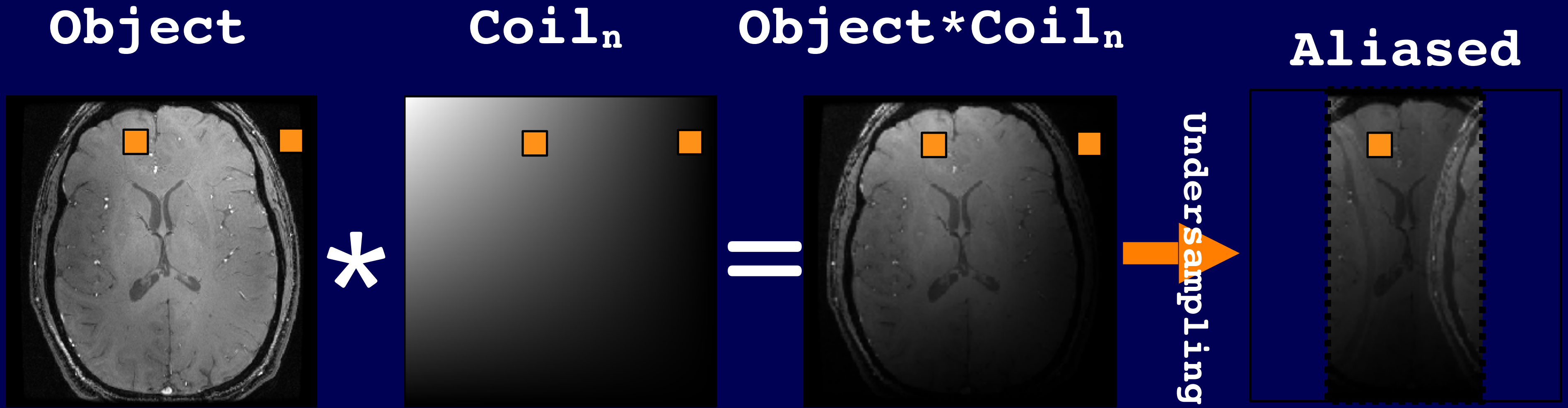
$$\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} C_1(0) & C_1(1) & C_1(2) & C_1(3) \\ C_1(0) & -C_1(1) & C_1(2) & -C_1(3) \\ C_2(0) & C_2(1) & C_2(2) & C_2(3) \\ C_2(0) & -C_2(1) & C_2(2) & -C_2(3) \end{pmatrix}^{-1} \begin{matrix} S_1(0) \\ S_1(2) \\ S_2(0) \\ S_2(2) \end{matrix}$$

Sensitivity Encoding Matrix

- A huge matrix!
  - 256*256*32 by 256*256
- Pseudo inverse can be simplified in Cartesian sampling
- For non-Cartesian scanning, conjugate gradient methods can be used to iteratively solve the inverse problem.
- Requires prior knowledge of coil sensitivity
  - Errors in coil sensitivity causes artifacts

Recommended Reading: “SENSE: sensitivity encoding for fast MRI”, Pruessmann et al, MRM 1999

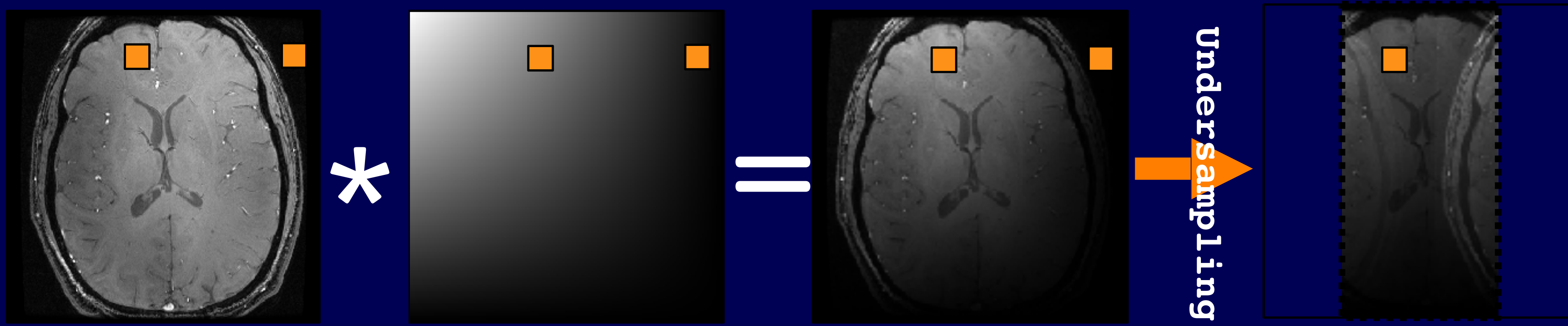
Cartesian SENSE



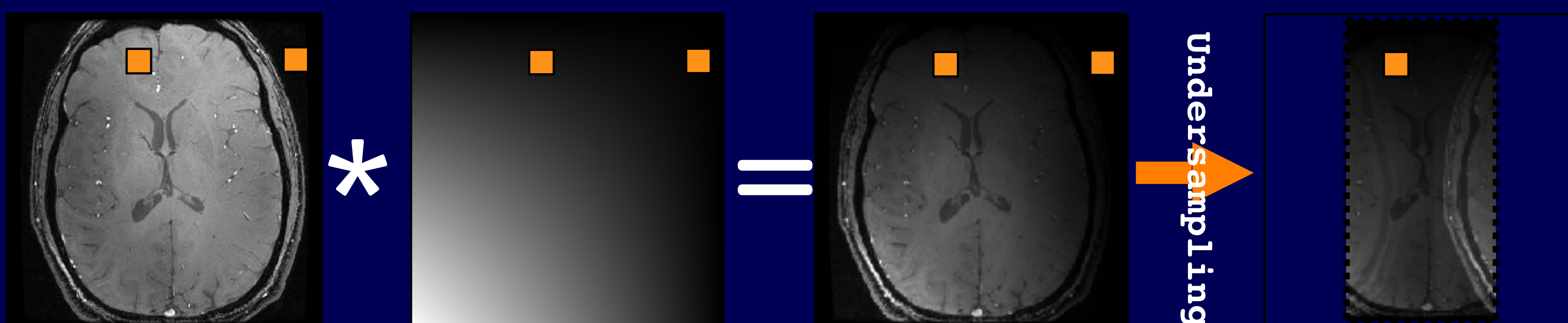
$$S_1(\vec{r}_1) = C_1(\vec{r}_1)\,I(\vec{r}_1) + C_1(\vec{r}_2)\,I(\vec{r}_2)$$

Cartesian SENSE

Object | $Coil_n$ | $Object*Coil_n$ | Aliased

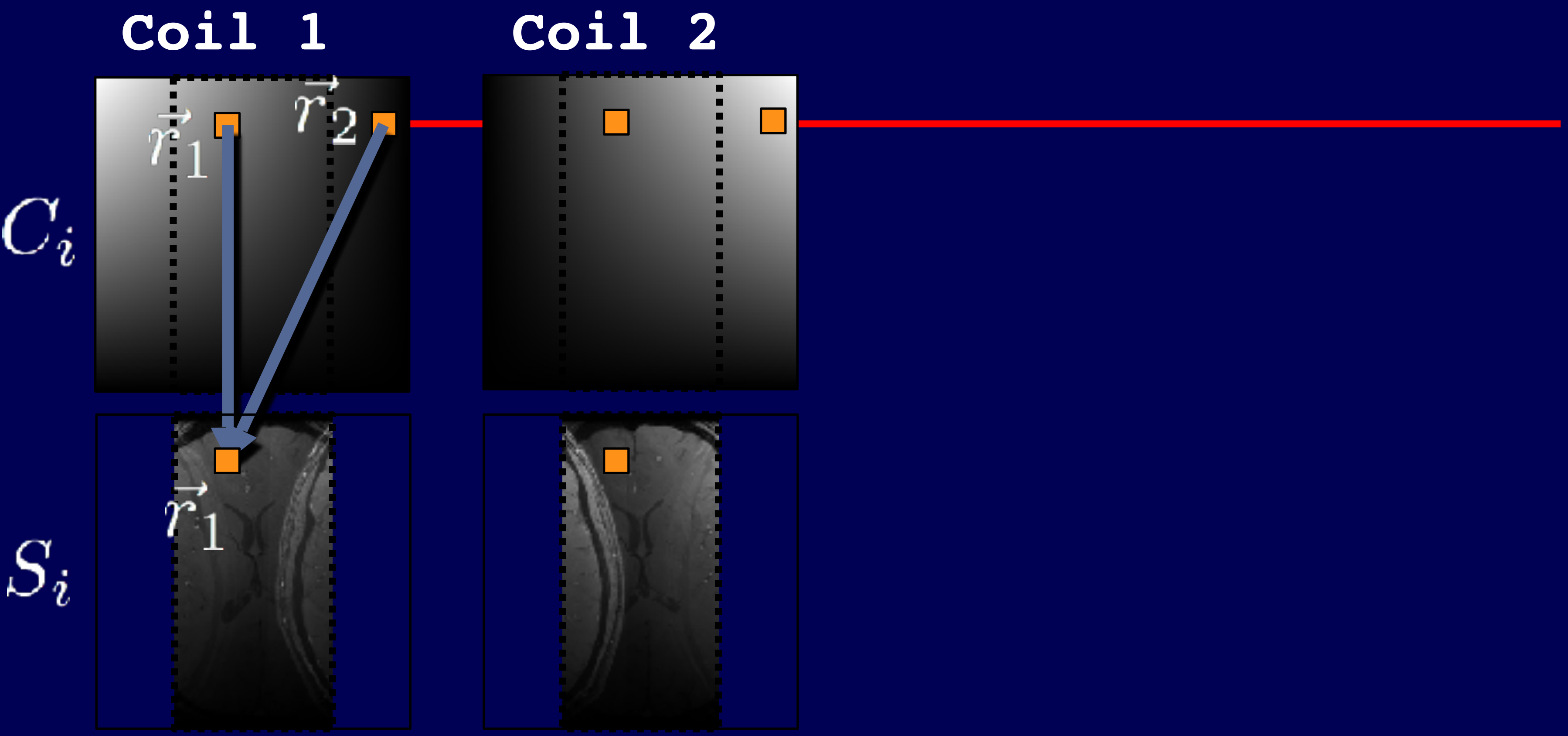


$$S_1(\vec{r}_1) = C_1(\vec{r}_1)\,I(\vec{r}_1) + C_1(\vec{r}_2)\,I(\vec{r}_2)$$
$$S_2(\vec{r}_1) = C_2(\vec{r}_1)\,I(\vec{r}_1) + C_2(\vec{r}_2)\,I(\vec{r}_2)$$

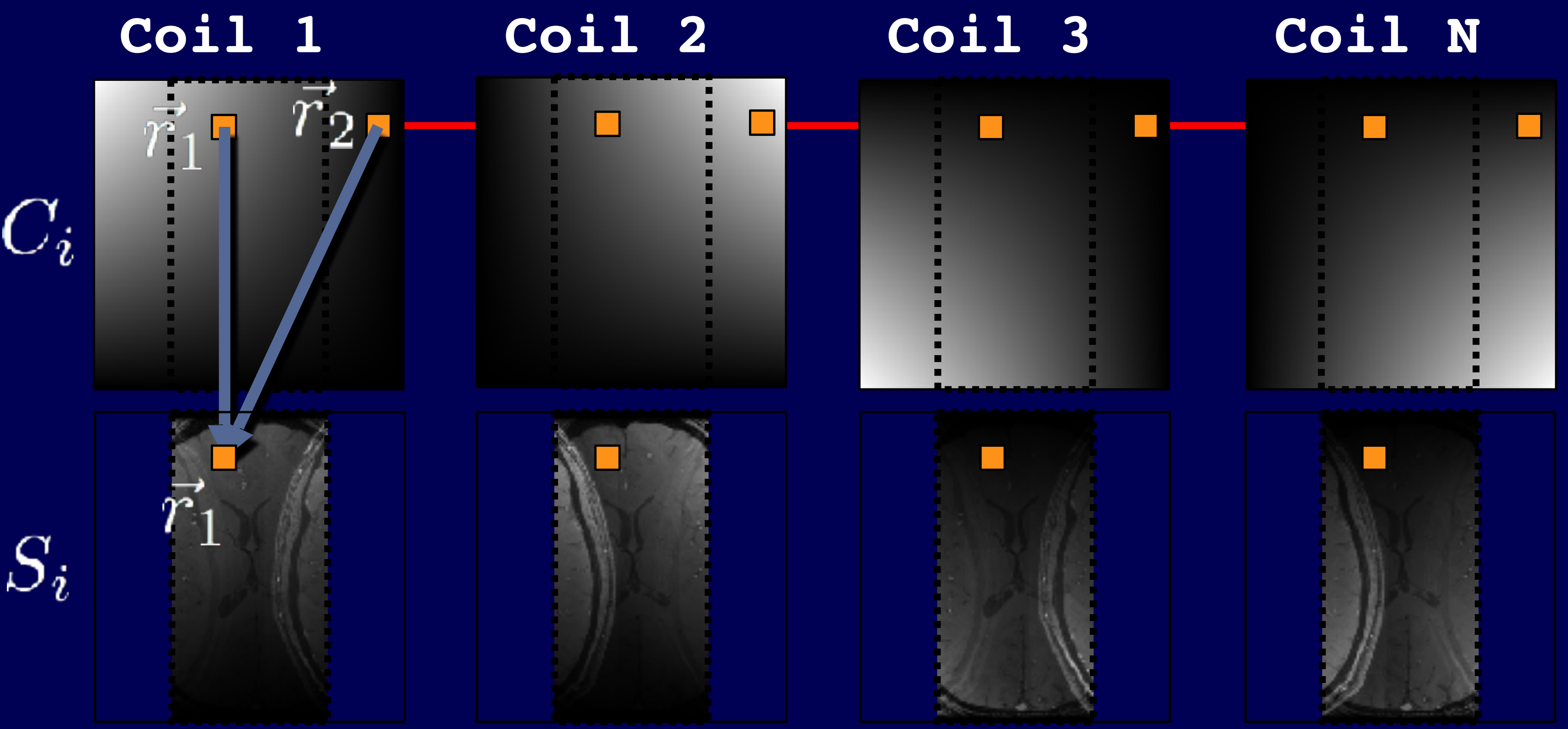


Coil 1

$$S_1(\vec{r}_1) = C_1(\vec{r}_1)\,I(\vec{r}_1) + C_1(\vec{r}_2)\,I(\vec{r}_2)$$



$$S_1(\vec{r}_1) = C_1(\vec{r}_1)\,I(\vec{r}_1) + C_1(\vec{r}_2)\,I(\vec{r}_2)$$
$$S_2(\vec{r}_1) = C_2(\vec{r}_1)\,I(\vec{r}_1) + C_2(\vec{r}_2)\,I(\vec{r}_2)$$



$$
\begin{aligned}
S_1(\vec{r}_1) &= C_1(\vec{r}_1)\, I(\vec{r}_1) + C_1(\vec{r}_2)\, I(\vec{r}_2) \\
S_2(\vec{r}_1) &= C_2(\vec{r}_1)\, I(\vec{r}_1) + C_2(\vec{r}_2)\, I(\vec{r}_2) \\
\vdots &= \vdots + \vdots \\
S_n(\vec{r}_1) &= C_n(\vec{r}_1)\, I(\vec{r}_1) + C_n(\vec{r}_2)\, I(\vec{r}_2)
\end{aligned}
$$

SENSE Rate-2

$$\begin{bmatrix} S_1(\vec{r}_1) \\ S_2(\vec{r}_1) \\ \vdots \\ S_n(\vec{r}_1) \end{bmatrix} = \begin{bmatrix} C_1(\vec{r}_1) & C_1(\vec{r}_2) \\ C_2(\vec{r}_1) & C_2(\vec{r}_2) \\ \vdots & \vdots \\ C_n(\vec{r}_1) & C_n(\vec{r}_2) \end{bmatrix} \begin{bmatrix} I(\vec{r}_1) \\ I(\vec{r}_2) \end{bmatrix}$$

Known [n x 1]

Known [n x 2]

Unknown [2 x 1]

$$\mathbf{I} = \mathbf{C}^{+}\mathbf{S}$$

$$\mathrm{C}^{+} = pseudoinverse\,(\mathrm{C})$$

SENSE and SNR

$$SNR_{SENSE} = \frac{SNR}{g\sqrt{R}}$$

- R - reduction or acceleration factor
  - Loss associated with scan time reduction
  - Typically ~1/2 N-coils
- g - geometry factor
  - Loss associated with coil correlation
  - For R=1, g=1
  - For R=2, g=~1.5-2
- SNR is spatially dependent
  - Higher in areas of aliasing

How Fast Can We Go?

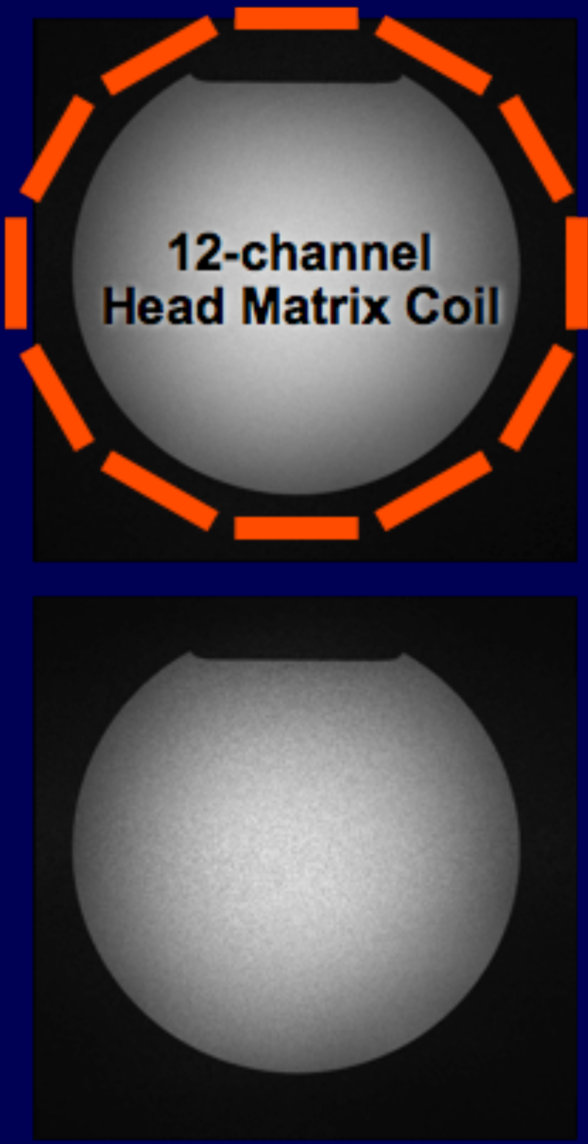
Sensitivity Encoding Matrix Conditioning

- Depends on several factors
  - Accuracy of coil sensitivity
  - K-space under-sampling pattern
  - Coil geometry and sensitivity

- Noise is amplified during inversion
  - G-factor

Parallel Imaging Tradeoffs

ref



PAT x 2

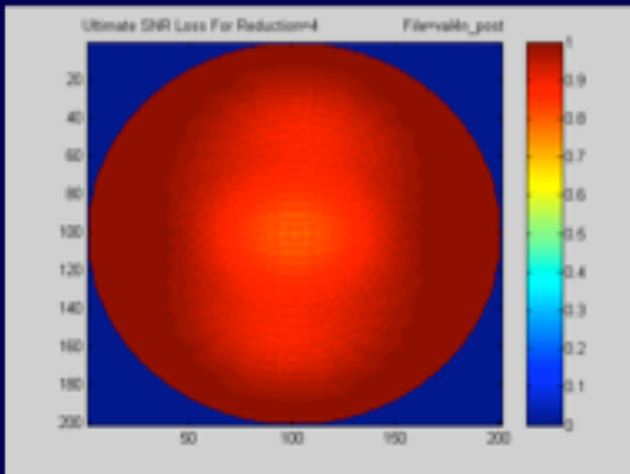
f_p = acceleration factor

g = coil geometry factor

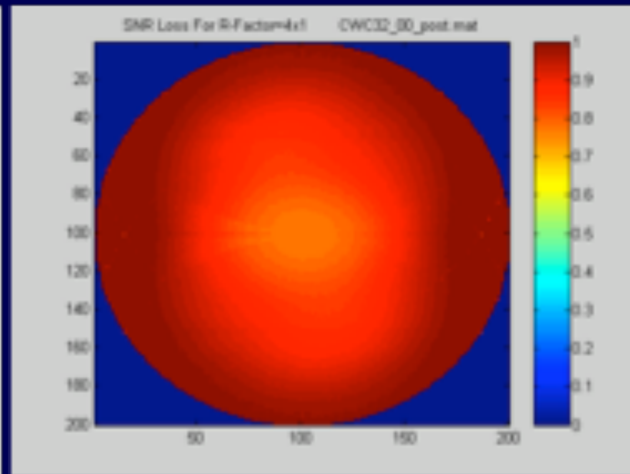
PAT x 3

PAT x 4

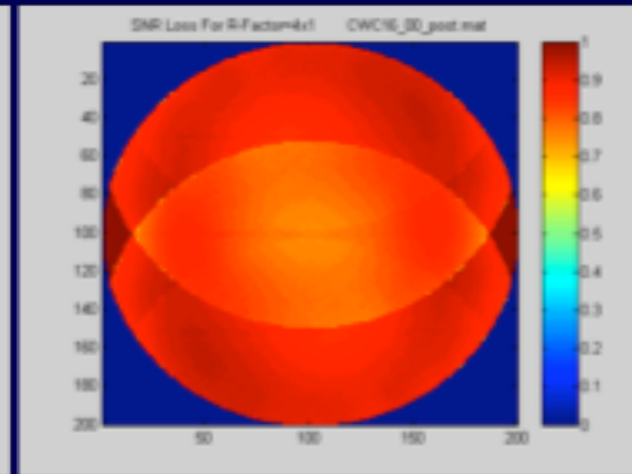
1/g-Map for Rate-4



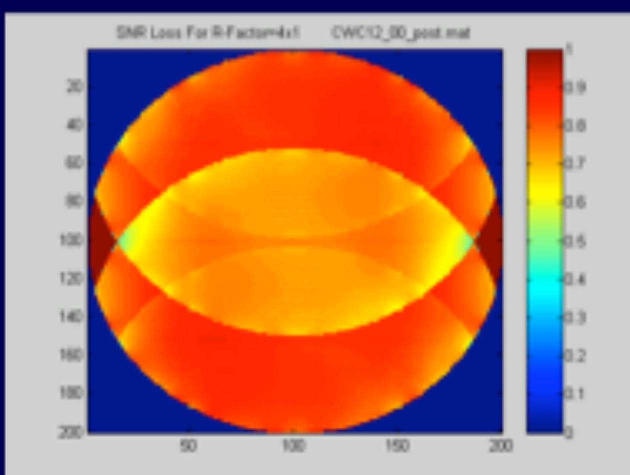
∞ elements



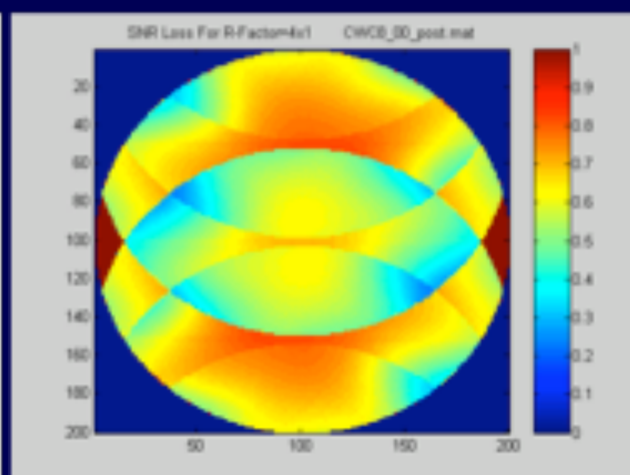
32 elements



16 elements



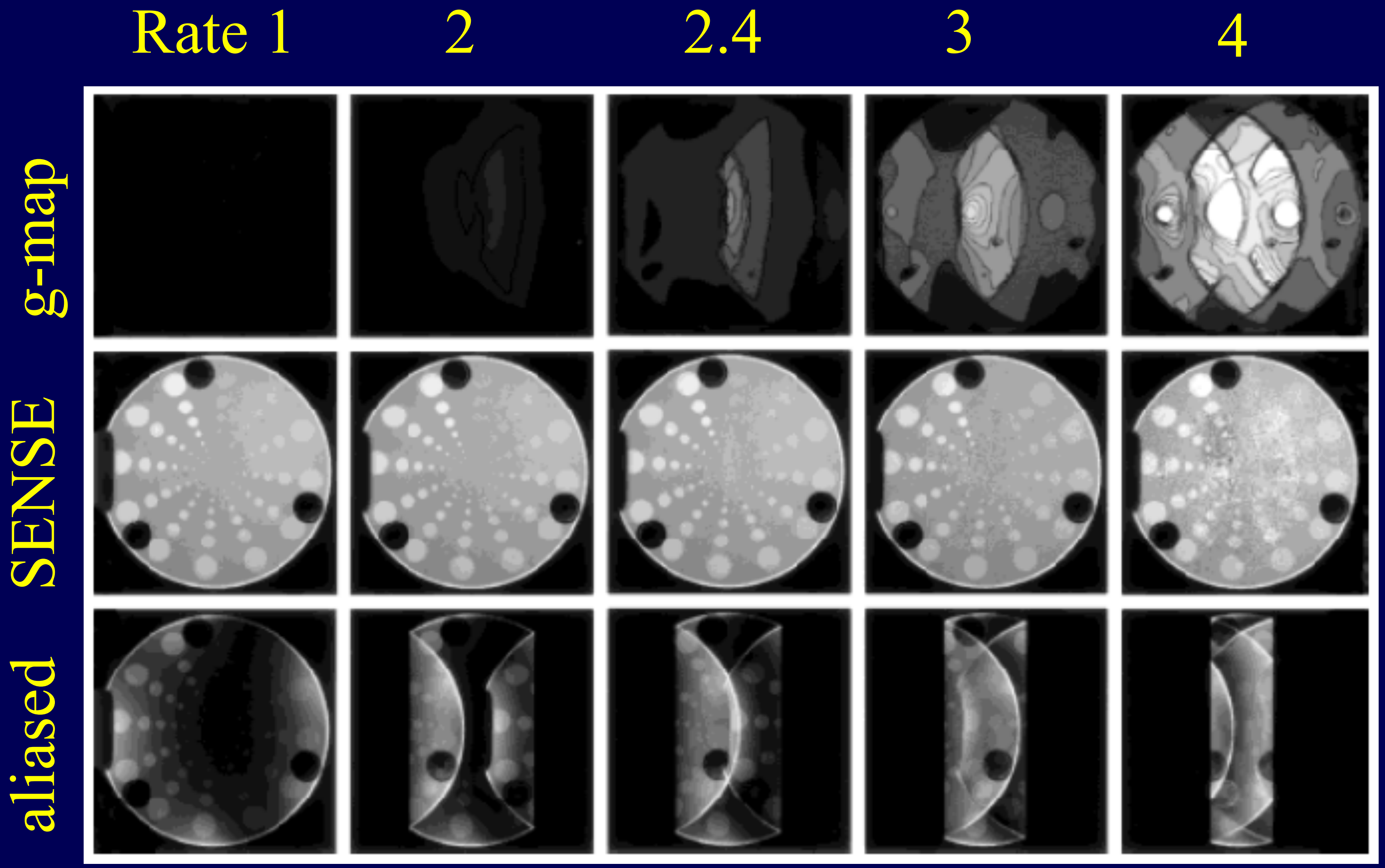
12 elements



8 elements

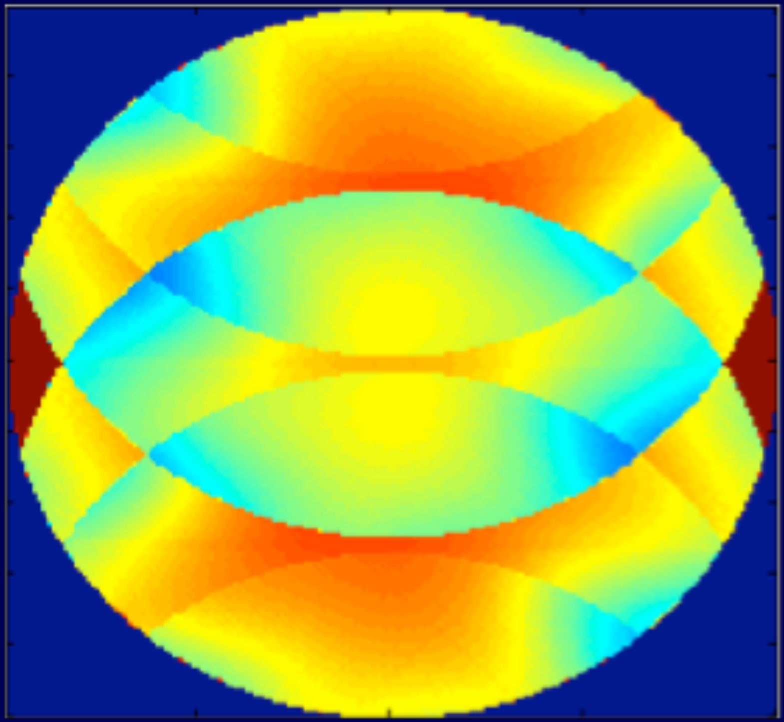
Relative
SNR
Scale

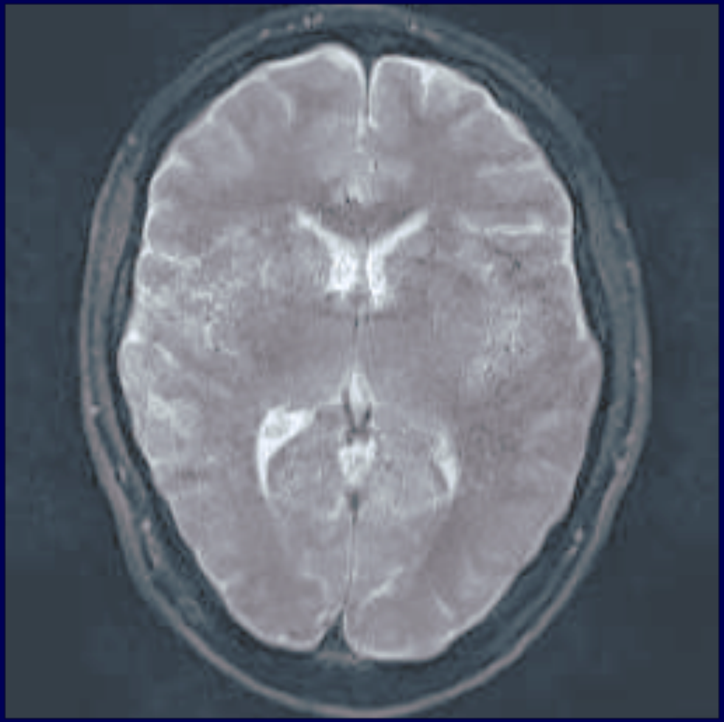
G-factor and its impact on image



Pruessmann et al, MRM 1999

1/g-factor map & Rate-4



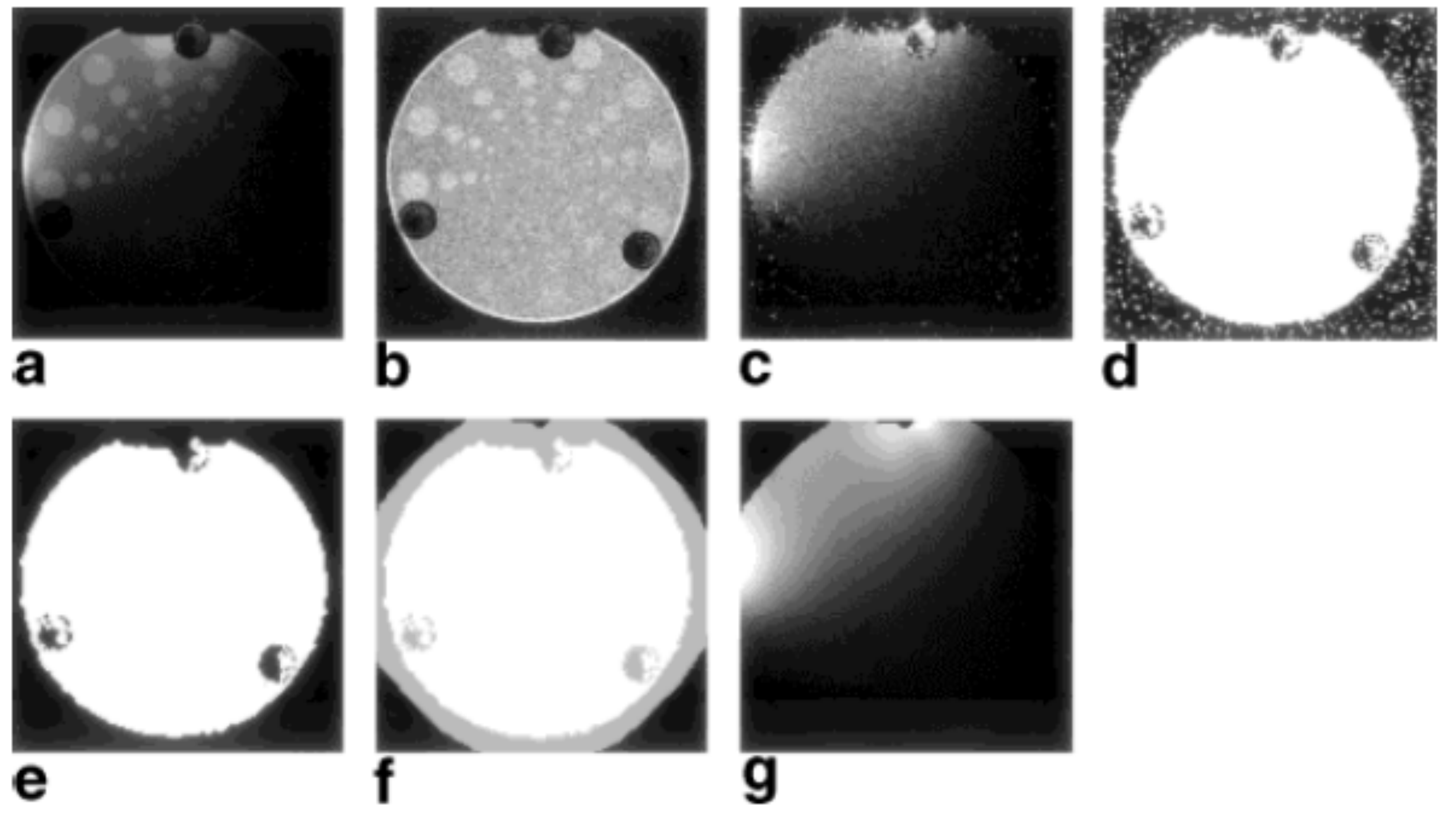


8-channel Head coil Rate-4 (tight FOV)

Outstanding Problems

- SNR optimization
  - Coil design
  - Reconstruction algorithms
- Estimation of ***true*** coil sensitivities

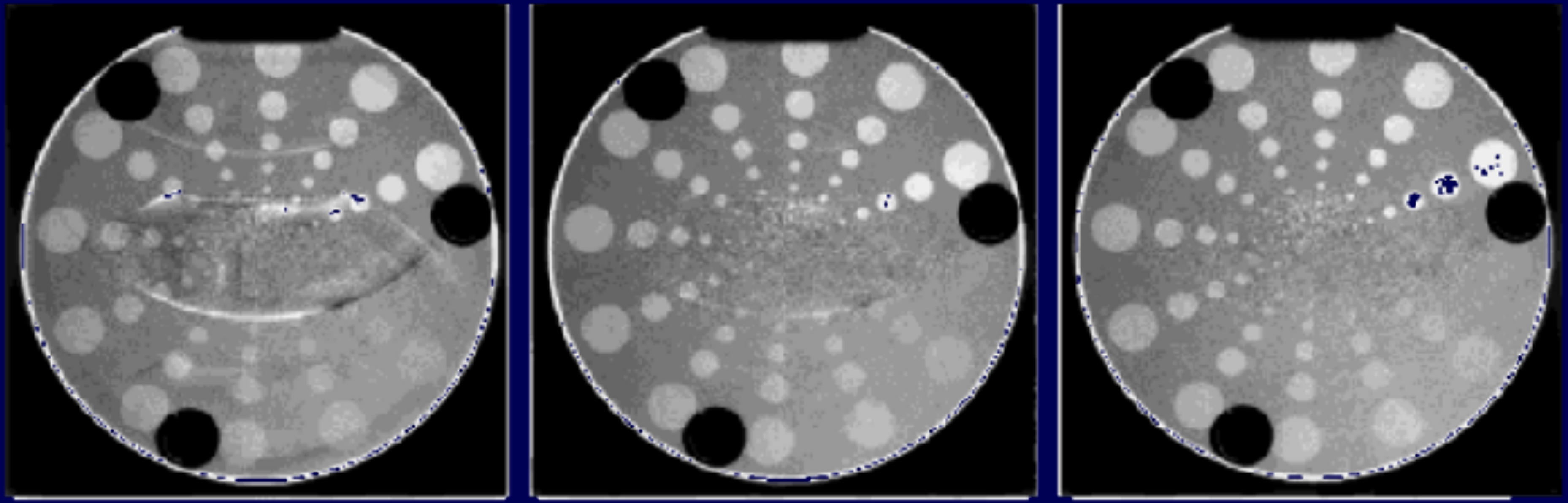
Coil Sensitivity Estimation



Pruessmann et al, MRM 1999

Dependence on coil sensitivity accuracy

- Images reconstructed using coil sensitivity maps calculated using different order P of polynomial fitting



P=0 P=1 P=2

Pruessmann et al, MRM 1999

K-space based parallel imaging methods

Synthesizing spatial harmonics

$$S_\gamma(k_y) = \int_y C_\gamma(y) M(y) e^{-i2\pi k_y y} dy$$

IF

$$C^{comp}(y) = \sum_\gamma n_\gamma C_\gamma(y) = e^{-i2\pi\Delta k_y y}$$

THEN

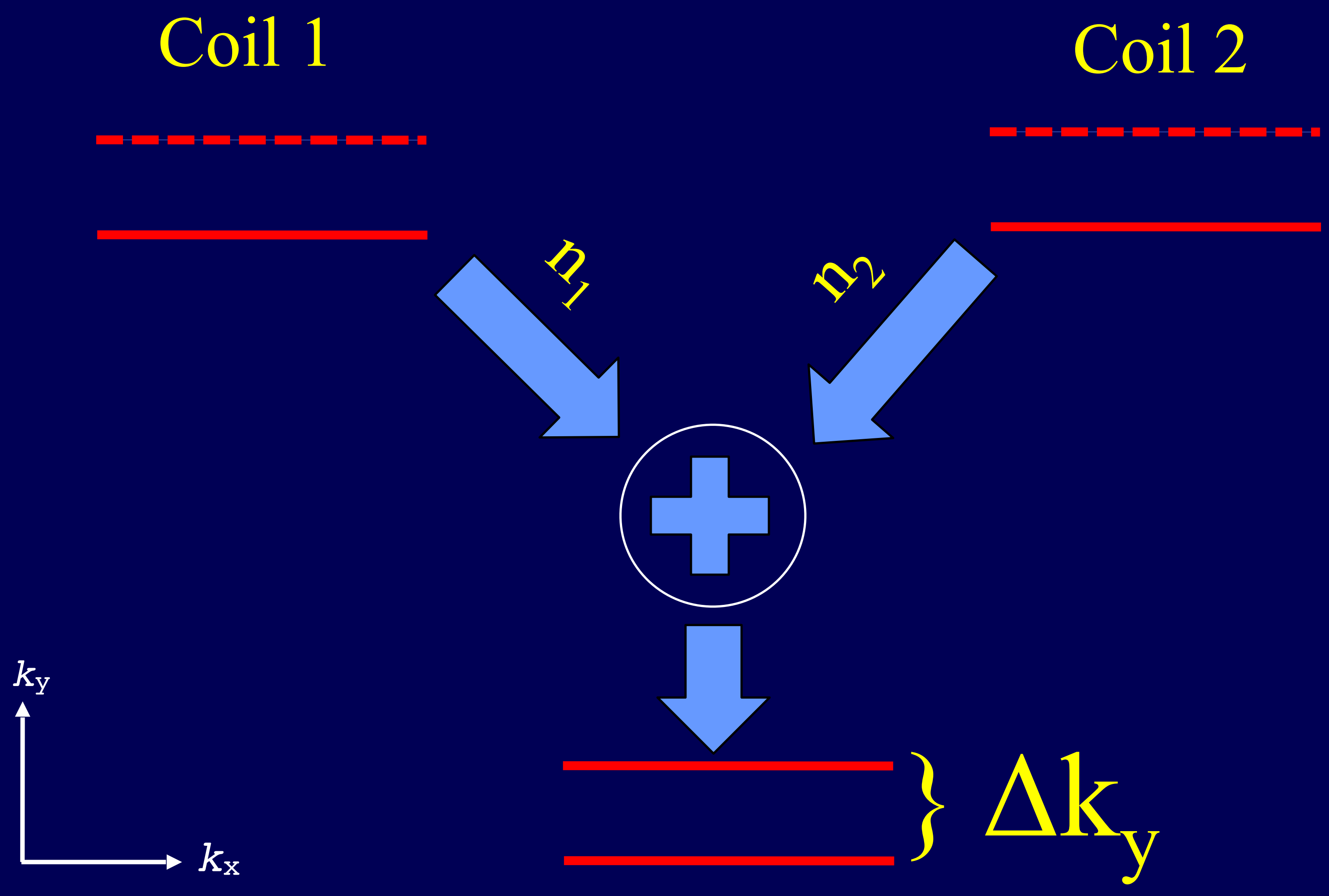
$$\sum_\gamma n_\gamma S_\gamma(k_y) = \sum_\gamma n_\gamma \int_y C_\gamma(y) M(y) e^{i2\pi k_y y} dy$$

$$= \int_y \left(\sum_\gamma n_\gamma C_\gamma(y) \right) M(y) e^{-i2\pi k_y \cdot y} dy = \int_y M(y) e^{-i2\pi(k_y + \Delta k_y) y}$$

$$= S(k_y + \Delta k_y)$$

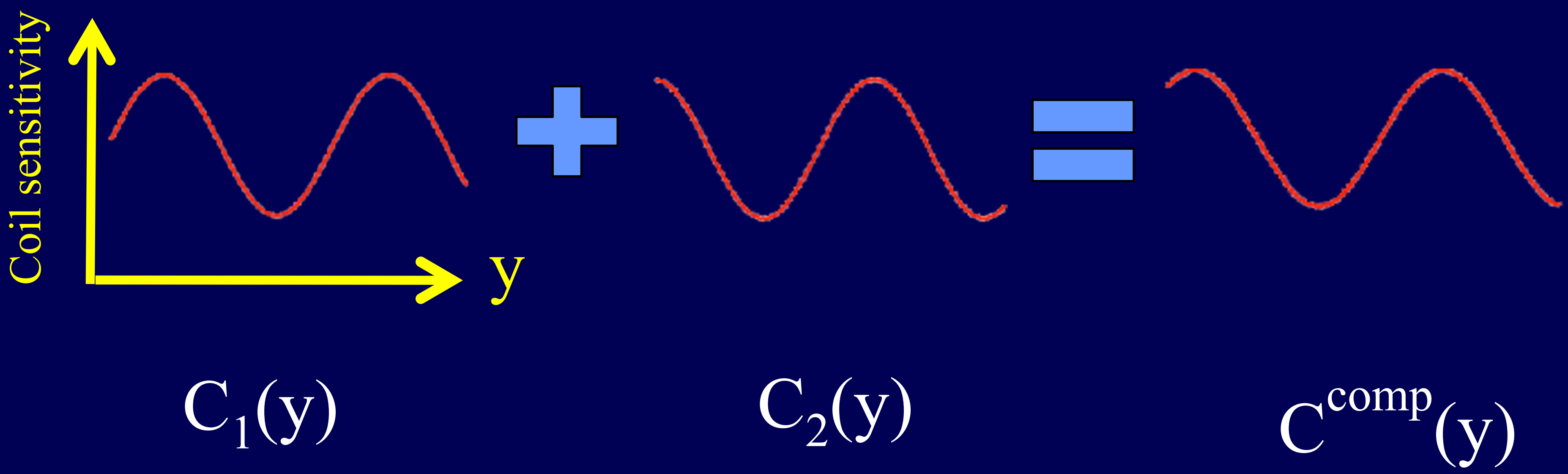
Use of Harmonics: Skipping k-space lines

$$\sum_{\gamma} n_{\gamma} S_{\gamma}\left(k_{y}\right) = S(k_{y} + \Delta k_{y})$$

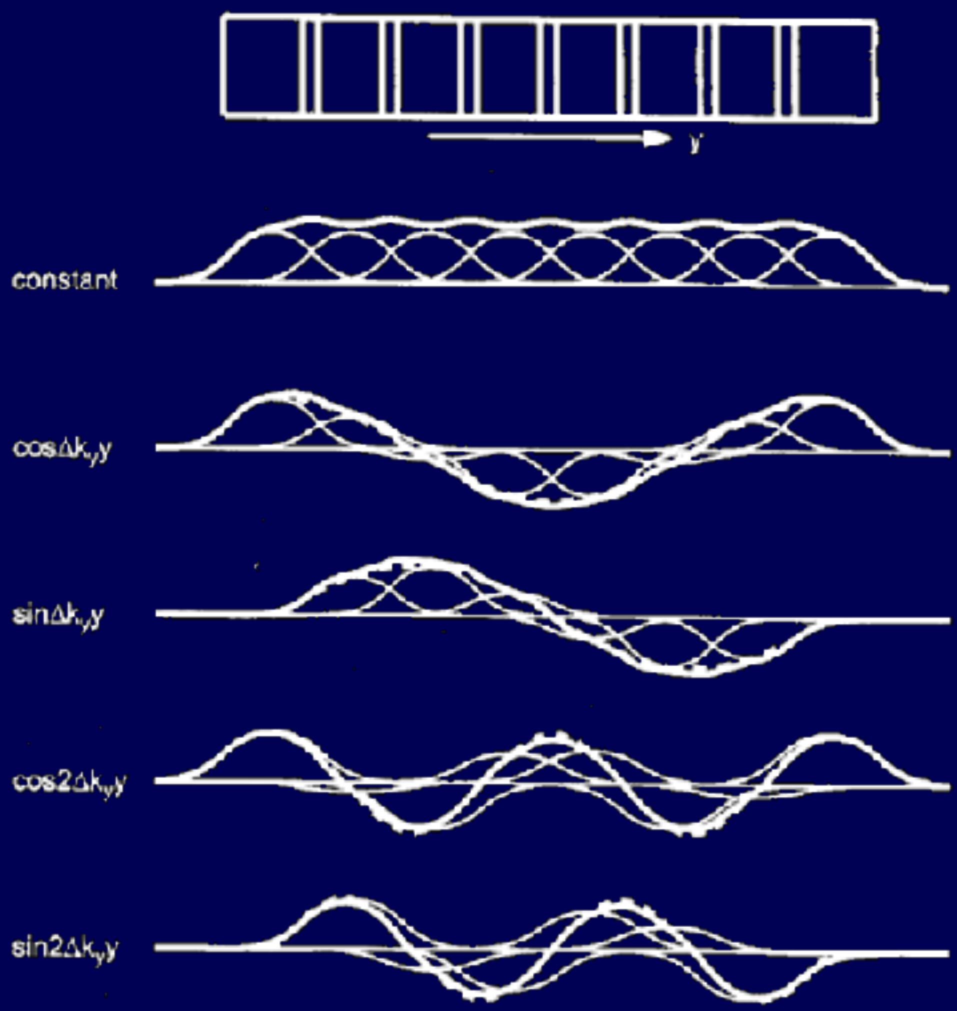


What frequency can we synthesize?

- Depends on the frequency component of coil sensitivities
- Extreme Example:

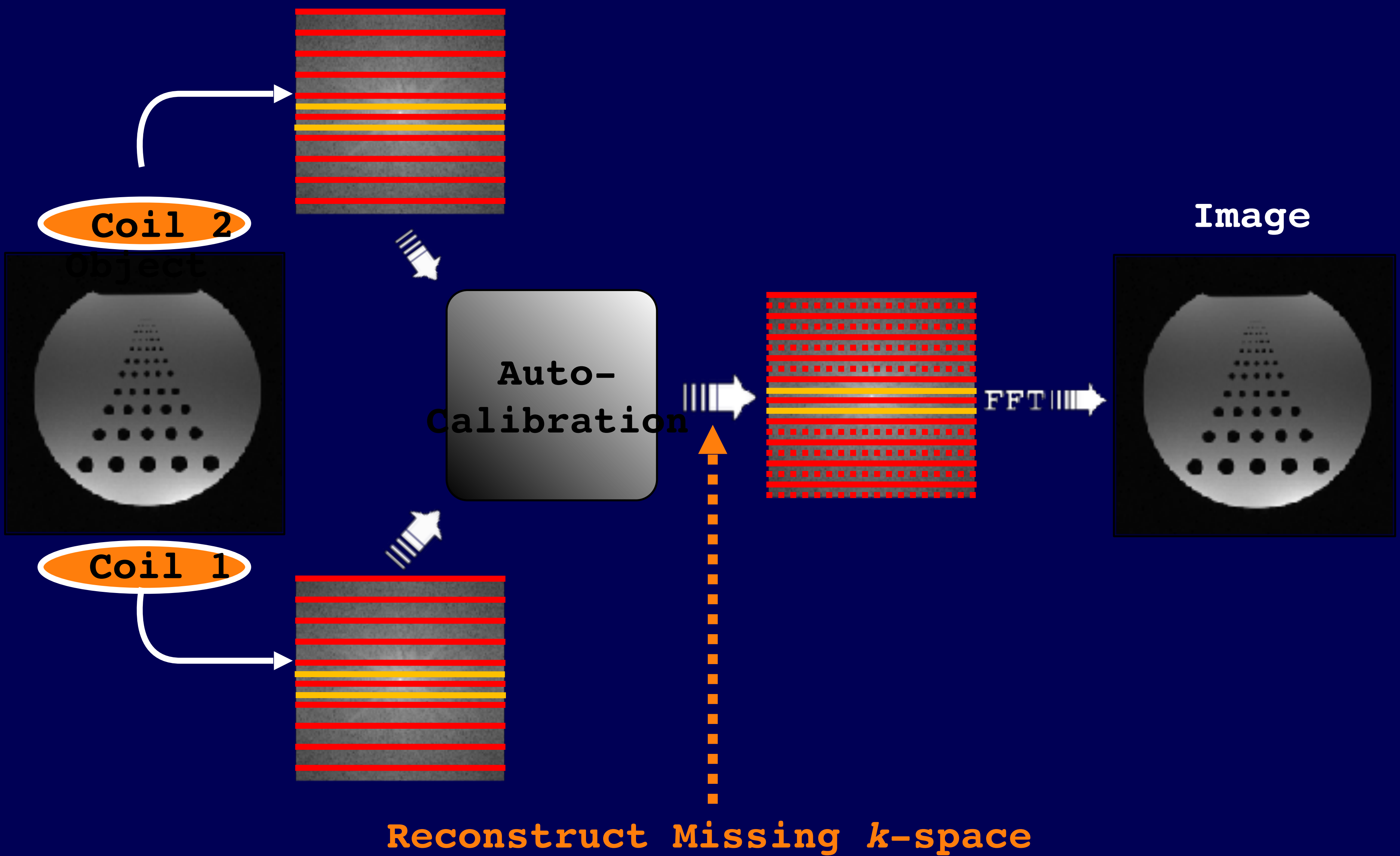


Spatial Harmonics

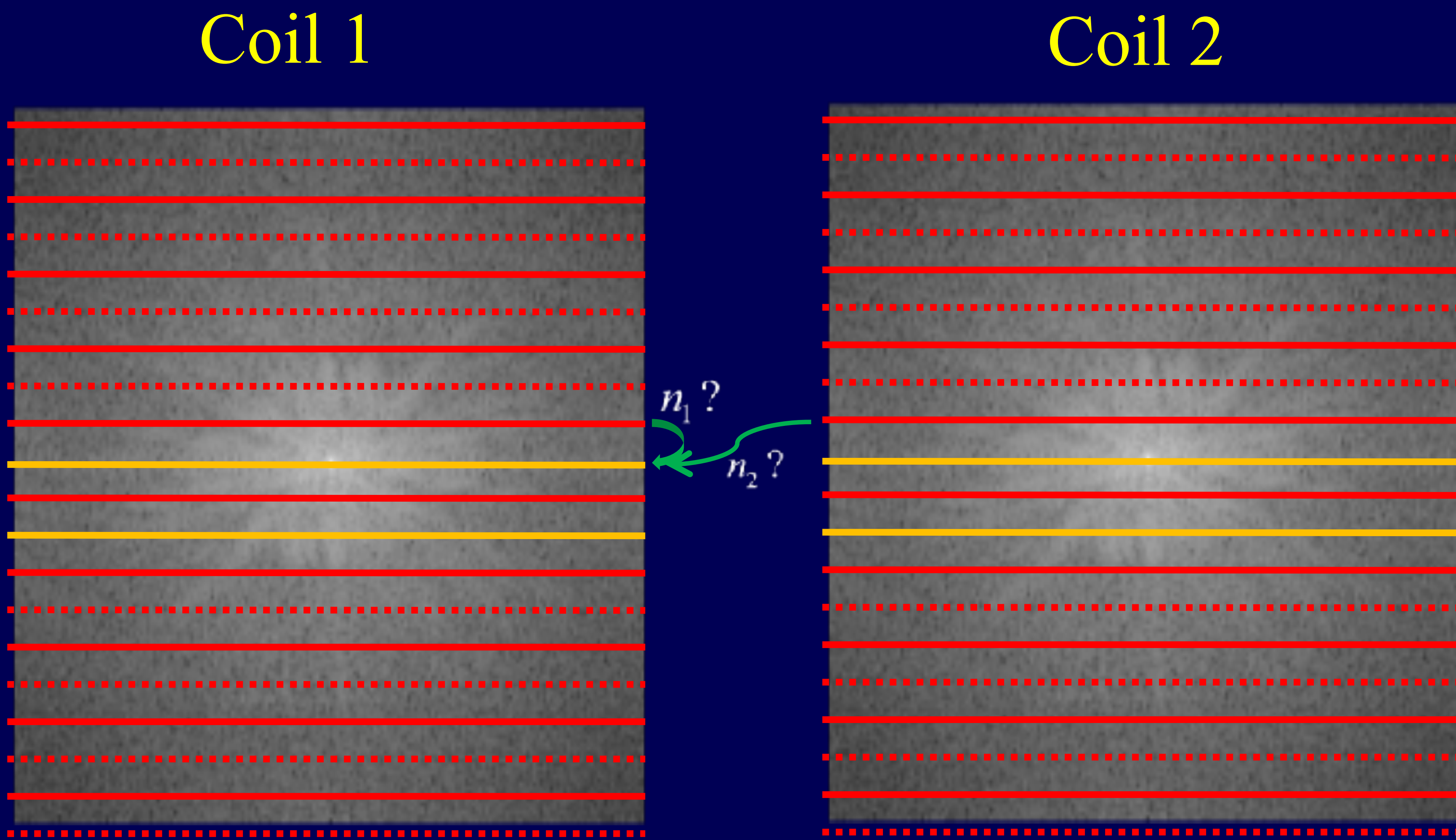


Sodickson et al, MRM 38:591-603

SMASH

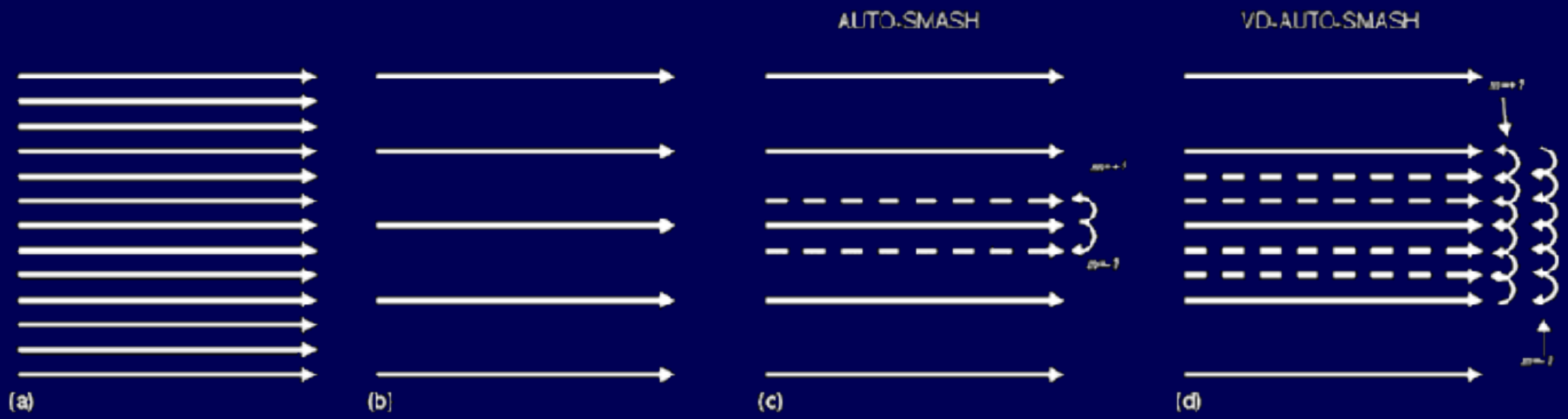

Coil 2
Object
Coil 1
Auto-
Calibration
FFT
Image
Reconstruct Missing *k*-space

Auto-Calibration



$$\sum_{\gamma} n_{\gamma} S_{\gamma}\left(k_{y}\right) = S(k_{y} + \Delta k_{y})$$

Variations of SMASH



Comparison b/w SENSE and SMASH

- SMASH is a special case of SENSE
  - Spatial harmonics allow for reduction of encoding matrix
- SMASH does not require direct measurement of coil sensitivity
  - Auto-calibrating
- SENSE fails when FOV < Object size

Parallel Imaging Summary

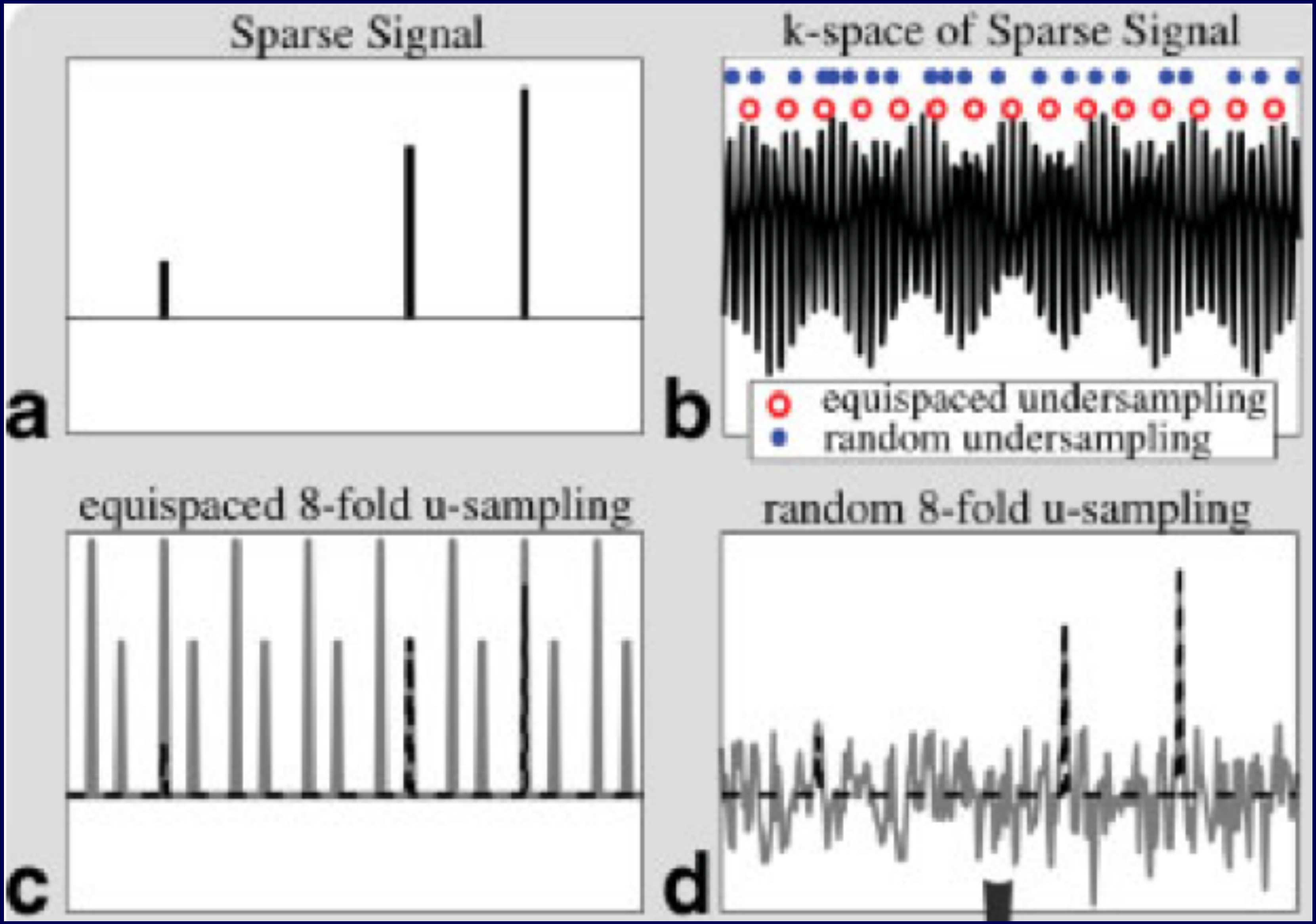
- Parallel imaging uses coil sensitivities to speed up MRI acquisition
- Cases for parallel imaging
  - Higher patient throughput, real-time imaging, imaging for interventions, motion suppression
- Cases against parallel imaging
  - SNR starving applications, imaging coil map problems

Compressed Sensing MRI

- CS is a method complimentary to parallel imaging to speed up image acquisitions
- Two requirements
  - Sparsity in a transform domain
  - Random under-sampling

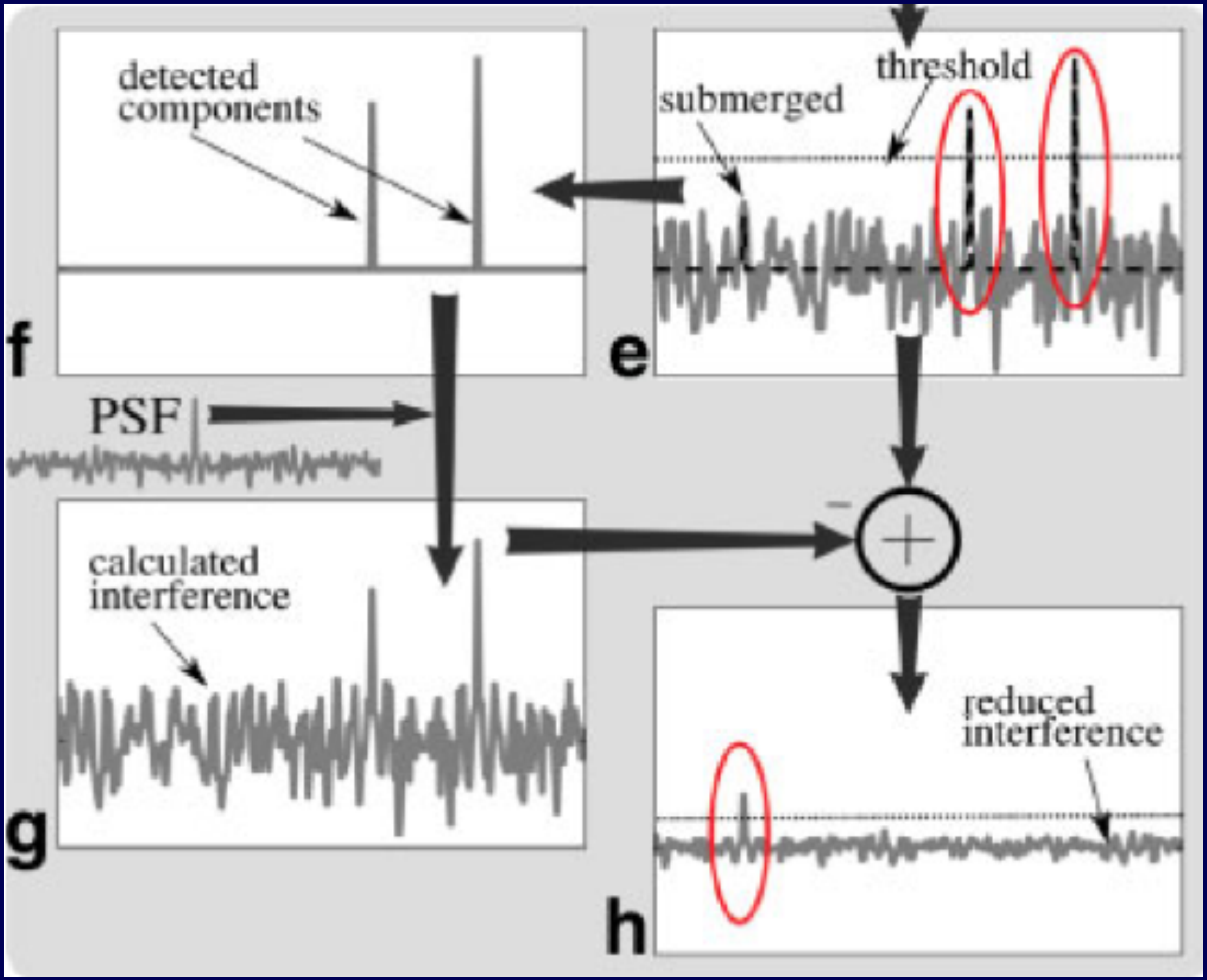
- To the board…

Introduction to CS



Lustig MRM 2007

Introduction to CS

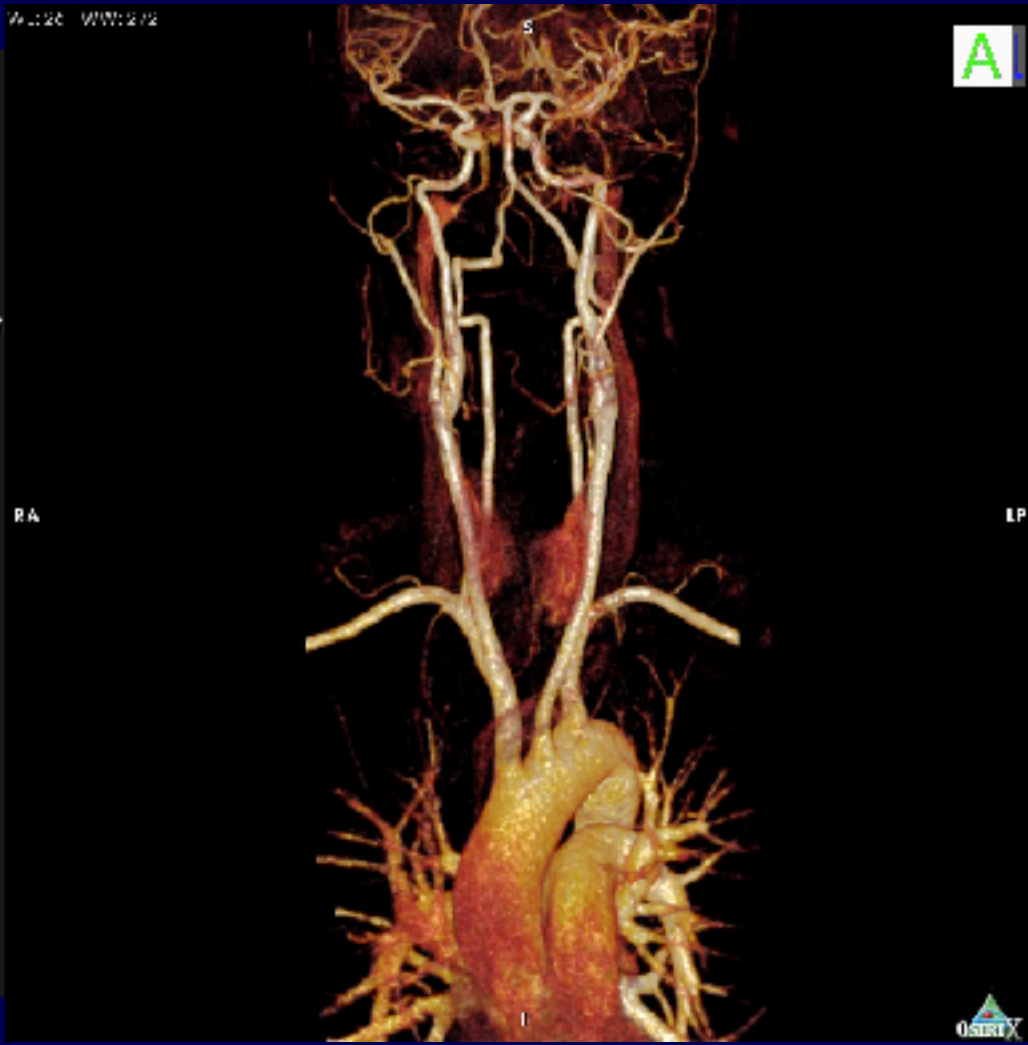


Lustig MRM 2007

Types of Sparsity

- In image domain
  - CE-MR Angiography
- In temporal domain
  - cine cardiac MRI
- In both temporal and image domain
  - Dynamic CE-MRA
  - DCE perfusion
- …

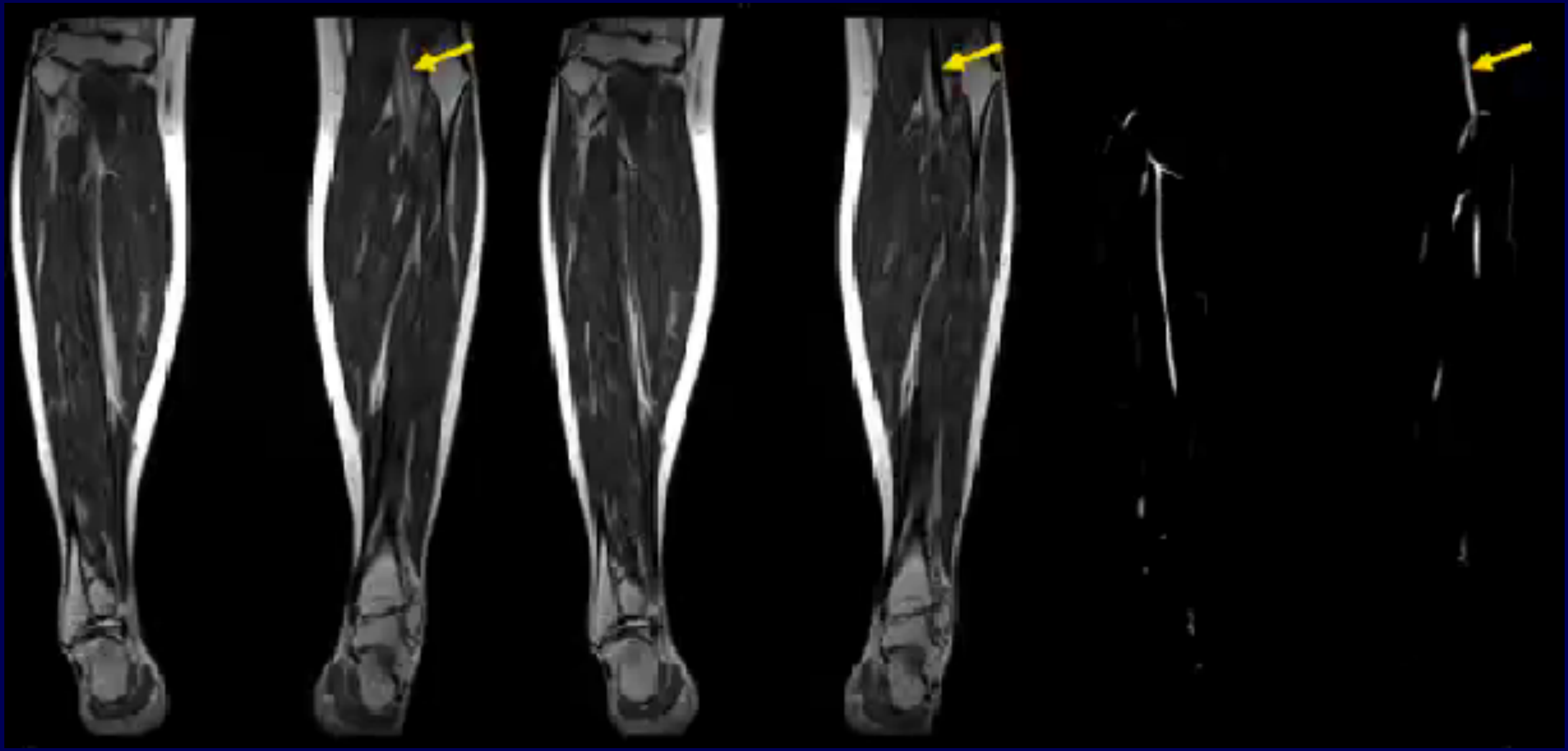
Sparsity in MRA images



DCE MRA and Perfusion

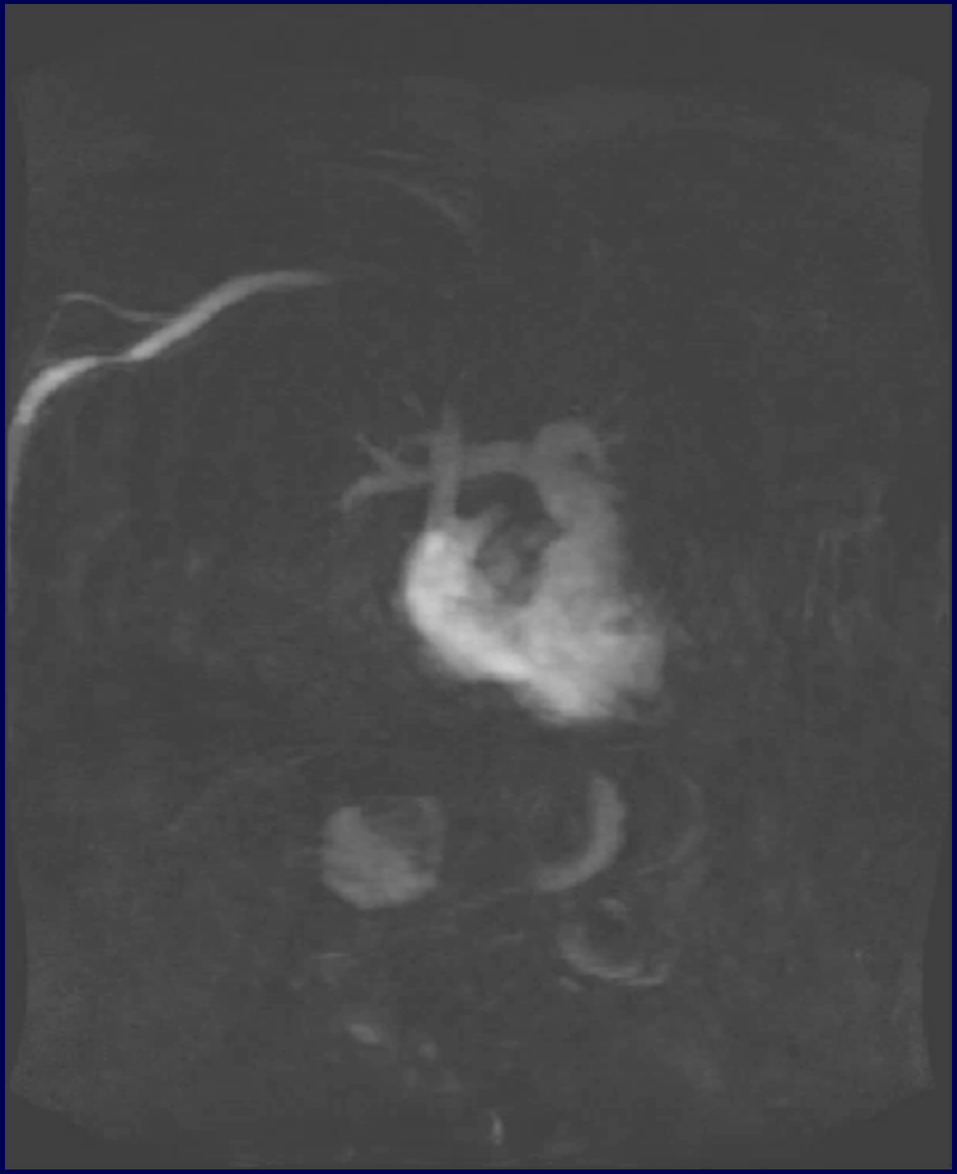
- Background Subtraction before CS
  - Enhanced sparsity, higher temp. resol.

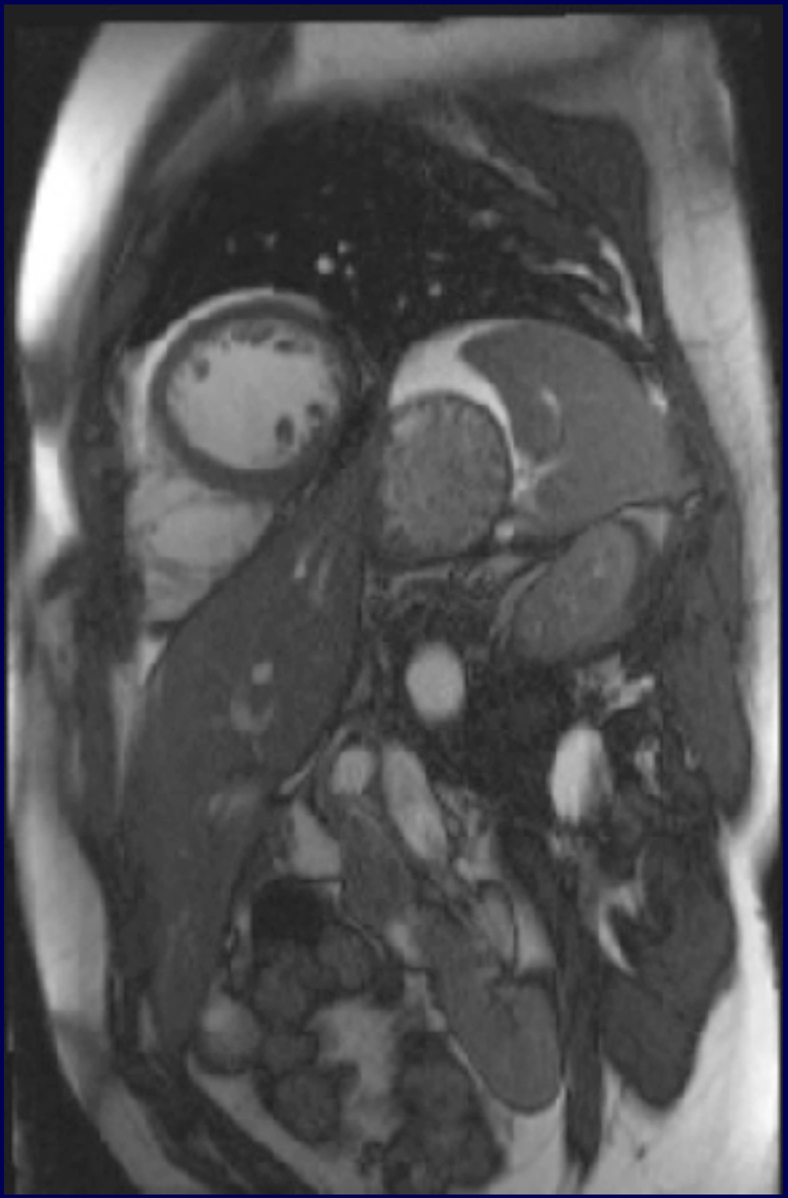
Systole − Diastole = Diff.



Storey, Lee, NYU, ISMRM 2010

Sparsity in Time





Questions?

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