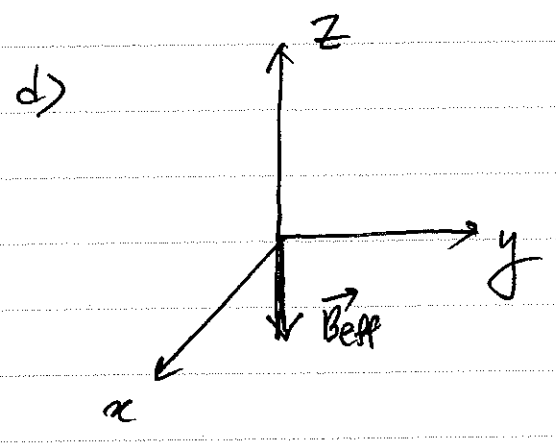
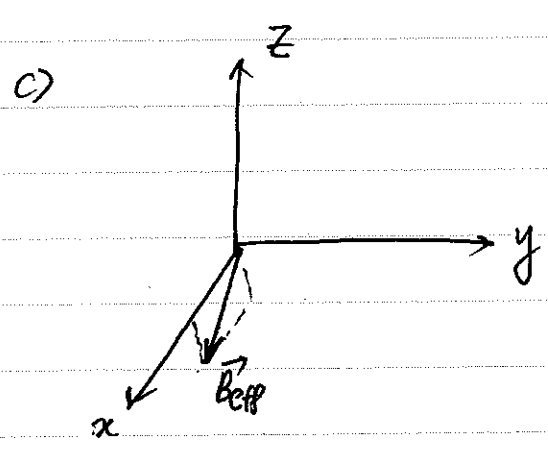
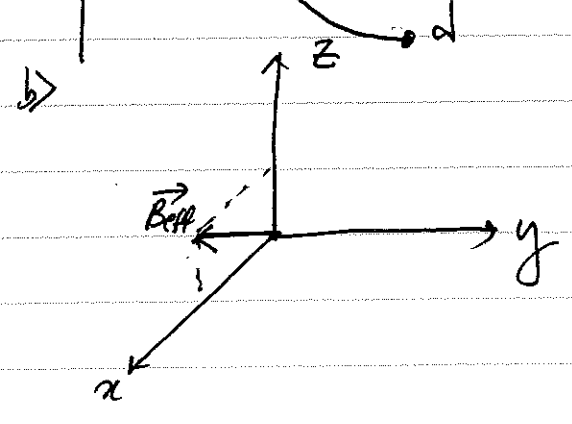
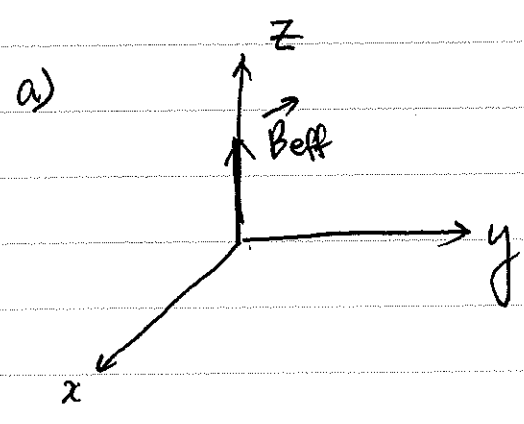
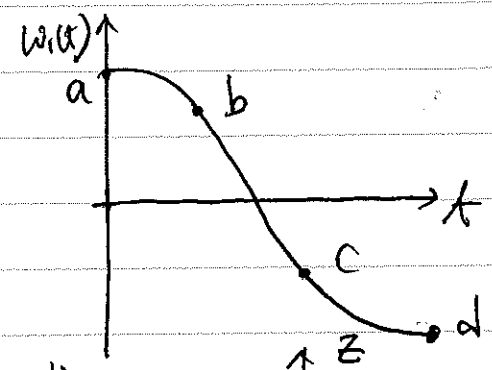
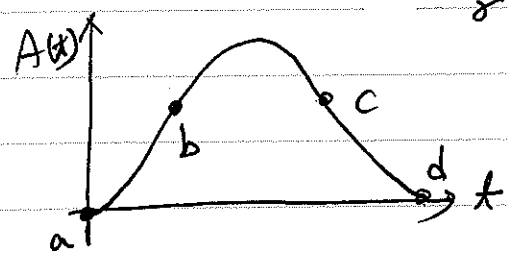


* Adiabatic RF pulse

$$B_1(t) = A(t) e^{-i \omega_1(t) \cdot t}$$

$$\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \frac{\omega_1(t)}{\gamma} \end{pmatrix}$$

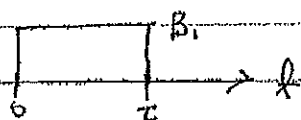


①

* Small tip-angle example

- consider a rectangular RF pulse (duration of τ)

$$B_1(t) = B_1 \cdot \Pi\left(\frac{t - \tau/2}{\tau}\right)$$



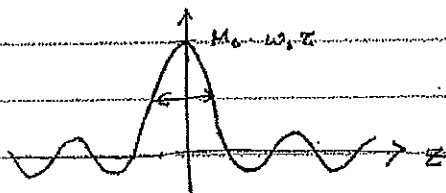
$$M_r(\tau, z) = i M_0 e^{-i \omega(z) \tau/2} \mathcal{F}_{1D} \left\{ \omega_1 \left(t + \frac{\tau}{2} \right) \right\} \Big|_{f = \frac{-i}{2\pi} (\omega_2 \cdot z)}$$

$$\omega_1 \left(t + \frac{\tau}{2} \right) = \underbrace{\gamma \cdot B_1}_{\omega_1} \cdot \Pi\left(\frac{t}{\tau}\right)$$

$$\mathcal{F}_{1D} \left\{ \Pi\left(\frac{t}{\tau}\right) \right\} = \tau \operatorname{sinc}(\tau \cdot f)$$

$$\Rightarrow M_r(\tau, z) = i M_0 e^{-i \omega(z) \tau/2} \cdot \underbrace{\omega_1 \tau}_{\text{canceled out by the refocusing pulse}} \operatorname{sinc}\left(\tau \cdot \frac{\gamma}{2\pi} G_2 \cdot z\right)$$

canceled out
by the refocusing pulse



$$\Delta z = \frac{1}{\frac{\gamma}{2\pi} G_2}$$

* Full analytical solution

②

$$\frac{d\vec{M}}{dt} = \begin{bmatrix} 0 & w(z) & 0 \\ -w(z) & 0 & w_1(t) \\ 0 & -w_1(t) & 0 \end{bmatrix} \vec{M}$$

$$M_r = M_x + i M_y \leftarrow \text{phasor representation}$$

$$\frac{dM_r}{dt} = -i w(z) M_r + i w_1(t) M_z$$

$$\frac{dM_z}{dt} = -i w_1(t) M_y$$

Solution \Rightarrow very difficult \Rightarrow using approximation

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1$$

$$M_z \approx M_0 \approx \text{constant}$$

$$\Rightarrow \frac{dM_z}{dt} = 0$$

$$\frac{dM_r}{dt} = -i w(z) M_r + i w_1(t) M_0$$

$$\frac{dy}{dx} + p(x)y = q(x)$$

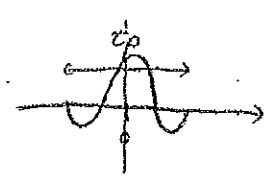
$$y = \frac{\int u(x) q(x) dx}{u(x)}, \text{ where } u(x) = \exp\left\{\int p(x) dx\right\}$$

③

$$\frac{d(M_r)^y}{dt} + \lambda \underbrace{W(z)}_{P(z)} (M_r)^y = \underbrace{\lambda W_1(t) M_0}_{R(z)}$$

$$M_r(t, z) = \lambda \cdot M_0 e^{-\lambda W(z) \cdot t} \int_0^t W_1(\tau) e^{-\lambda W(z) \tau} d\tau$$

Assume, the RF pulse is symmetric and peaks at $t=0$ such that the pulse ends at $t = \tau_p/2$, τ_p is the length of the RF pulse \rightarrow let $\tau' = t - \tau_p/2$



$$M_r(\tau_p) = \lambda M_0 e^{-\lambda \frac{W(z) \tau_p}{2}} \int_{-\tau_p/2}^{\tau_p/2} W_1(t + \frac{\tau_p}{2}) e^{-\lambda W(z) t} dt$$

assuming $W_1(t + \frac{\tau_p}{2}) = \begin{cases} 0 & \text{for } |t| > \frac{\tau_p}{2} \\ W_1(t + \frac{\tau_p}{2}) & \text{for } |t| < \frac{\tau_p}{2} \end{cases}$

$$\Rightarrow M_r(\tau, z) = \lambda M_0 e^{-\lambda W(z) \tau / 2} \int_{-ID}^{ID} \left\{ W_1\left(t + \frac{\tau}{2}\right) \right\} \Big|_{\tau = -\tau_p/2}^{\tau = \tau_p/2}$$

④

$$* \quad |M_{xy}(\tau, z)| = M_0 F_{1D} \left\{ \omega_1 \left(1 + \frac{\tau}{2} \right) \right\} \Big|_{f = -\frac{\partial}{\partial \tau} G_2 z}$$