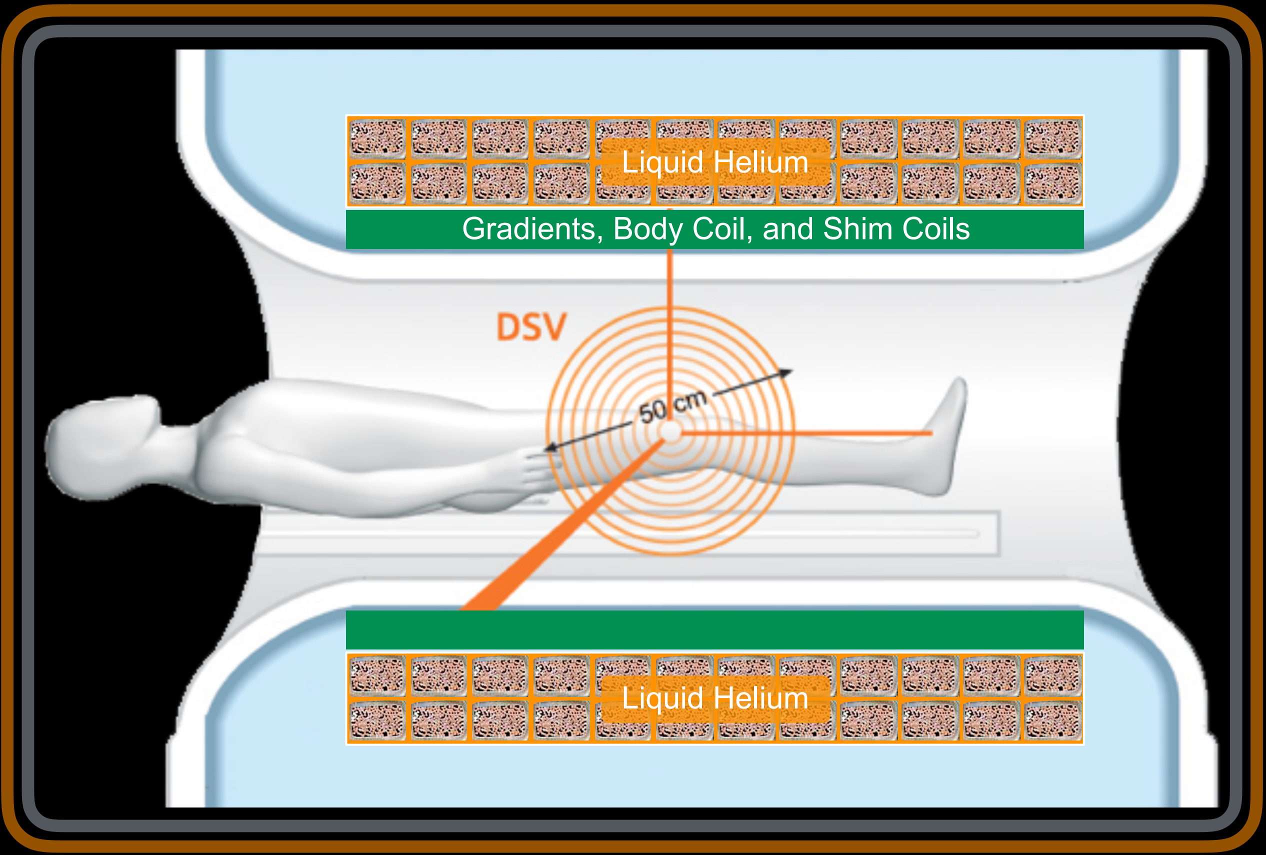
Bulk Magnetization and Nuclear Precession

Class Business

- **Matlab available via SEASNET**
  - http://www.seas.ucla.edu/acctapp
- **Website up and running**
  - http://mrrl.ucla.edu/education/m219/
  - Slides, video, code, reading, PDFs, etc.
  - Code available on website
    - Review code as needed
- **Meet with TAs for Matlab help.**

Lecture 1 - Summary

MRI uses a superconducting electromagnet!



Copper RF Shielding
Steel Magnetic Shielding

$$B = \mu I N L^{-1}$$

1.5T=4π×10^{-7}•**508 A**•235•1 m^{-1}

$$\vec{B}_0 = B_0 \vec{k}$$

Homogeneity – <4ppm peak-peak variation (6μT @ 1.5T!)

Lecture #1 Learning Objectives

- List several advantages and disadvantages of magnetic resonance imaging (MRI).
- Define the essential requirements for an MRI experiment.
- Describe the basic MRI magnet (B_0) design.
- Explain the importance of superconductivity.
- Be able to discuss several B_0-related safety and room design considerations.
- Write a mathematical expression for the B_0 field and discuss spatial and temporal homogeneity.
- Explain B_0 ramping and quenching.

Questions?

Bulk Magnetization and Nuclear Precession

Learning Objectives

- **Explain three B_0 principles and the importance of Zeeman splitting.**
- **Describe the importance of spin, charge, and mass to NMR.**
- **Define the equation of motion for an ensemble of spins.**
- **Differentiate free and forced precession in the laboratory and rotating frames.**
- **Learn to solve for the bulk magnetization dynamics under specific conditions.**

Dipoles to Images

$\vec{\mu}$	Magnetic Moment	
$\downarrow$		B_0
$\vec{M}$	Bulk Magnetization	
$\downarrow$		B_1
$\vec{M}_{xy}(t)$	Transverse Magnetization	
$\downarrow$		$G_{Encoding}$
$\vec{M}_{xy}(\vec{r}, t)$	Spatially Encoded Magnetization	
$\downarrow$		Coil
$V(t)$	Received Voltage	
$\downarrow$		PSD
$S(t)$	Complex Signal	
$\downarrow$		$G_{Decoding}$
$S\left(\vec{k}\right)$	k-space signal	
$\downarrow$		Reconstruction
$I(\vec{r})$	Image	

Main Field (B_0) - Principles

- B_0 is a strong magnetic field
  - >1.5T
  - Z-oriented

$$\vec{B}_0 = B_0\vec{k}$$

Eqn. 3.5

- B_0 generates bulk magnetization ($\vec{M}$)
  - More B_0, more

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

Eqn. 3.26

- B_0 forces $\vec{M}$ to precess
  - Larmor Equation

$$\omega = \gamma B$$

Eqn. 3.18

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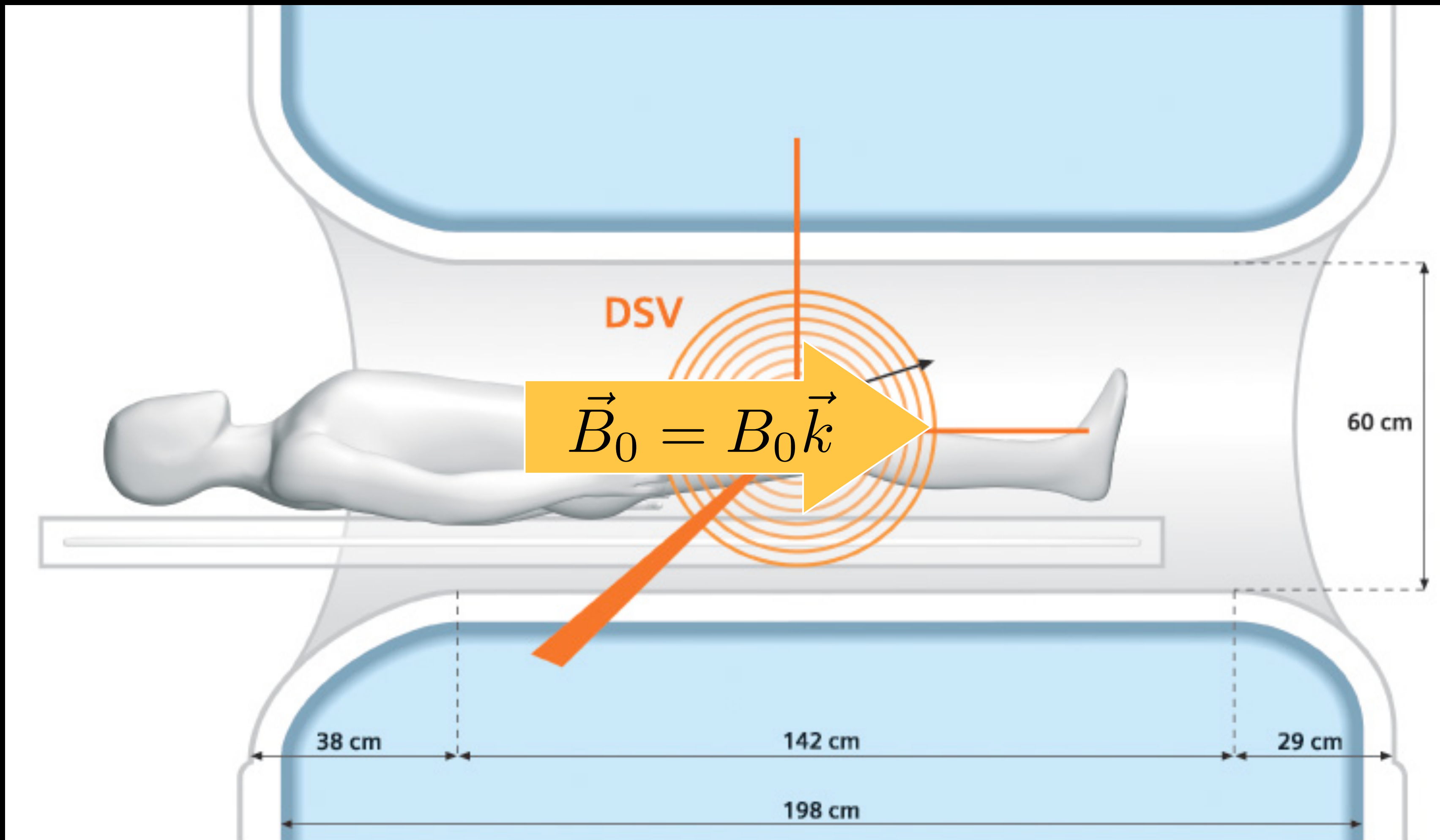
B_0 Field



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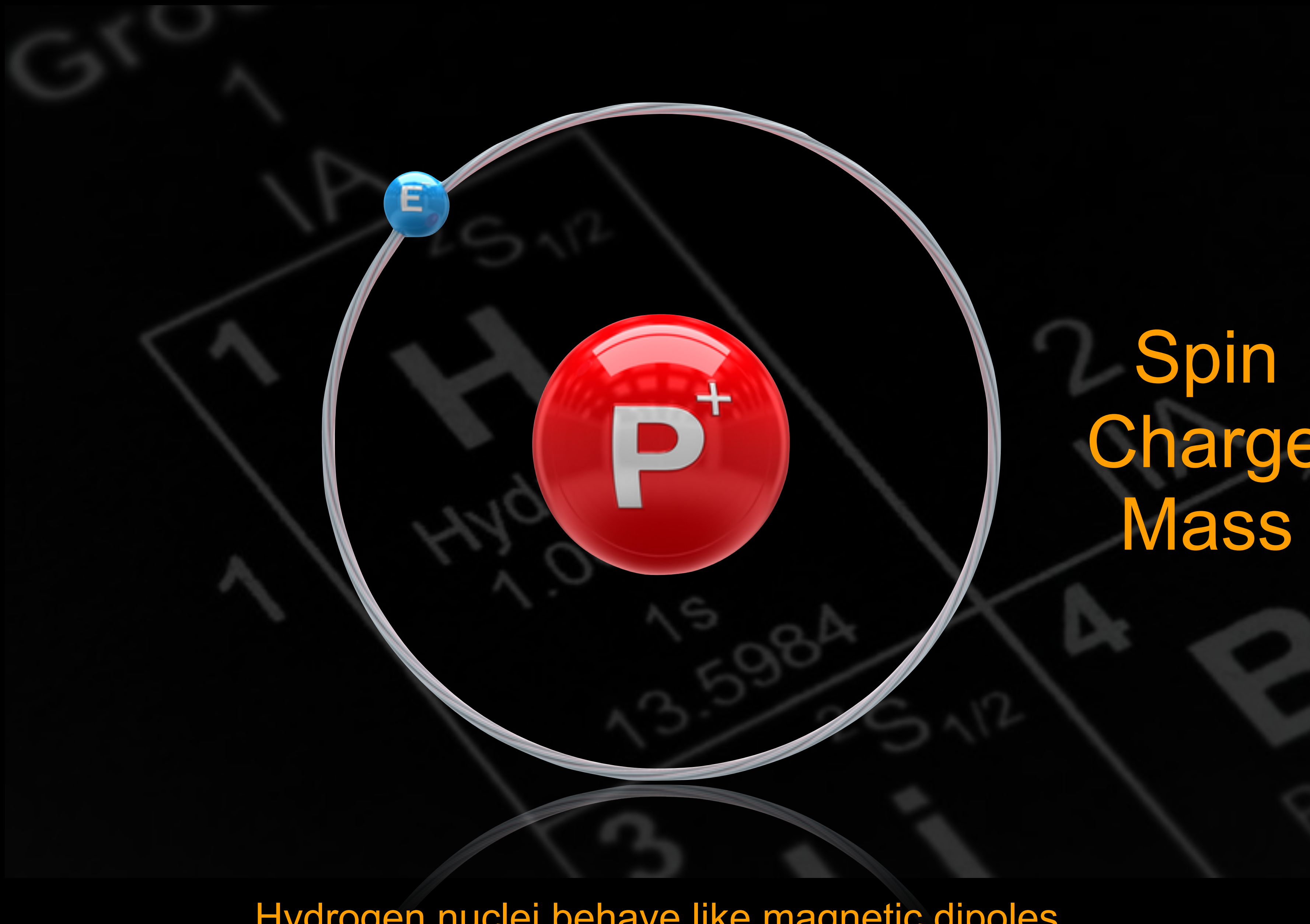
Hydrogen



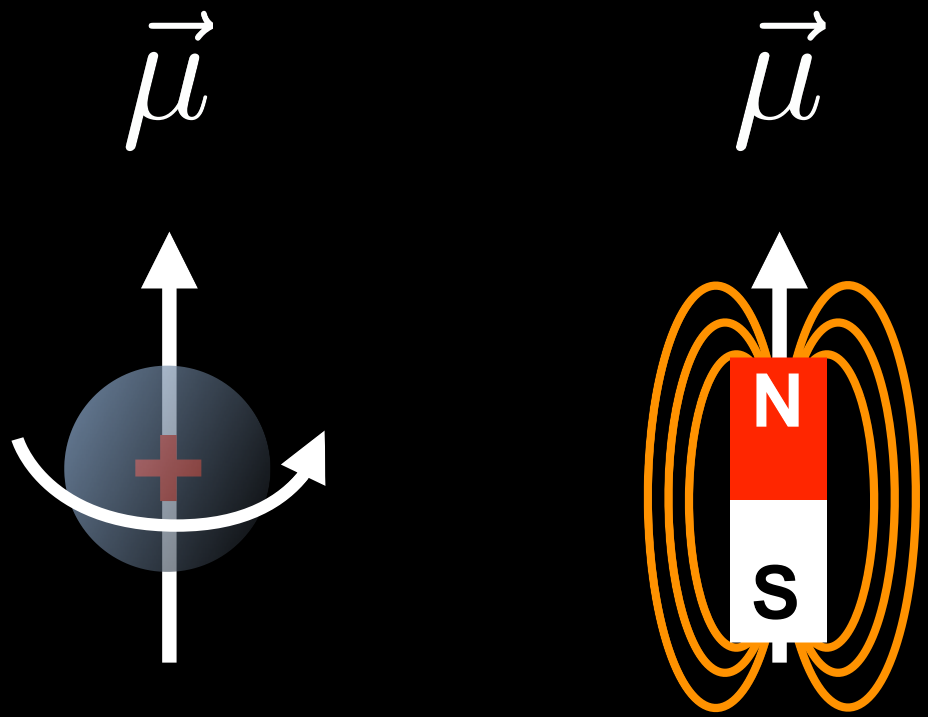
Spin
Charge
Mass

Hydrogen nuclei behave like magnetic dipoles.

Magnetic Dipole Moments

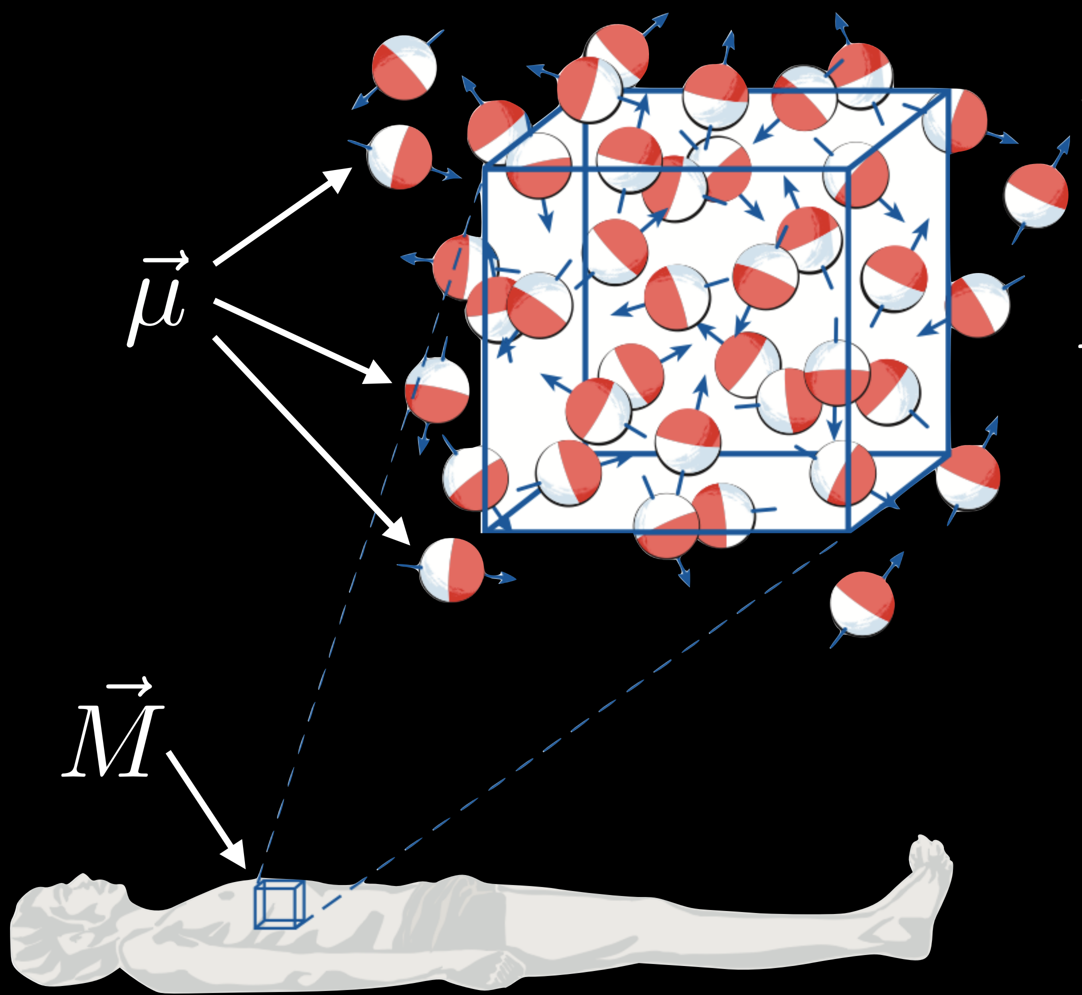
Spin + Charge ➟ Magnetic Moment ➟ $\vec{\mu}$ [J•T^{-1} *or* kg•m^2/s^2]

"a measure of the strength of the system's net magnetic source"
--http://en.wikipedia.org/wiki/Magnetic_moment



Hydrogen nuclei have magnetic dipole moments.

Bulk Magnetization



$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$ Eqn. 3.26

$N_{total}=0.24\times10^{23}$ spins in a 2x2x10mm voxel

But not all spins contribute to our measured signal...

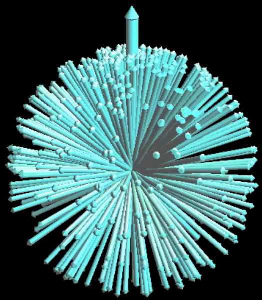
Equilibrium Bulk Magnetization

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$ Eqn. 3.26

$$\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$ Eqn. 3.36

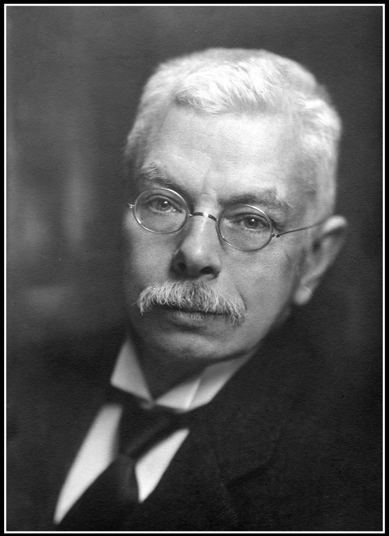
$$\vec{M}_z^0 = |\vec{M}| = \frac{\gamma^2\hbar^2 B_0 N_s}{4KT_s}$$ Eqn. 3.39

$$\vec{M}_x^0 = \vec{M}_y^0 = 0$$

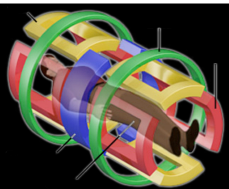


Bulk magnetization at equilibrium in a B_0 field.

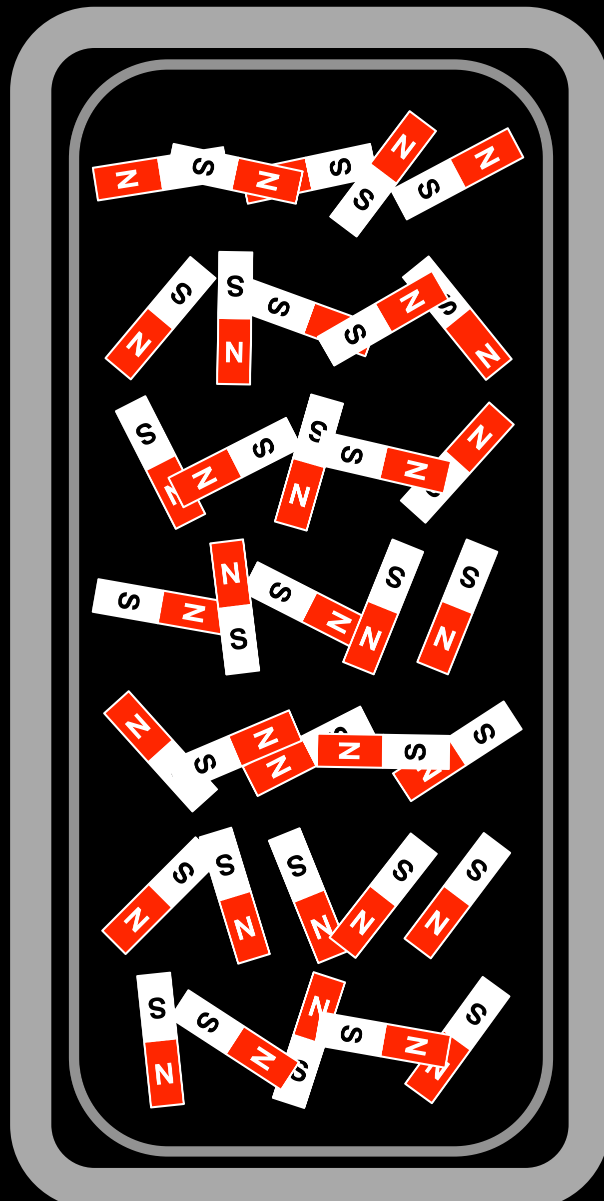
Zeeman Splitting



Pieter Zeeman
b. 25 May 1865
d. 9 Oct 1943



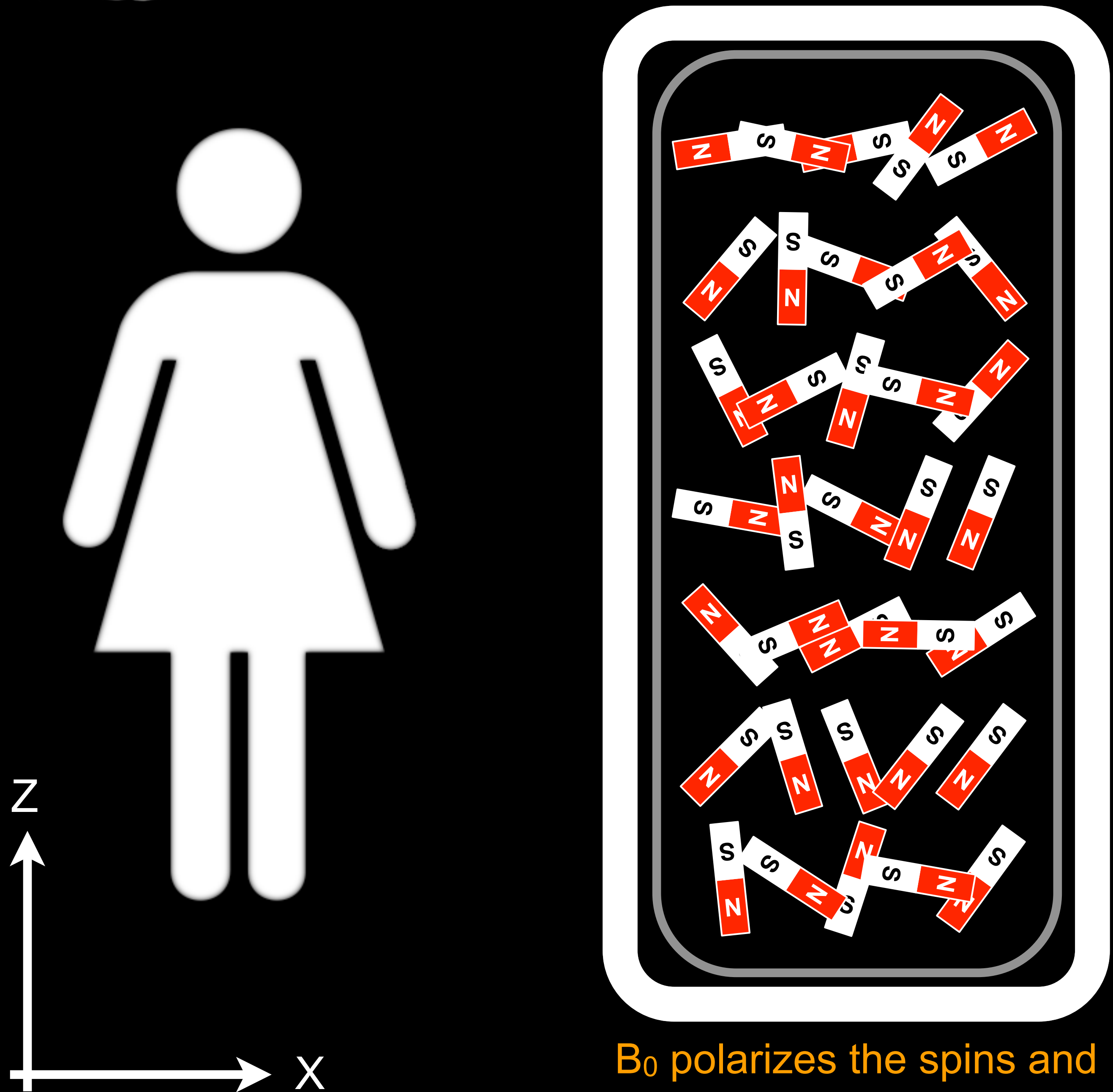
B_0 Field OFF



$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = 0$$

Spins point in all directions.

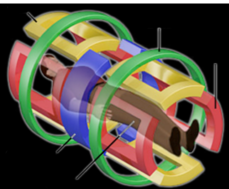
B_0 Field ON

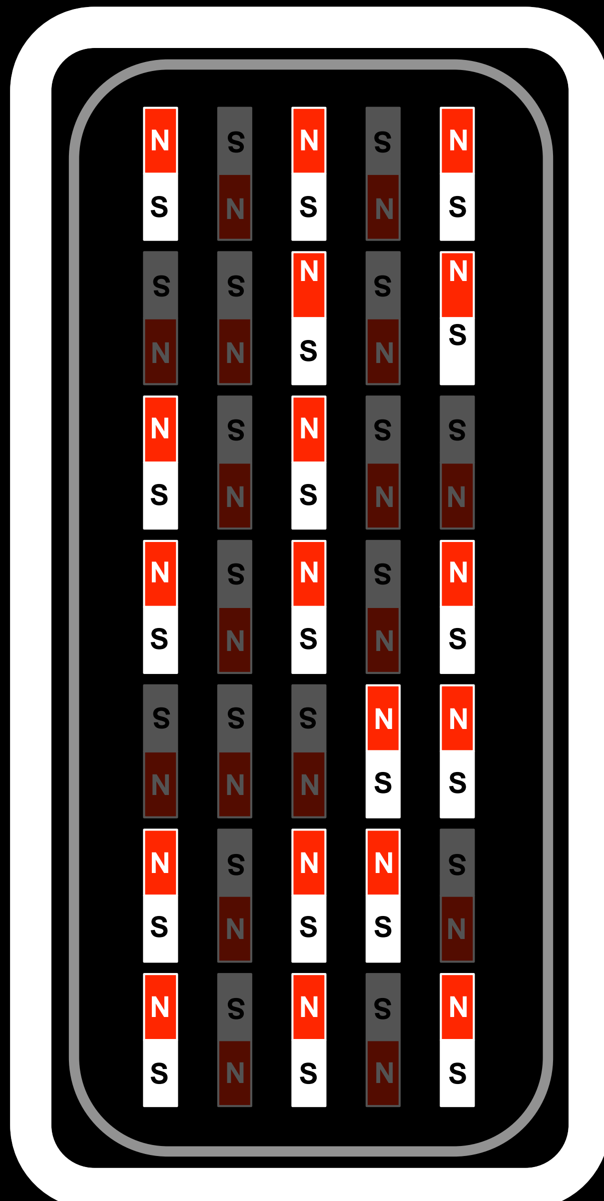


$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = M_z$$

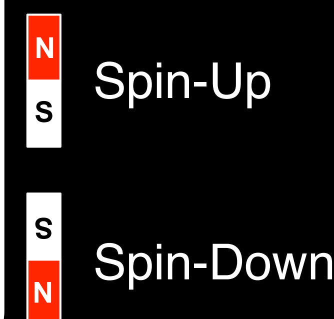
B_0 polarizes the spins and generates bulk magnetization.

B_0 Field ON



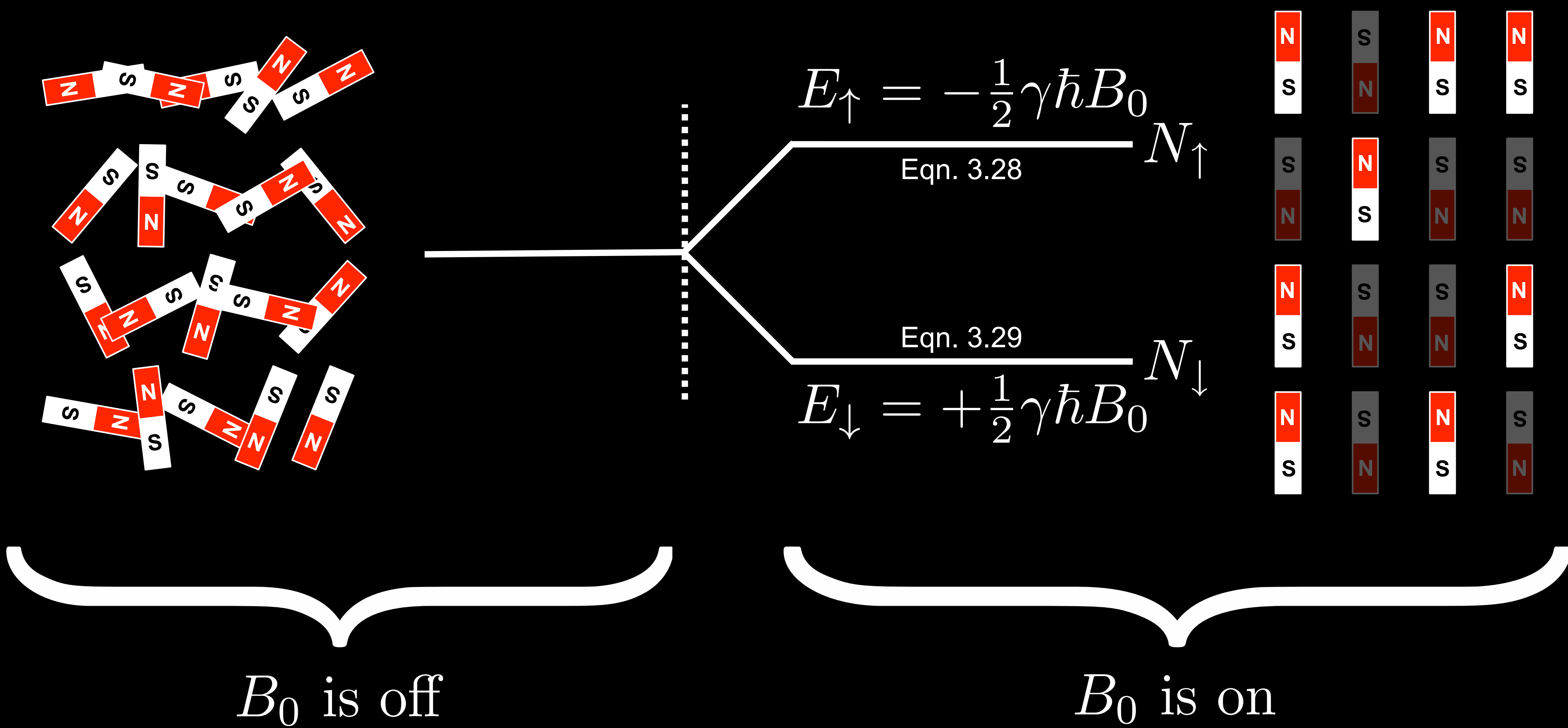


$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n = M_z$$



Only a very small number are spin-up relative to spin-down.

Zeeman Splitting



$N_{\uparrow}$ = Spin-Up State, Low Energy

$N_{\downarrow}$ = Spin-Down State, High Energy

Zeeman Splitting

$$\frac{N_\uparrow - N_\downarrow}{N_{total}} \approx \frac{\gamma h B_0}{2KT}$$ Eqn. 3.35

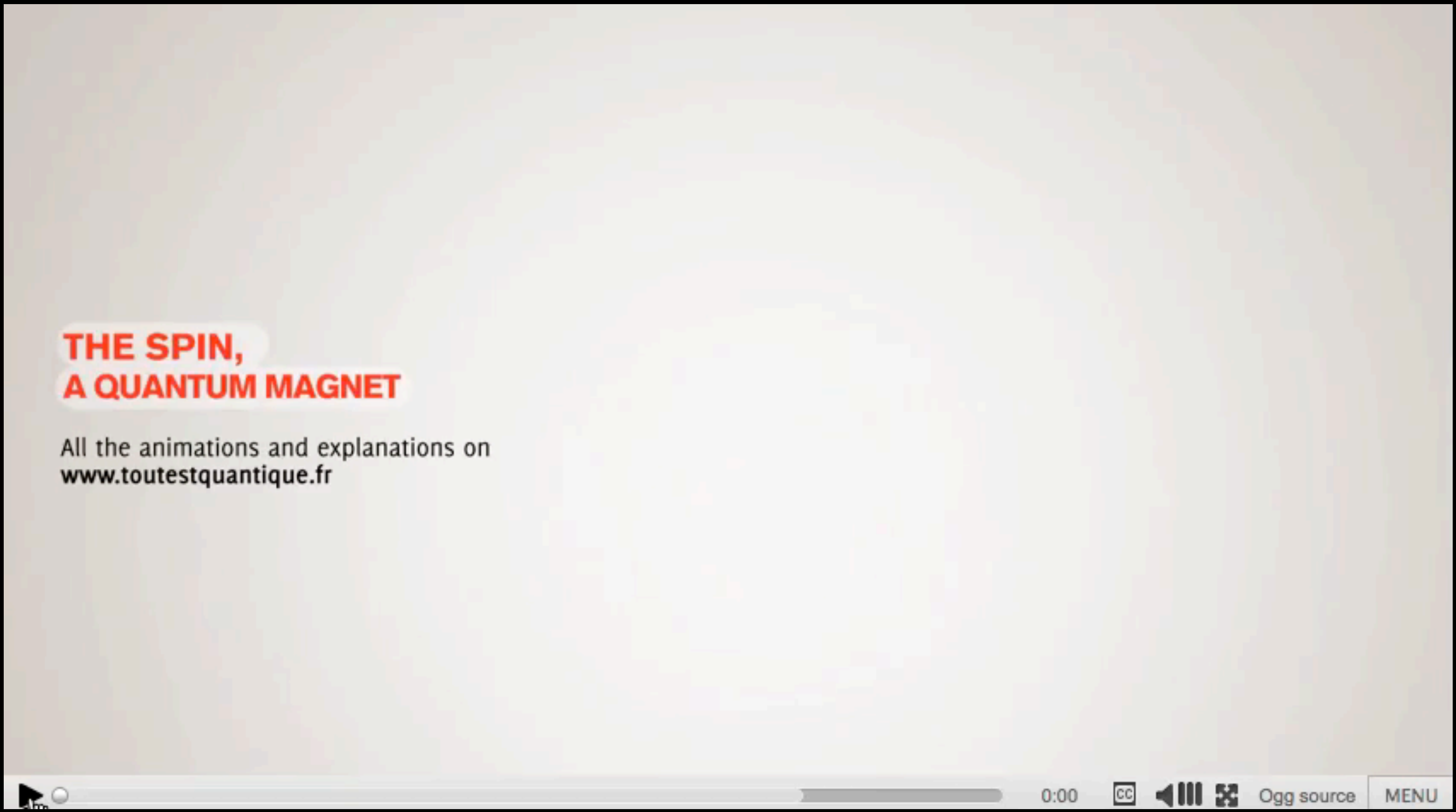
$$\begin{array}{lcl} \gamma & = & 42.58 \times 10^6 \text{ Hz/T} \\ h & = & 6.6 \times 10^-34 \text{ J} \cdot \text{s [Planck' Constant]} \\ T & = & 300\text{K (room temperature)} \\ K & = & 1.38 \times 10^{-23} \text{ J/K [Boltzmann Constant]} \\ B_0 & = & 1.5\text{T} \end{array}$$

$$\frac{N_\uparrow - N_\downarrow}{N_{total}} \approx \frac{42.58 \times 10^6 \cdot 6.6 \times 10^{-34} \cdot 1.5}{2 \cdot 1.38 \times 10^{-23} \cdot 300} \approx 4.5 \times 10^{-6}$$

Nuclear Spin

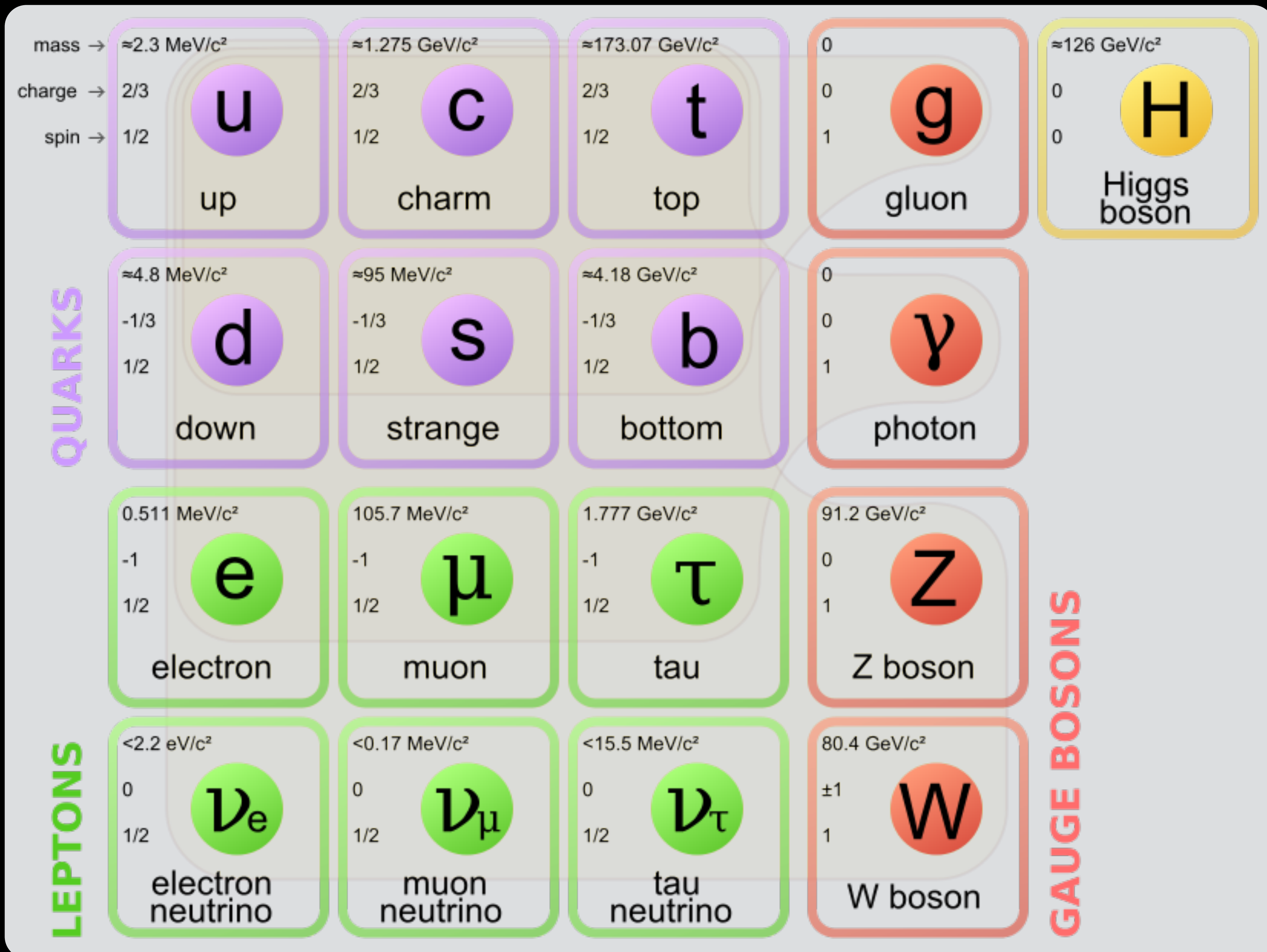
"*The concept of spin is difficult. It was forced upon scientists by the experimental evidence*."
– Malcolm Levitt in *Spin Dynamics*

How was spin first observed?



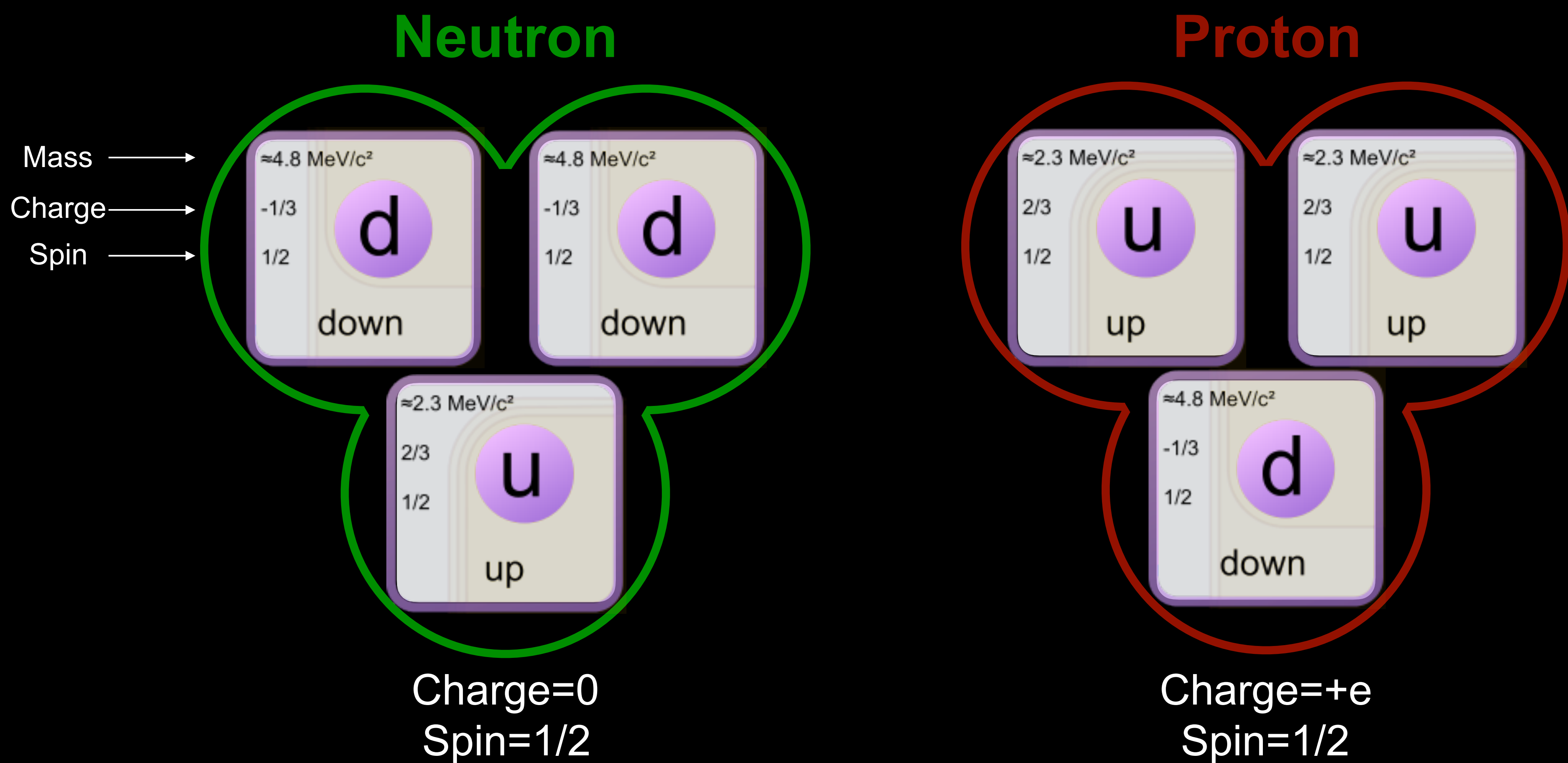
Otto Stern and Walther Gerlach performed the Stern–Gerlach experiment in Frankfurt, Germany in 1922.

The Standard Model



http://en.wikipedia.org/wiki/Standard_Model

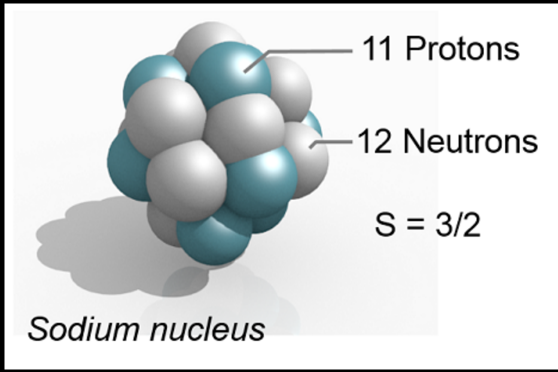
Nuclear Spin - Quarks



Spin Crisis!

Nuclear Spin Quantum Number (I)

- A nucleus is NMR active only if $I \neq 0$
- Nuclei with an odd mass number have ***half-integral spin***
  - Spin-1/2 – ^{1}H, ^{13}C, ^{15}N, ^{19}F, ^{31}P
  - Spin-3/2 – ^{23}Na
  - Spin-5/2 – ^{17}O
- Nuclei with an even mass number and an even charge number have ***zero spin***
  - ^{12}C and ^{16}O
- Nuclei with an even mass number, but an odd charge number have ***integral spin***
  - ^{2}H and ^{14}N



NMR Active Nuclei

Isotope	*Spin [I]*	Natural Abundance	Gyromagnetic Ratio [MHz/T]	Relative Sensitivity	Absolute Sensitivity
^{1}H	1/2	0.9980	42.57	1	9.98E-01
^{2}H	1	0.0160	6.54	0.015	2.40E-04
^{12}C	0	0.9890	---	---	---
^{13}C	1/2	0.0110	10.71	0.016	1.76E-04
^{14}N	1	0.9960	3.08	0.001	9.96E-04
^{15}N	1/2	0.0040	-4.32	0.001	4.00E-06
^{16}O	0	0.9890	---	---	---
^{17}O	5/2	0.0004	-5.77	0.029	1.16E-05
^{19}F	1/2	1.0000	40.05	0.83	8.30E-01
^{23}Na	3/2	1.0000	11.26	0.093	9.30E-02
^{31}P	1/2	1.0000	17.24	0.066	6.60E-02

The ***relative*** sensitivity is at constant magnetic field and equal number of nuclei.

– Using a factor of $\gamma^{\frac{11}{4}} I(I+1)$; ^{1}H is the reference standard.

The ***absolute*** sensitivity is the relative sensitivity multiplied by natural abundance.

P. Callaghan & http://www.cryst.bbk.ac.uk/PPS2/projects/schirra/html/nuclei.htm

Gyromagnetic Ratio

- Gyromagnetic Ratio
  - Physical constant
  - Unique for each NMR active nuclei
  - Ratio of the magnetic moment to the angular momentum

$$\vec{\mu} = \gamma \vec{S}$$

  - Governs the frequency of ***precession***
  - Gamma vs. Gamma-bar

$$\bar{\gamma} = \gamma / 2\pi$$

What are the implications of spin?

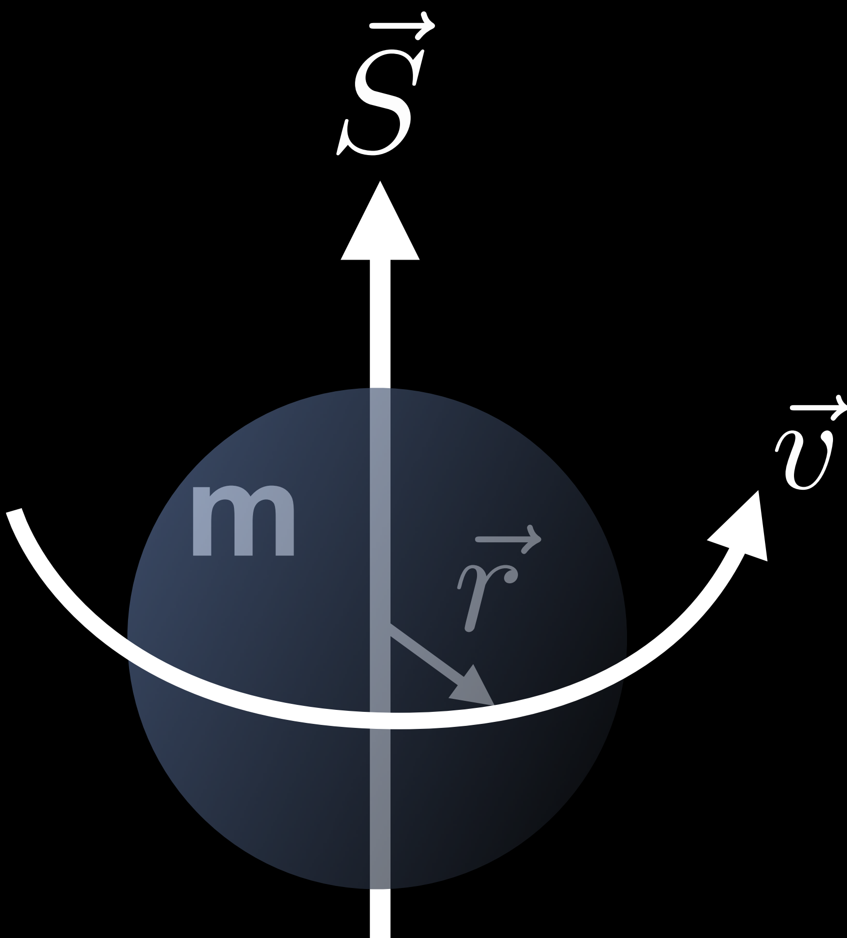
Spin + Mass and Spin + Charge ➠ NMR

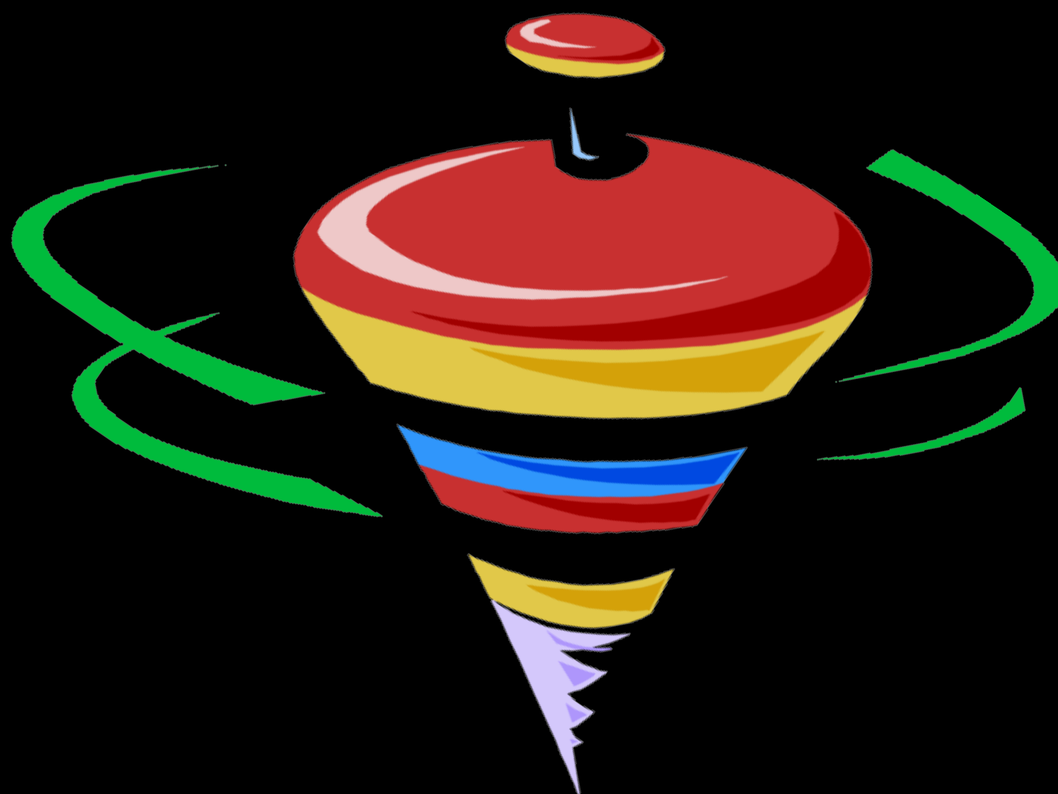
Nuclear Precession

Movie Courtesy of Donald Plewes @ U. Toronto

Spin Angular Momentum

Spin + Mass ➟ Spin Angular Momentum ➟ $\vec{S}$ [kg·m^{2}s^{-1}]

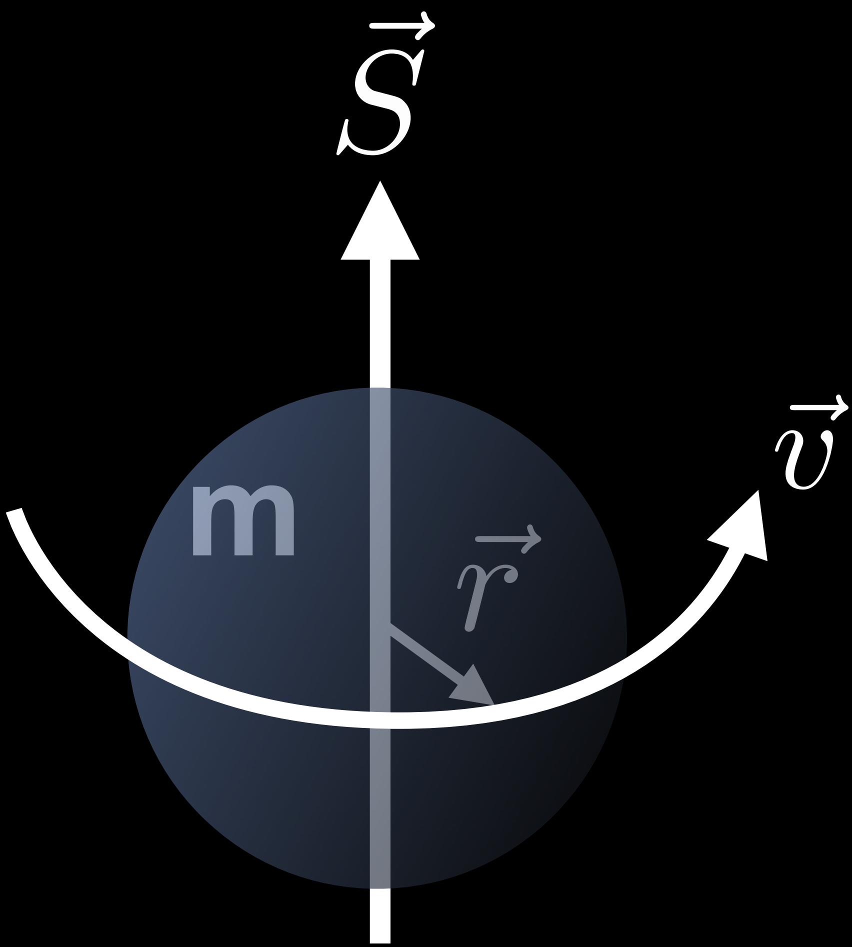




Hydrogen nuclei have spin angular momentum.

Spin Angular Momentum

Spin + Mass ➠ Spin Angular Momentum ➠ $\vec{S}$ [kg·m^{2}s^{-1}]



$$\vec{S} = \vec{r} \times \vec{p}$$
$$= \vec{r} \times m\vec{v}$$

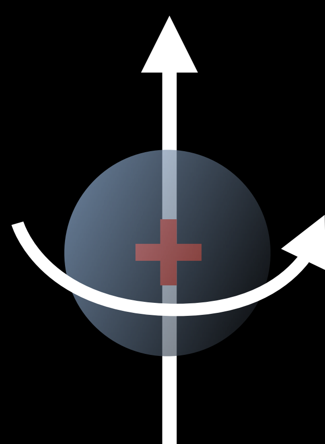
Hydrogen nuclei have spin angular momentum.

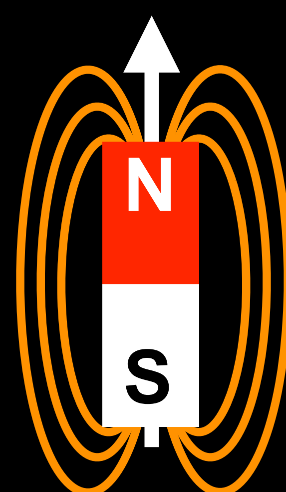
Magnetic Dipole Moments

Spin + Charge ➟ Magnetic Moment ➟ $\vec{\mu}$ [J•T^{-1} *or* kg•m^2/s^2]

“a measure of the strength of the system's net magnetic source”
--http://en.wikipedia.org/wiki/Magnetic_moment

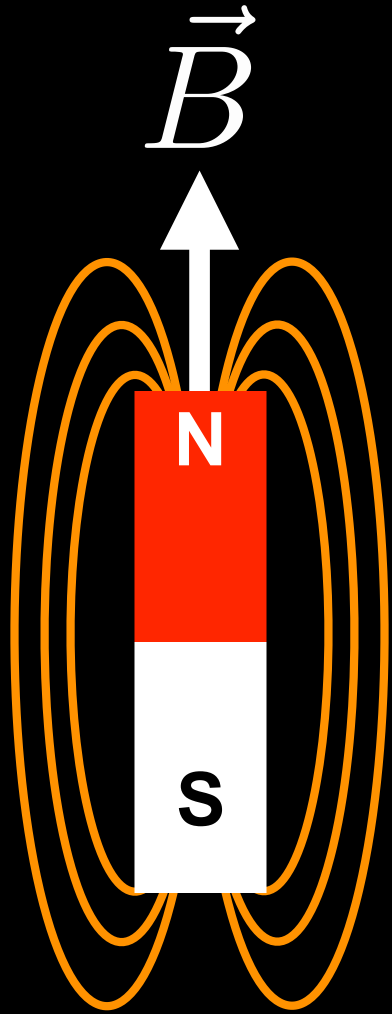
$\vec{\mu}$ $\vec{\mu}$





Hydrogen nuclei have magnetic dipole moments.

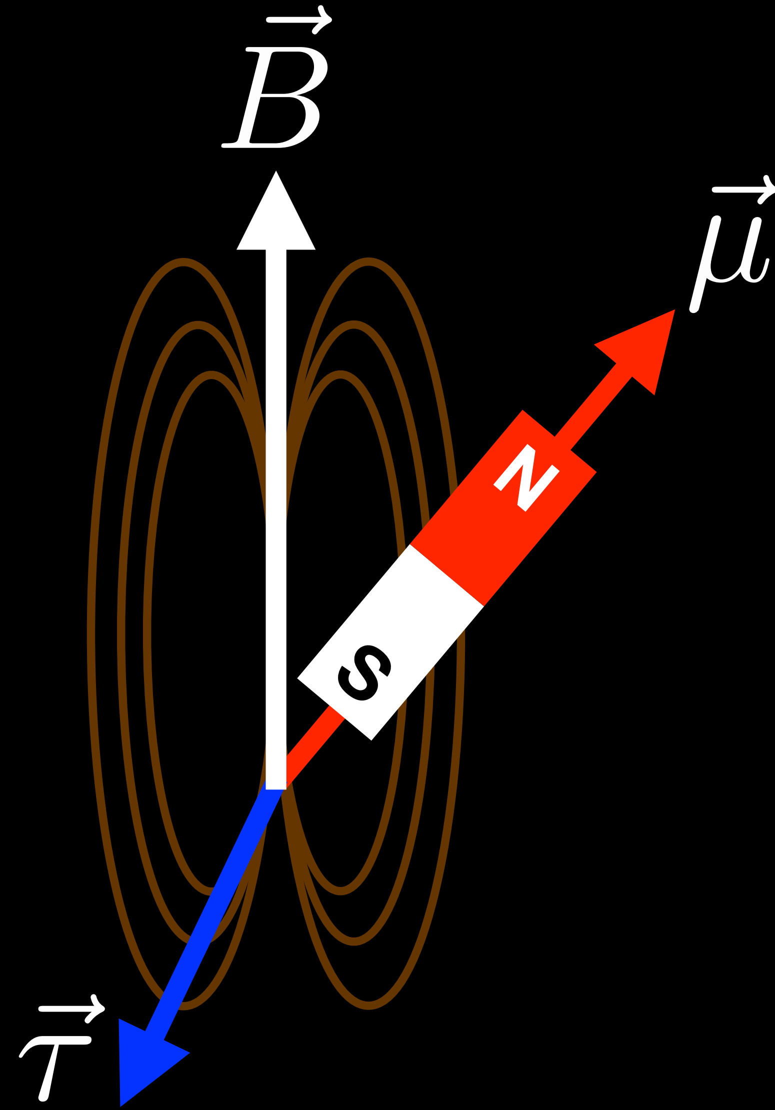
B-Field



"vector field which can exert a magnetic force on moving electric charges and on magnetic dipoles"
--http://en.wikipedia.org/wiki/Magnetic_field

Magnetic Dipole in a B-Field

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



B_0 exerts a torque on the ^1H magnetic dipole moment.

David Geffen School of Medicine

UCLA Radiology

Main Field (B_0) - Principles

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- B_0 forces $\vec{M}$ to precess
  - Larmor Equation

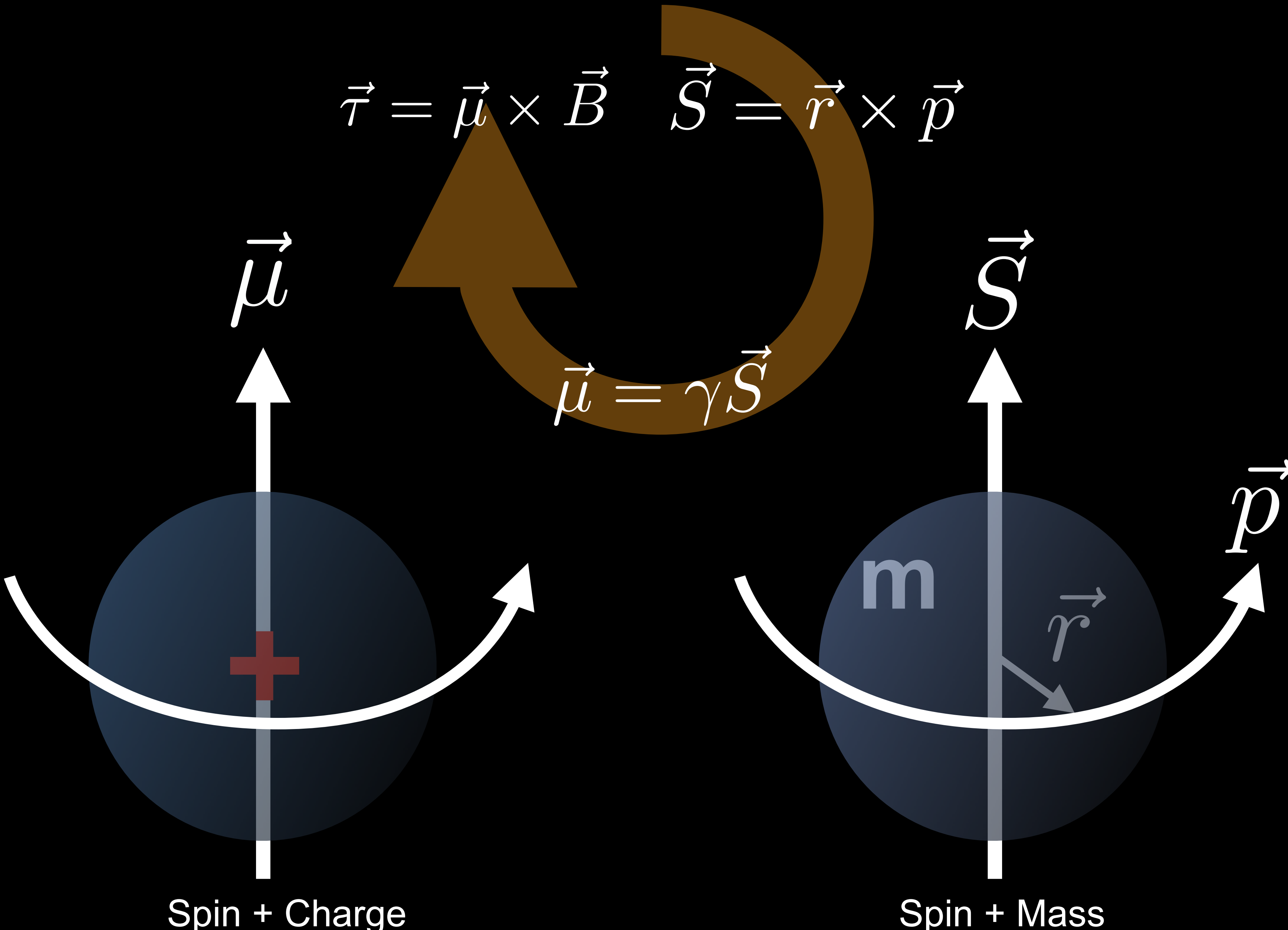
$$\omega = \gamma B$$ Eqn. 3.18

Spin vs. Precession

- **Spin**
  - Intrinsic form of angular momentum
  - Quantum mechanical phenomena
  - No classical physics counterpart
    - Except by hand-waving analogy…
- **Precession**
  - Spin+Mass+Charge give rise to precession

So where does the Larmor equation come from?

Magnetic Moments & Angular Momentum



Spin + Mass and Spin + Charge ➠ NMR

To the board...

Equation of Motion for the Bulk Magnetization

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats)
[Classical Description]

What is a general solution?

Free & Forced Precession

Free vs. Forced Precession

Free Precession – Precession of the bulk magnetization vector about the static magnetic field after a pulse excitation. Free precession of the transverse magnetization at the Larmor frequency is responsible for the detectable NMR signal.

– *Liang & Lauterbur p. 375*

Forced Precession – Precession of the bulk magnetization about the excitation RF field.

– *Liang & Lauterbur p. 374*

Four Special Cases...

- **Laboratory Frame**
  - Coordinate system anchored to scanner
  - 1) *Free Precession* in the lab frame
  - 2) *Forced Precession* in the lab frame

- **Rotating Frame**
  - Coordinate system anchored to spin system
  - 3) *Free Precession* in the rotating frame
  - 4) *Forced Precession* in the rotating frame

- **...all without relaxation. We assume:**
  - a) Relaxation time constants are “really” long

  OR

  - b) Time scale of event is << relaxation time constant

Free Precession In The Laboratory Frame Without Relaxation

Rotations & Euler’s Formula

Vectors

- A ***vector*** ($\vec{v}$) describes a physical quantity (e.g. bulk magnetization or velocity) at a point in space and time and has a magnitude (positive real number), a direction, and physical units.

- To define a vector we need a ***basis***:

$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

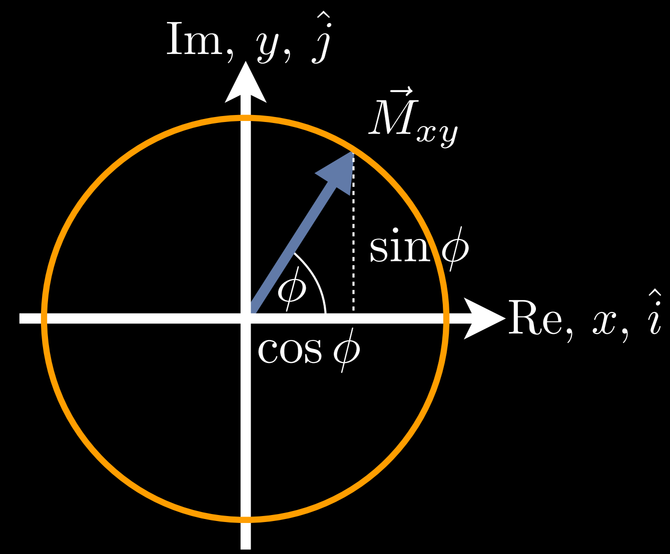
- A 3D ***vector*** has components:

$$\vec{M} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

2D Vectors - Euler's Formula

- Euler's formula provides a compact representation of a 2D vector using a complex exponential:

$$e^{i\phi} = \cos\phi + i\sin\phi$$



$\vec{M}_{xy} = M_x\hat{i} + M_y\hat{j}$	Vector components
$= M_x + iM_y$	Complex components
$= \lvert\vec{M}_{xy}\rvert\cos\phi\,\hat{i} + \lvert\vec{M}_{xy}\rvert\sin\phi\,\hat{j}$	Trigonometric components
$= \lvert\vec{M}_{xy}\rvert\cos\phi + i\lvert\vec{M}_{xy}\rvert\sin\phi$	Complex trigonometric components
$= \lvert\vec{M}_{xy}\rvert e^{i\phi}$	Euler's notation

Euler's formula is mathematically convenient.
There is nothing explicitly *imaginary* about M_{xy}.

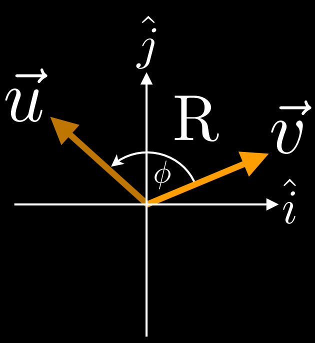
Rotations

- ***Rotations*** (R) are vector valued orthogonal transformations that preserve the magnitude of vectors and the angles between them.
- The simplest rotation matrix is the ***identity*** matrix:

$$\mathrm{R} = \mathrm{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ therefore } \vec{v} = \mathrm{I}\vec{v}$$

- More simply, R transforms (rotates) one vector to another:

$$\vec{u} = \mathrm{R}\vec{v}$$



Rotations

$\hat{i}$ ends up here

$\hat{j}$ ends up here

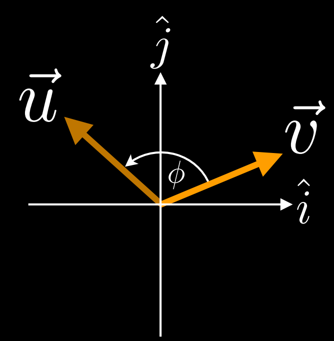
$\hat{k}$ does not change

Magnitude of rotation

$$R_z^{\phi} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Axis (phase) of rotation

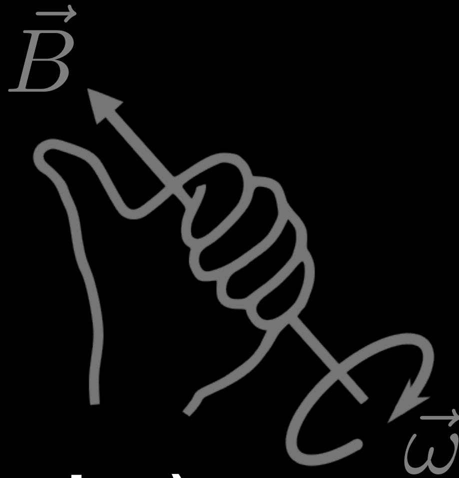
$$\vec{u} = R\vec{v}$$



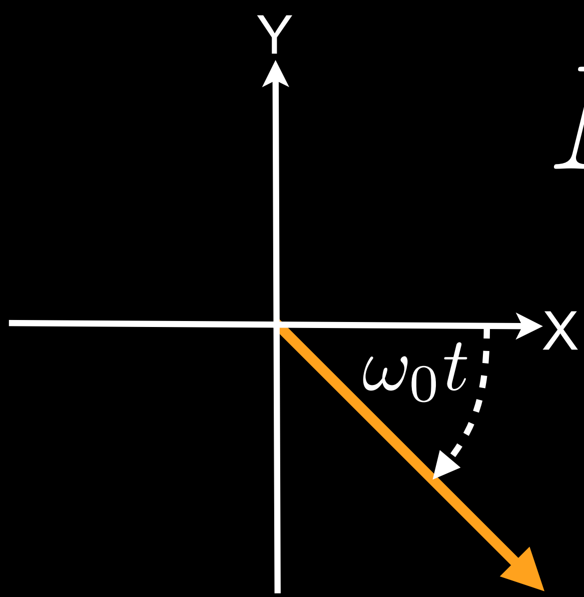
To the board...

Free Precession In The Laboratory Frame Without Relaxation

$$\mathbf{R}_z(\omega_0 t) = \begin{bmatrix} \cos\omega_0 t & \sin\omega_0 t & 0 \\ -\sin\omega_0 t & \cos\omega_0 t & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Precession is left-handed (clockwise).



$$\vec{M}(t) = \mathbf{R}_z(\omega_0 t)\vec{M}^0$$

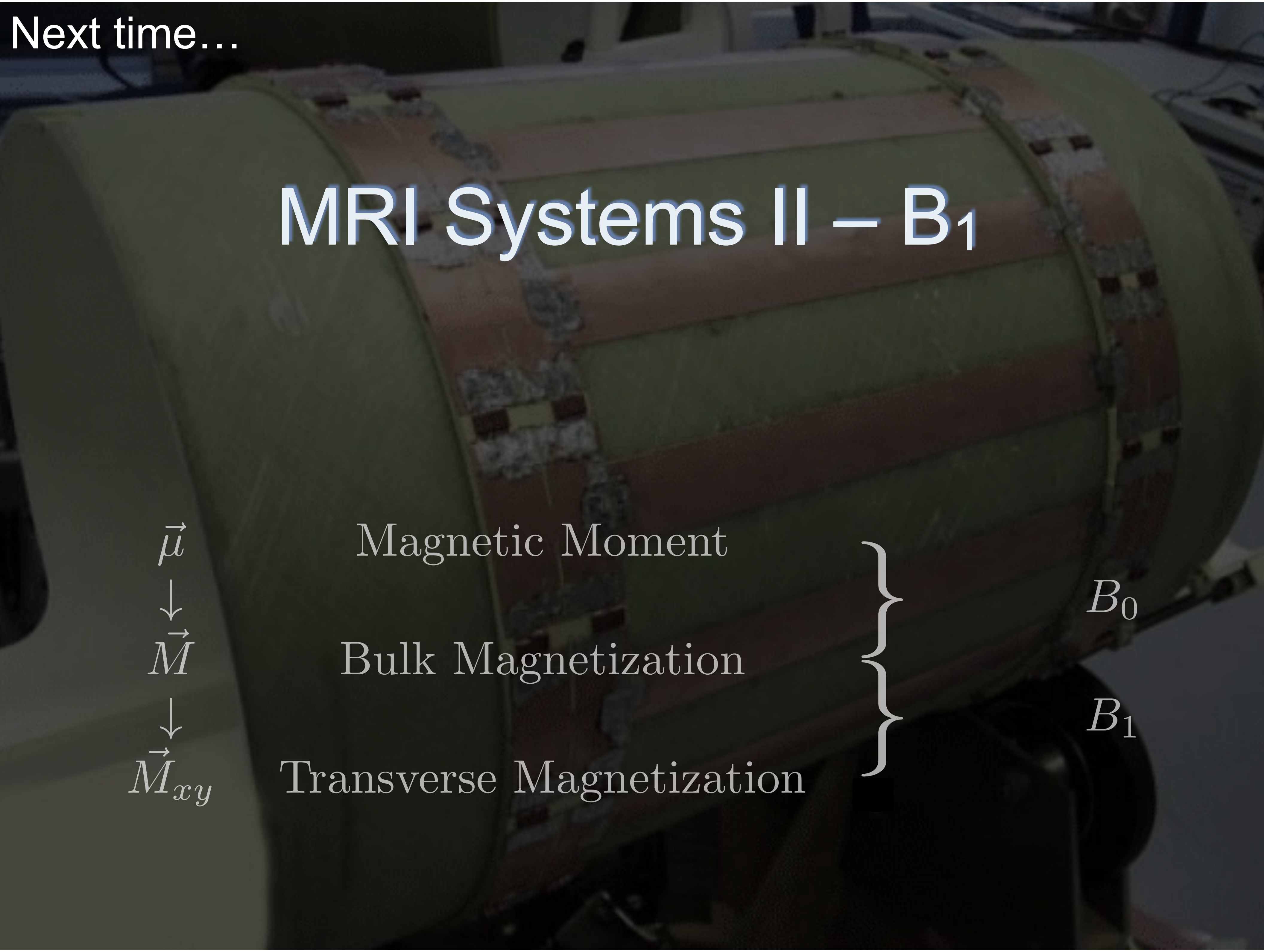
$$\Updownarrow$$

$$\omega_0 = \gamma B_0$$

Free Precession In The Laboratory Frame Without Relaxation

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \left(\vec{B_0} \right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

To The Board…


Next time…
MRI Systems II – B_1
$\vec{\mu}$ Magnetic Moment
$\downarrow$
$\vec{M}$ Bulk Magnetization
$\downarrow$
$\vec{M}_{xy}$ Transverse Magnetization
B_0
B_1

Thanks

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