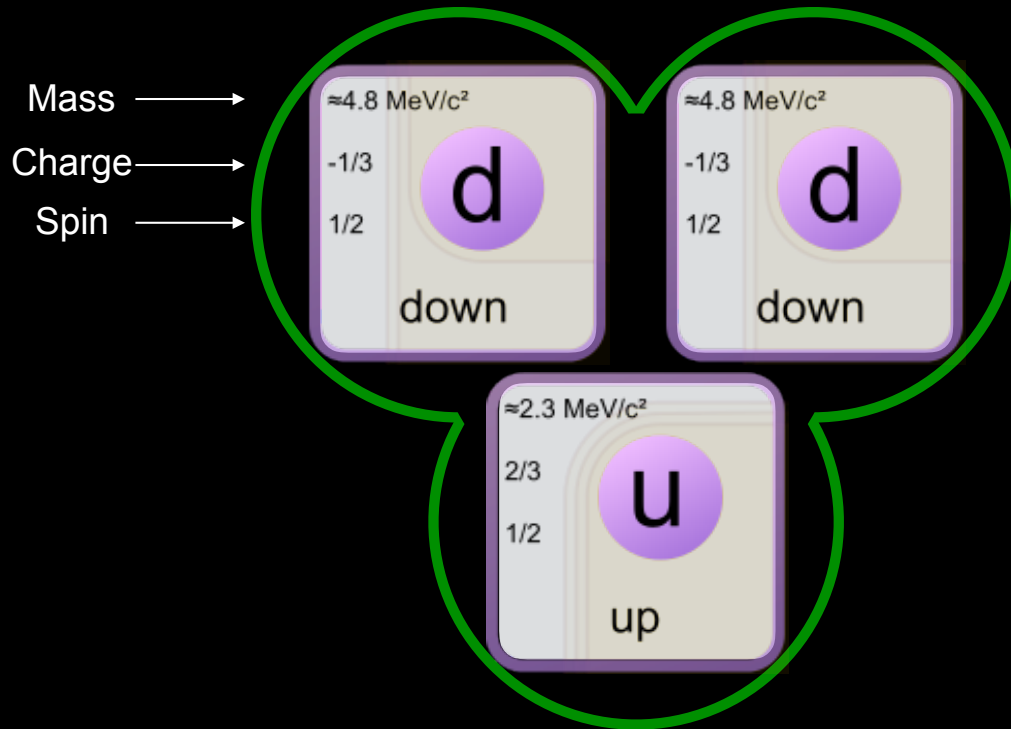


MRI Systems II – B_1



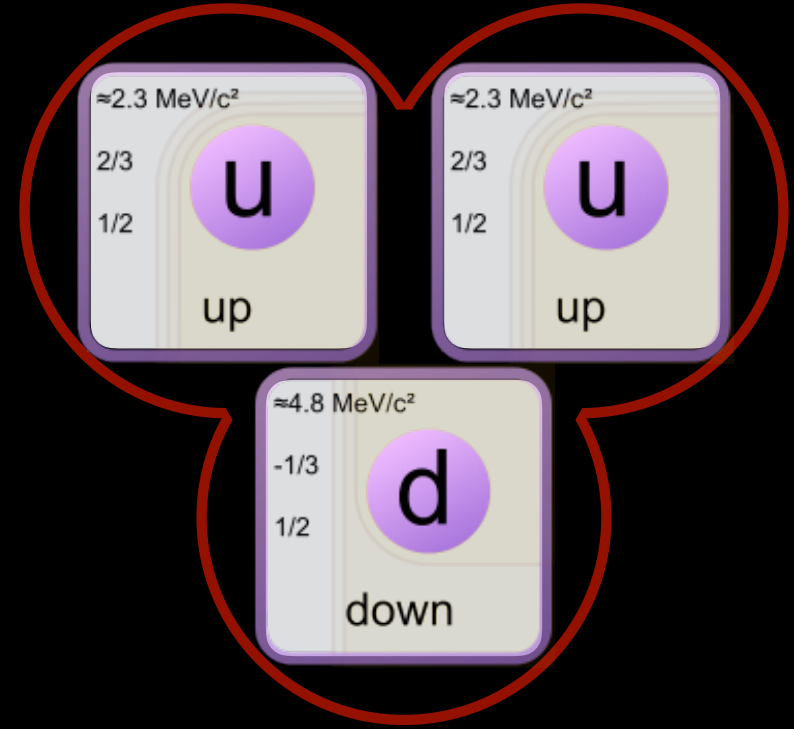
Spin+Charge \Rightarrow Magnetic Moment?

Neutron



Charge=0
Spin=1/2

Proton



Charge=+e
Spin=1/2

What about the neutron? It has no charge, but it has spin!

Lecture #2 Learning Objectives

- Explain three B_0 principles and the importance of Zeeman splitting.
- Describe the importance of spin, charge, and mass to NMR.
- Define the equation of motion for an ensemble of spins.
- Differentiate free and forced precession in the laboratory and rotating frames.
- Learn to solve for the bulk magnetization dynamics under specific conditions.

Lecture 2 - Summary

$$\vec{\tau} = \vec{\mu} \times \vec{B} \quad \vec{S} = \vec{r} \times \vec{p}$$

$$\vec{\tau} = \frac{d\vec{S}}{dt} \quad \vec{\mu} = \gamma \vec{S}$$

$$\frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

Equation of Motion for a Magnetic Dipole

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$

$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$

$$M_z(t) = M_z^0$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of Motion for the bulk magnetization.

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma (\vec{B}_0)$$

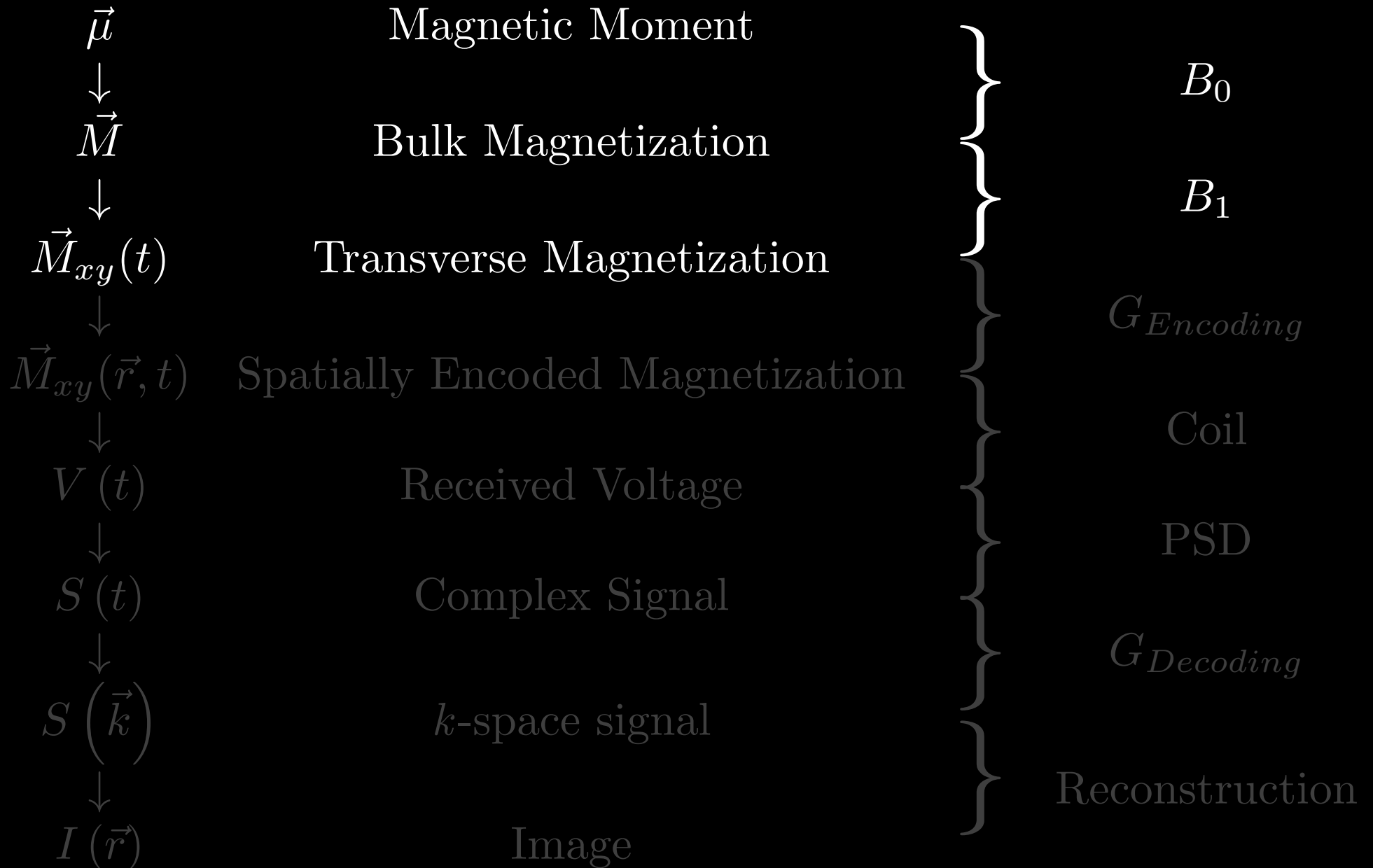
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$\vec{B}_0 = B_0 \vec{k}$$

Lecture 2 - Summary

- Free Precession in the Laboratory Frame
- **Forced Precession in the Laboratory Frame**
 - Coordinate system anchored to scanner
- **Free Precession in the Rotating Frame**
- **Forced Precession in the Rotating Frame**
 - Coordinate system anchored to spin system
- **...all without relaxation.**
 - a) Relaxation time constants are “really” long
 - b) Time scale of event is \ll relaxation time constant

Dipoles to Images



MRI Systems II – B_1



Lecture #3 Learning Objectives

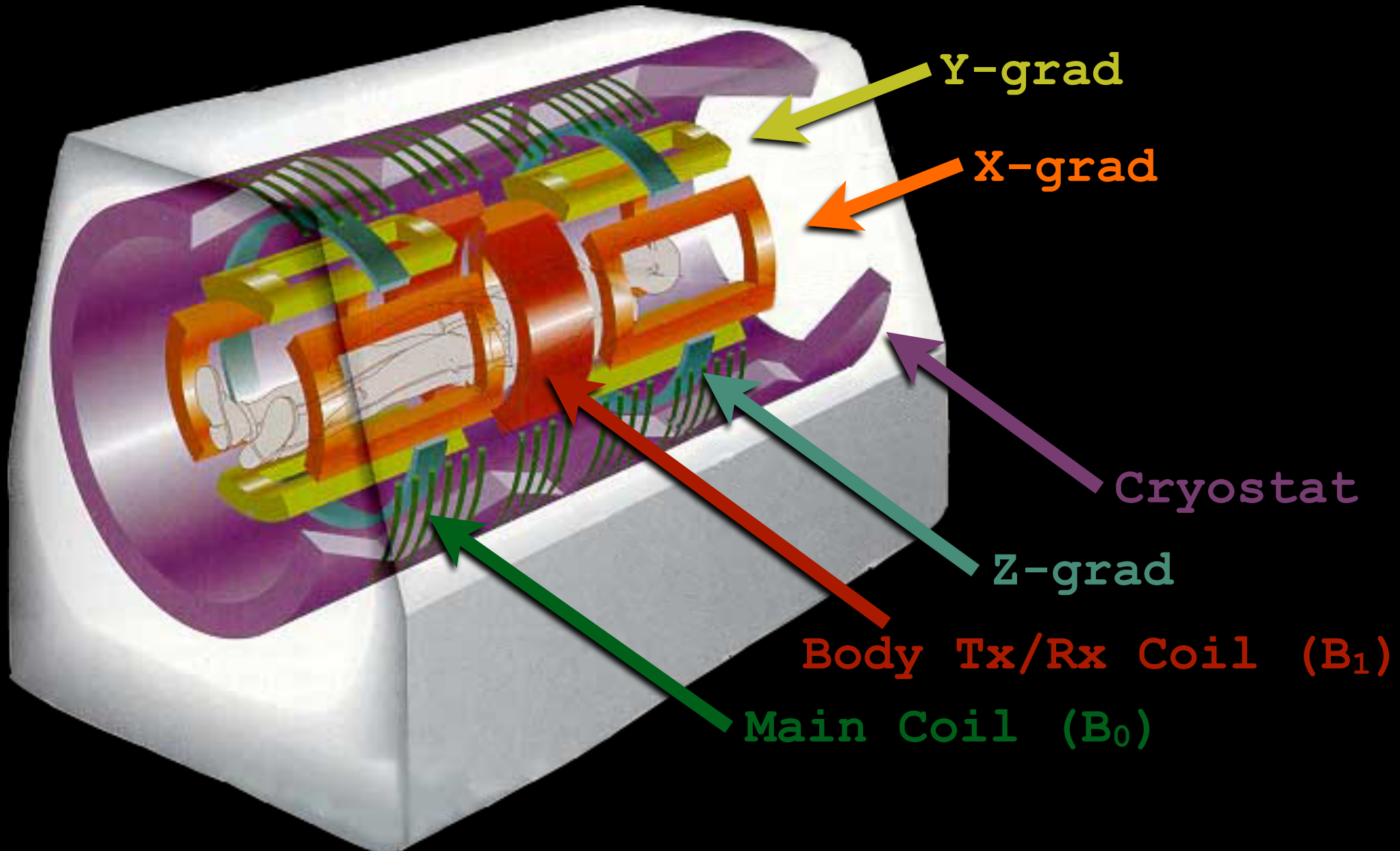
- Distinguish spin, precession, and nutation.
- Appreciate that any B-field acts on the the spin system.
- Understand the advantage of a circularly polarized RF B-field.
- Differentiate the lab and rotating frames.
- Define the equation of motion in the lab and rotating frames.
- Know how to compute the flip angle from the B1-envelope function.
- Understand how to apply the RF hard pulse matrix operator.

B₁ Field - RF Pulse

- B₁ is a
 - radiofrequency (**RF**)
 - 42.58MHz/T (63MHz at 1.5T)
 - short duration **pulse** (~0.1 to 5ms)
 - small amplitude
 - <30 μT
 - circularly polarized
 - rotates at Larmor frequency
 - magnetic field
 - perpendicular to B₀

RF Birdcage Coil

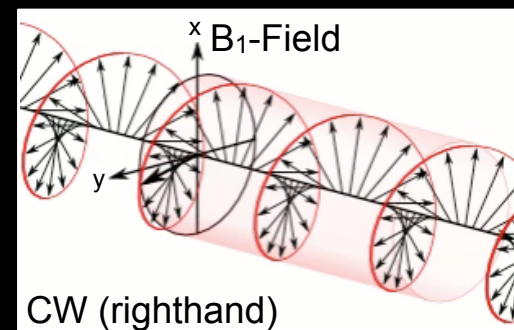
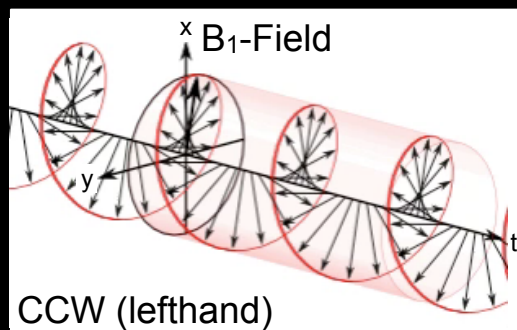
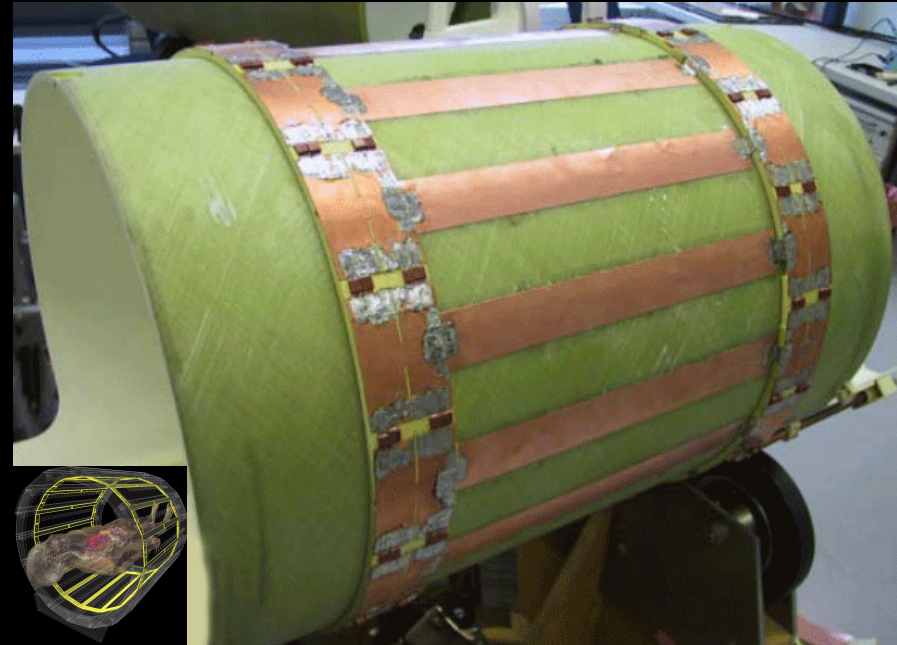
MRI Hardware

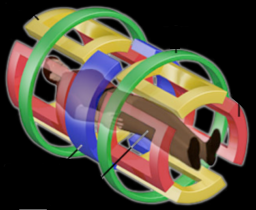


RF Birdcage Coil

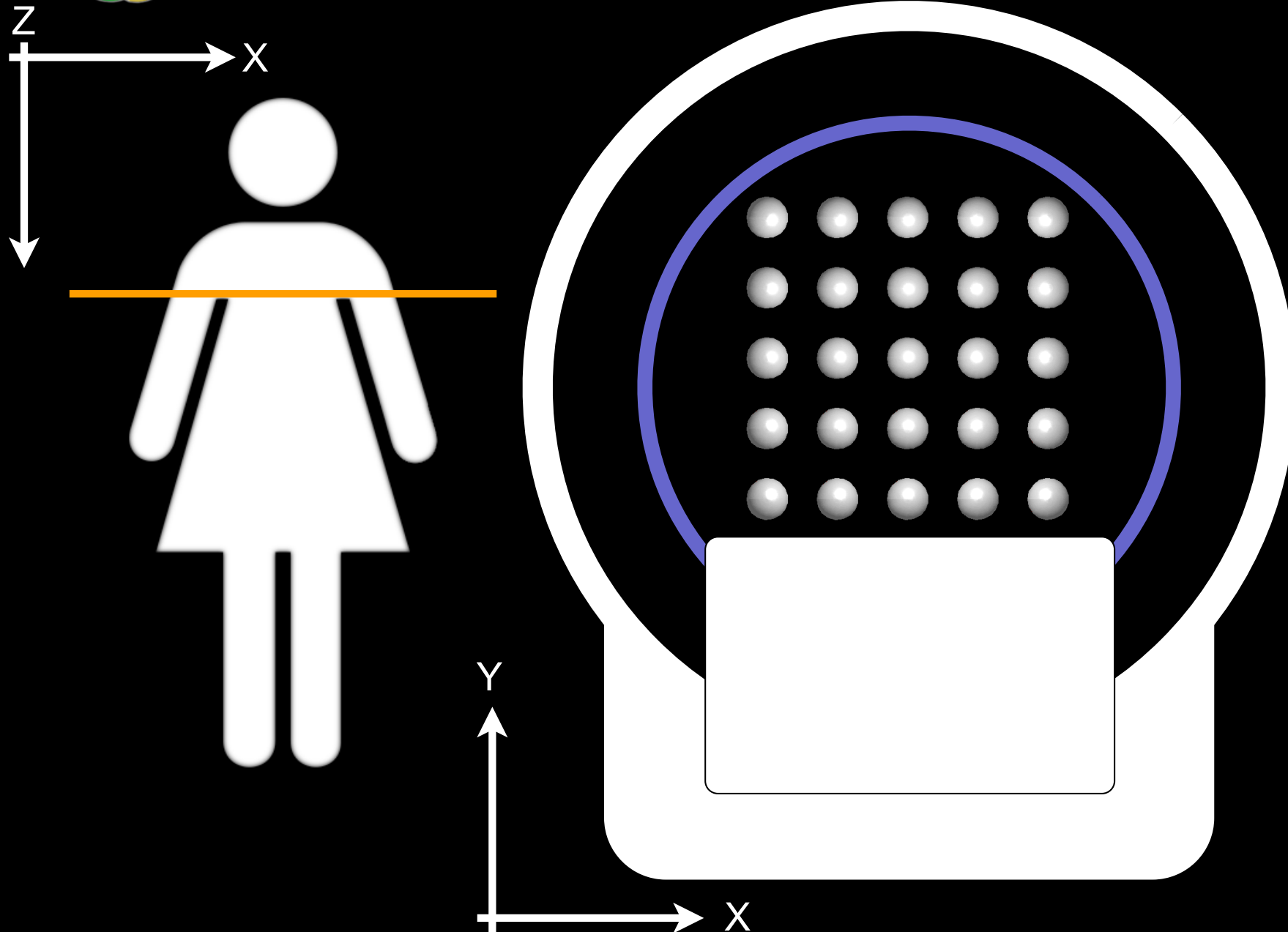
- **Most common design**
- **Highly efficient**
 - Nearly all of the fields produced contribute to imaging
- **Very uniform field**
 - Especially radially
 - Decays axially
 - **Uniform sphere if $L \approx D$**
- **Generates a “quadrature” field**
 - Circular polarization

Body Tx/Rx Coil (B_1)





RF Excitation - Lab Frame



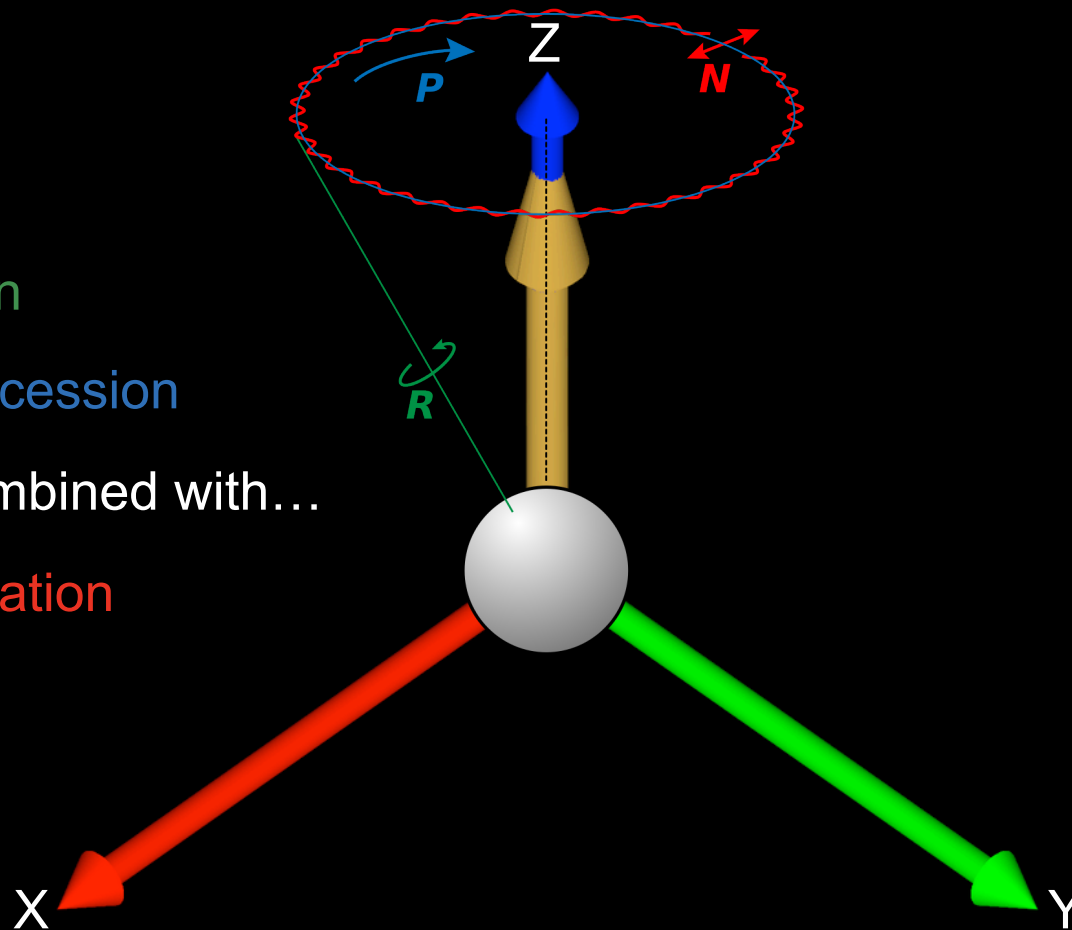
RF Excitation - Lab Frame

^1H has intrinsic Spin

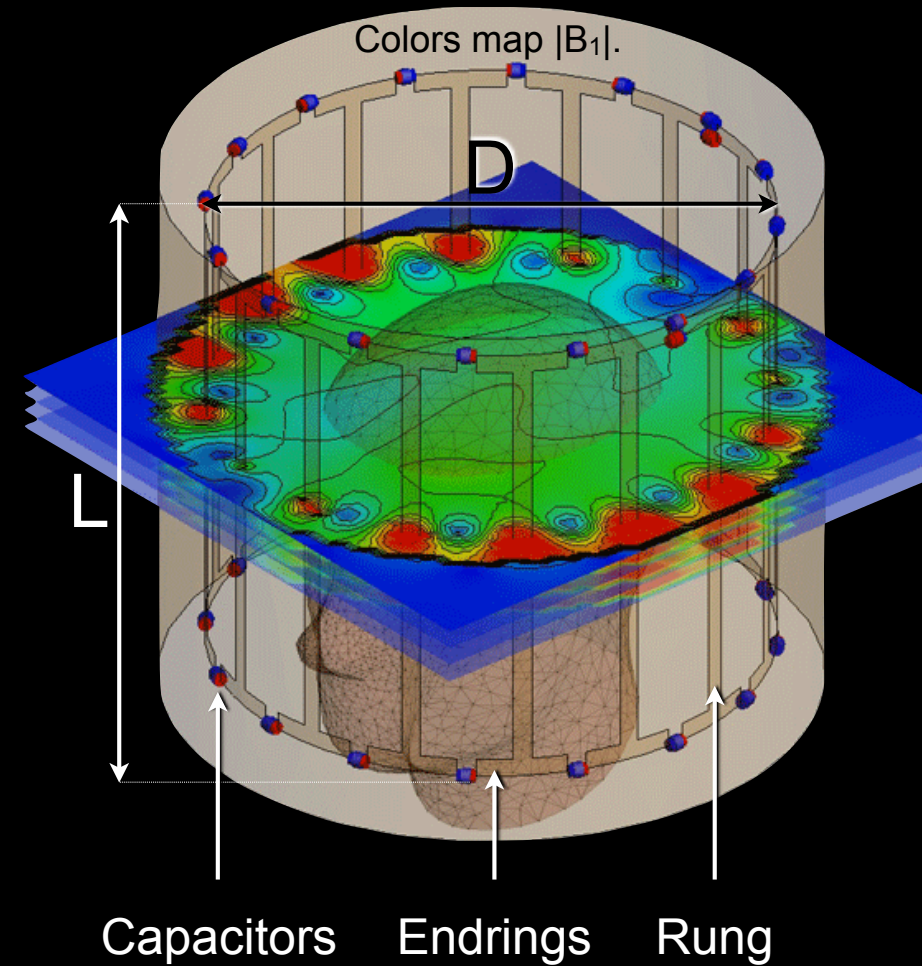
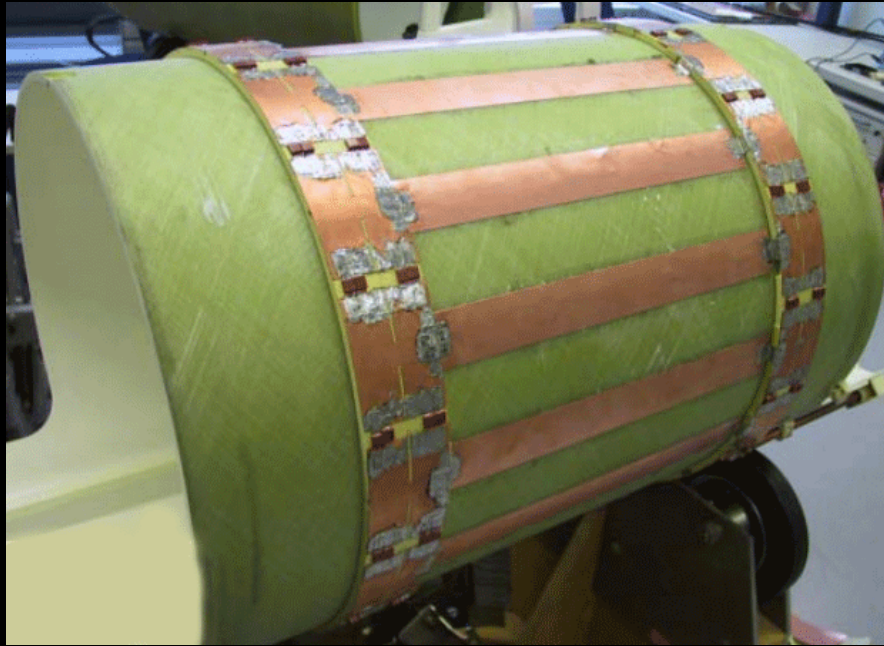
$\omega_0 = \gamma B_0$ Precession

Combined with...

$\omega_1 = \gamma B_1$ Nutation



RF Birdcage Coil

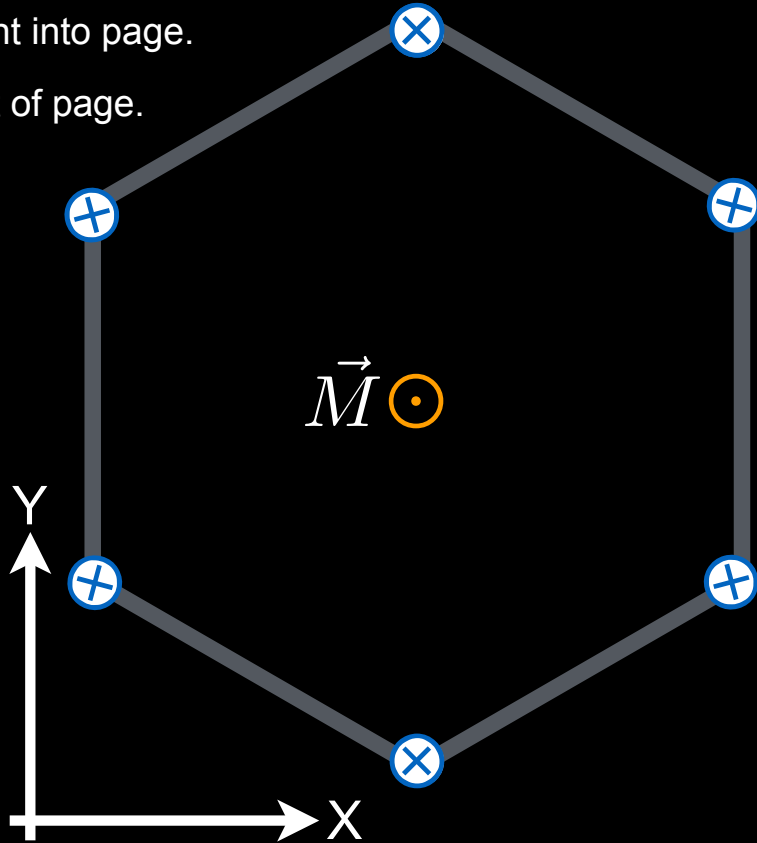


Birdcage coils are used to generate **low SAR [W/kg] circularly polarized RF B₁-fields**.

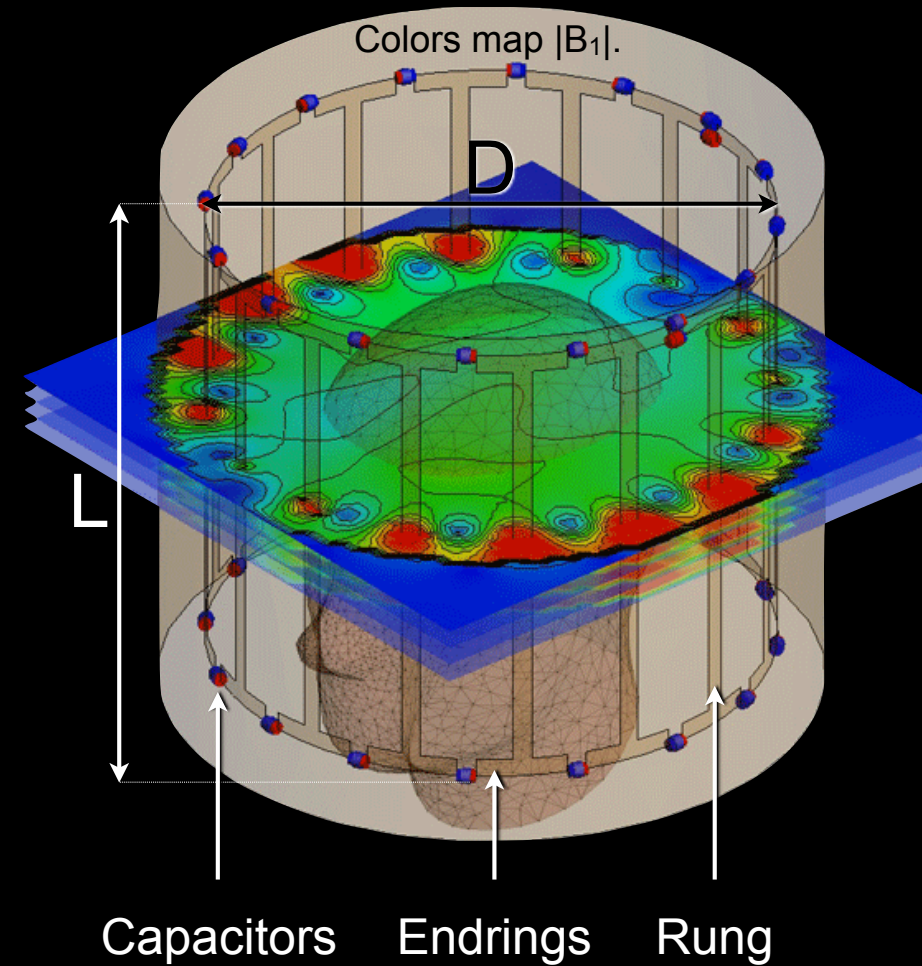
RF Birdcage Coil

⊗ Current into page.

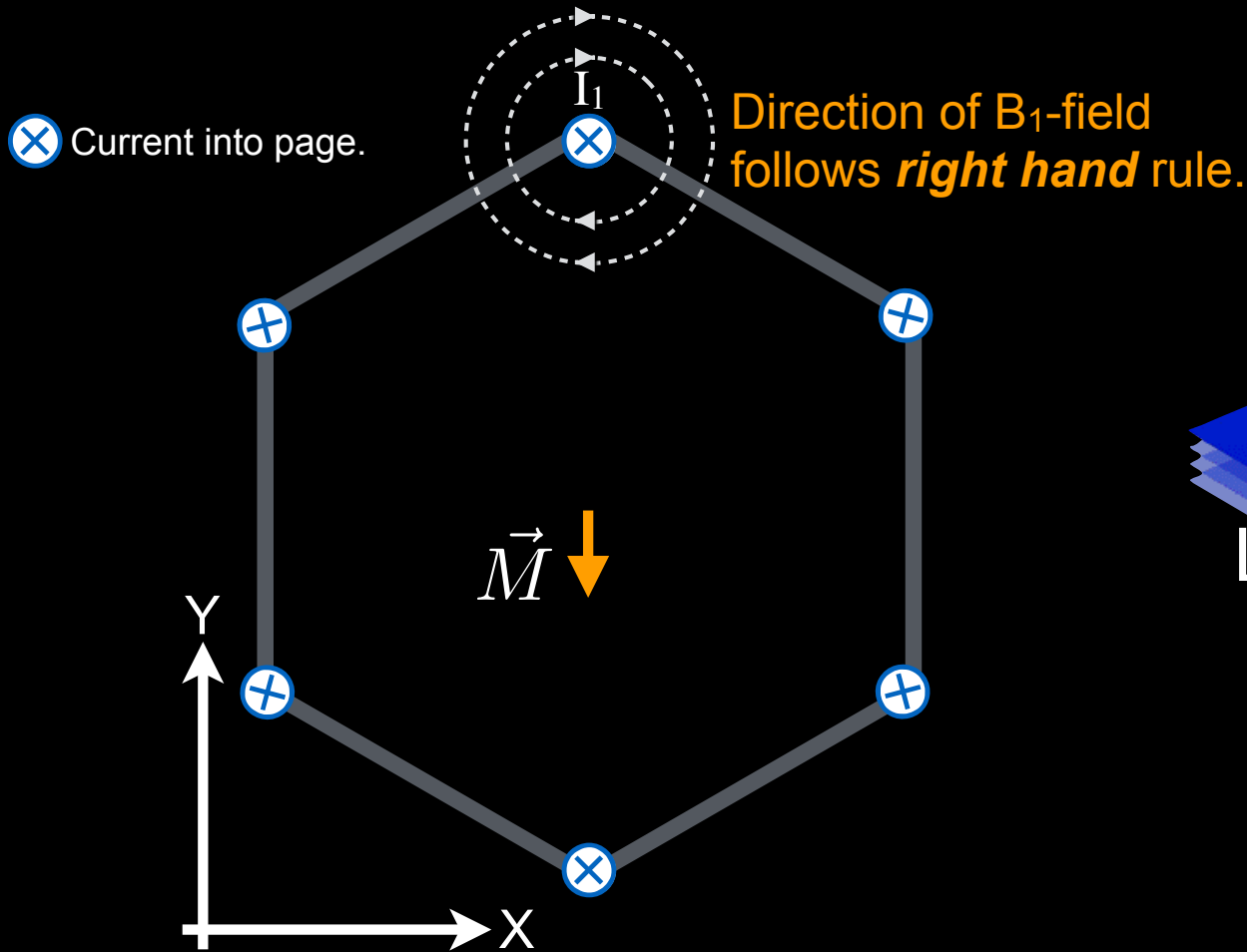
⊙ \vec{M} out of page.



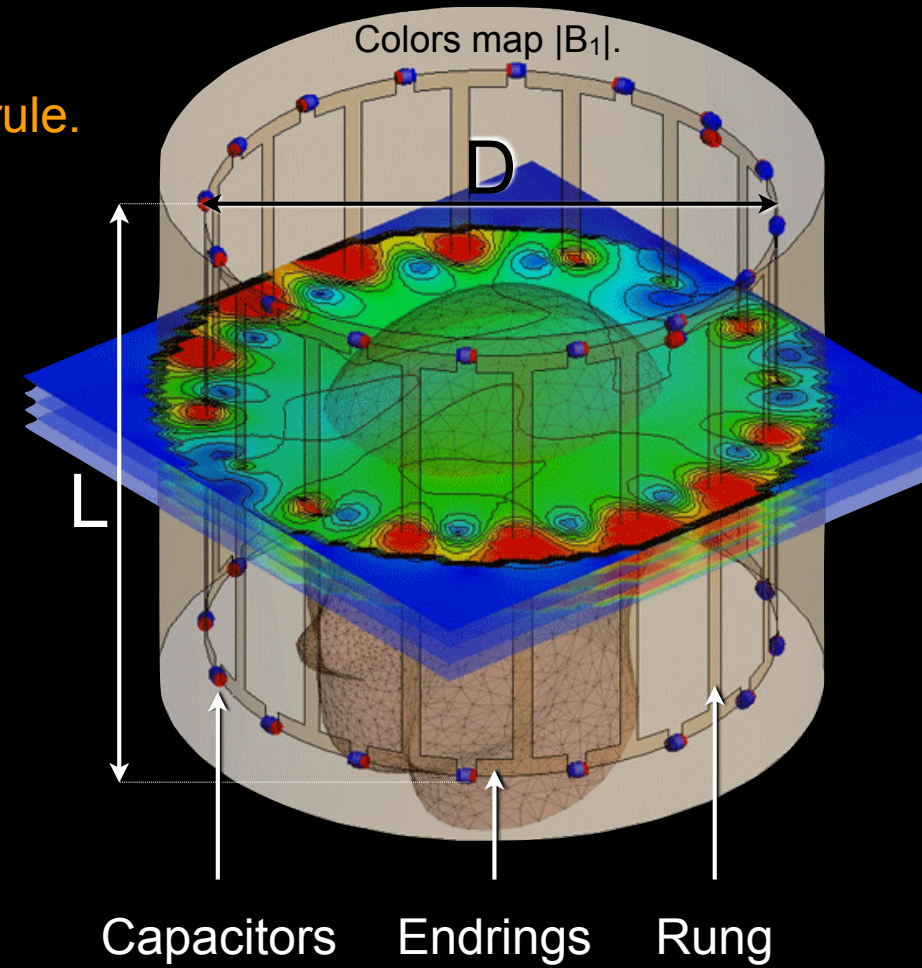
In the absence of any applied RF the bulk magnetization is oriented along the z-axis.



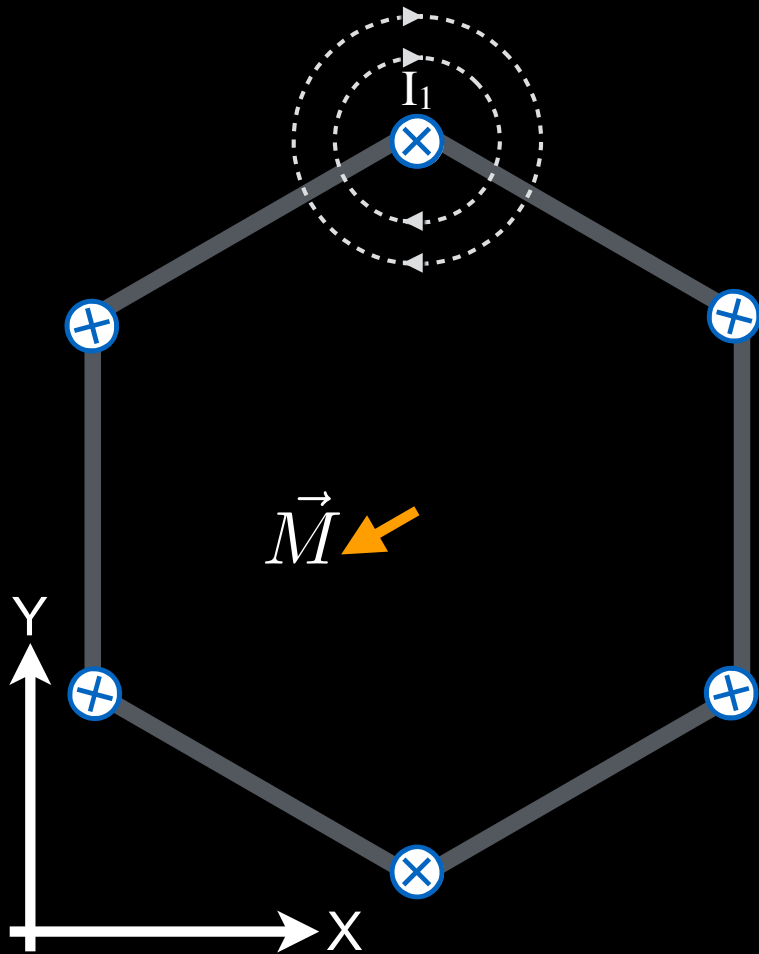
RF Birdcage Coil



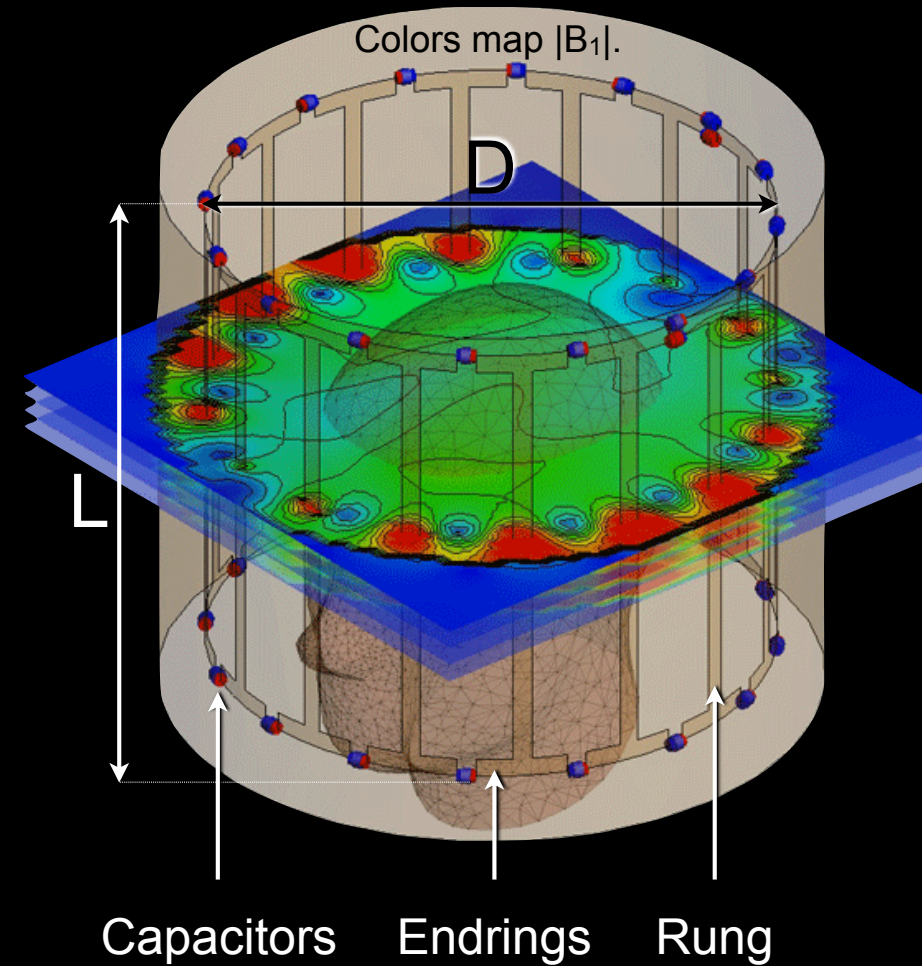
A current (I_1) induces a *left-handed* nutation about the B_1 -field.



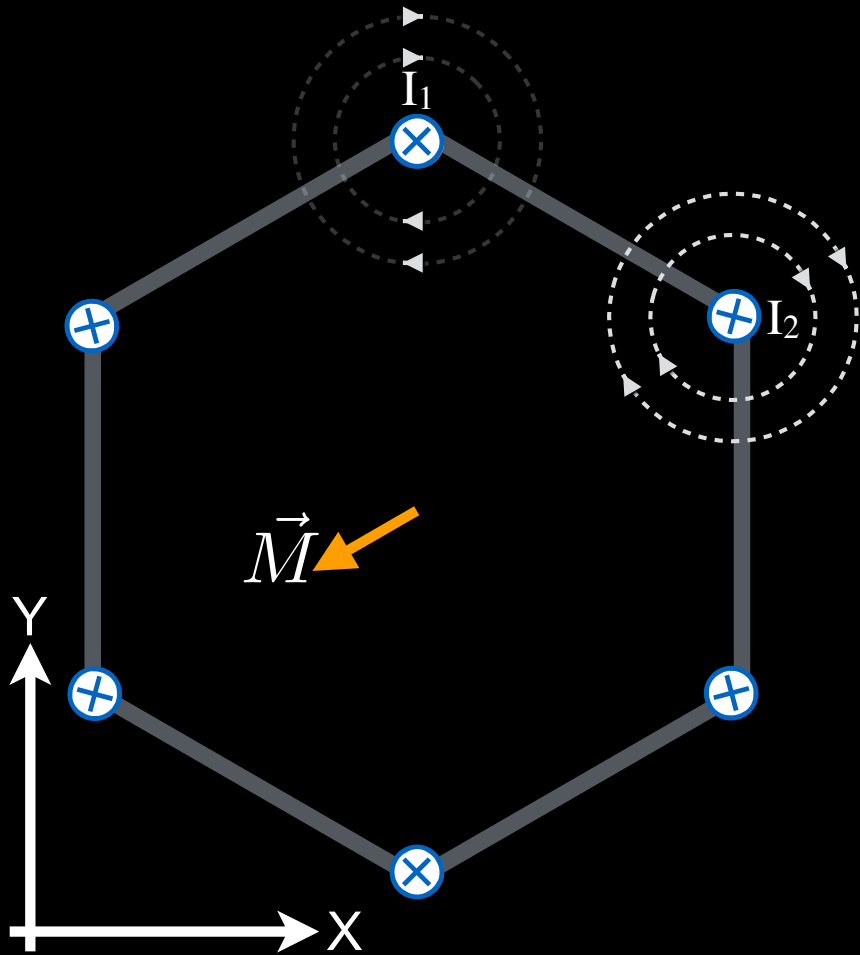
RF Birdcage Coil



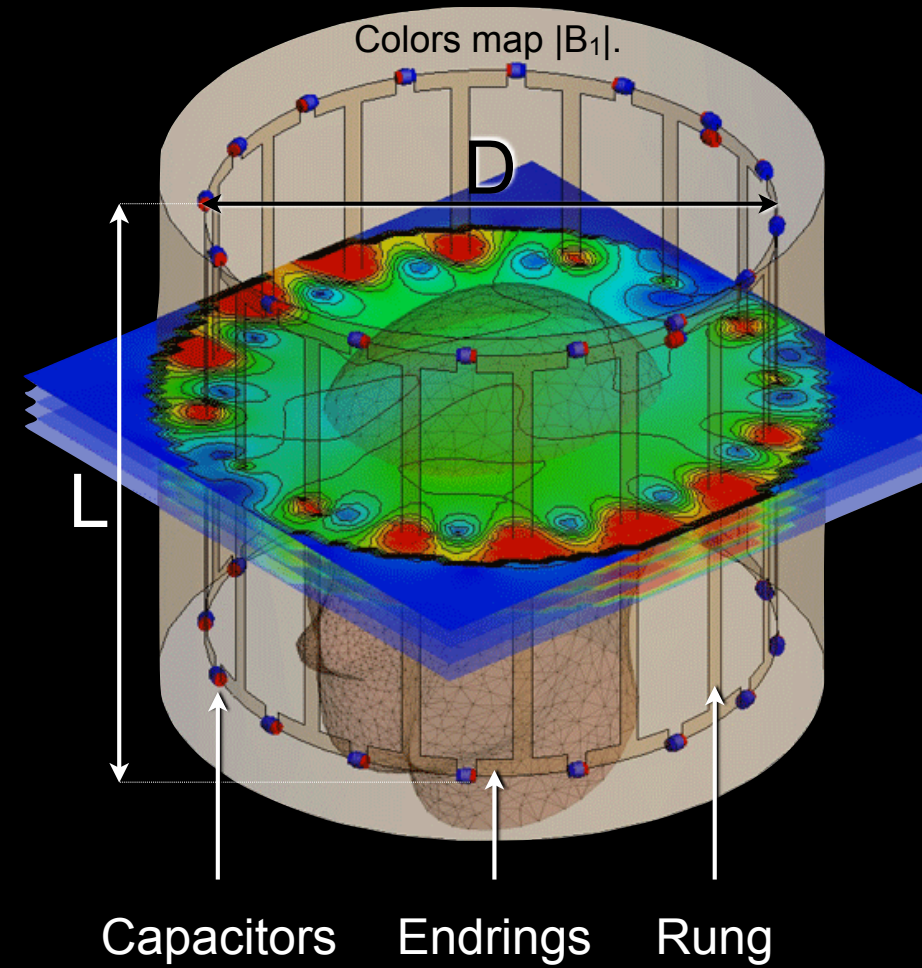
Precession from B_0 advances the spin clockwise (*left hand rule*).



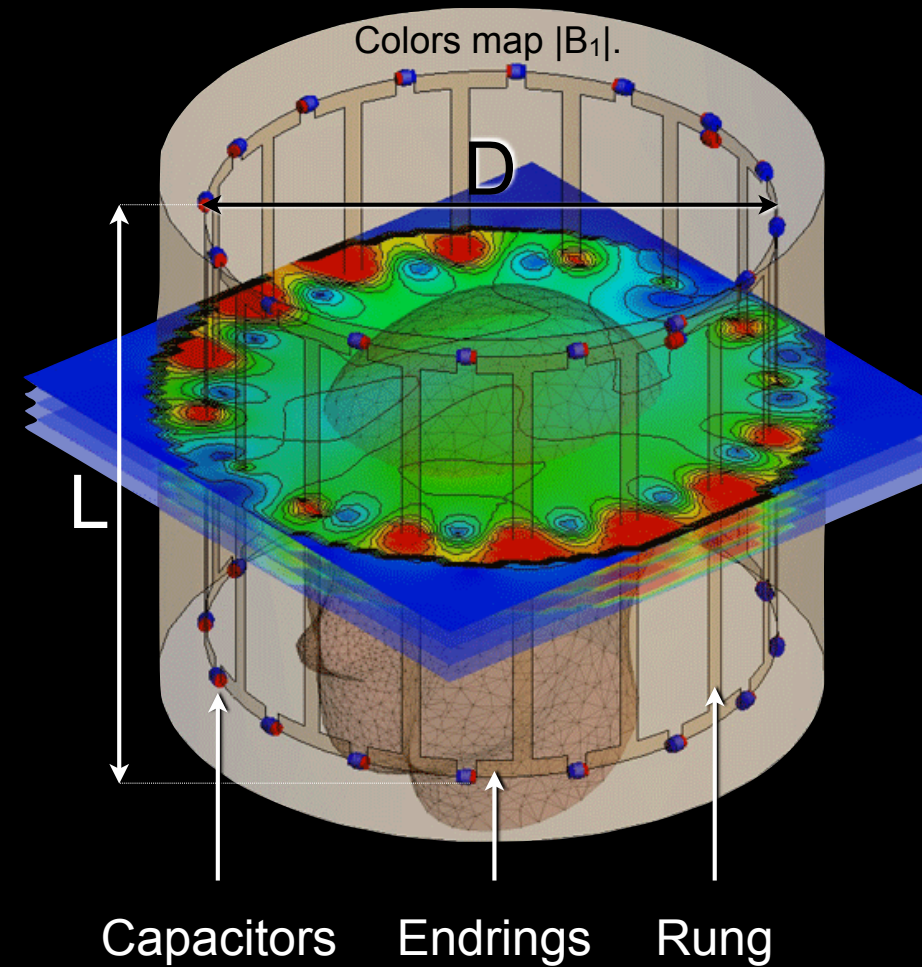
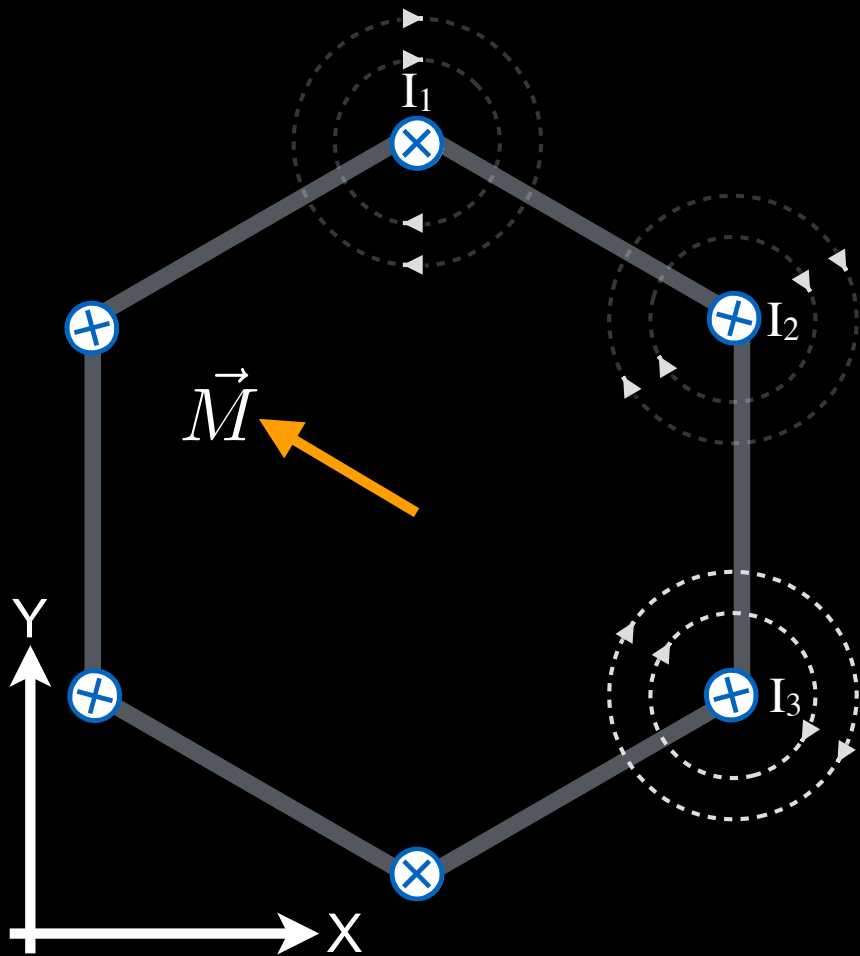
RF Birdcage Coil



B_1 nutation from I_2 generates more M_{xy} .



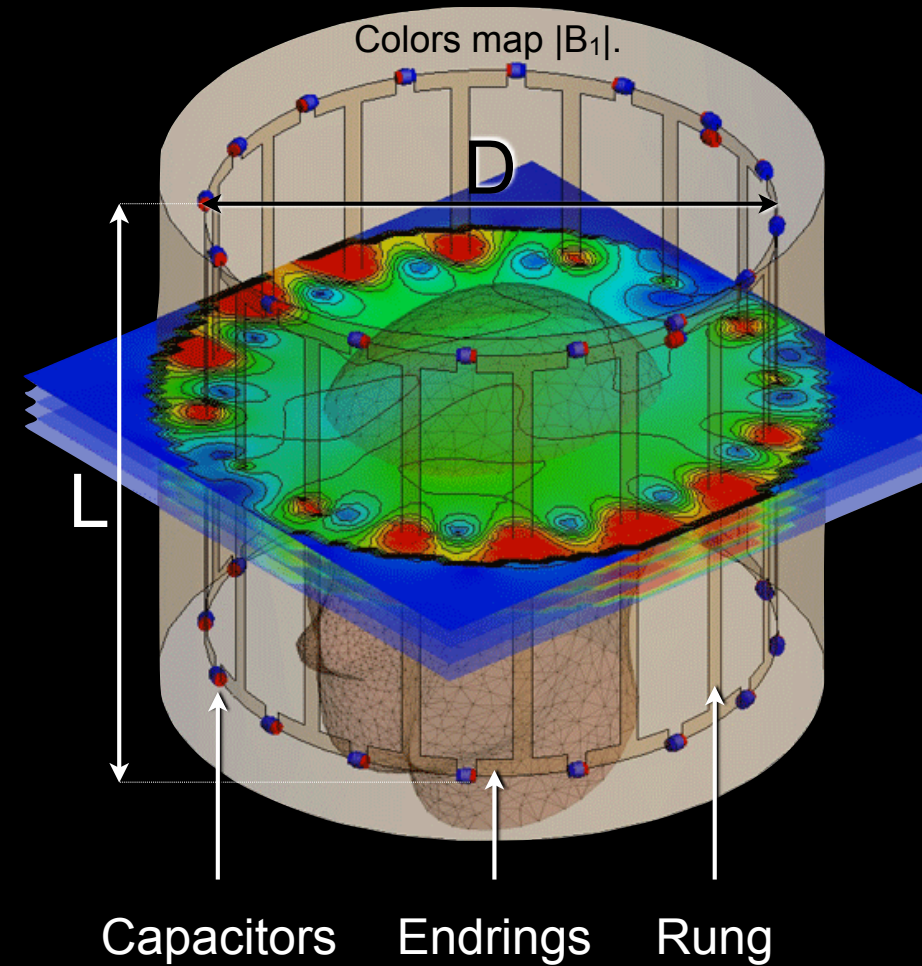
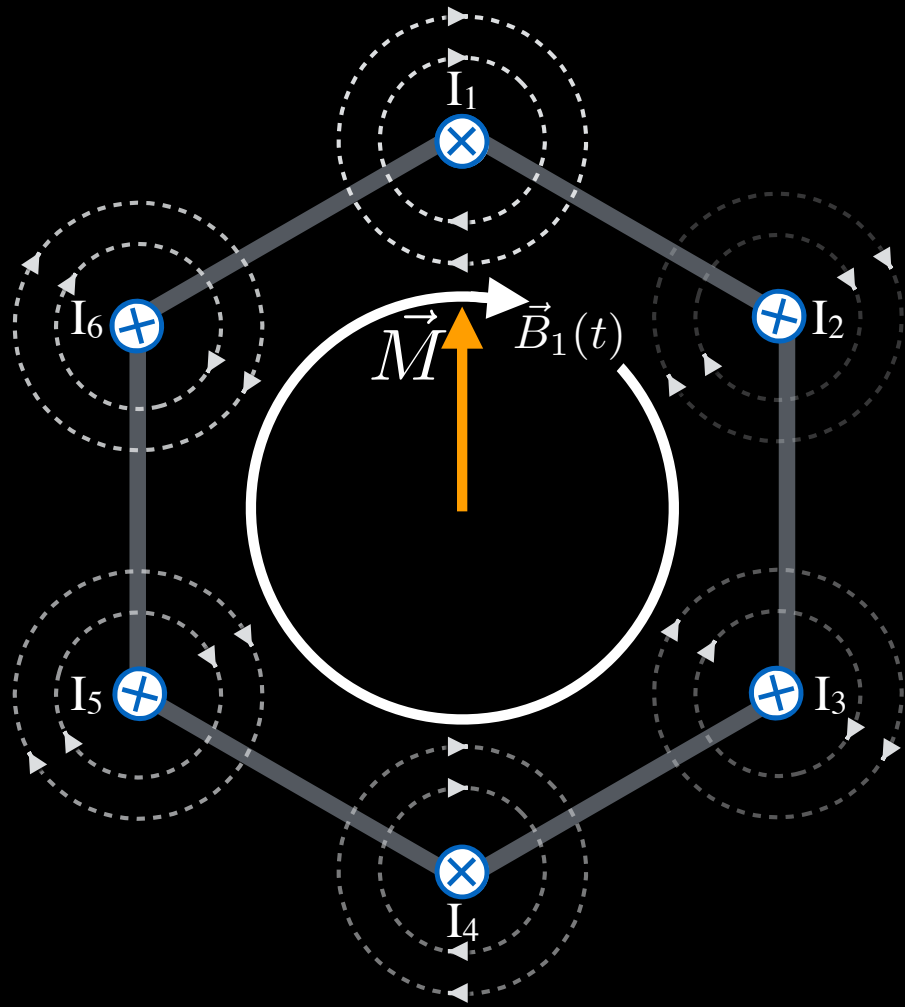
RF Birdcage Coil



$$I_n(t) = I_0 \sin \left(\omega_{RF} t - \frac{2\pi(n-1)}{N_{Rungs}} \right)$$

Current in the n^{th} rung.
Creates a CW B_1 -field.

RF Birdcage Coil



$$I_n(t) = I_0 \sin \left(\omega_{RF} t - \frac{2\pi(n-1)}{N_{Rungs}} \right)$$

Current in the n^{th} rung.
Creates a CW B_1 -field.

Consider reading Chp. 16.3 in Haacke.

B₁ Inhomogeneity

B₁ Inhomogeneity: Imperfect B₁ amplitude as a function of spatial position.

Sources:

- Hardware imperfections.
- Conductivity & permittivity of subject/object [1].
- Wavelength effects.

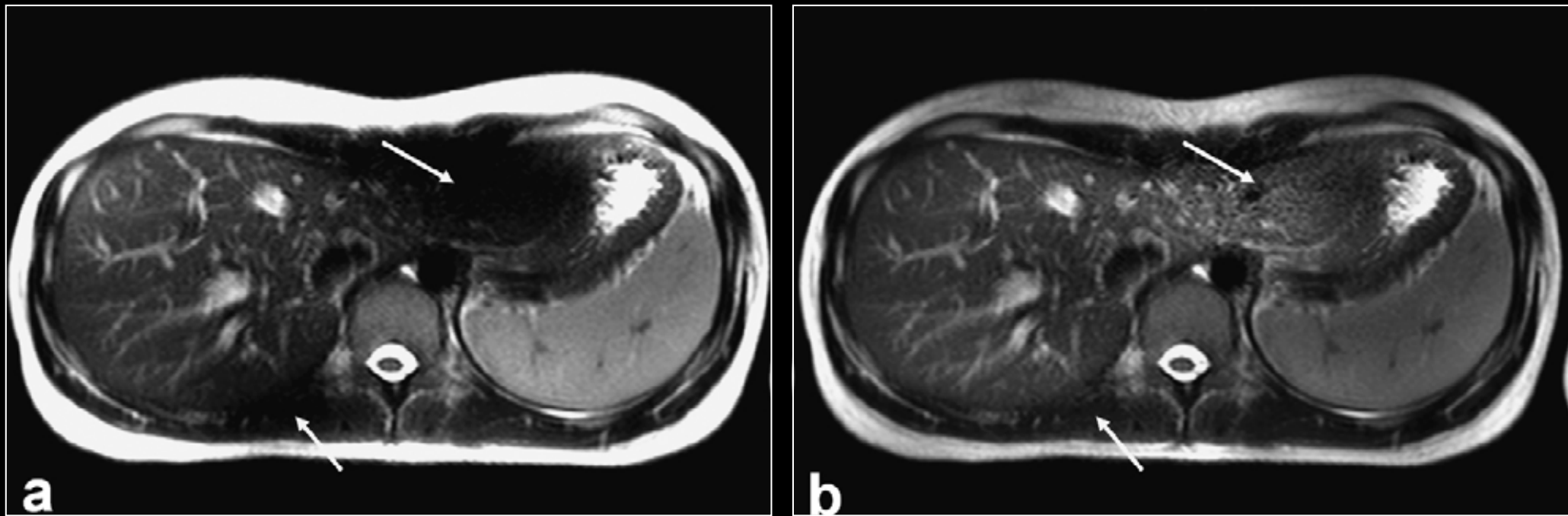
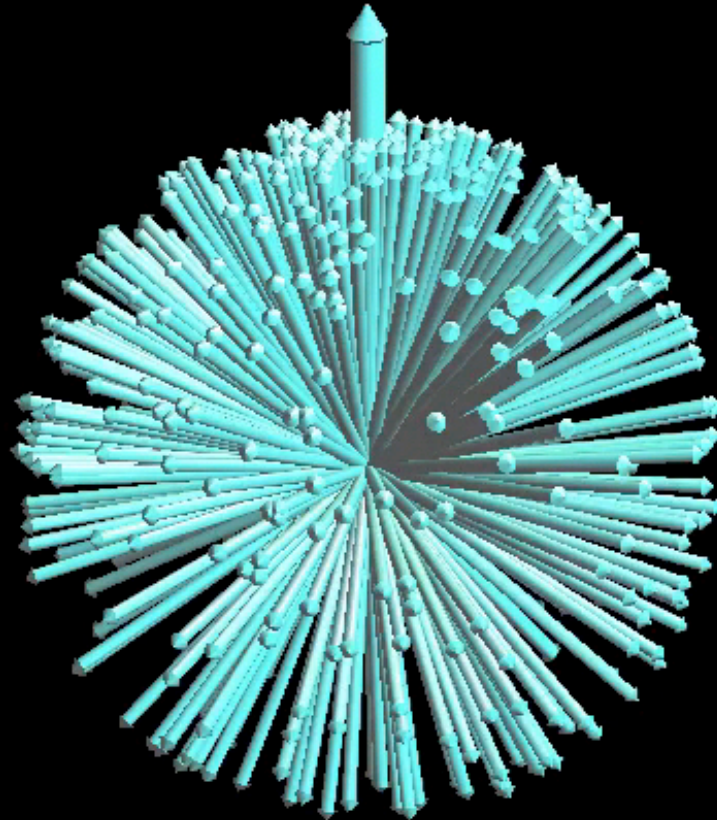


Fig. 5. Signal loss due to **inhomogeneous flip-angle distribution** at 3T. (a) Wavelength effects result in reduced signal intensity in the abdomen (arrows). (b) This effect can in some cases be reduced by manually increasing the RF-transmitter amplitude (here by 50%) and by applying image post-processing filters to obtain more uniform image intensities. Images courtesy of W. Horger, Siemens Medical Solutions, Germany [2]

Resonance

Ensemble of Precessing Spins



“The equilibrium magnetization is stationary, so even though the individual spins are precessing, there is no net emission of radio waves in equilibrium.”

Resonance

- Quantum Physics
 - Electromagnetic radiation of frequency ω_{RF} carries energy that induces a coherent transition of spins from N_{\uparrow} to N_{\downarrow} .
- Classical Physics
 - $\vec{B}_1(t)$ rotates in the same manner as the precessing spins.
 - Coherently “pushes” on bulk magnetization.

Resonance Condition (Quantum)

$$\Delta E = E_{\downarrow} - E_{\uparrow} = \hbar\gamma B_0 \quad E_{RF} = \hbar\omega_{RF}$$

Zeeman Splitting

Planck's Law



$$\hbar\gamma B_0 = \hbar\omega_{RF}$$



$$\omega_{RF} = \gamma B_0 = \omega_0$$

Resonance Condition

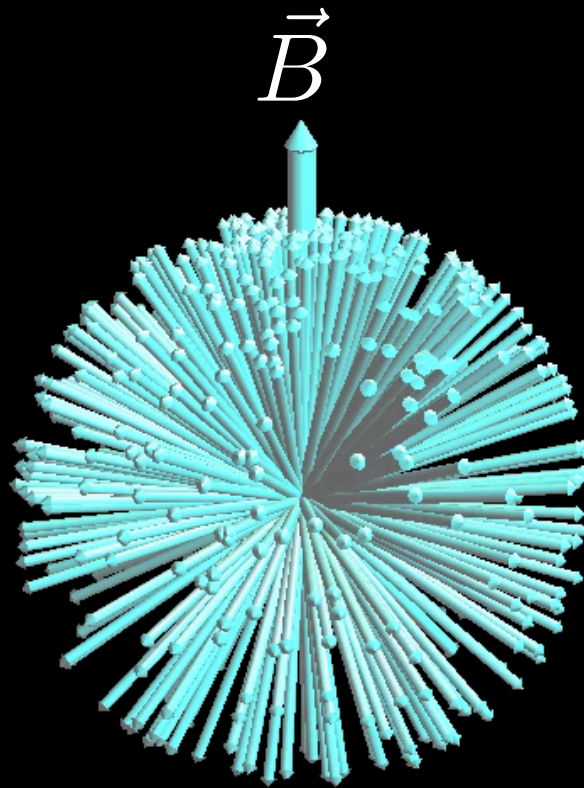
Resonance requires that the frequency of the RF energy (ω_{RF}) match the frequency of precession (ω_0).

Resonance Condition (Classical)

“Establishment of a phase coherence among these ‘randomly’ precessing spins in a magnetized spin system is referred to as *resonance*.”

– Liang & Lauterbur p.69

$$N_{\uparrow} \approx N_{total} \times (1 + 2.25 \times 10^{-6})$$



$$N_{\downarrow} \approx N_{total} \times (1 - 2.25 \times 10^{-6})$$

SAR, Polarization, and B_1 Safety

SAR Limitations

- **Specific Absorption Rate**
 - Measure of the rate of energy absorption during exposure to a RF electromagnetic field
 - Measured in units of [W/kg]
- High-field (>1.5T) imaging with high flip angles (>45-90°) can be challenging.

$$\text{SAR} \propto \omega_0^2 B_1^2 \propto B_0^2 \alpha^2$$

SAR Limits

Limit	Whole-Body Average	Head Average	Head, Trunk Local SAR	Extremities Local
IEC (6-minute average)				
Normal (all patients)	2 W/kg (0.5°C)	3.2 W/kg	10 W/kg	20 W/kg
First level (supervised)	4 W/kg (1°C)	3.2 W/kg	10 W/kg	20 W/kg
Second level (IRB approval)	4 W/kg (>1°C)	>3.2 W/kg	>10 W/kg	>20 W/kg
Localized heating limit	39°C in 10 g	38°C in 10 g		40°C in 10 g
FDA	4 W/kg for 15 min	3 W/kg for 10 min	8 W/kg in 1g for 10 min	12 W/kg in 1g for 5 min

Basic RF Pulse - Linear Polarized

$$\vec{B}_1(t) = 2B_1^e(t) \cos(\omega_{RF}t + \theta) \vec{i}$$

$B_1^e(t)$

pulse envelope function

ω_{RF}

excitation carrier frequency

θ

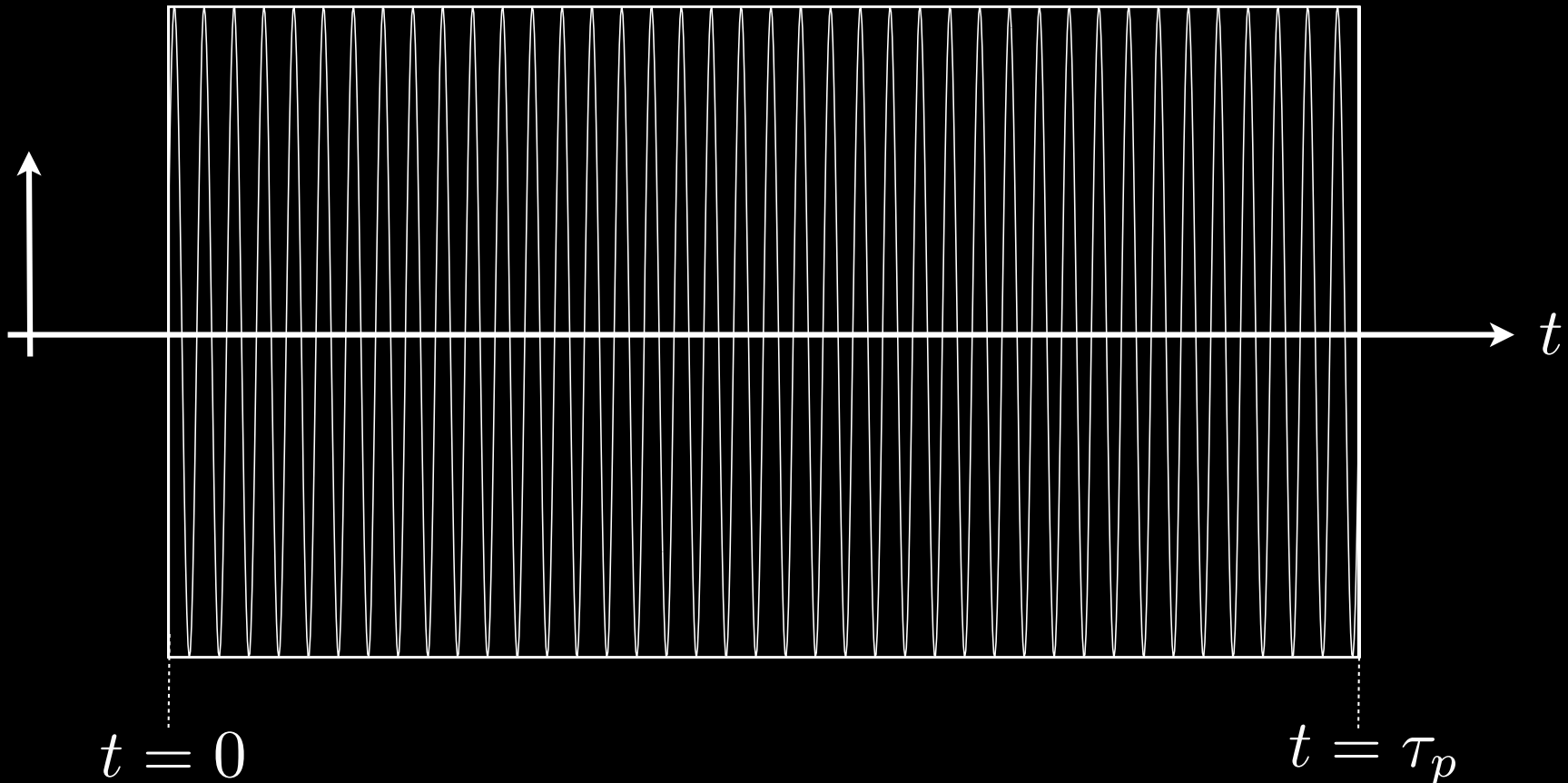
initial phase angle

\vec{i}

linearly polarized

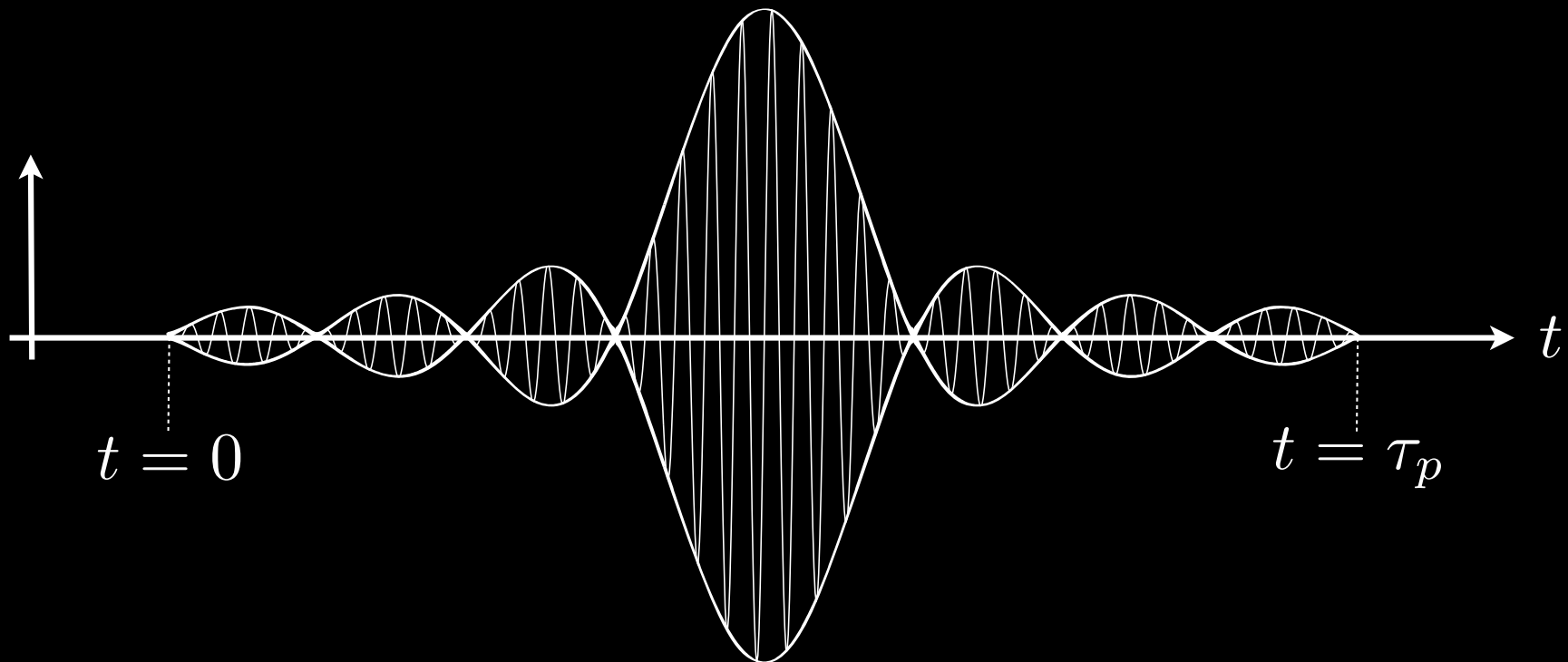
Rect Envelope Function

$$B_1^e(t) = B_1 \square\left(\frac{t - \tau_p/2}{\tau_p}\right) = \begin{cases} B_1, & 0 \leq t \leq \tau_p \\ 0, & \textit{otherwise} \end{cases}$$



Sinc Envelope Function

$$B_1^e(t) = \begin{cases} B_1 \text{sinc} [\pi f_\omega (t - \tau_p/2)], & 0 \leq t \leq \tau_p \\ 0, & \textit{otherwise} \end{cases}$$



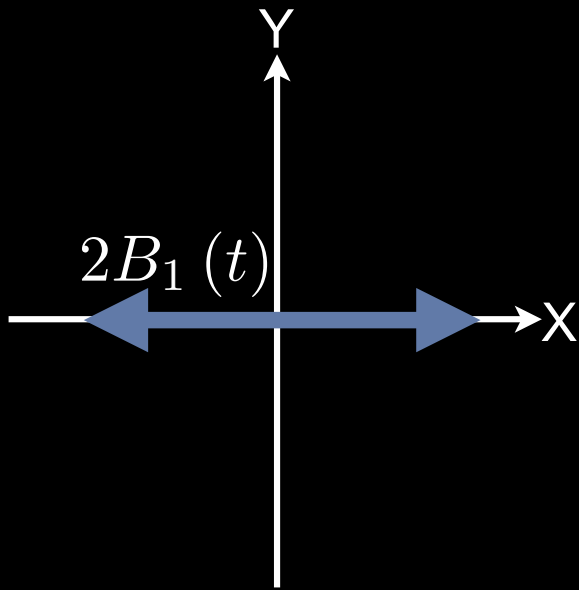
Circular vs. Linear Polarization

- **Linear Polarization**
 - Simple, cheap
 - Higher RF power
- **Circular Polarization**
 - Generated with a quadrature RF transmitter coil
 - More complex & more expensive
 - Reduced RF power deposition

Linearly Polarized Fields

Linear Polarization

$$2B_1^e(t) \cos(\omega_{RF}t) \hat{i}$$



Arrow indicates direction of B-field.

Circularly Polarized Fields

Linear Polarization

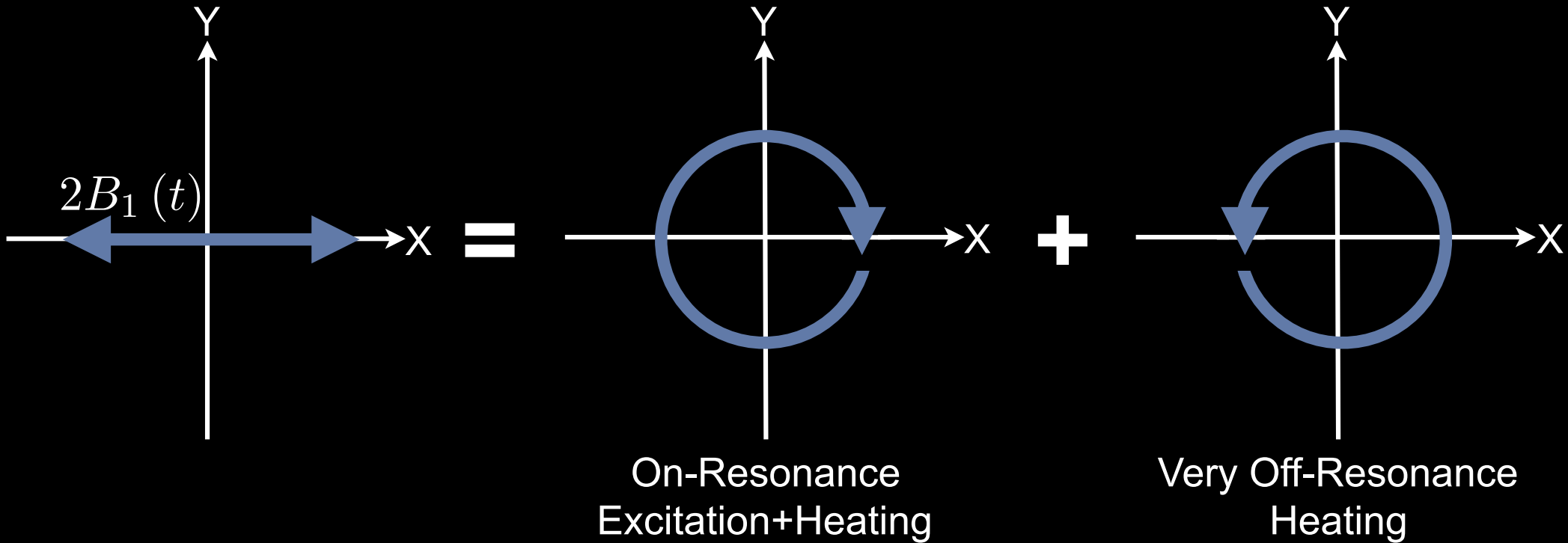
$$2B_1^e(t) \cos(\omega_{RF}t) \hat{i}$$

CW Circular Polarization

$$= B_1^e(t) [\cos(\omega_{RF}t) \hat{i} - \sin(\omega_{RF}t) \hat{j}]$$

CCW Circular Polarization

$$+ B_1^e(t) [\cos(\omega_{RF}t) \hat{i} + \sin(\omega_{RF}t) \hat{j}]$$



Arrow indicates direction of B-field.

Circularly Polarized Fields

Linear Polarization

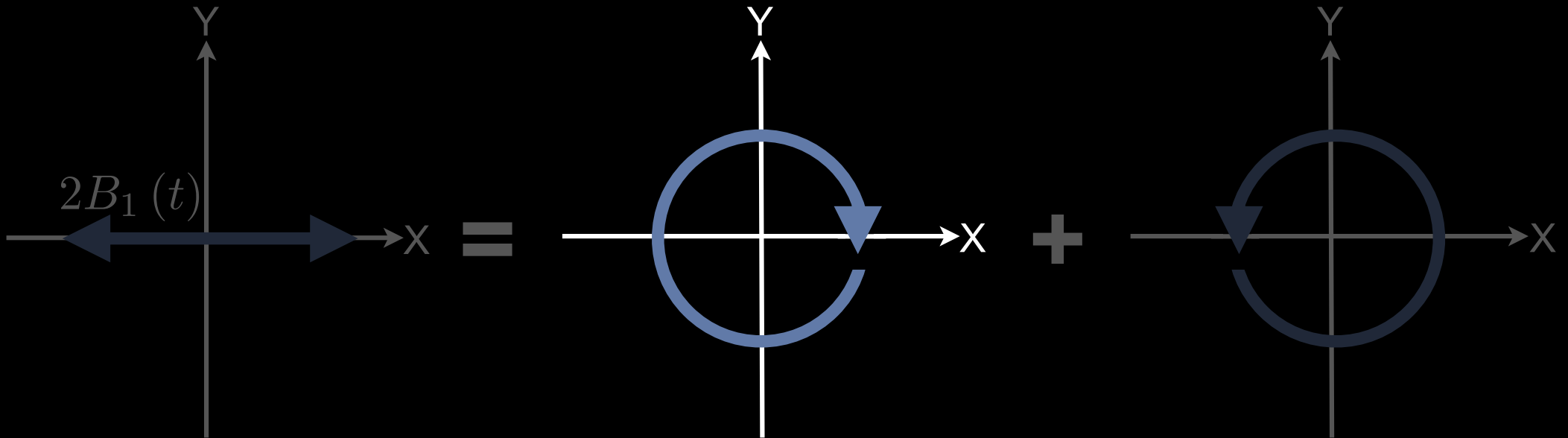
$$2B_1^e(t) \cos(\omega_{RF}t) \hat{i}$$

CW Circular Polarization

$$= B_1^e(t) [\cos(\omega_{RF}t) \hat{i} - \sin(\omega_{RF}t) \hat{j}]$$

CCW Circular Polarization

$$+ B_1^e(t) [\cos(\omega_{RF}t) \hat{i} + \sin(\omega_{RF}t) \hat{j}]$$



On-Resonance
Excitation+Heating

Very Off-Resonance
Heating

First Generation MRI
Systems Used
Linear Polarization

Modern MRI Systems
Only Use CW Circular
Polarization

Modern MRI
Systems Don't Apply
The CCW Field

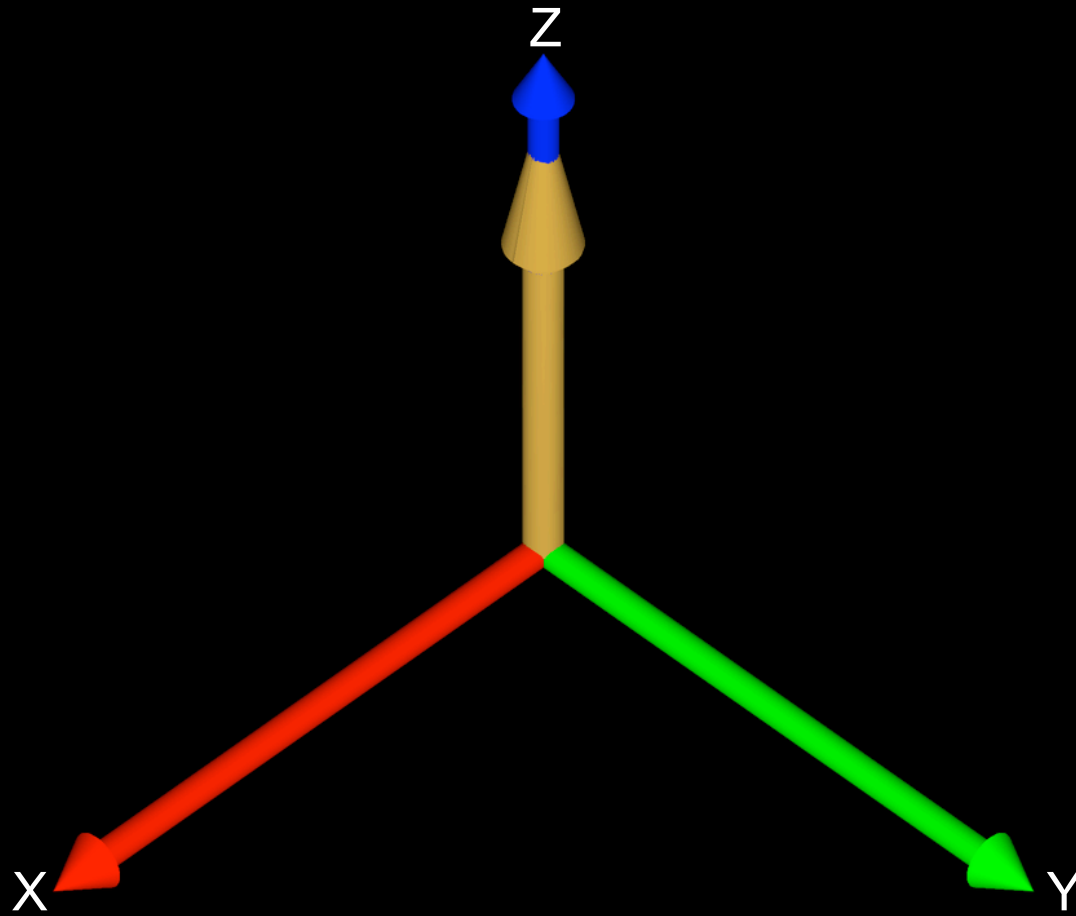
Arrow indicates direction of B-field.

Forced Precession in the Laboratory Frame without Relaxation

Four Special Cases...

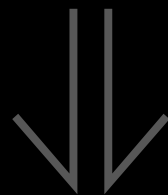
- Free Precession in the Laboratory Frame
- **Forced Precession in the Laboratory Frame**
 - Coordinate system anchored to scanner
- Free Precession in the Rotating Frame
- **Forced Precession in the Rotating Frame**
 - Coordinate system anchored to spin system
- **...all without relaxation.**
 - a) Relaxation time constants are “really” long
 - b) Time scale of event is \ll relaxation time constant

Forced Precession - Lab Frame



Forced Precession in the Laboratory Frame without Relaxation

$$\begin{aligned} \frac{d\vec{M}}{dt} &= \vec{M} \times \gamma \left(\vec{B}_0 + \vec{B}_1 \right) \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ \gamma B_{1,x}^e(t) & \gamma B_{1,y}^e(t) & \gamma B_0 \end{vmatrix} \end{aligned}$$

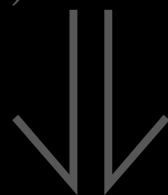


$$\frac{dM_x}{dt} = \gamma B_0 M_y - \gamma B_{1,y}^e(t) M_z$$

$$\frac{dM_y}{dt} = -\gamma B_0 M_x + \gamma B_{1,x}^e(t) M_z$$

$$\frac{dM_z}{dt} = \gamma B_{1,y}^e(t) M_x - \gamma B_{1,x}^e(t) M_y$$

Complex
Coupling



Forced Precession in the Laboratory Frame without Relaxation

$$\left. \begin{aligned} \frac{dM_x}{dt} &= \gamma B_0 M_y - \gamma B_{1,y}^e(t) M_z \\ \frac{dM_y}{dt} &= -\gamma B_0 M_x + \gamma B_{1,x}^e(t) M_z \\ \frac{dM_z}{dt} &= \gamma B_{1,y}^e(t) M_x - \gamma B_{1,x}^e(t) M_y \end{aligned} \right\} \begin{array}{l} \text{Complex} \\ \text{Coupling} \end{array}$$

$$\Downarrow$$
$$B_1(t) = B_1^e(t) \left[\cos(\omega_{RF}t + \theta) \hat{i} - \sin(\omega_{RF}t + \theta) \hat{j} \right]$$

Forced Precession in the Laboratory Frame without Relaxation

$$\left. \begin{aligned} \frac{dM_x}{dt} &= \gamma B_0 M_y - \gamma B_{1,y}^e(t) M_z \\ \frac{dM_y}{dt} &= -\gamma B_0 M_x + \gamma B_{1,x}^e(t) M_z \\ \frac{dM_z}{dt} &= \gamma B_{1,y}^e(t) M_x - \gamma B_{1,x}^e(t) M_y \end{aligned} \right\} \begin{array}{l} \text{Complex} \\ \text{Coupling} \end{array}$$



$$B_1(t) = B_1^e(t) \left[\cos(\omega_{RF}t + \theta) \hat{i} - \sin(\omega_{RF}t + \theta) \hat{j} \right]$$



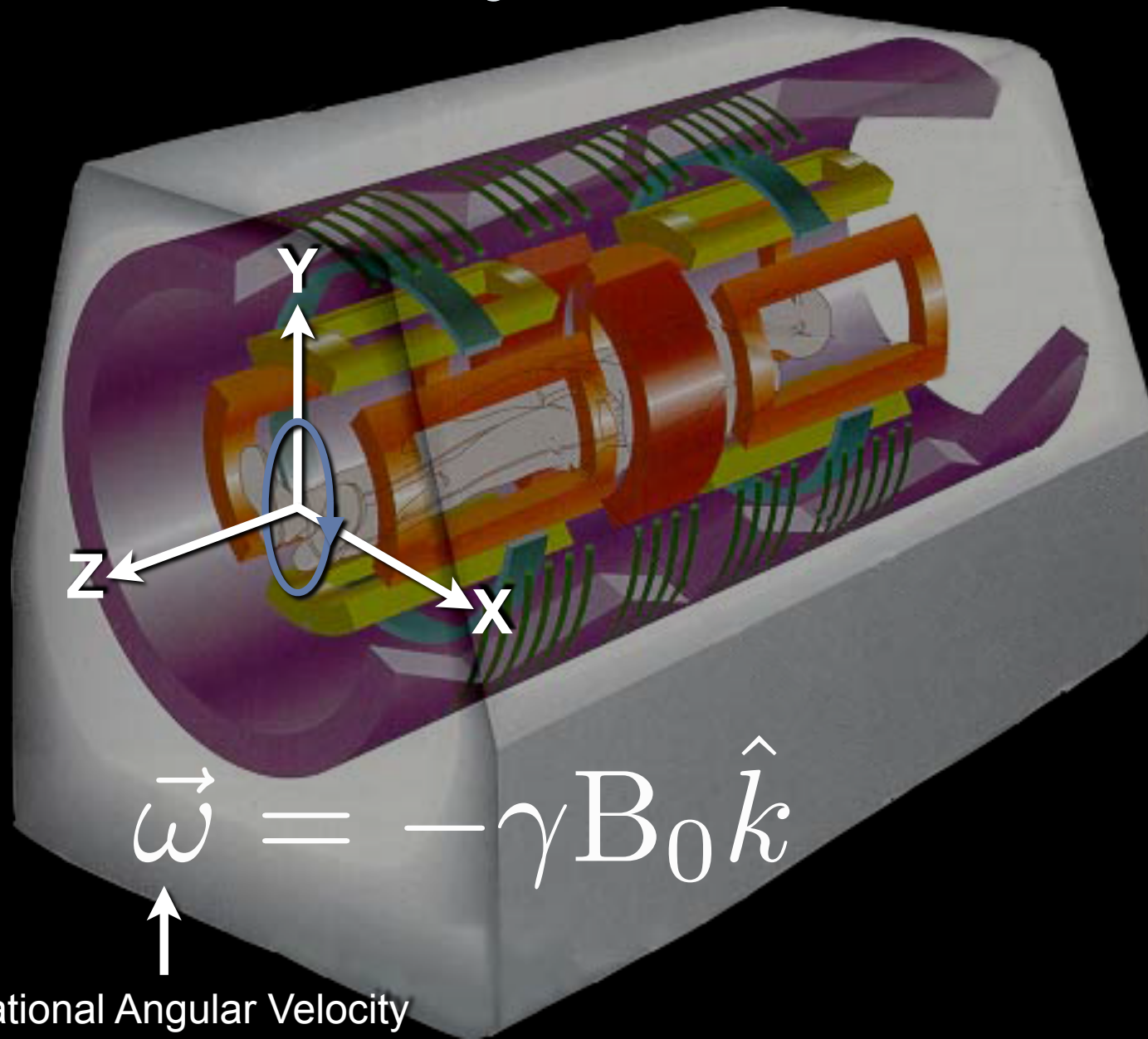
$$\frac{dM_x}{dt} = \gamma B_0 M_y + \gamma B_1^e(t) \sin(\omega_{RF}t + \theta) M_z$$

$$\frac{dM_y}{dt} = -\gamma B_0 M_x + \gamma B_1^e(t) \cos(\omega_{RF}t + \theta) M_z$$

$$\frac{dM_z}{dt} = -\gamma B_1^e(t) \sin(\omega_{RF}t + \theta) M_x - \gamma B_1^e(t) \cos(\omega_{RF}t + \theta) M_y$$

Rotating Coordinate Frame

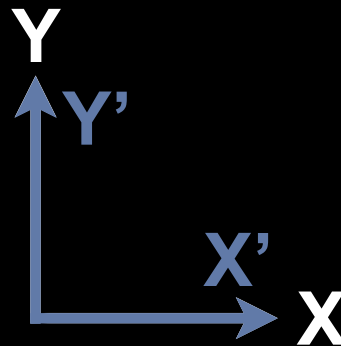
Laboratory Coordinates



Rotational Angular Velocity

Rotating Frame Coordinates

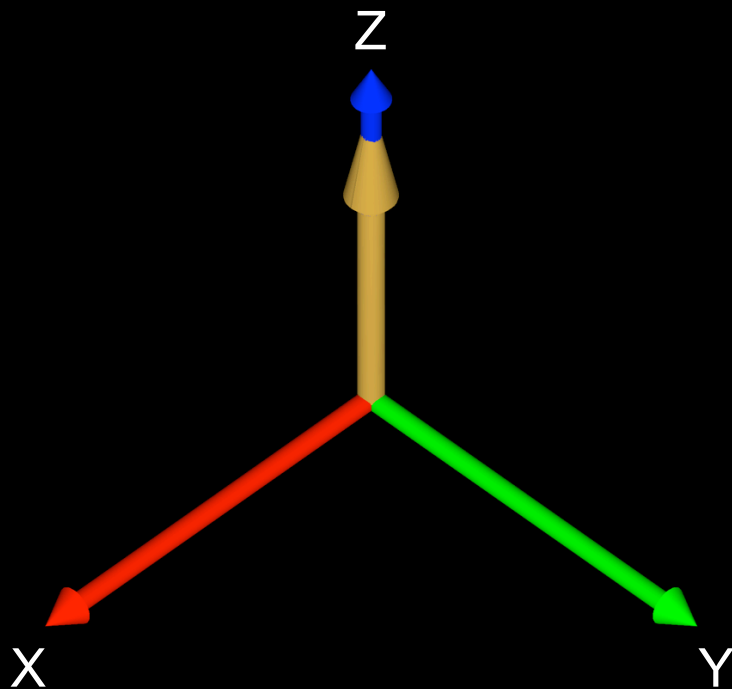
- Simplifies the mathematics of MRI
- If the rotational frequency of the rotating frame (x' - y') is matched to the bulk magnetization's precessional frequency, then rotational motion of the bulk magnetization is “removed” or demodulated.
- The rotating frame's transverse (x' y') plane rotates clockwise (left-handed) at frequency ω .



Lab vs. Rotating Frame

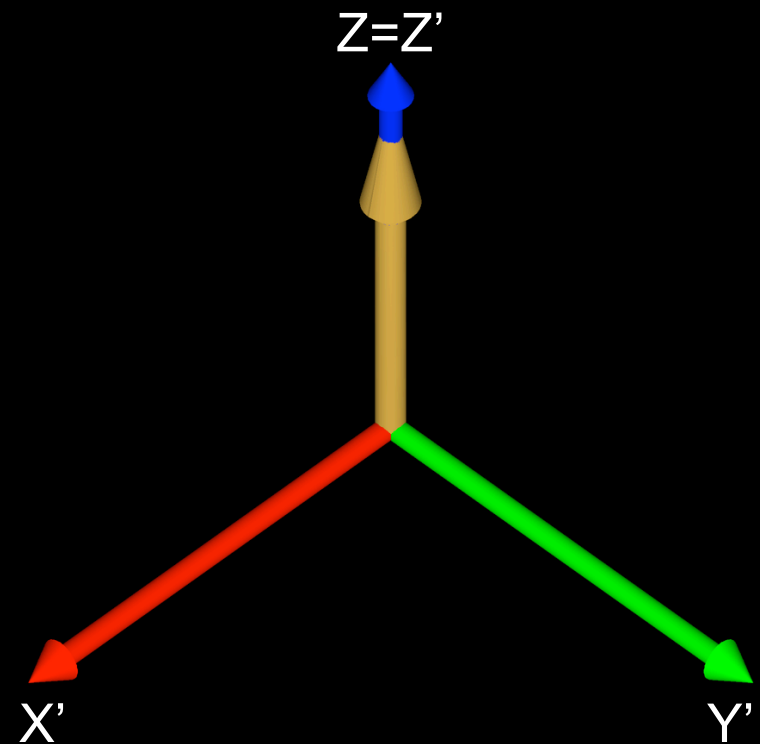
- The rotating frame simplifies the mathematics and permits more intuitive understanding.

Laboratory Frame



Spins Precess

Rotating Frame



Observer Precesses

Note: Both coordinate frames share the same z-axis.

Relationship Between Lab and Rotating Frames

Rotating Frame

Laboratory Frame

$$\begin{array}{lcl}
 \hat{i}' & \equiv & \cos(\omega t) \hat{i} - \sin(\omega t) \hat{j} \\
 \hat{j}' & \equiv & \sin(\omega t) \hat{i} + \cos(\omega t) \hat{j} \\
 \hat{k}' & \equiv & \hat{k}
 \end{array}
 \qquad
 \begin{array}{lcl}
 \hat{i} & \equiv & \cos(\omega t) \hat{i}' + \sin(\omega t) \hat{j}' \\
 \hat{j} & \equiv & -\sin(\omega t) \hat{i}' + \cos(\omega t) \hat{j}' \\
 \hat{k} & \equiv & \hat{k}'
 \end{array}$$

Note: Both coordinate frames share the same z-axis.

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix}$$

Bulk magnetization components in the rotating frame.

$$\vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix}$$

Applied B-field components in the rotating frame.

$$\begin{array}{l}
 B_{z'} \equiv B_z \\
 M_{z'} \equiv M_z
 \end{array}$$

Note: B-field and bulk magnetization z-components are equivalent in the two frames.

Equation of Motion

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

Equation of motion for an ensemble of spins (isochromats).
[Laboratory Frame]

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \left(\frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot} \right)$$

Equation of motion for an ensemble of spins (isochromats).
[Rotating Frame]

$$\vec{B}_{eff} \equiv \frac{\vec{\omega}_{rot}}{\gamma} + \vec{B}_{rot}$$

↑
Effective B-field that M experiences in the rotating frame.

↑
Fictitious field that demodulates the apparent effect of B_0 .

↑
Applied B-field in the rotating frame.

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Four Special Cases...

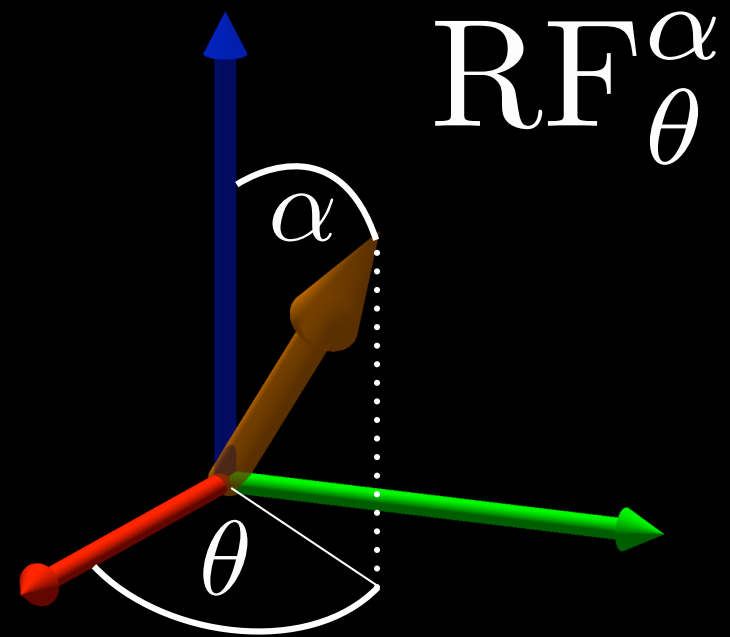
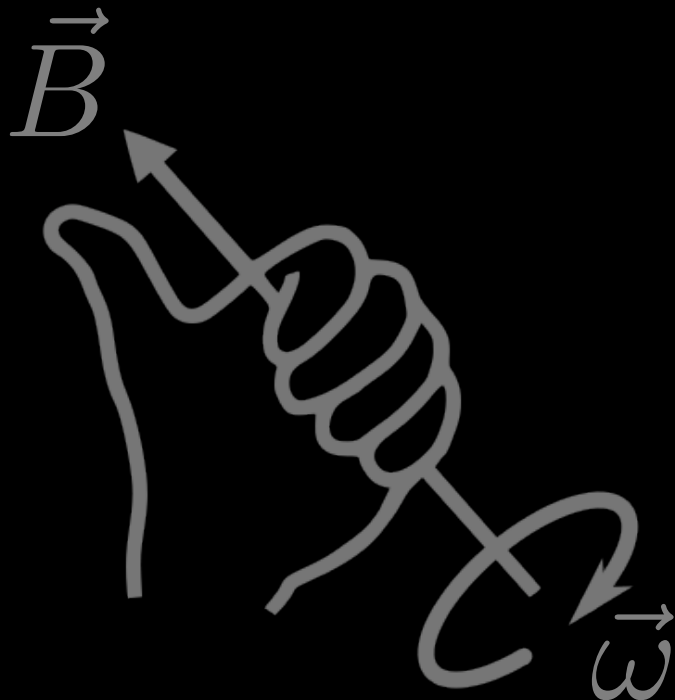
- Free Precession in the Laboratory Frame
- Forced Precession in the Laboratory Frame
 - Coordinate system anchored to scanner
- Free Precession in the Rotating Frame
- Forced Precession in the Rotating Frame
 - Coordinate system anchored to spin system
- **...all without relaxation.**
 - a) Relaxation time constants are “really” long
 - b) Time scale of event is \ll relaxation time constant

To The Board...

Mathematics of Hard RF Pulses

Parameters & Rules for RF Pulses

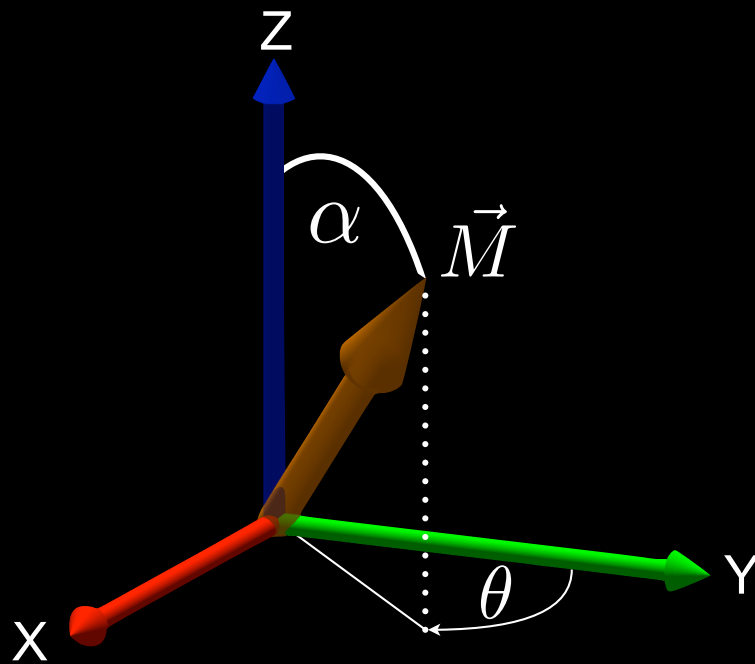
- RF pulses have a “flip angle” (α)
 - RF fields induce **left-hand** rotations
 - All B-fields do this for **positive** γ
- RF pulses have a “phase” (θ)
 - Phase of 0° is about the x-axis
 - Phase of 90° is about the y-axis



RF Flip Angle

Flip Angle

- “Amount of rotation of the bulk magnetization vector produced by an RF pulse, with respect to the direction of the static magnetic field.”
 - Liang & Lauterbur, p. 374

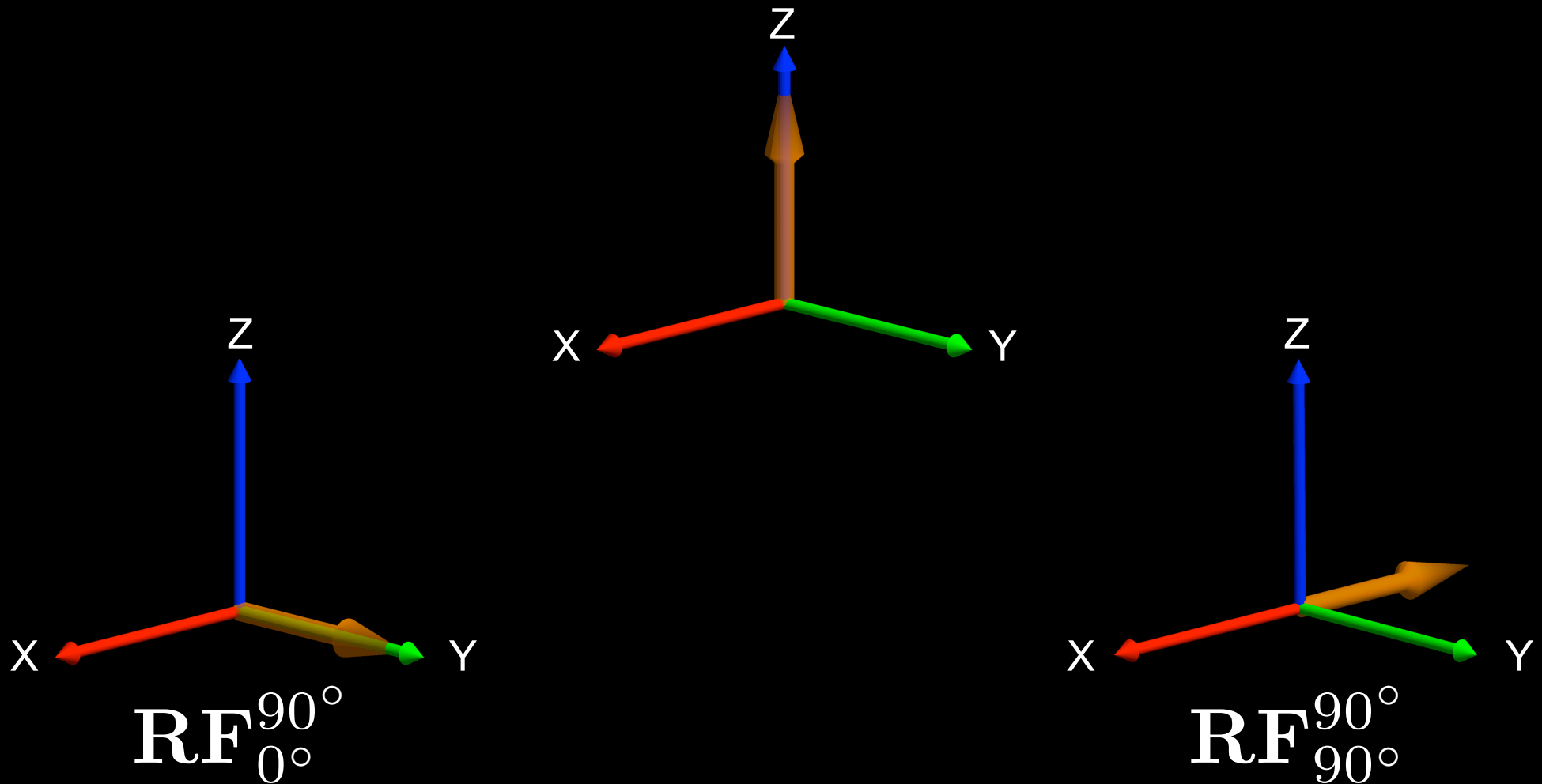


$$\omega_1 = \gamma B_1$$

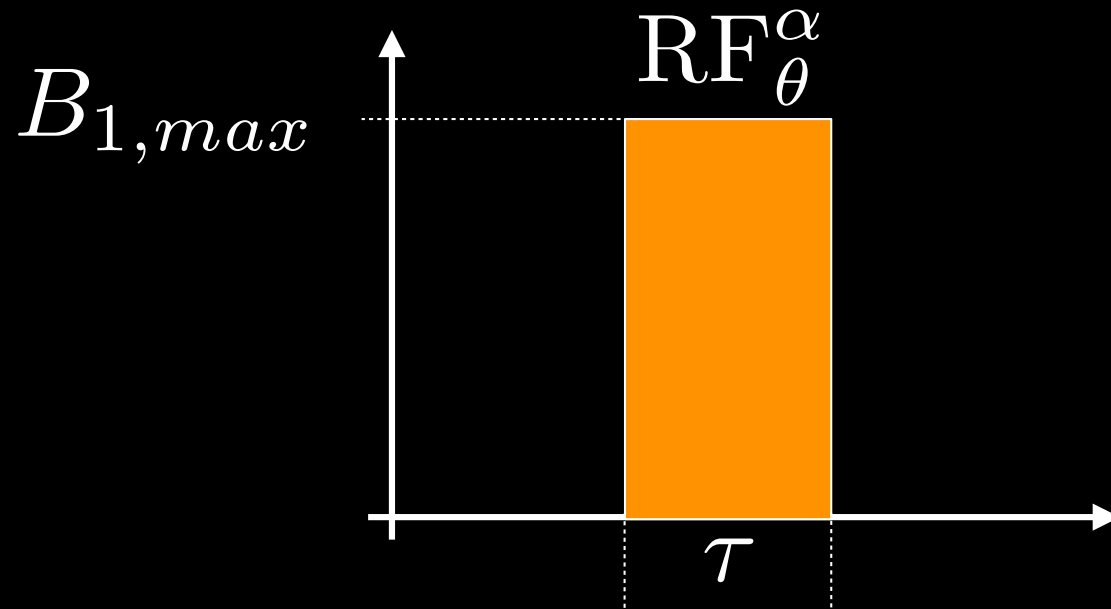
B-fields induce precession!

Rules for RF Pulses

$\mathbf{RF}^{\alpha} \rightarrow$ Flip Angle
 $\theta \rightarrow$ Phase



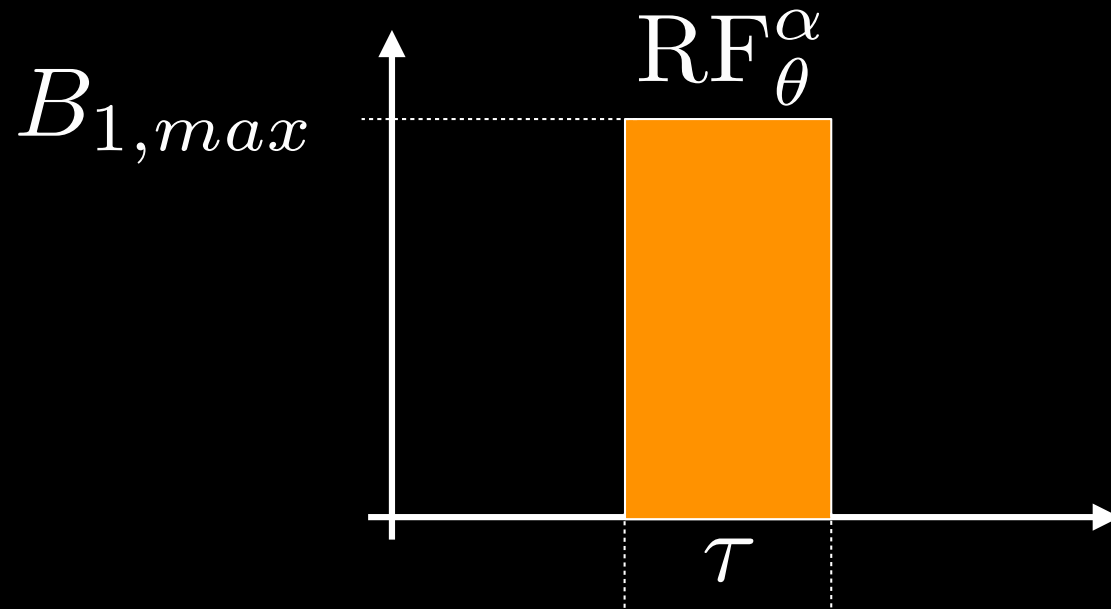
How to determine α ?



$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t) dt$$

- Rules:
- 1) Specify α
 - 2) Use $B_{1,max}$ if we can
 - 3) Shortest duration pulse

How to determine α ?

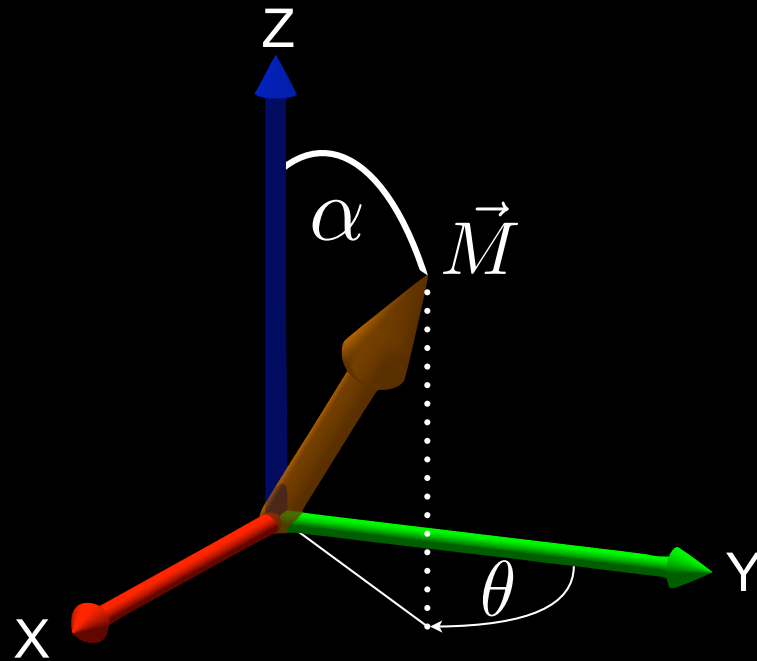


$$\alpha = \gamma \int_0^{\tau_p} B_1^e(t) dt$$

$$\tau = \frac{\alpha}{\gamma B_{1,max}} = \frac{\pi/2}{2\pi \cdot 42.57 \text{ Hz}/\mu\text{T} \cdot 60 \mu\text{T}} = 0.098 \text{ ms}$$

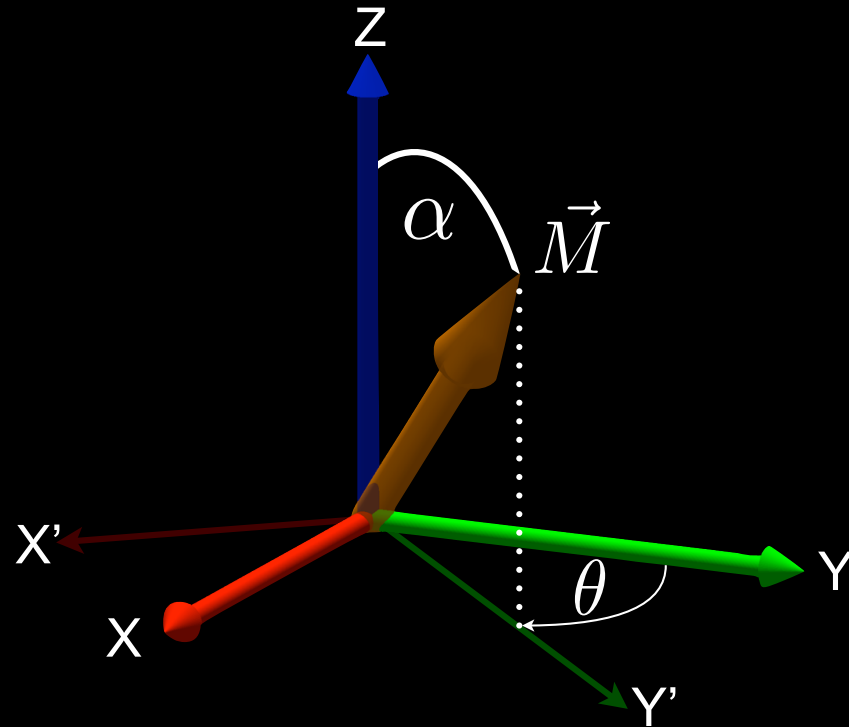
RF Phase

Bulk Magnetization in the Lab Frame



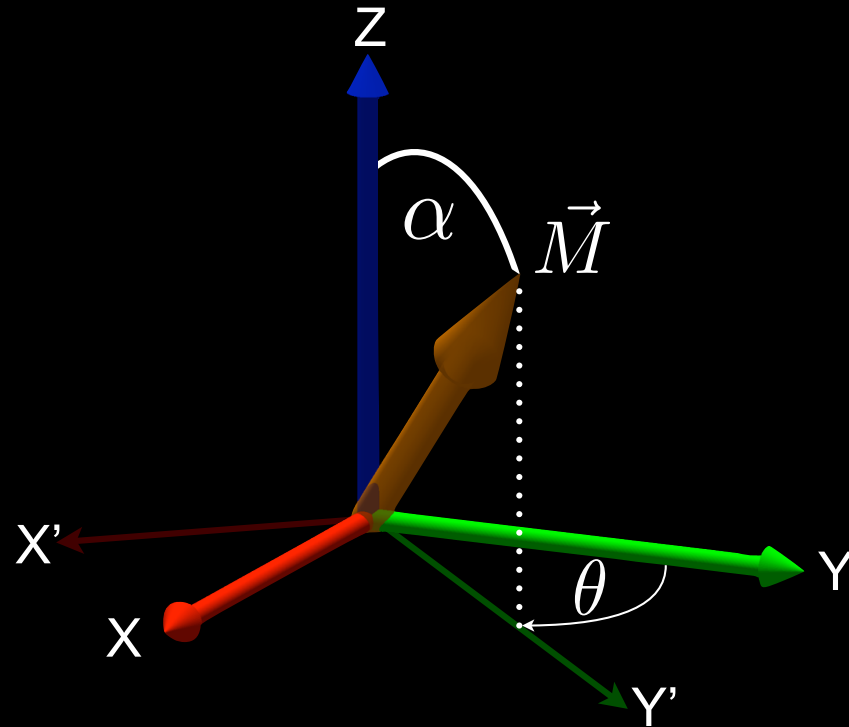
How do we mathematically account for α and θ ?

Change of Basis (θ)



$$\mathbf{R}_Z(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

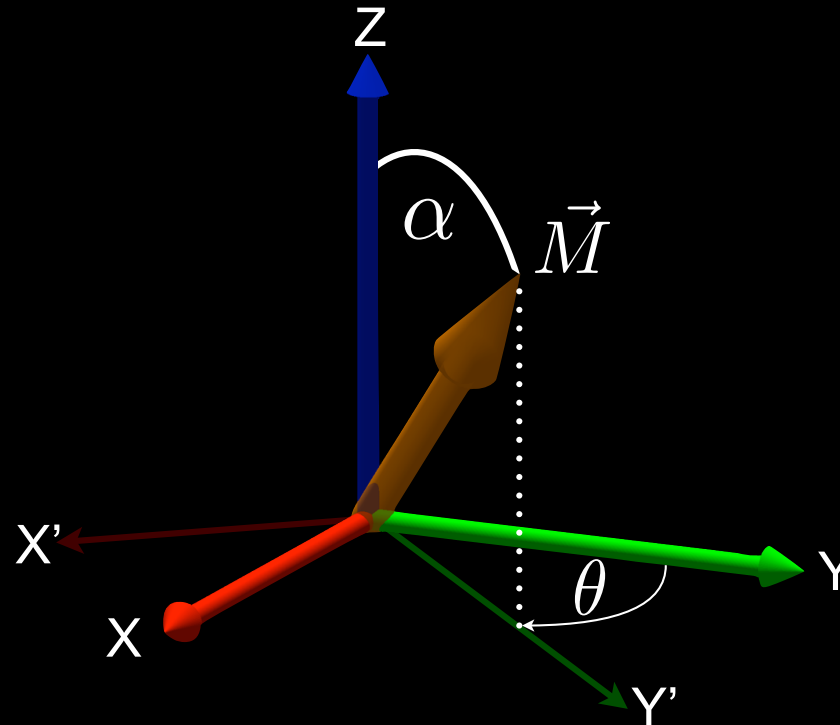
Rotation by Alpha



$$\mathbf{R}_{X'}(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

Rotate M by α about x' -axis.

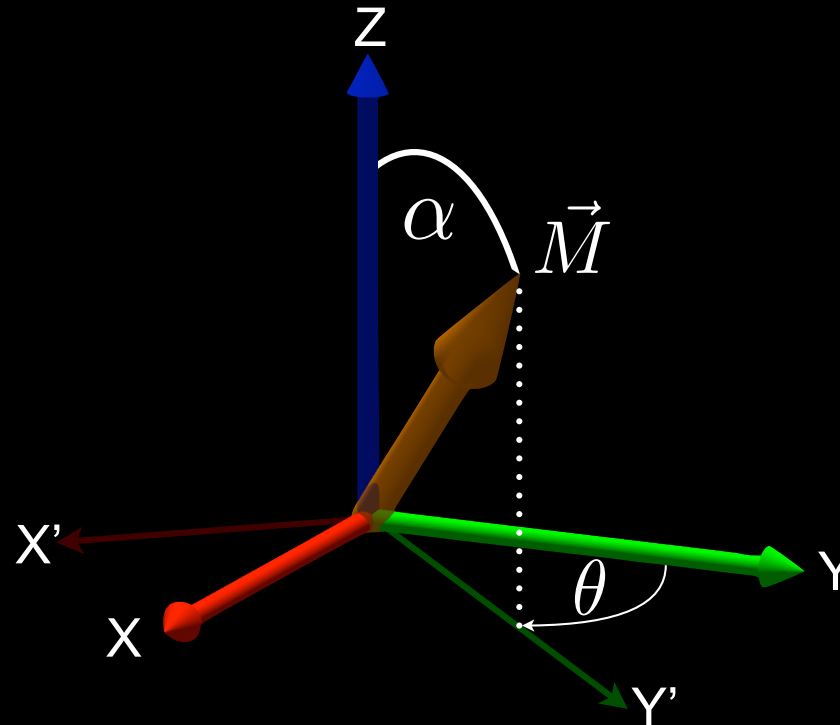
Change of Basis (- θ)



$$\mathbf{R}_Z(-\theta) = \begin{bmatrix} \cos(-\theta) & \sin(-\theta) & 0 \\ -\sin(-\theta) & \cos(-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate back to the lab frame's x-axis and y-axis.

RF Pulse Operator



$$\mathbf{R}_\theta^\alpha = \mathbf{R}_Z(-\theta) \mathbf{R}_X(\alpha) \mathbf{R}_Z(\theta)$$

$$= \begin{bmatrix} c^2\theta + s^2\theta c\alpha & c\theta s\theta - c\theta s\theta c\alpha & -s\theta s\alpha \\ c\theta s\theta - c\theta s\theta c\alpha & s^2\theta + c^2\theta c\alpha & c\theta s\alpha \\ s\theta s\alpha & -c\theta s\alpha & c\alpha \end{bmatrix}$$

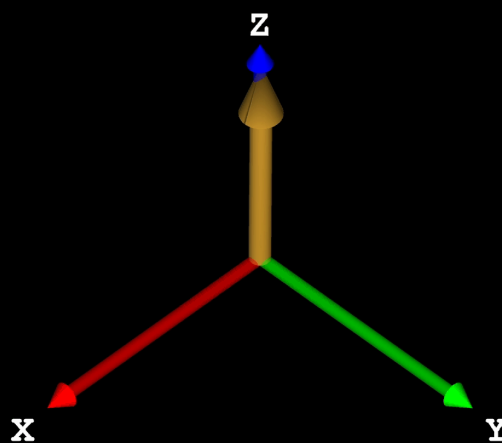
Types of RF Pulses

Types of RF Pulses

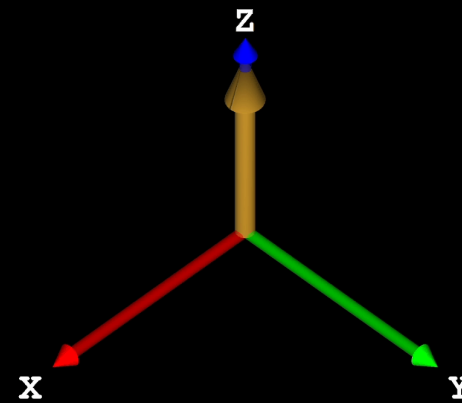
- **Excitation Pulses**
- **Inversion Pulses**
- **Refocusing Pulses**
- **Saturation Pulses**
- **Spectrally Selective Pulses**
- **Spectral-spatial Pulses**
- **Adiabatic Pulses**

Excitation Pulses

- Tip M_z into the transverse plane
- Typically $200\mu\text{s}$ to 5ms
- **Non-uniform across slice thickness**
 - Imperfect slice profile
- **Non-uniform within slice**
 - Termed B_1 inhomogeneity
 - Non-uniform signal intensity across FOV



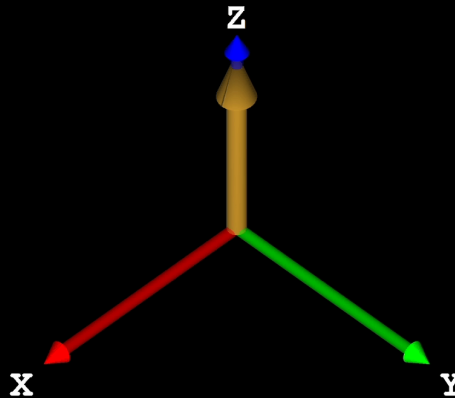
90° Excitation Pulse



Small Flip Angle Pulse

Inversion Pulses

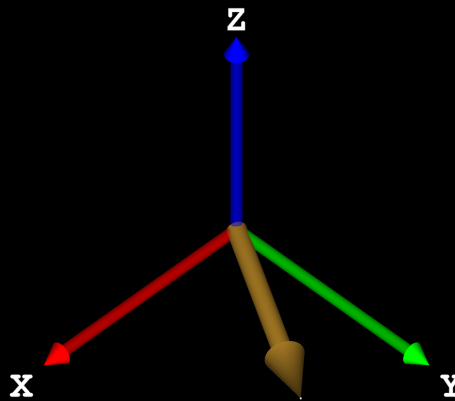
- **Typically, 180° RF Pulse**
 - non-180° that still results in $-M_z$
- **Invert M_z to $-M_z$**
 - Ideally produces no M_{xy}
- **Hard Pulse**
 - Constant RF amplitude
 - Typically non-selective
- **Soft (Amplitude Modulated) Pulse**
 - Frequency/spatially/spectrally selective
- **Typically followed by a crusher gradient**



180° Inversion Pulse

Refocusing Pulses

- **Typically, 180° RF Pulse**
 - Provides optimally refocused M_{XY}
 - Largest **spin echo** signal
- **Refocus spin dephasing due to**
 - imaging gradients
 - local magnetic field inhomogeneity
 - magnetic susceptibility variation
 - chemical shift
- **Typically followed by a crusher gradient**



180° Refocusing Pulse

Thanks



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