Motion for magnetic dipole ①

$$* \quad \frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

$$\vec{M} = \sum_{n=1}^{N_{total}} \vec{\mu}_n$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$ $\Rightarrow$ motion for bulk magnetization

$$\vec{B}_0 = B_0 \hat{k}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$\frac{dM_x}{dt} = M_y \cdot \gamma B_0$$

$$\frac{dM_y}{dt} = -M_x \gamma B_0$$

$$\frac{dM_z}{dt} = \emptyset$$

$$M_x(t) = M_x^o \cos(\gamma B_o t) + M_y^o \sin(\gamma B_o t)$$
$$M_y(t) = -M_x^o \sin(\gamma B_o t) + M_y^o \cos(\gamma B_o t)$$
$$M_z(t) = M_z^o$$

where $\vec{M}^o = \begin{bmatrix} M_x^o \\ M_y^o \\ M_z^o \end{bmatrix}$

or

$$\begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} = \begin{bmatrix} \cos(\gamma B_o t) & \sin(\gamma B_o t) & 0 \\ -\sin(\gamma B_o t) & \cos(\gamma B_o t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_x^o \\ M_y^o \\ M_z^o \end{bmatrix}$$

$$\vec{M} = R_z(\gamma B_o t) \cdot \vec{M}_o$$

$$\left[\begin{array}{l} \omega_o = \gamma B_o \\ \vec{\omega} = \gamma \vec{B} = \gamma B_o \hat{k} \end{array} \right.$$

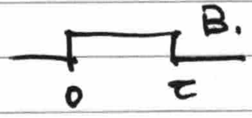
$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \Longrightarrow \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_1(t) = B_1^e(t) \left[\cos(\omega_{RF} t + \theta)\hat{i} - \sin(\omega_{RF} t + \theta)\hat{j}\right]$$

$$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{\omega}_{rot}}{\gamma}$$

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega_{RF} \end{pmatrix}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1^e \cos\theta \\ B_1^e \sin\theta \\ B_o - \frac{\omega_{RF}}{\gamma} \end{pmatrix}$$



$$B_1(t) = B_1, \quad 0 \le t \le \tau$$

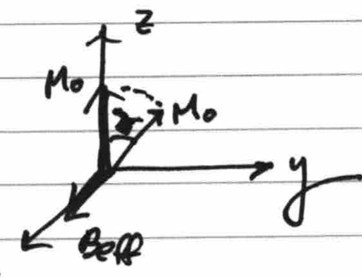
$$\vec{B}_{eff} = \begin{pmatrix} B_1 \\ 0 \\ 0 \end{pmatrix} \quad \text{at on-resonance}$$

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\Rightarrow \vec{M}_{rot}(t) = R_x(\gamma B_1 t) \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ M_0 \sin(\gamma B_1 t) \\ M_0 \cos(\gamma B_1 t) \end{bmatrix}$$

Graphically,



flip angle $\alpha = \gamma B_1 \tau$

* Numbers

$\alpha = 90° = \frac{\pi}{2}$, $\tau = 1\,ms$

$$\frac{\pi}{2} = \gamma B_1 \, 1\,ms \Rightarrow B_1 \simeq 0.06\,G = 6\,\mu T$$