* Magnetic fields
  - B_0 large static field
  - B_1 radio frequency field
  - $G_{x,y,z}$ gradient fields

$\boxed{B_0}$ without B_0 → spin oriented randomly

$$\sum \vec{\mu} = 0 = \vec{M}$$

↑

"net or bulk" magnetization

With B_0 → 2 things happen

a) Polarization

$$\sum \vec{\mu} \neq 0 = \vec{M}$$ alignment of spins

$$= M_z \hat{k}$$

along $\hat{z}$ direction, "longitudinal"

In the presence of B_0, 2 energy status

n_+, parallel

n_-, antiparallel

* Boltzmann distribution $\frac{n_-}{n_+} = e^{-\Delta E/kT}$

b) Resonance

at equilibrium, $\vec{M} \parallel \vec{B}$

If $\vec{M} \nparallel \vec{B}$, precession will occur.

From classical mechanics,

torque experienced by $\vec{\mu}$

$$= \vec{\mu} \times \vec{B} = \frac{d}{dt}\left(\begin{matrix}\text{angular}\\ \text{momentum}\end{matrix}\right) = \frac{d}{dt}(\hbar \vec{I})$$

$$\vec{\mu} = \gamma \hbar \vec{I}$$

$$\Rightarrow \frac{d\vec{\mu}}{dt} = \vec{\mu} \times \gamma \vec{B}$$

$\downarrow$ collection of spins

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad , \quad \vec{B} = \vec{B_0} = B_0 \hat{k}$$

$\Rightarrow \vec{M}$ will precess about the axis of $\vec{B}$ at an angular frequency of $\gamma |B|$.

* frequency of precession

$$f = \frac{\gamma}{2\pi} B \quad \text{Hz} \quad ; \text{ Larmor frequency}$$

For ^1H, $\frac{\gamma}{2\pi} = \gamma^{+} = 42.575 \ \frac{MHz}{T}$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}$$

let $\vec{B} = B_0 \cdot \hat{k}$

$$\frac{d\vec{M}}{dt} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ M_x & M_y & M_z \\ 0 & 0 & \gamma B_0 \end{vmatrix}$$

$$\frac{dM_x}{dt} = M_y \cdot \gamma B_0 \quad , \quad \frac{M_y}{dt} = -M_x \cdot \gamma B_0 \quad , \quad \frac{dM_z}{dt} = 0$$

$$\frac{dM_x^2}{dt^2} = \frac{dM_y}{dt} \cdot \gamma B_0 \quad , \quad \frac{dM_y^2}{dt^2} = -\frac{dM_x}{dt} \cdot \gamma B_0$$

$$= -(\gamma B_0)^2 \cdot M_x \qquad\qquad = -(\gamma B_0)^2 M_y$$

Assume, $M_x(t) = A\cos(\gamma B_0 t) + B\sin(\gamma B_0 t)$

$$\vec{M}^0 = \begin{bmatrix} M_x^0 \\ M_y^0 \\ M_z^0 \end{bmatrix}$$

$$M_x(t=0) = A = M_x^0$$

$$\frac{dM_x}{dt} = -A\gamma B_0 \sin(\gamma B_0 t) + B\gamma B_0 \cos(\gamma B_0 t)$$

$$= M_y \cdot \gamma B_0$$

$$M_y(t=0) = B = M_y^0$$

Finally,

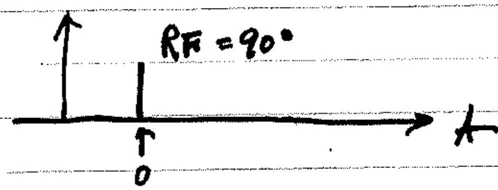
$$M_x(t) = M_x^0 \cos(\gamma B_0 t) + M_y^0 \sin(\gamma B_0 t)$$
$$M_y(t) = -M_x^0 \sin(\gamma B_0 t) + M_y^0 \cos(\gamma B_0 t)$$
$$M_z(t) = M_z^0$$

or,

$$\begin{bmatrix} M_x(t) \\ M_y(t) \\ M_z(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\gamma B_0 t) & \sin(\gamma B_0 t) & 0 \\ -\sin(\gamma B_0 t) & \cos(\gamma B_0 t) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_z(\gamma B_0 t)} \begin{bmatrix} M_x^0 \\ M_y^0 \\ M_z^0 \end{bmatrix}$$

$$\omega = \gamma B_0$$
$$\vec{\omega} = \gamma \vec{B_0} = \gamma B_0 \hat{k}$$

Ex1>



$$\vec{M}(0_-) = \begin{bmatrix} 0 \\ 0 \\ M_z^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ M_0 \end{bmatrix}$$

$$\vec{M}(0_+) = \begin{bmatrix} M_0 \\ 0 \\ 0 \end{bmatrix}$$ ← Immediately after a 90° RF pulse

$$\vec{M}(t) = R_z(\gamma B_0 t) \begin{bmatrix} M_0 \\ 0 \\ 0 \end{bmatrix}$$ ← $\vec{M}(t)$ after the 90° RF pulse

$$M_x(t) = M_0 \cos(\gamma B_0 t)$$
$$M_y(t) = -M_0 \sin(\gamma B_0 t)$$
← free precession

$$M_z(t) = 0$$