

* TBW = 4 , pulse duration = 1 ms

⇒ BW = 4 kHz

f = - (f/2π) Gz · z
4.257 kHz/G

to make Δz = 1 cm

Gz = Δf / Δz · 1 / (f/2π) = 4 / 1 · 1 / 4.257

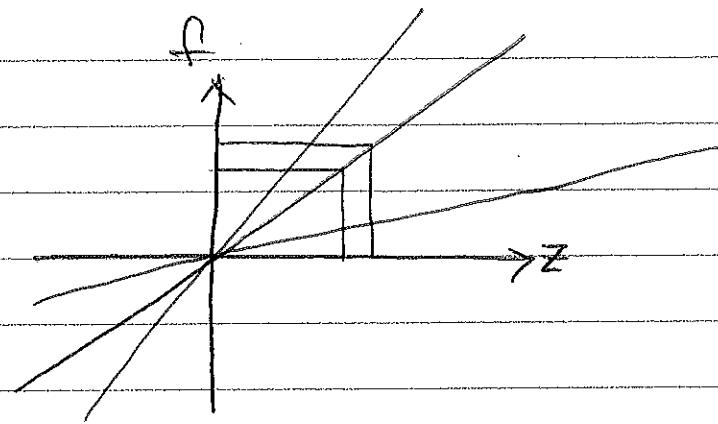
* TBW = 16 , pulse duration = 1 ms

⇒ BW = 16 kHz

To make Δz = 1 cm

Gz = 16 / 1 · 1 / 4.257

* Δf = - (f/2π) Gz · Δz



* Full analytical solution

⑤

$$\frac{d\vec{M}}{dt} = \begin{bmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{bmatrix} \vec{M}$$

$$M_r = M_x + i M_y \quad \leftarrow \text{phasor representation}$$

$$\frac{dM_r}{dt} = -i \omega(z) M_r + i \omega_1(t) M_z$$

$$\frac{dM_z}{dt} = -i \omega_1(t) M_y$$

Solution \Rightarrow very difficult \Rightarrow using approximation

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1$$

$$M_z \approx M_0 \approx \text{constant}$$

$$\Rightarrow \frac{dM_z}{dt} = 0$$

$$\frac{dM_r}{dt} = -i \omega(z) M_r + i \omega_1(t) M_0$$

$$\frac{dy}{dx} + p(x)y = q(x)$$

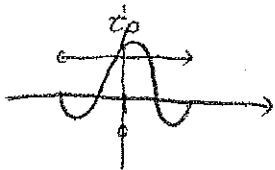
$$y = \frac{\int u(x) q(x) dx}{u(x)}, \quad \text{where } u(x) = \exp\left\{\int p(x) dx\right\}$$

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$$\frac{dM_r}{dz} + \gamma W(z) M_r = \gamma W_1(z) M_0$$

$$M_r(t, z) = \gamma \cdot M_0 e^{-\gamma W(z) \cdot t} \int_0^t W_1(\tau) e^{-\gamma W(z) \tau} d\tau$$

Assume, the RF pulse is symmetric and peaks at $t=0$ such that the pulse ends at $t = \tau_p/2$, τ_p is the length of the RF pulse \rightarrow let $\tau' = t - \tau_p/2$



$$M_r(\tau_p) = \gamma M_0 e^{-\frac{\gamma W \tau_p}{2}} \int_{-\tau_p/2}^{\tau_p/2} W_1(t + \frac{\tau_p}{2}) e^{-\gamma W t} dt$$

assuming $W_1(t + \frac{\tau_p}{2}) = \begin{cases} 0 & \text{for } |t| > \frac{\tau_p}{2} \\ W_1(t + \frac{\tau_p}{2}) & \text{for } |t| < \frac{\tau_p}{2} \end{cases}$

$$\Rightarrow M_r(\tau, z) = \gamma M_0 e^{-\gamma W(z) \tau/2} \int_{-ID}^{ID} \left\{ W_1(t + \frac{\tau}{2}) \right\} dt$$

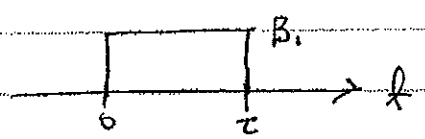
$f = -f(z)$
 $= -(\sigma/\pi) \sqrt{z \cdot z}$

* $|M_{xy}(\tau, z)| = M_0 \mathcal{F}_{1D} \left\{ w_1 \left(t + \frac{\tau}{2} \right) \right\} \Big|_{f = -\frac{\sigma}{2a} G_2 z}$

* Small tip-angle example

- consider a rectangular RF pulse (duration of τ)

$$B_1(t) = B_1 \cdot \Pi\left(\frac{t - \tau/2}{\tau}\right)$$



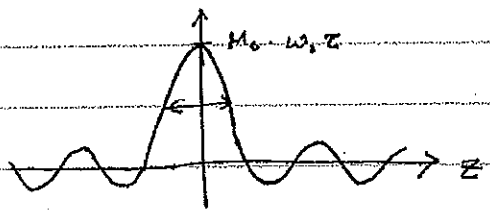
$$M_r(\tau, z) = \lambda M_0 e^{-iW(z)\tau/2} \mathcal{F}_{1D} \left\{ w_1 \left(t + \frac{\tau}{2} \right) \right\} \Big|_{f = \frac{\gamma}{2\pi} Gz \cdot z}$$

$$w_1 \left(t + \frac{\tau}{2} \right) = \underbrace{\gamma \cdot B_1}_{w_1} \cdot \Pi\left(\frac{t}{\tau}\right)$$

$$\mathcal{F}_{1D} \left\{ \Pi\left(\frac{t}{\tau}\right) \right\} = \tau \text{sinc}(\tau \cdot f)$$

$$\Rightarrow M_r(\tau, z) = \lambda M_0 e^{-iW(z)\tau/2} \cdot \underbrace{w_1 \tau}_{\text{canceled out by the refocusing pulse}} \text{sinc}\left(\tau \cdot \frac{\gamma}{2\pi} Gz \cdot z\right)$$

canceled out by the refocusing pulse



$$\Delta z = \frac{1}{\frac{\tau}{2\pi} \gamma \cdot Gz}$$