Compressed Sensing & Artificial Intelligence

M229 Advanced Topics in MRI

Kyung Sung, Ph.D.

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Class Business

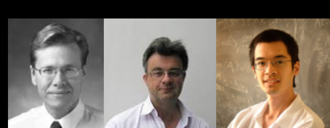
- Final project abstract due on 6/7 Friday
- Final project presentation on 6/13 (9-3pm)
- Guest Lecturers:
  - Dr. Debiao Li (6/4)
  - Dr. Xiaodong Zhong (6/6)

Today's Topics

- Compressed Sensing
  - Compressibility or Sparsity
  - Incoherent Measurement
  - Reconstruction
- CS-MRI Examples
-

Fast MRI Techniques

- Many reconstruction methods minimize aliasing artifacts by exploiting *__information redundancy__* (or *__prior knowledge__*)
  - Parallel imaging
  - Compressed sensing



Donoho, IEEE TIT, 2006
Candes et al., Inverse Problems, 2007

What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

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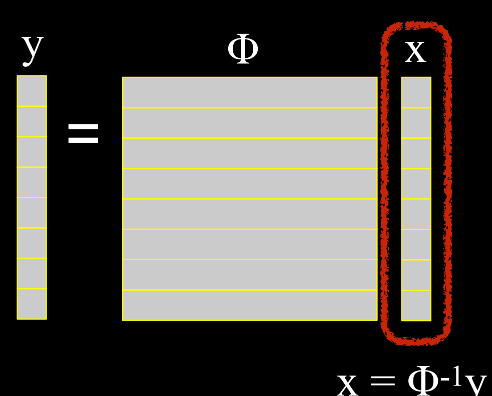
8 Equations
8 Unknowns

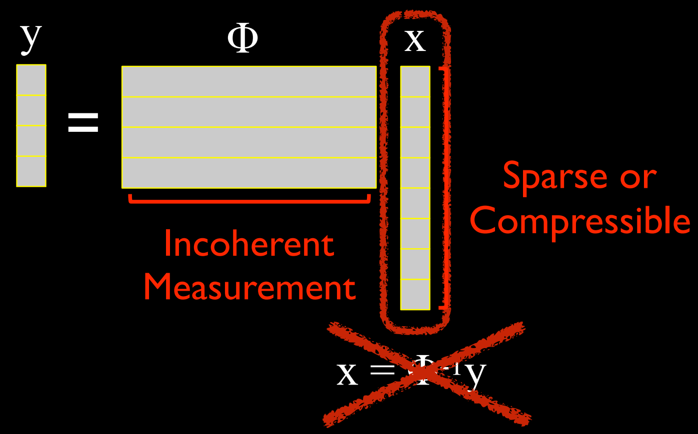


$x = \Phi^{-1} y$

What is Compressed Sensing?

- CS is about acquiring a sparse signal in a most efficient way (subsampling) with the help of an incoherent projecting basis

4 Equations
8 Unknowns



What is Compressed Sensing?

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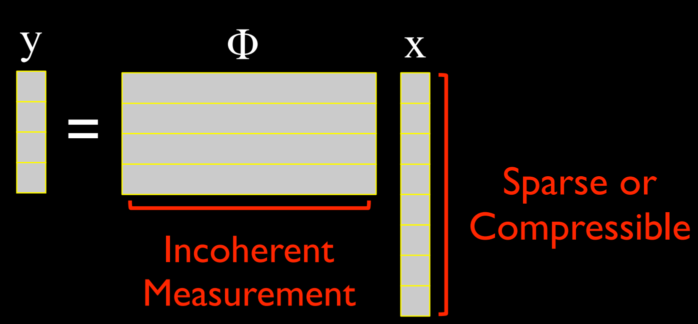
4 Equations
8 Unknowns



We still can find 8 unknowns!

Math Background

L0-norm ($|x|_0$): a number of non-zero coefficients

L1-norm ($|x|_1$): a sum of absolute values of coefficients

L2-norm ($|x|_2$): a sum of squared values of coefficients

$$x = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \quad x = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 3 \end{bmatrix}$$

Simple Example

$$\underbrace{\begin{bmatrix} 0 \\ 1 \\ 1 \\ 4 \end{bmatrix}}_{y} = \underbrace{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}}_{\Phi} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}}_{x}$$

Simple Example

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}} \begin{matrix} \\ 0 \\ 0 \\ \\ \\ \\ \end{matrix}$$

$x_2 + x_3 = 0$

Simple Example

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} x_1 \\ 0 \\ 0 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix}} \begin{matrix} 0 \\ \\ \\ 0 \\ 1 \\ \\ \end{matrix}$$

$x_4 + x_5 = 1$

$x_1 + x_2 + x_5 = 1$ ($x_2 \to 0$)

Simple Example

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ x_6 \end{pmatrix}} \quad 3$$

$$\overset{1}{x_5} + x_6 = 4$$

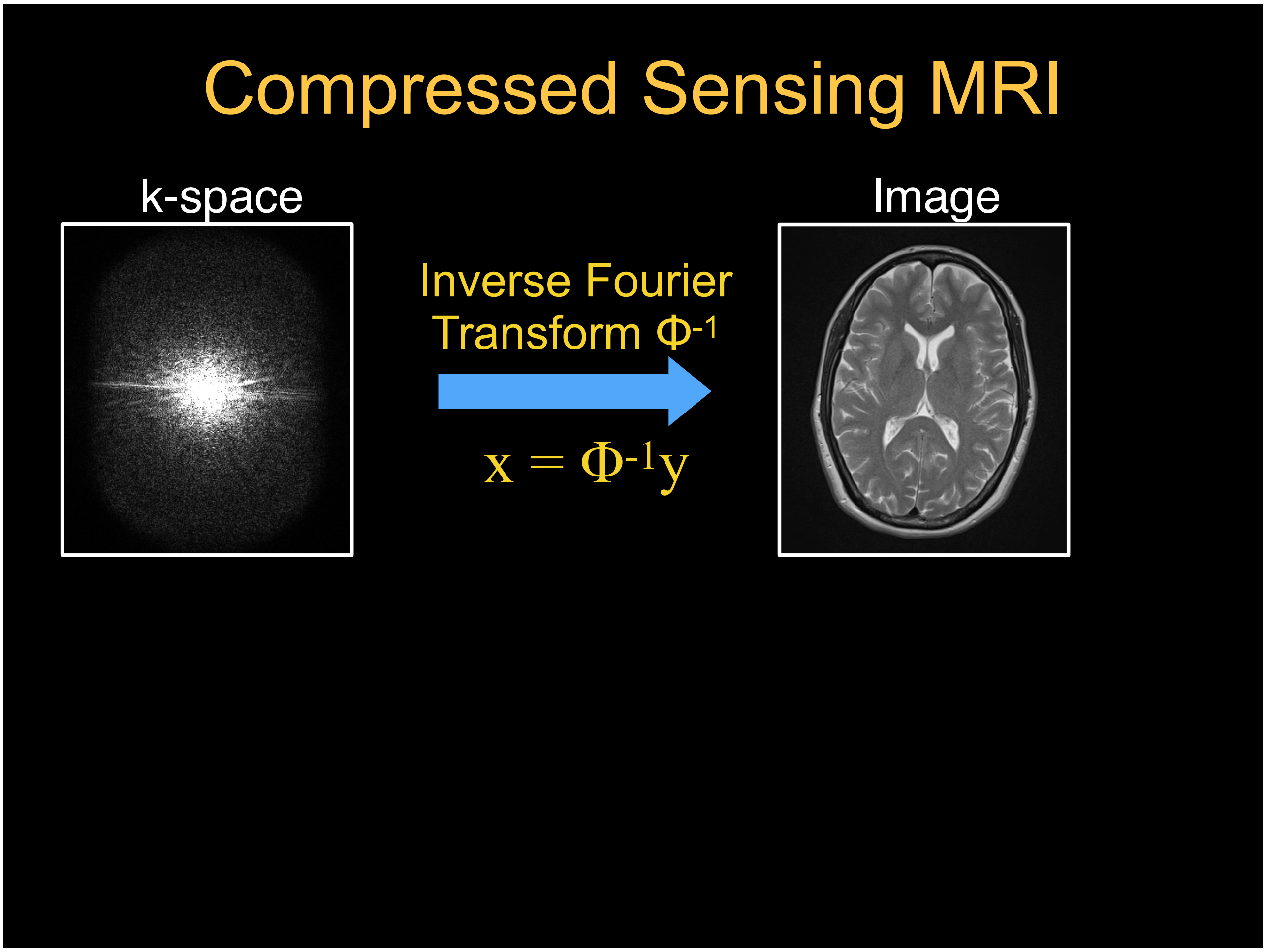
Simple Example

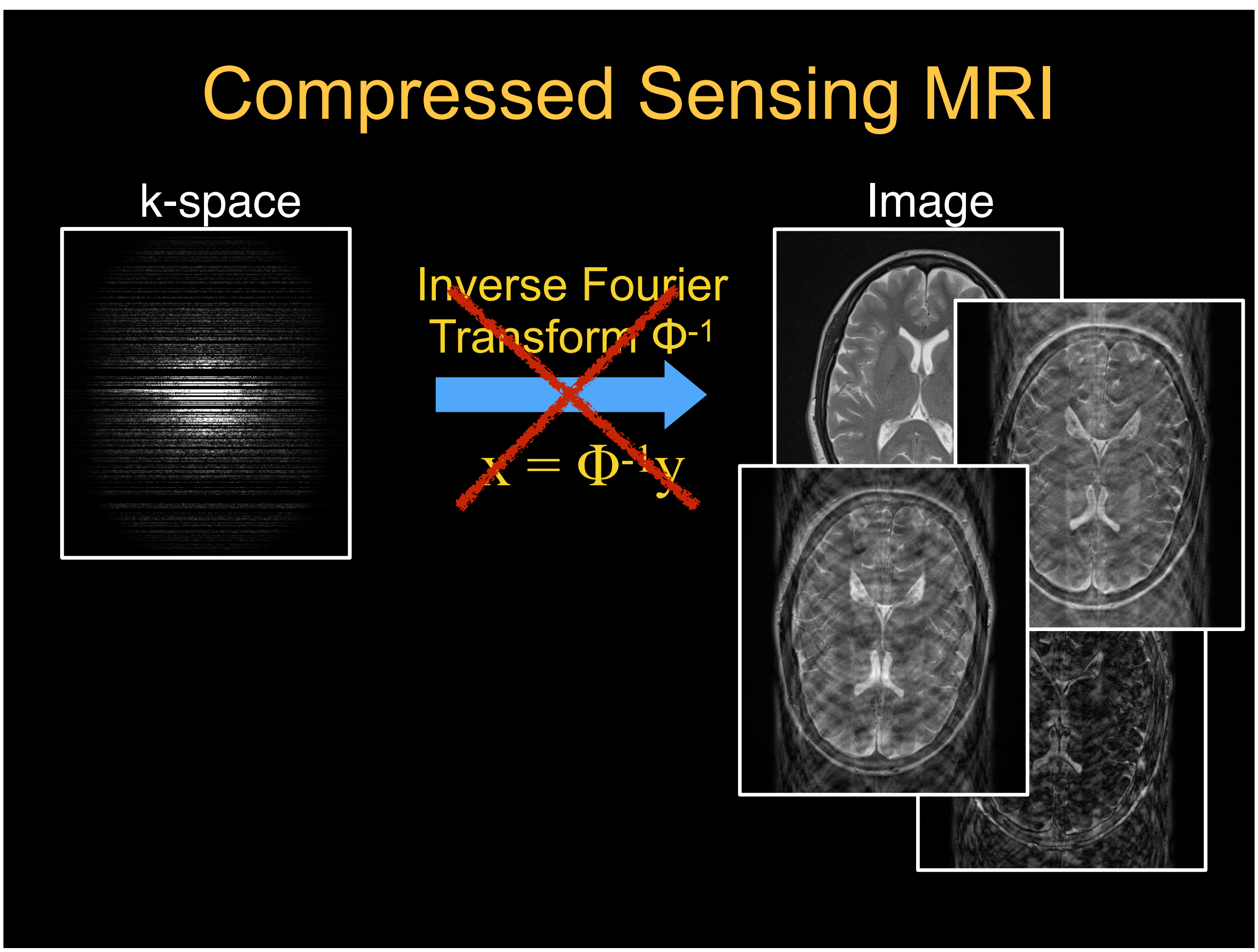
2) This should be "smart"

1) This should be "sparse"

$$\underset{y}{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix}} = \underset{\Phi}{\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}} \underset{x}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 3 \end{pmatrix}}$$

3) Reconstruction should be "feasible"


Compressed Sensing MRI
k-space
Image
Inverse Fourier
Transform Φ-1
x = Φ-1y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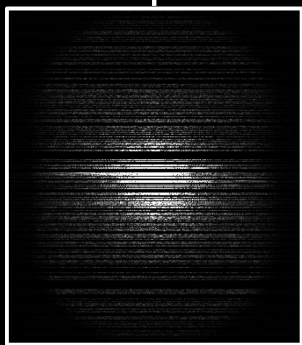
Compressed Sensing MRI

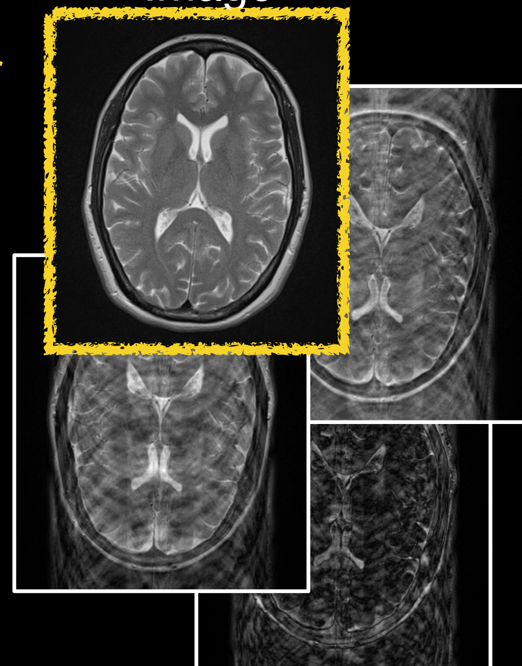
k-space



Inverse Fourier Transform Φ^{-1}

$x = \Phi^{-1}y$

Image



Choose the most compressible image matching data (*systematic optimization*)

Systematic Optimization

- Assuming *sparsity* and *incoherence* are provided, an image can be recovered with highly undersampled data by:

minimize $|\Psi x|_1$, subject to $y = \Phi x$

Sparse Transform (e.g., Wavelet Transform)

Randomly Undersampled Fourier Transform

Systematic Optimization

- Assuming *sparsity* and *incoherence* are provided, an image can be recovered with highly undersampled data by:

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Sparse Transform (e.g., Wavelet Transform)

Randomly Undersampled Fourier Transform

- We can relax the minimization by using regularization,

minimize $F(x): |y - \Phi x|_2^2 + \lambda |\Psi x|_1$

Regularization Parameter

Three Tenets of CS

minimize $F(x): |y - \Phi x|_2^2 + R(x)$

Data Consistency

Compressibility Constraint

- Three key elements of Compressed Sensing:

Compressibility

Incoherence

Nonlinear Reconstruction

Compressibility Constraint

minimize $F(x): |y - \Phi x|^2_2 + \underline{R(x)}$

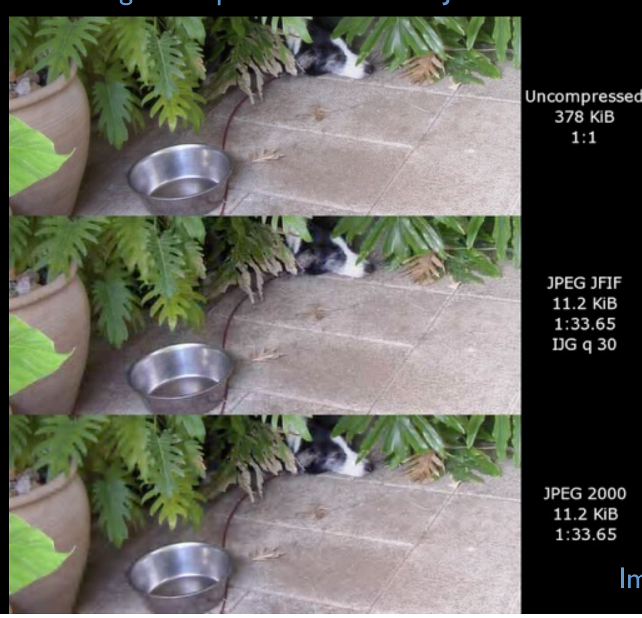
Compressibility Constraint

- $R(x) = \lambda|x|_1$ (*Identity Transform*)
- $R(x) = \lambda|\Psi x|_1$ (*Wavelet Transform*)
- $R(x) = \lambda H(x)$ (*Total Variation*)
- $R(x) = \lambda|x|_*$ (*Rank or Nuclear Norm*)
- Many more…

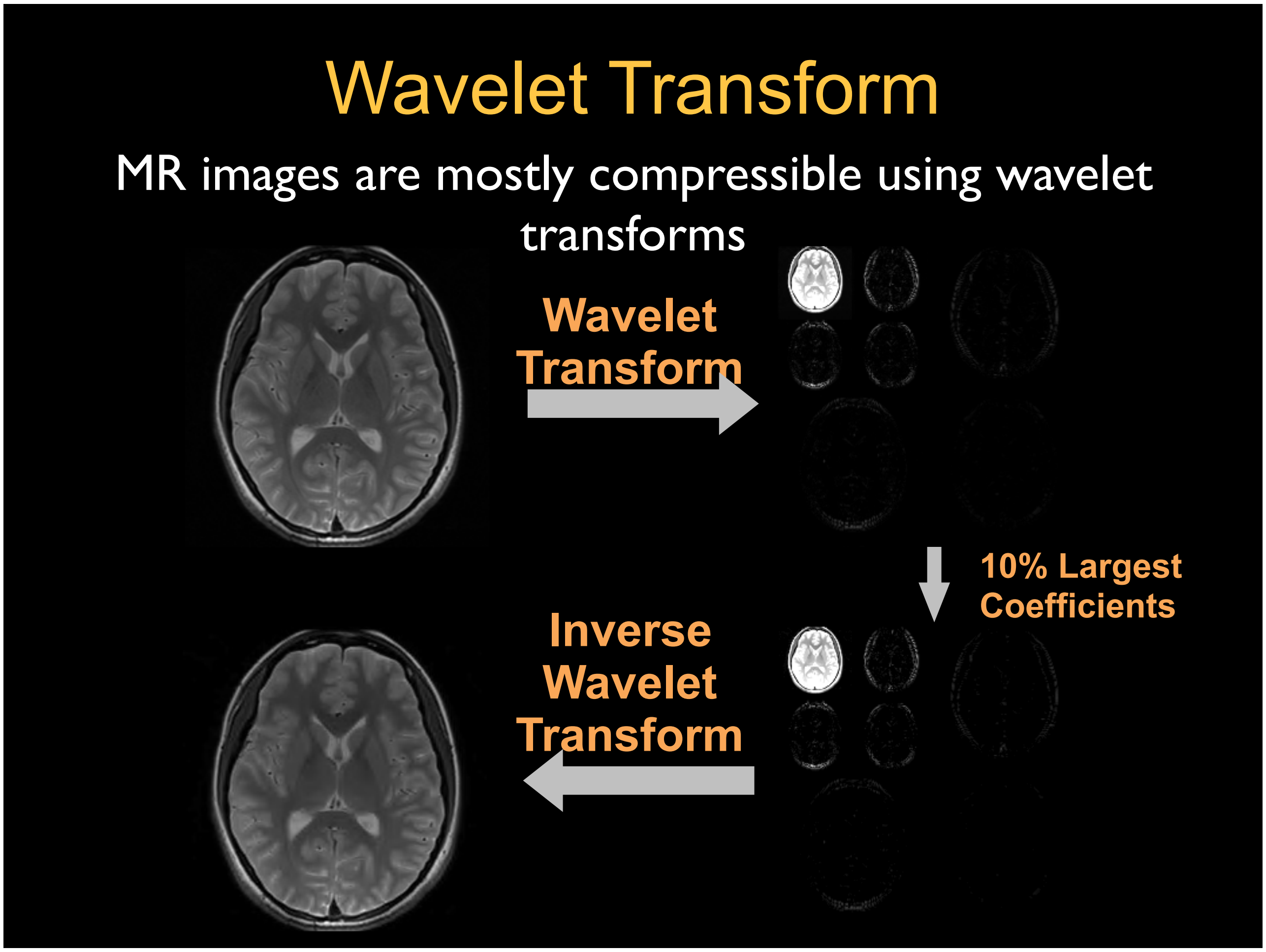
Wavelet Transform

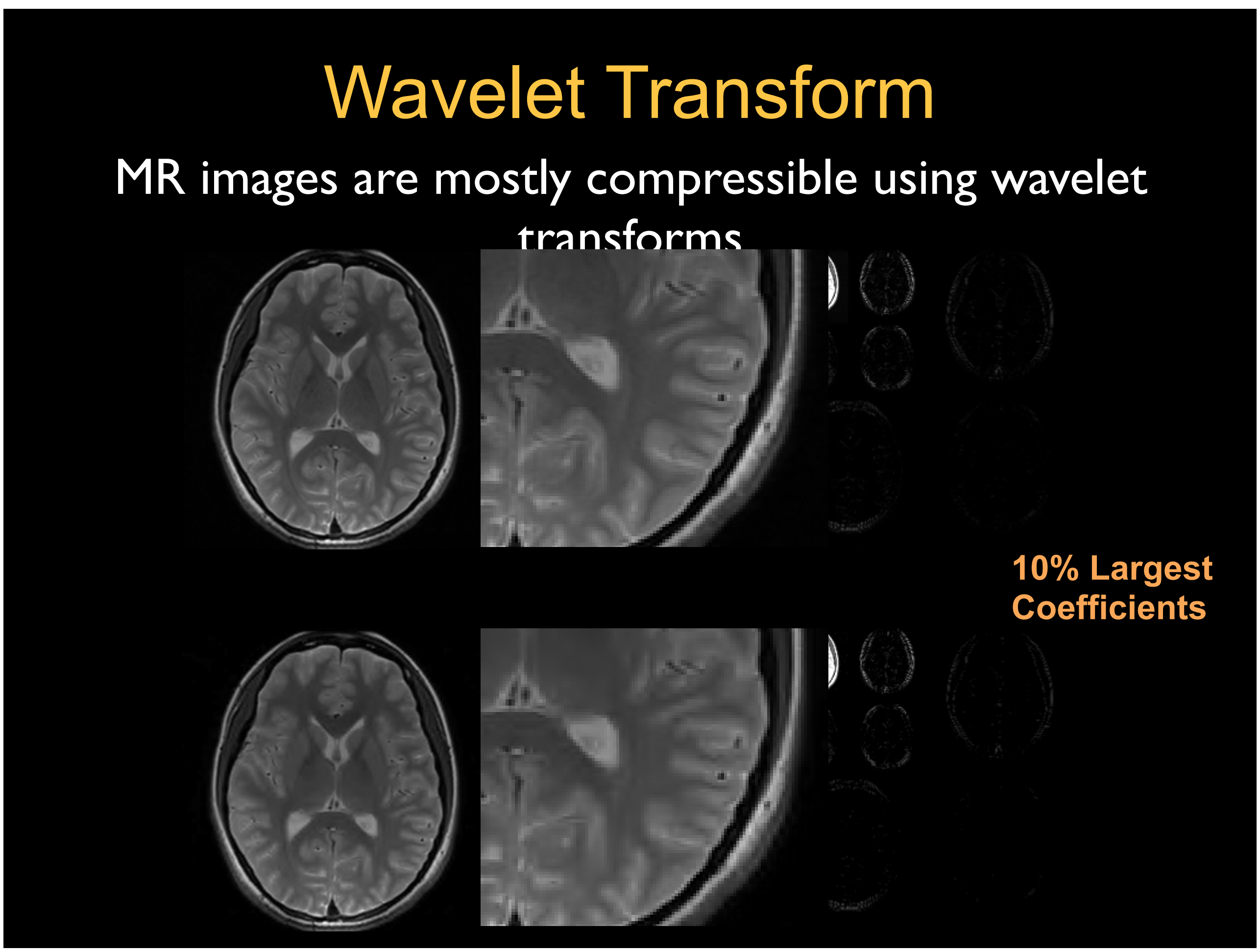
- Natural images are compressible using wavelet transforms

Image Compression Standard: JPEG2000



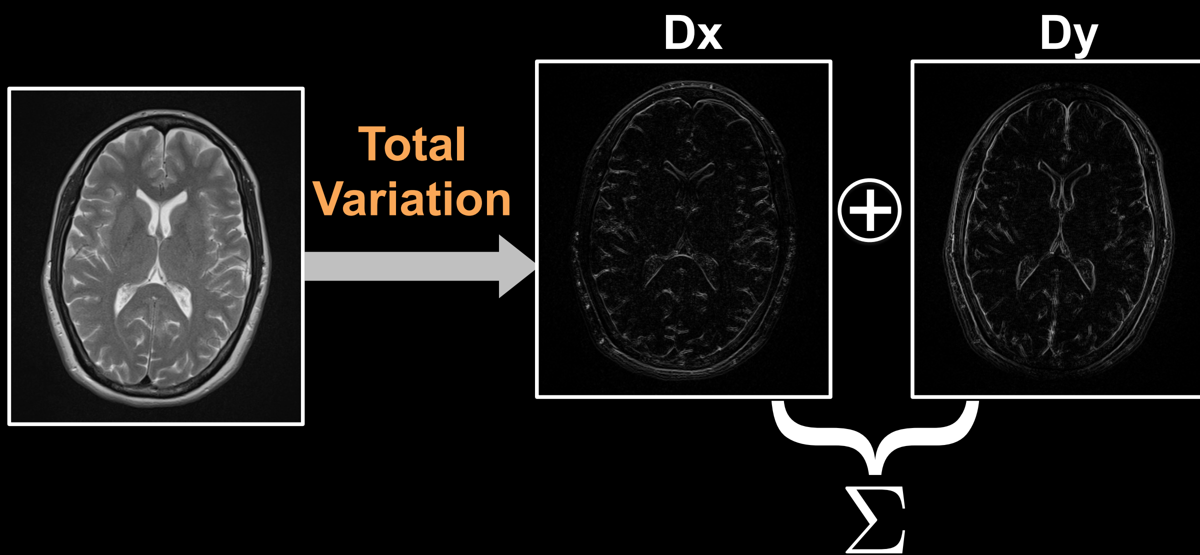
Images from Wikipedia


Wavelet Transform
MR images are mostly compressible using wavelet transforms
Wavelet Transform
10% Largest Coefficients
Inverse Wavelet Transform


Wavelet Transform
MR images are mostly compressible using wavelet transforms
10% Largest Coefficients

Total Variation

$$H(x) = \sum_{i,j} \sqrt{\underbrace{|x_{i+1,j} - x_{i,j}|^2}_{Dx} + \underbrace{|x_{i,j+1} - x_{i,j}|^2}_{Dy}}$$



Total Variation

Limitations / Considerations

- Define reconstruction domain and exploit information redundancy (or prior knowledge)
  - More apparent when MRI is repeated on a same object (e.g., repeating with different time points, flip angles, TEs, etc)
- Be aware of underlying assumptions of each constraint
  - Wavelet / TV denoising
- Consistent compressibility is desirable to easily anticipate reconstruction quality

Limitations / Considerations

- High vs. low computational complexities
  - Wavelet transform
  - Total Variation
  - Nuclear norm
- Multiple compressibility constraints vs. single constraint
  - Reconstruction quality
  - Reconstruction stability

CS Reconstruction

- Assuming *sparsity* and *incoherence* are provided, an image can be recovered with highly undersampled data by:

$$\text{minimize } |\Psi x|_1, \text{ subject to } \quad y = \Phi x$$

Sparse Transform (e.g., Wavelet Transform) — Randomly Undersampled Fourier Transform

- We can relax the minimization by using regularization,

$$\text{minimize } F(x): |y - \Phi x|_2^2 + \lambda |\Psi x|_1$$

Regularization Parameter

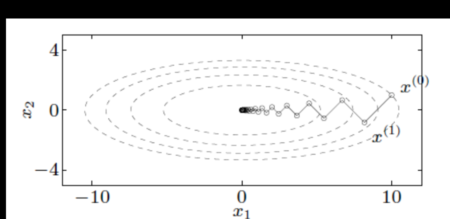
- When λ carefully chosen, unconstrained minimization becomes identical to original minimization

Solving L1 Minimization

- How can we solve this?

$$\text{Minimize}\{ f(x) = |y - \Phi x|_2^2 + \lambda |\Psi x|_1 \}$$

- Review of convex optimization:



General descent method.

given a starting point $x \in \mathbf{dom}\, f$.
repeat
1. Determine a descent direction Δx.
2. *Line search.* Choose a step size $t > 0$.
3. *Update.* $x := x + t\Delta x$.

until stopping criterion is satisfied.

– A choice for search direction (Δx) can be different (e.g. gradient decent method, Newton's method, etc)

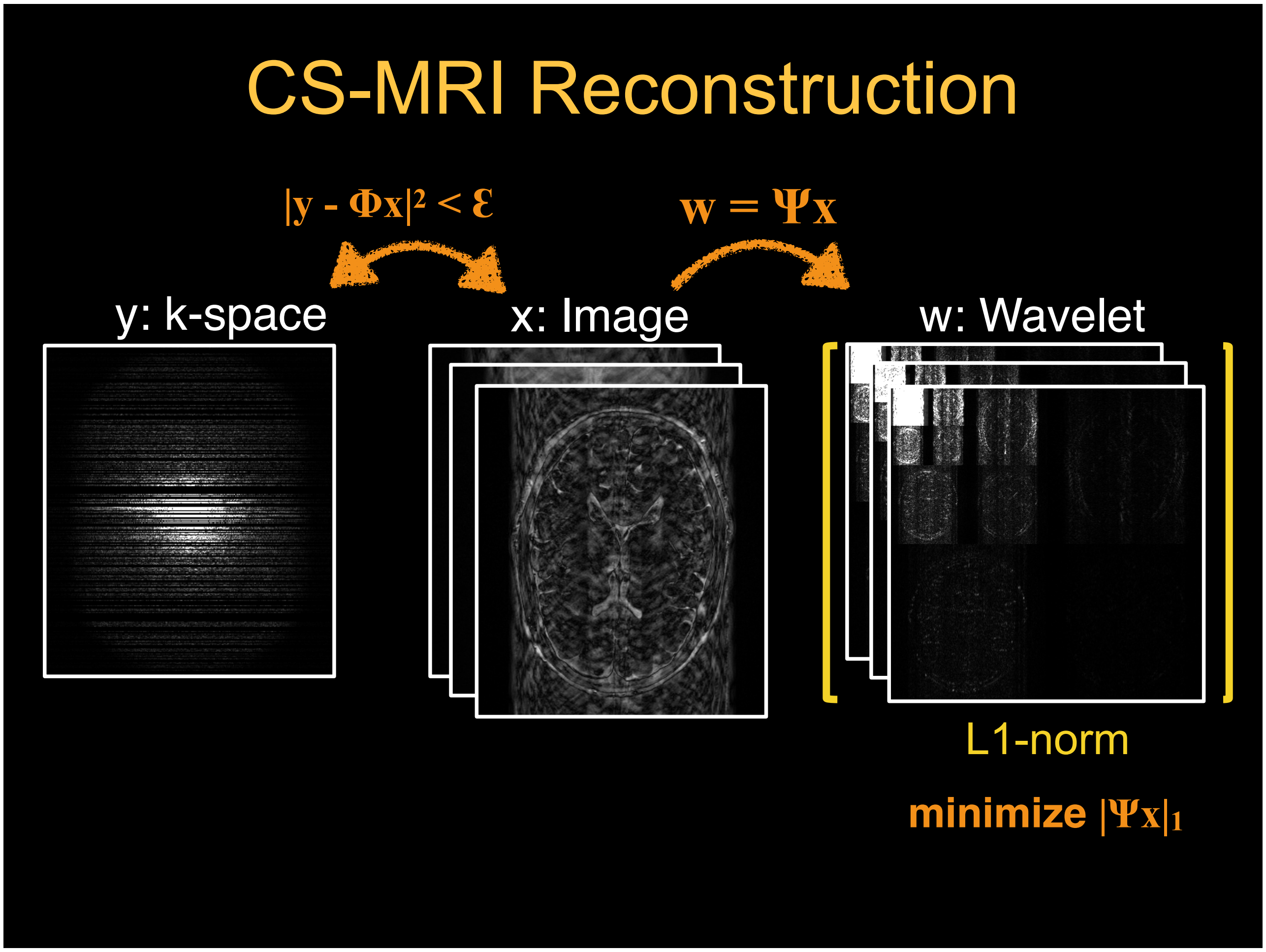
CS-MRI Reconstruction

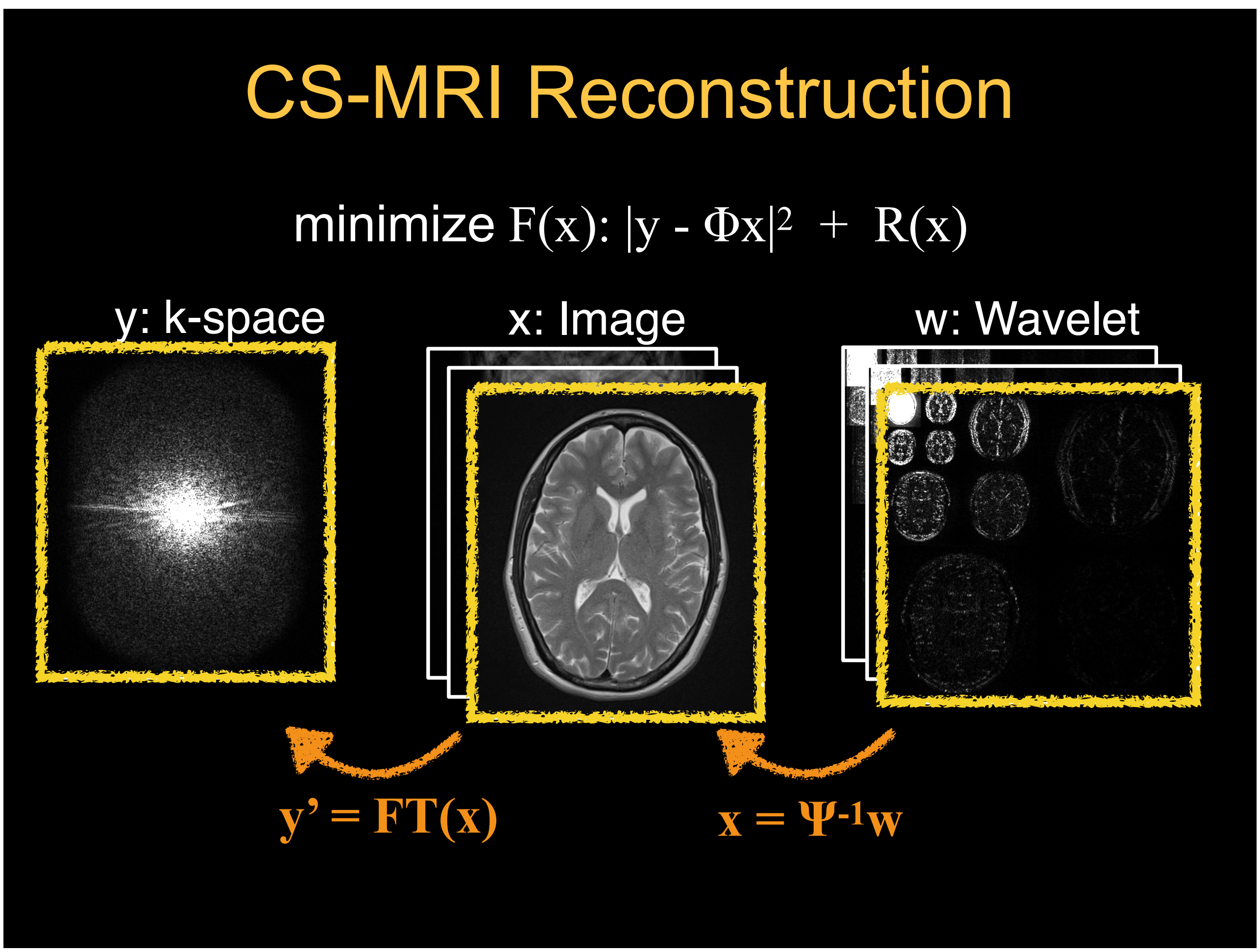
minimize $F(x): |y - \Phi x|_2^2 + R(x)$

- Minimizing F(x) is non-trivial since R(x) is not differentiable
  - Linear programming is challenging due to high computational complexity
- Simple gradient-based algorithms have been developed:
  - Re-weighted L1 / FOCUSS
  - IST / IHT / AMP / FISTA
  - Split Bregman / ADMM

I.F. Gorodnitsky, et al., J. Electroencephalog. Clinical Neurophysiol. 1995 Daubechies I, et al. Commun. Pure Appl. Math. 2004
Elad M, et al. in Proc. SPIE 2007
T. Goldstein, S. Osher, SIAM J. Imaging Sci. 2009

To the board ...


CS-MRI Reconstruction
$|y - \Phi x|^2 < \varepsilon$
$w = \Psi x$
y: k-space
x: Image
w: Wavelet
L1-norm
minimize $|\Psi x|_1$


CS-MRI Reconstruction
minimize $F(x)$: $|y - \Phi x|^2 + R(x)$
y: k-space
x: Image
w: Wavelet
$y' = FT(x)$
$x = \Psi^{-1} w$

Summary So Far…

minimize $F(x)$: $|y - \Phi x|_2^2 + R(x)$

Data Consistency — Compressibility Constraint

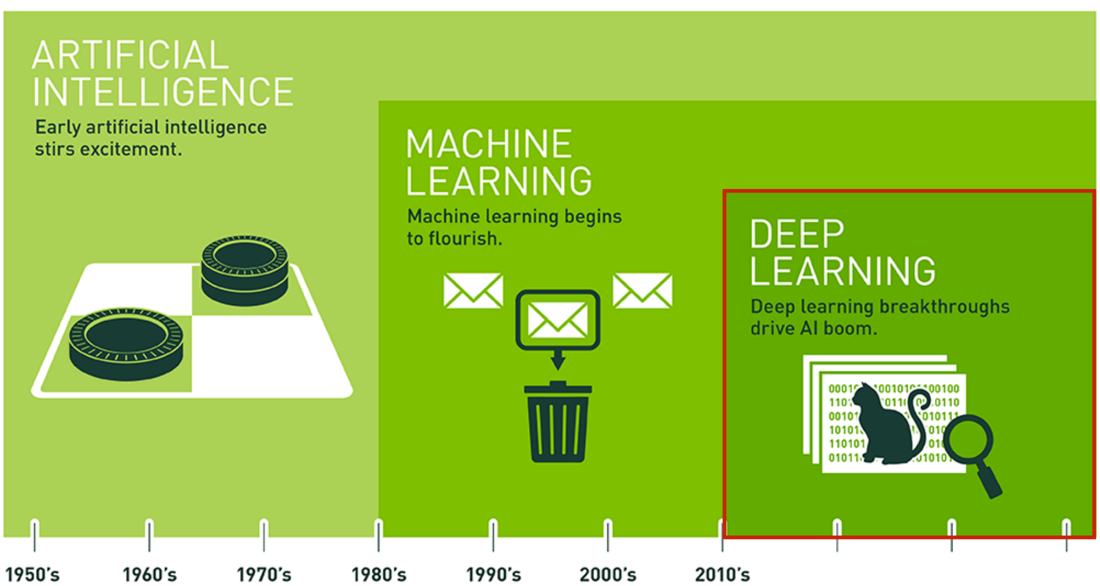
Reconstruction Domain
Compressibility Constraint
Incoherent Measurement
Reconstruction

State-of-the-Art CS-MRI

- Reducing possible reconstruction failure
  - Improve sparse transformations
  - Develop k-space undersampling schemes
- Integrating CS with DL/parallel imaging
  - Develop compatible undersampling patterns
  - Develop reconstruction methods

State-of-the-Art CS-MRI

- Methods to evaluate CS reconstructed images
  - RMSE / SSIM / Mutual Information
- Reducing reconstruction time
  - Reduce computational complexity
  - Parallelize reconstruction problems
- Developing stable reconstruction algorithms
  - Minimize / avoid the number of regularization parameters



Since an early flush of optimism in the 1950s, smaller subsets of artificial intelligence – first machine learning, then deep learning, a subset of machine learning – have created ever larger disruptions.

From nvidia.com

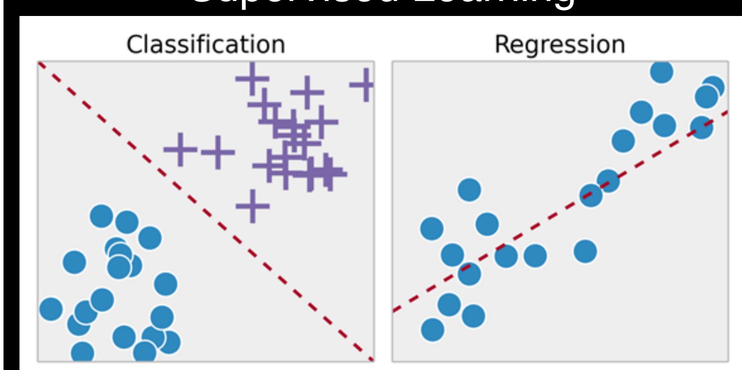
Types of Machine Learning

Machine Learning

- **Supervised Learning** — Task driven (Regression /Classification)
- **Unsupervised Learning** — Data driven (Clustering)
- **Reinforcement Learning** — Algorithm learns to react to an environment

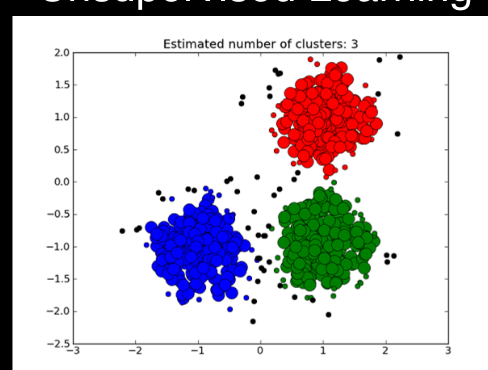
Types of Machine Learning

Supervised Learning



Discriminative Model

Unsupervised Learning



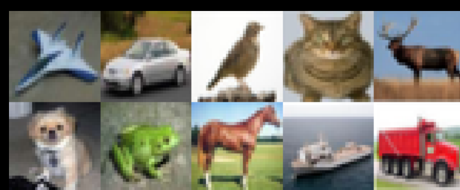
Generative Model

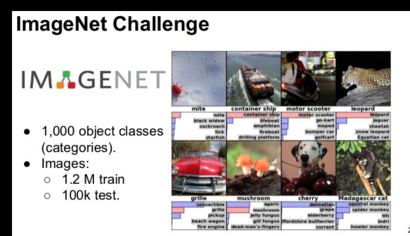
Common Datasets for Deep Learning

- MNIST: 60,000 images, hand writing digits (1998)
- CIFAR-10: 60,000 images 10 classes of

Not trivial to build medical imaging database with a high number of images and accurate labeling

- ImageNet: 1,300,000 high-res images, 1,000 classes of object (2012)





Key Design Considerations

1. Define clear clinical questions
2. Design deep learning models
   - Supervised vs. unsupervised learning
   - Descriptive vs. generative modeling
3. Consider potential limitations
   - Limited amount of training and testing data
   - Uncertainties in labeling

MRI Applications

- Regression
  - Prediction of a continuous variable from input
- Segmentation
- Classification
- Reconstruction
- Generative (create new images based on current)

Summary

- CS-MRI has a lot of potential but is not a magic box!
- Always remember key components of CS:

Reconstruction Domain

Compressibility (or Sparsity)

Incoherent Measurement

Reconstruction

Thanks!

Kyung Sung, PhD

ksung@mednet.ucla.edu

http://kyungs.bol.ucla.edu