

RF Pulse Design

RF Pulses / Adiabatic Pulses

M229 Advanced Topics in MRI

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Class Business

- Homework 1 is due on 4/24
 - Sevgi will hold office hours on 4/17 and 4/24 (8:30-10:30am)

Outline

- Review of RF pulses
- Adiabatic passage principle
- Adiabatic inversion

Review of RF Pulses

Notation and Conventions

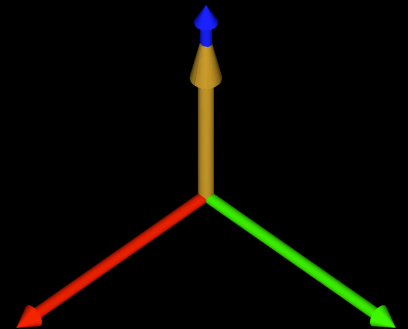
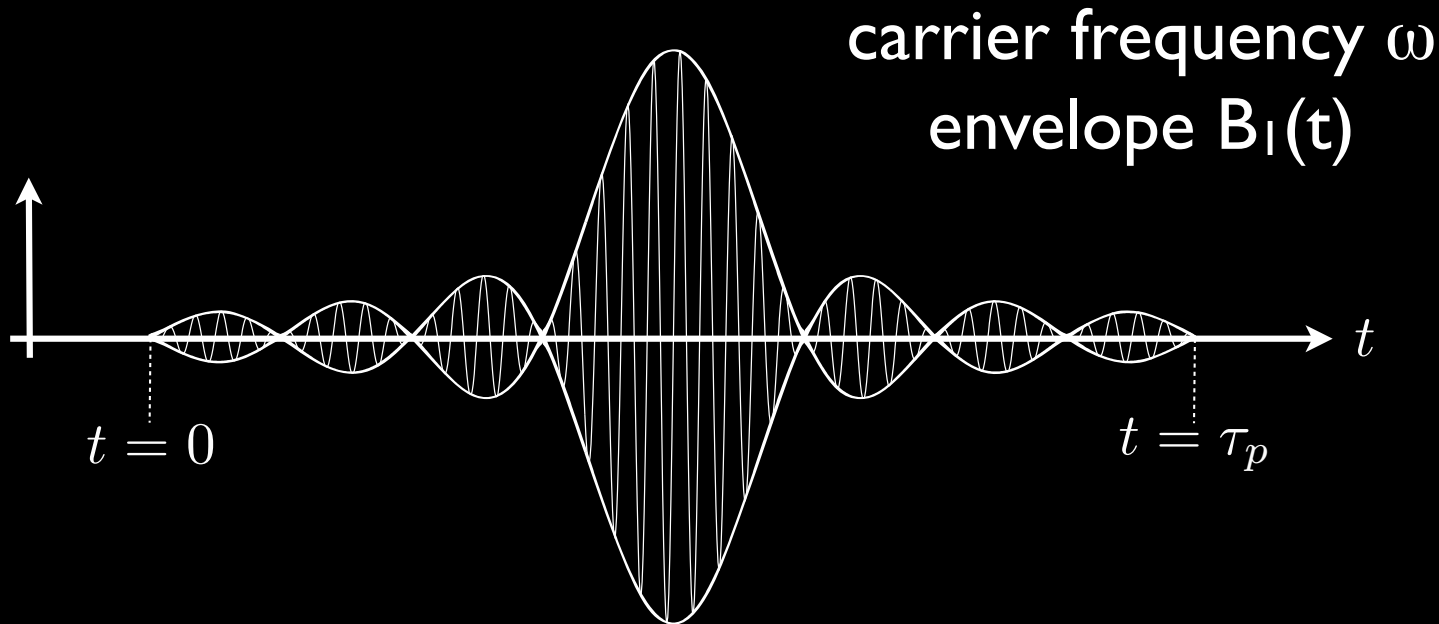
$$\vec{B} = B_0 \hat{k} + B_1(t) [\cos \omega t \hat{i} - \sin \omega t \hat{j}]$$

- ω = carrier frequency
- ω_0 = resonant frequency
- $B_1(t)$ = complex valued envelop function

RF Pulse - Excitation

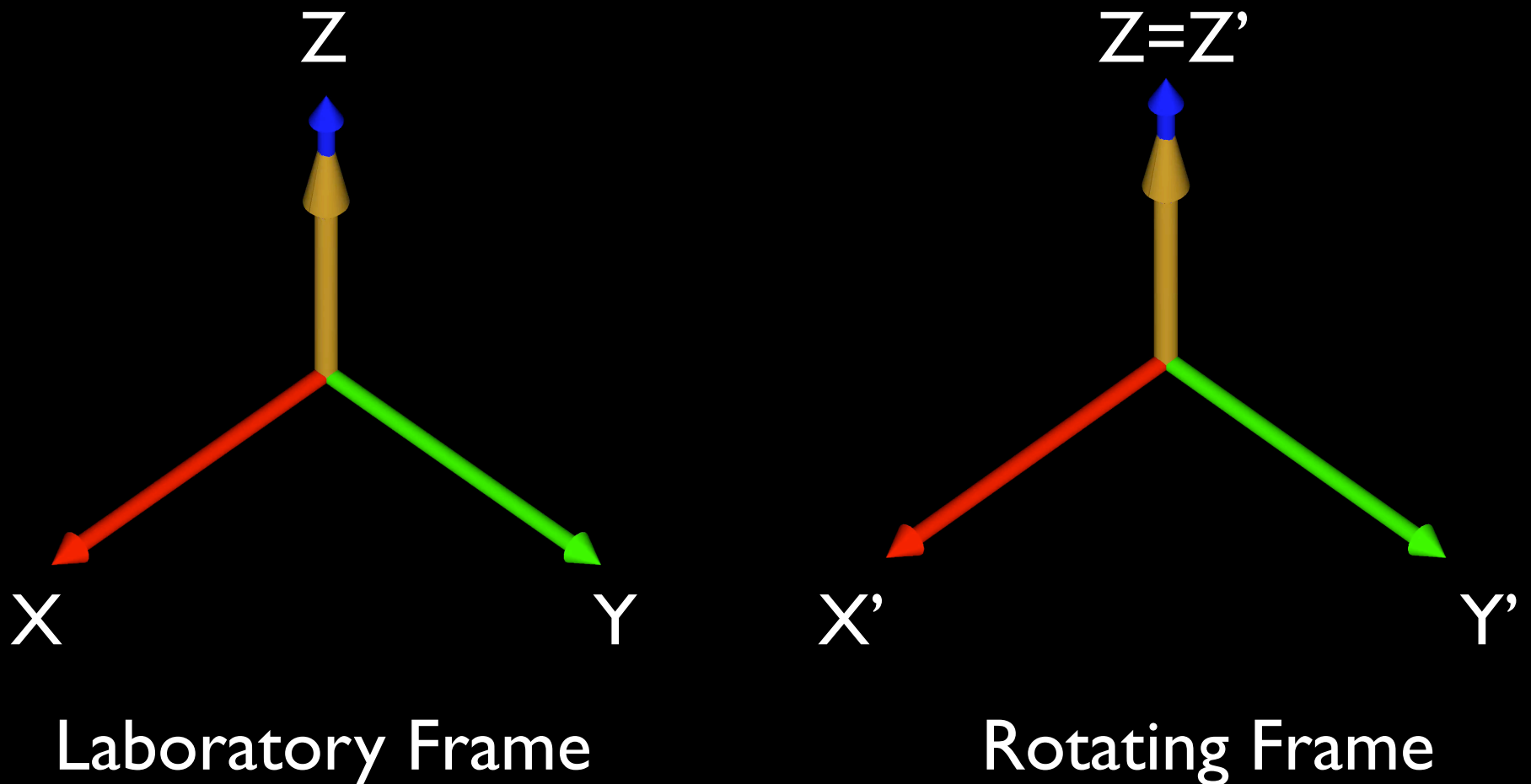
$$\vec{B} = B_0 \hat{k} + B_1(t) [\cos \omega t \hat{i} - \sin \omega t \hat{j}]$$

$$B_1(t) \cdot [\cos(\omega t) \hat{i} - \sin(\omega t) \hat{j}]$$



Lab vs. Rotating Frame

- The rotating frame simplifies the mathematics and permits more intuitive understanding.



Rotating Frame

Rotating Frame Definitions

$$\vec{M}_{rot} \equiv \begin{bmatrix} M_{x'} \\ M_{y'} \\ M_{z'} \end{bmatrix} \quad \vec{B}_{rot} \equiv \begin{bmatrix} B_{x'} \\ B_{y'} \\ B_{z'} \end{bmatrix} \quad \begin{array}{l} B_{z'} \equiv B_z \\ M_{z'} \equiv M_z \end{array}$$

$$\vec{M}_{lab}(t) = R_Z(\omega_0 t) \cdot \vec{M}_{rot}(t)$$

$$\vec{B}_{lab}(t) = R_Z(\omega_0 t) \cdot \vec{B}_{rot}(t)$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B} \quad \longrightarrow \quad \frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

Bloch Equation (Rotating Frame)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{\omega}_{rot}}{\gamma}$ fictitious field

$$\vec{\omega}_{rot} = \begin{pmatrix} 0 \\ 0 \\ -\omega \end{pmatrix}$$

Bloch Equation (Rotating Frame)

$$\vec{B}_{eff} = \vec{B}_{rot} + \frac{\vec{\omega}_{rot}}{\gamma}$$

$$\vec{B}_{lab} = \begin{pmatrix} B_1(t) \cos \omega_0 t \\ B_1(t) \sin \omega_0 t \\ B_0 \end{pmatrix} \quad \vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ B_1(t) \\ B_0 \end{pmatrix}$$

Assume real-valued $B_1(t)$

$$\vec{B}_{rot} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 \end{pmatrix} \quad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix}$$

To the board ...

Bloch Equation with Gradient

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} \end{pmatrix} \rightarrow \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$$

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0} - \frac{\omega}{\gamma} + G_z z \end{pmatrix}$

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \quad \omega_1(t) = \gamma B_1(t)$$

To the board ...

B1 Variations

- In MRI, B1 field is not always uniform across the imaging volume
- B1 inhomogeneity can cause:
 - Image shading
 - Incomplete saturation (e.g. in fat suppression)
 - Incomplete inversion (e.g. CSF suppression, myocardium suppression in cardiac scar imaging)
 - Inaccurate/imprecise quantification in T1 mapping

B1 Variations

- It is highly desirable if we can excite tissue homogeneously and produce a uniform flip angle throughout

→ Adiabatic Pulses!

“Adiabatic pulses are a special class of RF pulses that can excite, refocus or invert magnetization vectors uniformly, even in the presence of a spatially nonuniform B1 field.”

Adiabatic Passage Principle

Adiabatic Pulses

- A special class of RF pulses that can achieve uniform flip angle
- Flip angle is independent of the applied B1 field

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Slice profile of an adiabatic pulse is obtained using Bloch simulations
- Can be used for excitation, inversion and refocusing

Adiabatic vs. Non-Adiabatic Pulses

Adiabatic Pulses:

$$\theta \neq \int_0^T B_1(\tau) d\tau$$

- Amplitude and frequency/phase modulation
- Long duration (8-12 ms)
- Higher B1 amplitude (>12 μT)
- Generally NOT multi-purpose (inversion pulse cannot be used for refocusing, etc.)

Non-Adiabatic Pulses:

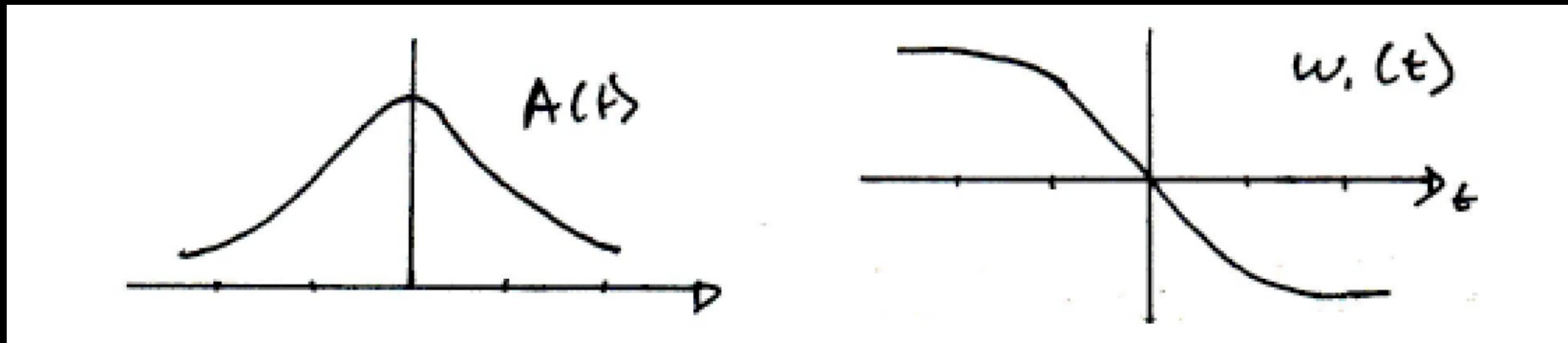
$$\theta = \int_0^T B_1(\tau) d\tau$$

- Amplitude modulation, constant carrier frequency (constant phase)
- Short duration (0.3 ms to 1 ms)
- Lower B1 amplitude
- Generally multi-purpose

Adiabatic Pulses

- Frequency modulated pulses:

$$B_1(t) = \underbrace{A(t)}_{\text{envelop}} \exp^{-i \int \underbrace{\omega_1(t')}_{\text{frequency sweep}} dt'}$$



- Or phase modulation:

$$B_1(t) = A(t) \exp^{-i\phi(t)}$$

Bloch Equation (at on-resonance)

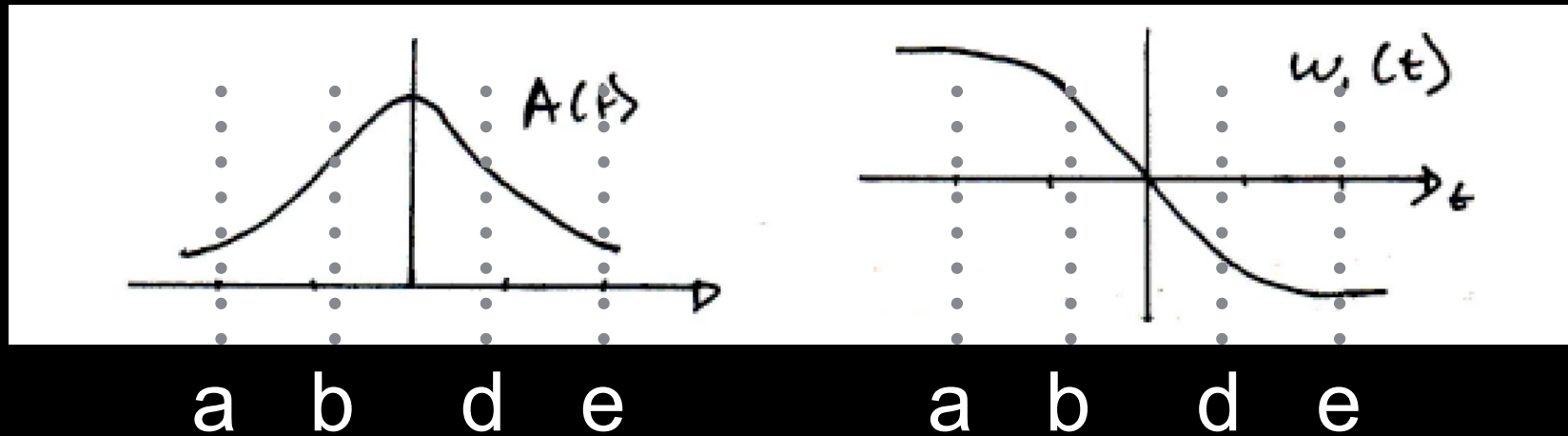
$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} A(t) \\ 0 \\ \cancel{B_0} \frac{\omega}{\gamma} + \frac{\omega_1(t)}{\gamma} \end{pmatrix}$

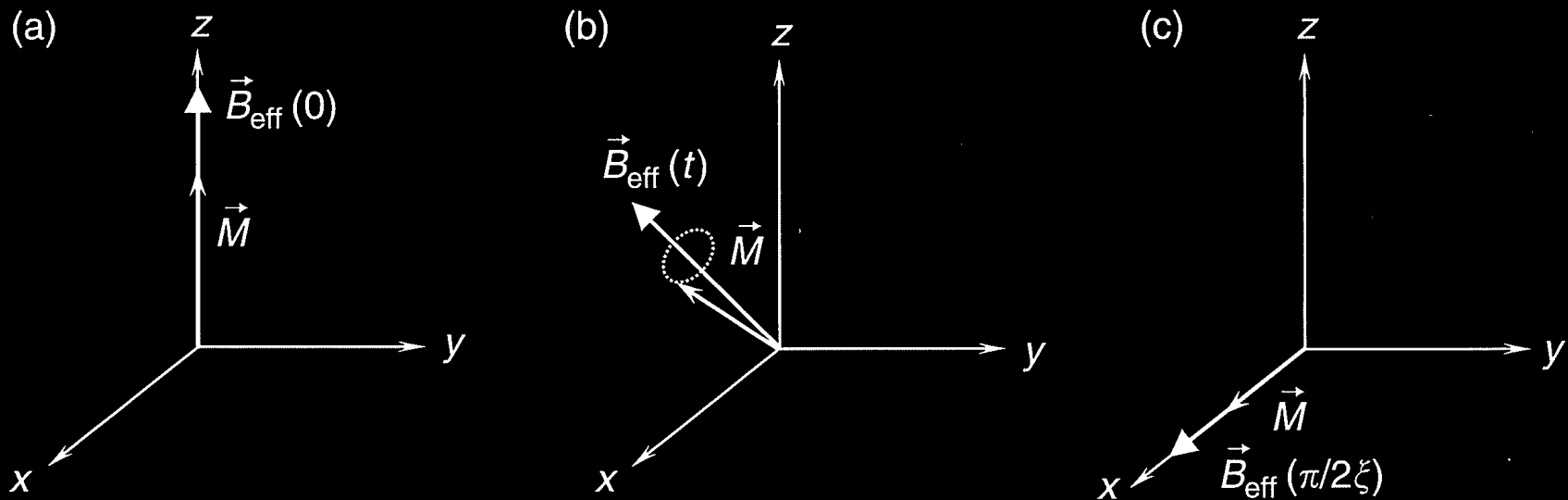
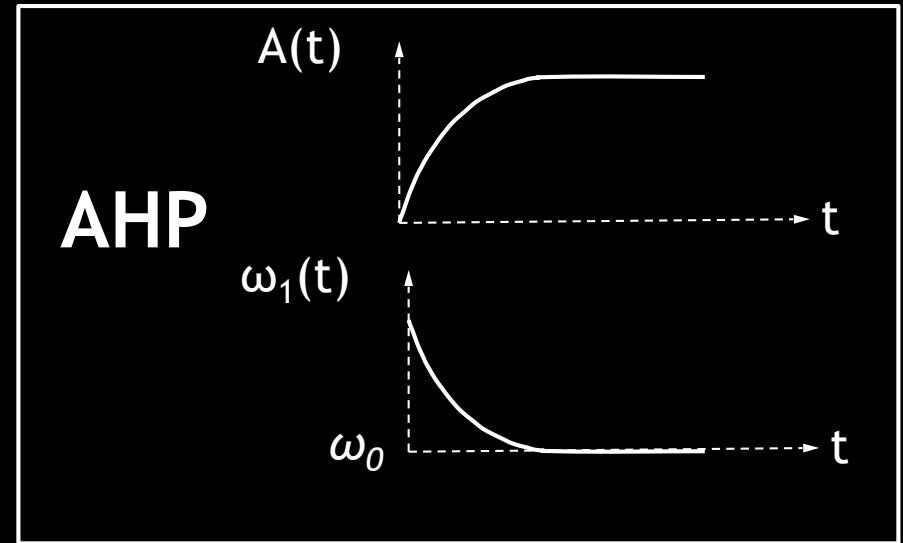
$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega_1(t) & 0 \\ -\omega_1(t) & 0 & \gamma A(t) \\ 0 & -\gamma A(t) & 0 \end{pmatrix} \vec{M}$$

Magnetization Plot



To the board ...

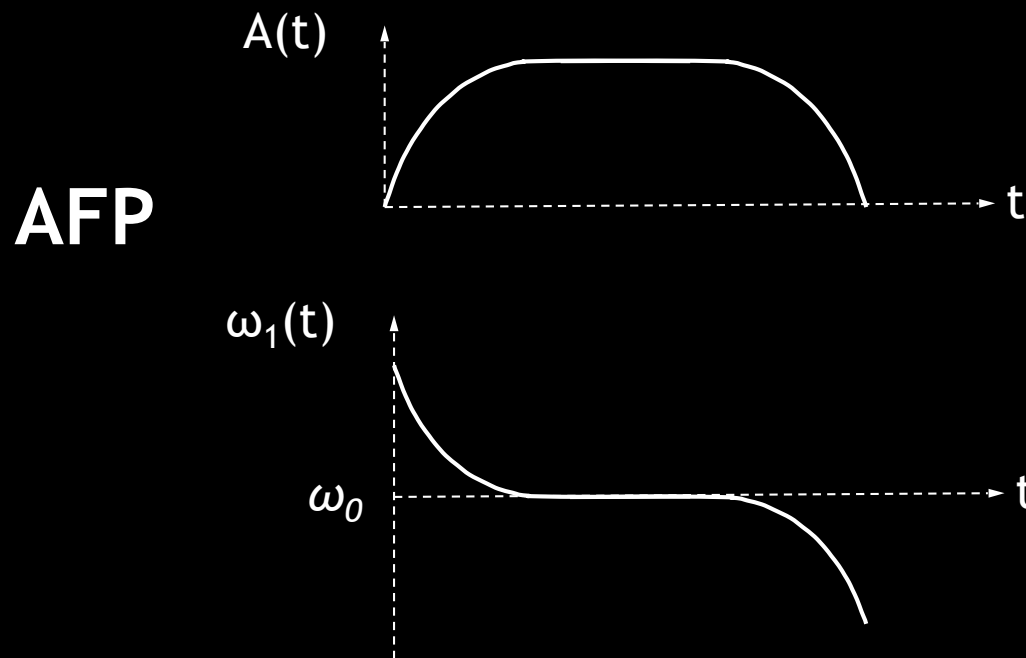
Adiabatic Excitation



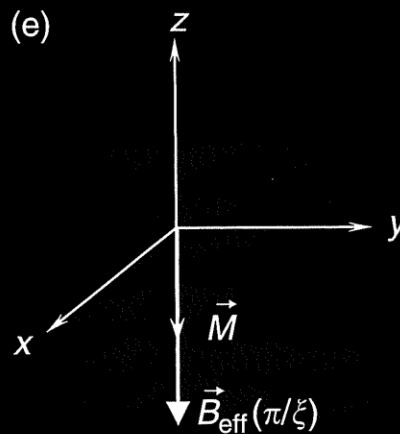
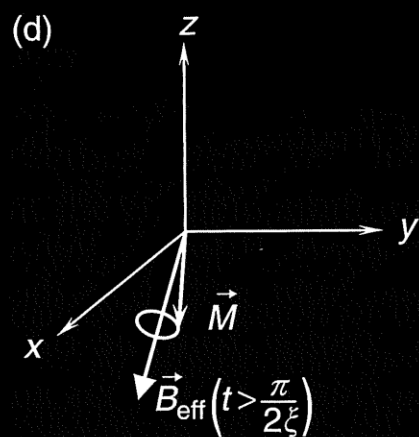
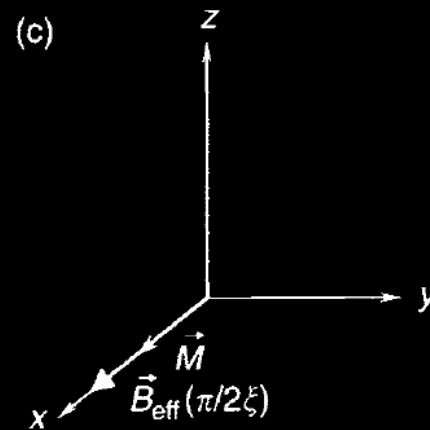
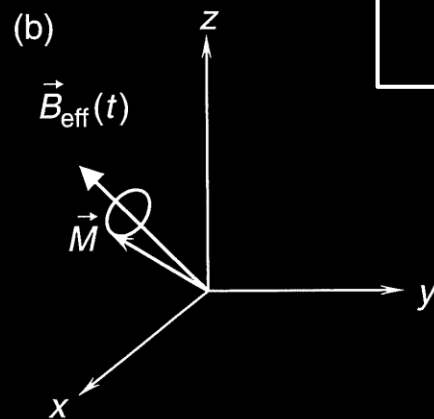
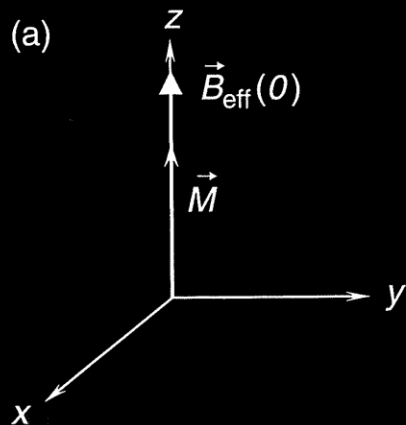
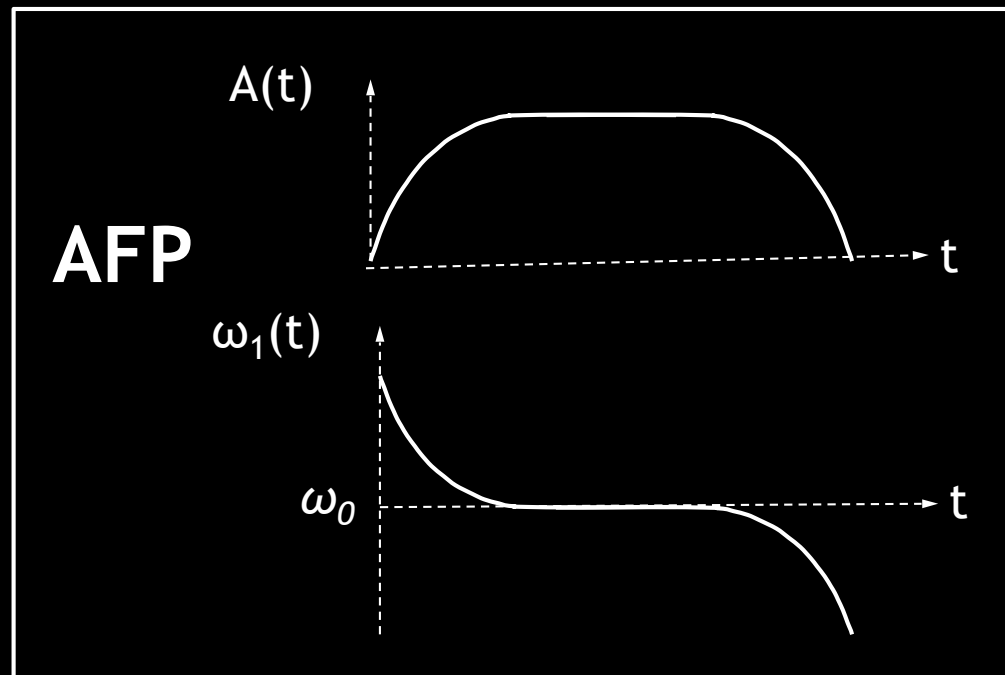
- At the end of the pulse, all the magnetization is in the transverse plane \rightarrow so we have adiabatic excitation!
- This is also called an **adiabatic half passage (AHP)**

Adiabatic Inversion

- An adiabatic inversion requires an adiabatic full passage (AFP) pulse:



Adiabatic Inversion



Adiabatic Inversion

Design of Adiabatic Inversion

- General equation for an adiabatic pulse:

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

- Many different types of adiabatic pulses can be designed by choosing different amplitude and frequency modulation functions
- The most famous one is...

The Hyperbolic Secant Inversion Pulse!

Hyperbolic Secant Pulse Equations

$$B_1(t) = A(t) \exp^{-i \int \omega_1(t') dt'}$$

where

$$A(t) = A_0 \operatorname{sech}(\beta t)$$

$$\omega_1(t) = -\mu \beta \tanh(\beta t)$$

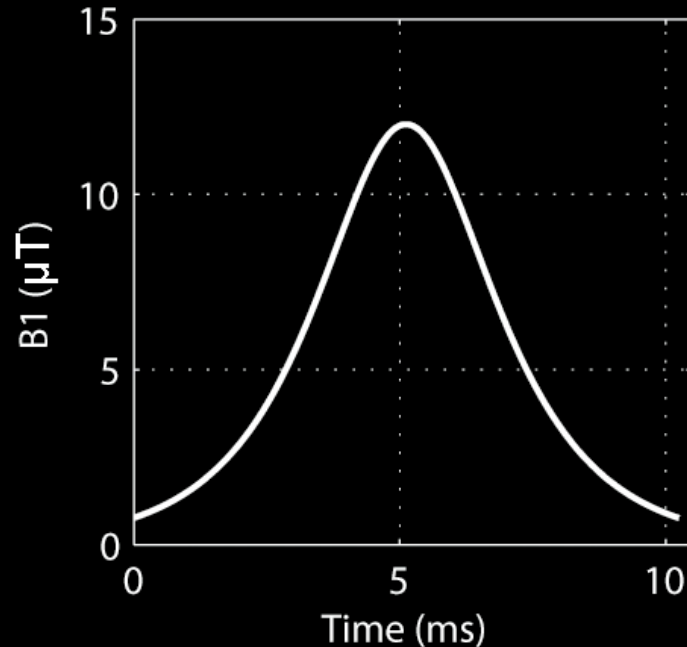
A_0 : peak amplitude (μT)

β : frequency modulation parameter (rad/s)

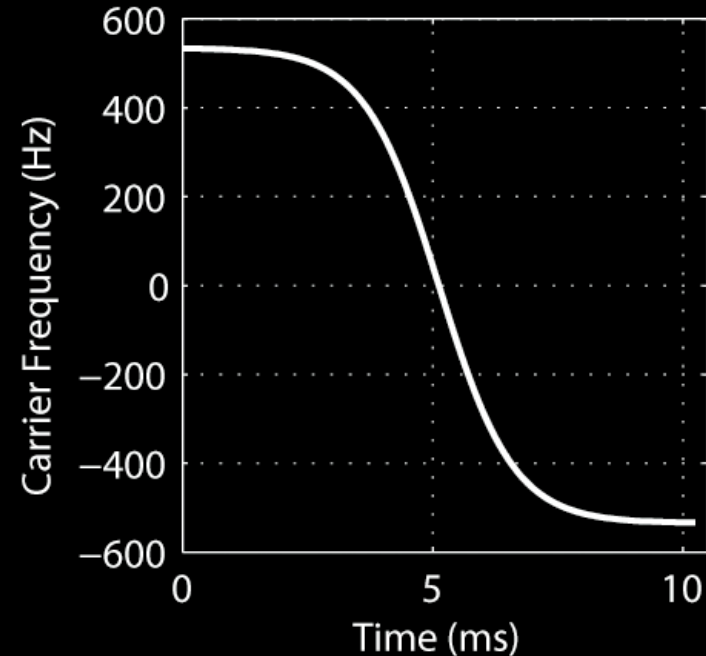
μ : phase modulation parameter (dimensionless)

Hyperbolic Secant Pulse Example

Amplitude Modulation, $A(t)$



Frequency Modulation, $\omega_1(t)$



Pulse Parameters:

$$A_0 = 12 \mu\text{T}$$

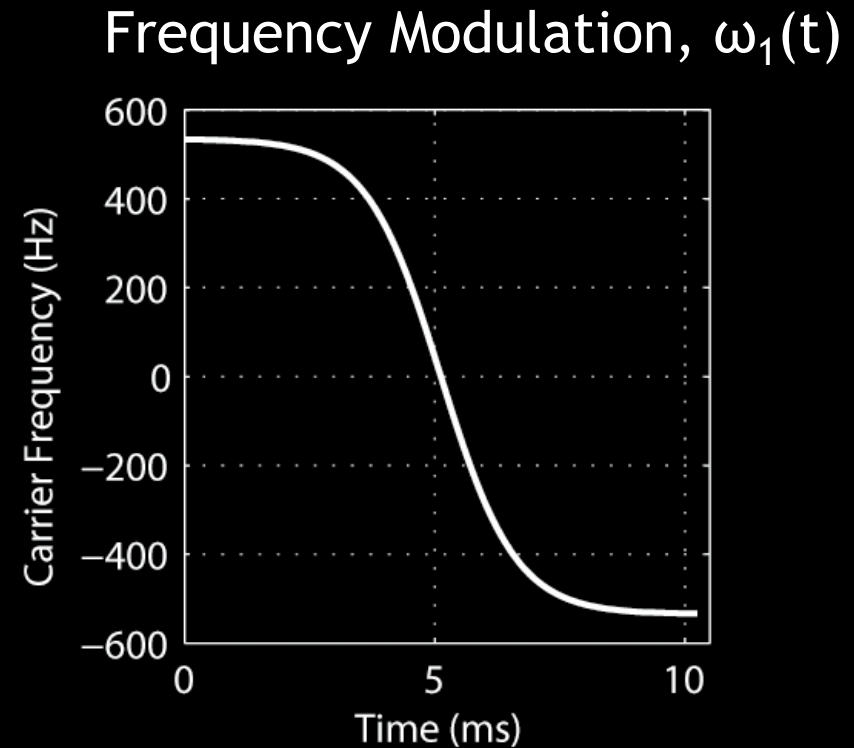
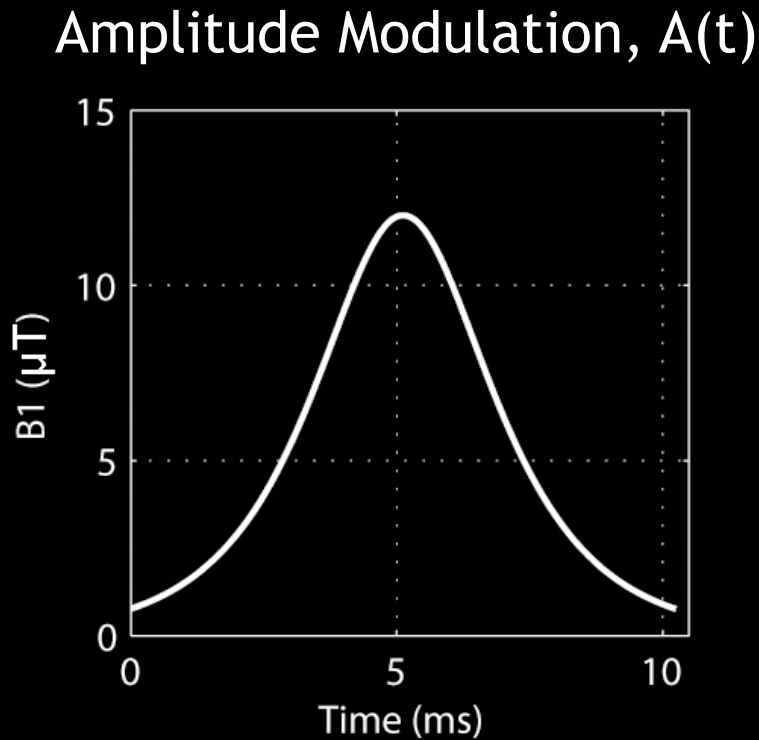
$$\mu = 5$$

$$\beta = 672 \text{ rad/s}$$

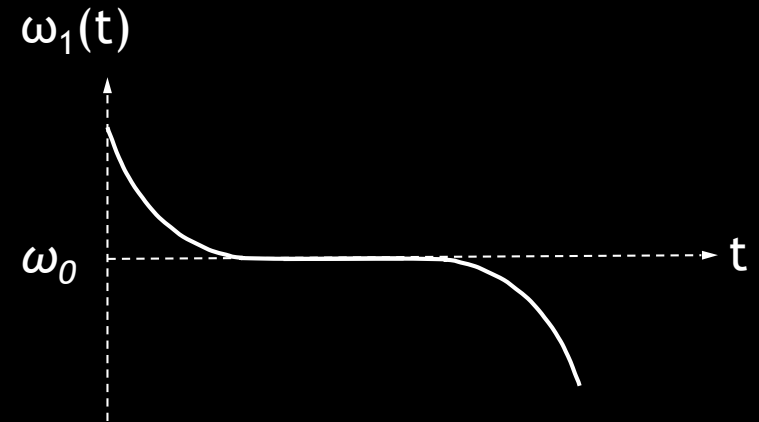
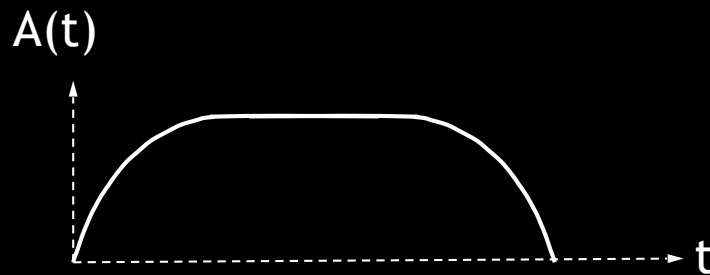
$$\text{Duration} = 10.24 \text{ ms}$$

Comparing Hyperbolic Secant with an AFP Example

Hyperbolic Secant Pulse



General Adiabatic Full Passage pulse



Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	A(t)	$\omega_1(t)$
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2} + \frac{1}{\sqrt{\beta}} \tan^{-1}(\sqrt{\beta}\tau)$
HS	$\text{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$
Gauss ^c	$\exp\left(-\frac{\beta^2\tau^2}{2}\right)$	$\frac{\text{erf}(\beta\tau)}{\text{erf}(\beta)}$
Hanning	$\frac{1+\cos(\pi\tau)}{2}$	$\tau + \frac{4}{3\pi} \sin(\pi\tau) \left[1 + \frac{1}{4} \cos(\pi\tau) \right]$
HSn ^c (n=8)	$\text{sech}(\beta\tau^n)$	$\int \text{sech}^2(\beta\tau^n) d\tau$
Sin40 ^d (n=40)	$1 - \left \sin^n\left(\frac{\pi\tau}{2}\right) \right $	$\tau - \int \sin^n\left(\frac{\pi\tau}{2}\right) \left(1 + \cos^2\left(\frac{\pi\tau}{2}\right) \right) d\tau$

Some Examples of Other Adiabatic Inversion Pulses

Pulse Name	A(t)	$\omega_1(t)$
Lorentz	$\frac{1}{1+\beta\tau^2}$	$\frac{\tau}{1+\beta\tau^2} + \frac{1}{\sqrt{\beta}} \tan^{-1}(\sqrt{\beta}\tau)$
HS	$\text{sech}(\beta\tau)$	$\frac{\tanh(\beta\tau)}{\tanh(\beta)}$

The shape of the inversion profile depends on the choice A(t) and $\omega_1(t)$!

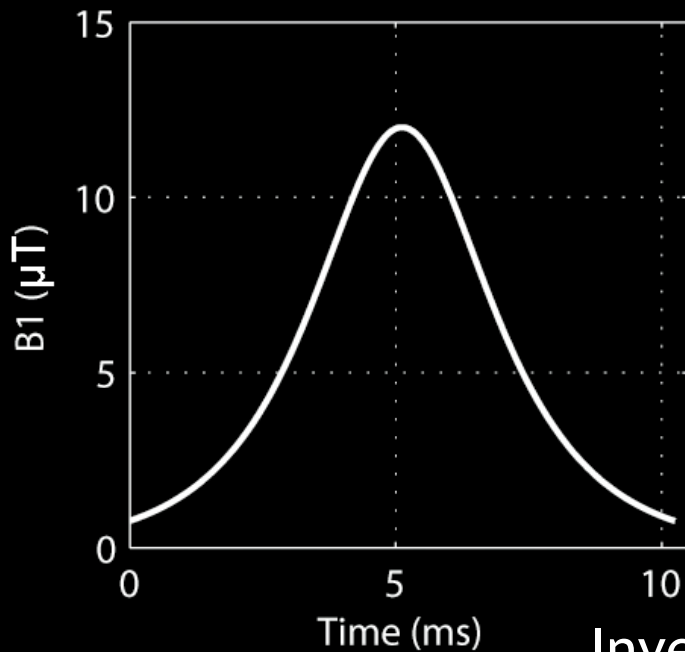
HSn^c (n=8) $\text{sech}(\beta\tau^n)$ $\int \text{sech}^2(\beta\tau^n) d\tau$

Sin40^d (n=40) $1 - \left| \sin^n\left(\frac{\pi\tau}{2}\right) \right|$ $\tau - \int \sin^n\left(\frac{\pi\tau}{2}\right) \left(1 + \cos^2\left(\frac{\pi\tau}{2}\right)\right) d\tau$

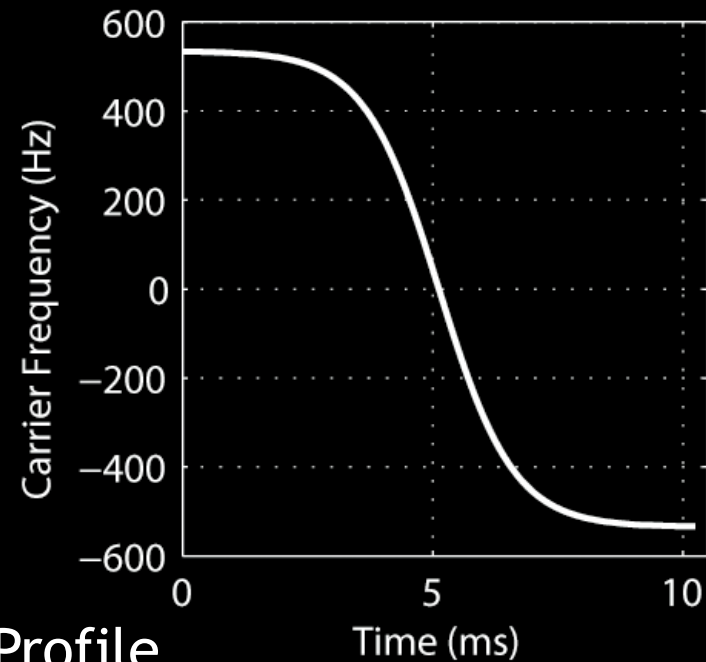
What Will Inversion Profile Look Like?

Hyperbolic Secant
Pulse

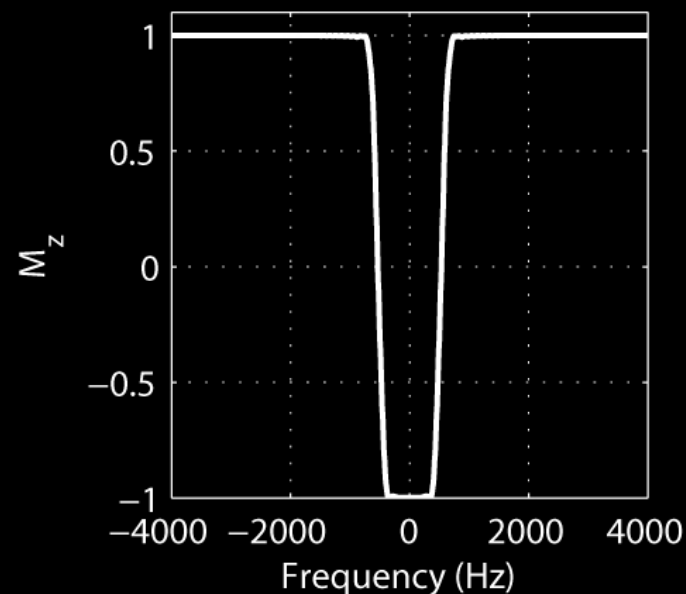
Amplitude Modulation, $A(t)$



Frequency Modulation, $\omega_1(t)$



Inversion Profile



Inversion Profiles

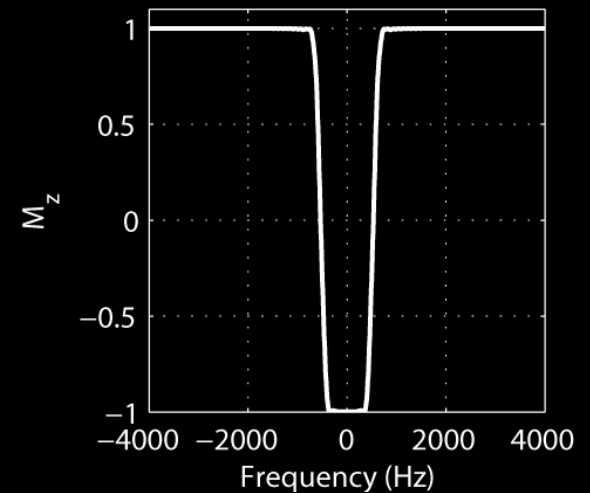
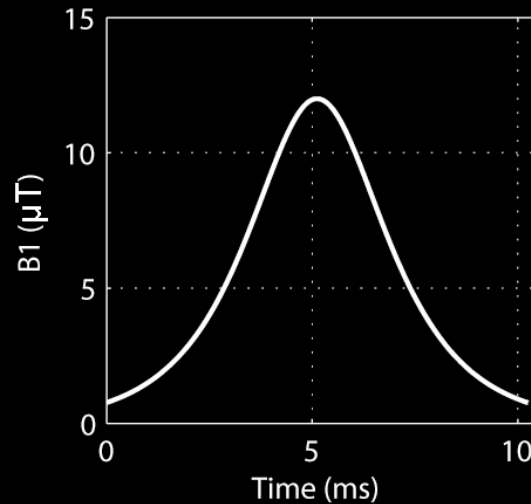
- The inversion profile typically calculated using Bloch simulation of the RF pulse (will be covered later) shows us the inversion efficiency and RF bandwidth
- The inversion efficiency depends strongly on the B1 amplitude (as well as pulse duration, T1, T2 and pulse shape)
- For the hyperbolic secant pulse,

$$\text{RF spectral bandwidth} = \mu\beta$$

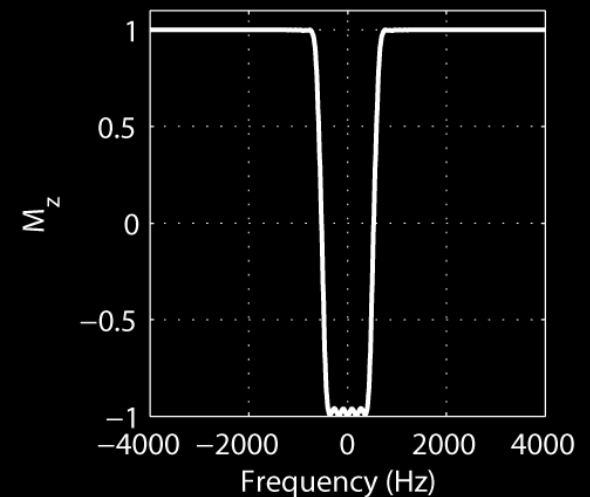
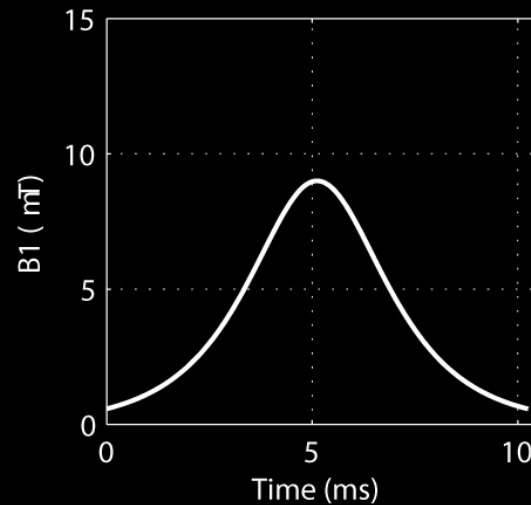
$$B_{1\text{max}} \gg (\beta\sqrt{\mu})/\gamma \quad (B_1 \text{ threshold for adiabaticity})$$

Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
 $B1_{\max} = 12 \mu\text{T}$

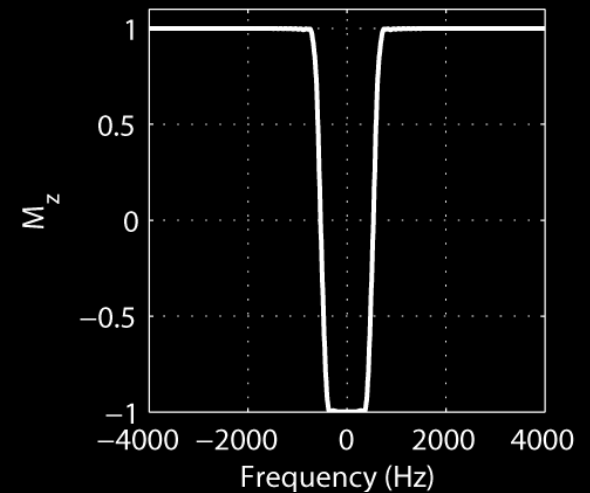
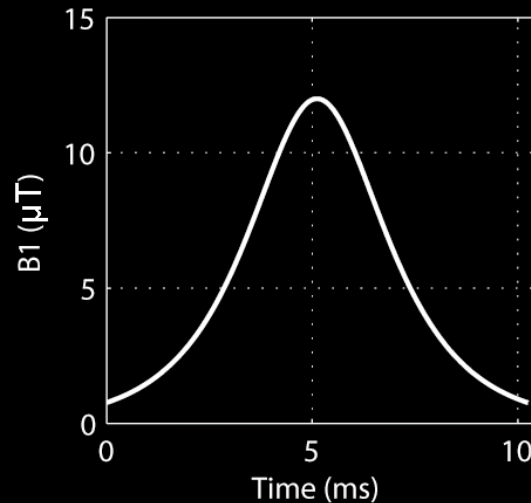


75% Attenuated Pulse
 $B1_{\max} = 9 \mu\text{T}$

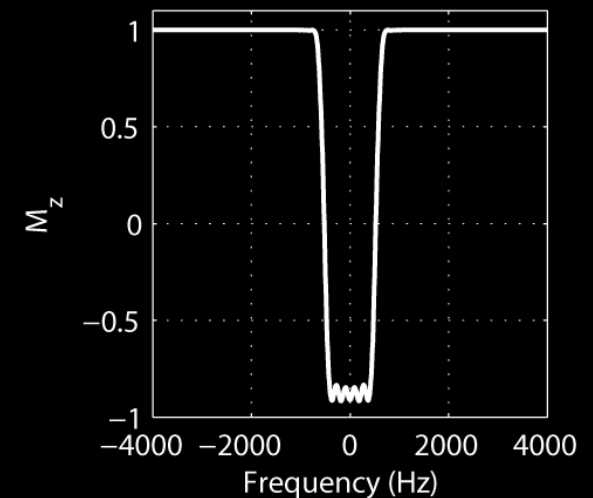
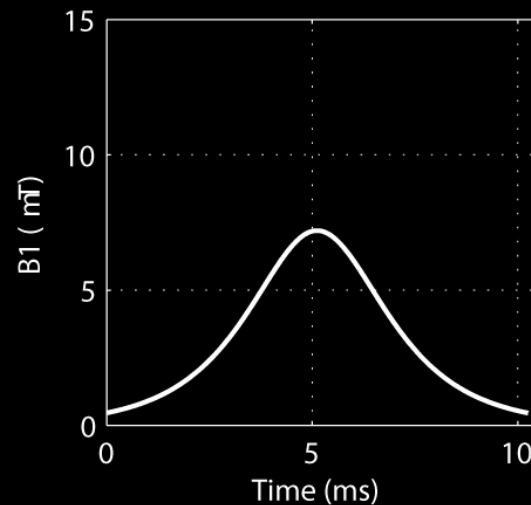


Hyperbolic Secant: Adiabatic Property

Original Pulse (100%)
 $B1_{\max} = 12 \mu\text{T}$



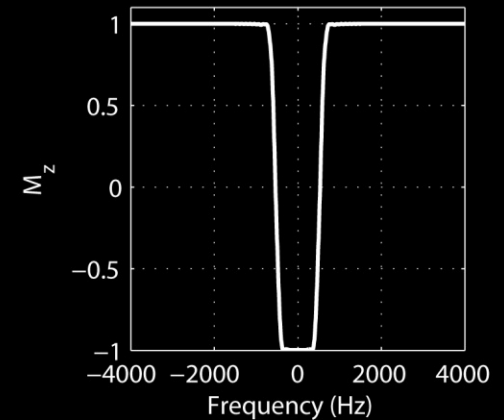
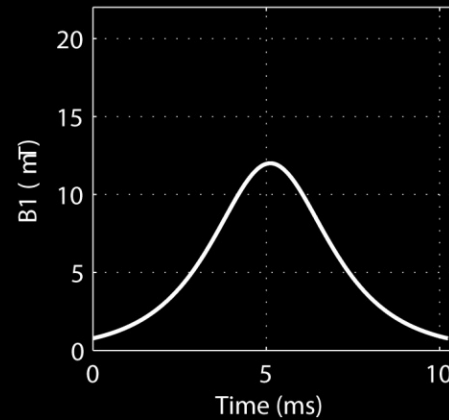
60% Attenuated Pulse
 $B1_{\max} = 7.2 \mu\text{T}$



$B1$ Threshold $\approx 6 \mu\text{T}$

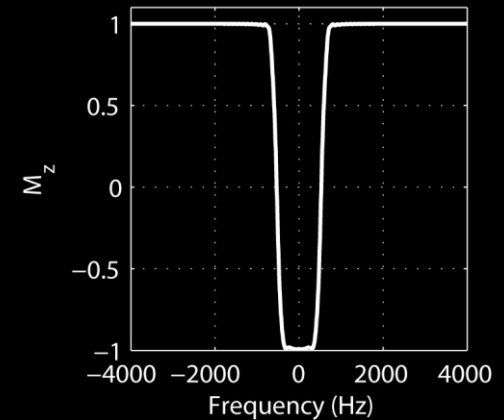
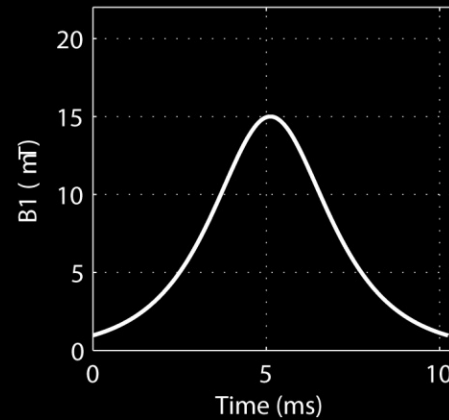
Original Pulse (100%)

$$B_1 = 12 \mu\text{T}$$



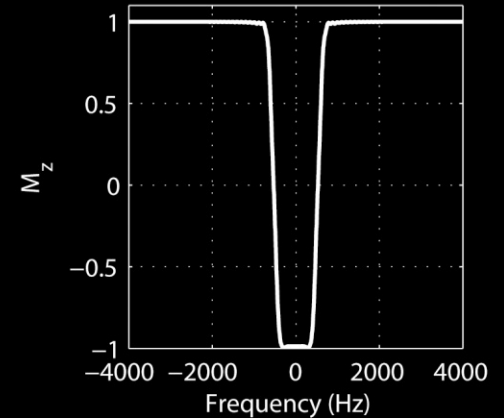
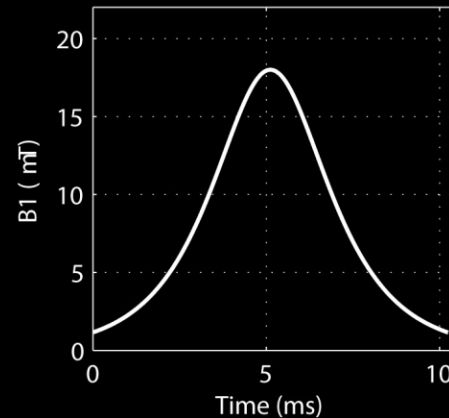
125% Amplified Pulse

$$B_1 = 15 \mu\text{T}$$



150% Amplified Pulse

$$B_1 = 18 \mu\text{T}$$



Comments

- Many envelope/modulation functions work
- If a range of adiabaticity is required, optimization can help reduce pulse length
- Hyperbolic Sech needs to be truncated, which can affect the overall performance

Thank You!

- Further reading

- Read "Adiabatic Refocusing Pulses" p.200-212
- Tannus et al., "Adiabatic Pulses", NMR in Biomedicine, Vol. 10, 423-434 (1997)

- Acknowledgments

- John Pauly's EE469b (RF Pulse Design for MRI)

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