RF Pulse Design

Multi-dimensional Excitation I

M229 Advanced Topics in MRI

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Today's Topics

- Review of adiabatic pulses
- Applications of adiabatic pulses
- Small tip approximation
- Excitation k-space interpretation

Summary for Adiabatic Pulses

Adiabatic Pulses

- Flip Angle $\neq \int_0^T B_1(t)dt$
- Amplitude and frequency modulation
- Long duration (8-12 ms)
- High B1 amplitude (>12 μT)
- Generally NOT multi-purpose (inversion pulses cannot be used for refocusing, etc.)

Non-adiabatic Pulses

- Flip Angle $= \int_0^T B_1(t)dt$
- Amplitude modulation with constant carrier frequency
- Short duration (0.3-1 ms)
- Low B1 amplitude
- Generally multi-purpose (inversion pulses can be used for refocusing, etc.)

Bloch Equation

$$\frac{d\vec{M}}{dt} = \vec{M} \times \gamma \vec{B}_{eff}$$

Non-selective vs. Selective Excitation

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} \end{pmatrix} \qquad \vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}}{\gamma} + G_z z \end{pmatrix}$$

Adiabatic Pulses

$$\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 - \frac{\omega_{RF}(t)}{\gamma} \end{pmatrix}$$

```matlab
%%% User inputs:
mu = 5;    % Phase modulation parameter [dimensionless]
beta1 = 672;    % Frequency modulation parameter [rad/s]
pulseWidth = 10.24;    % RF pulse duration [ms]
A0 = 0.12;    % Peak B1 amplitude [Gauss].

%%%%%%

nSamples = 512;          % number of samples in the RF pulse
dt = pulseWidth/nSamples/1000;  % time step, [seconds]
tim_sech = linspace(-pulseWidth/2,pulseWidth/2,nSamples)./1000';
% time scale to calculate the RF waveforms in seconds.

% Amplitude modulation function B1(t):
B1 = A0.* sech(beta1.*tim_sech);

% Carrier frequency modulation function w(t):
w = -mu.*beta1.*tanh(beta1.*tim_sech)./(2*pi);
% The 2*PI scaling factor at the end converts the unit from rad/s to Hz

% Phase modulation function phi(t):
phi = mu .* log(sech(beta1.*tim_sech));

% Put together complex RF pulse waveform:
rf_pulse = B1 .* exp(1i.*phi);

% Generate a time scale for the Bloch simulation:
tim_bloch = [0:(nSamples-1)]*dt;
```

Applications of Adiabatic Pulses

Adiabatic Pulses

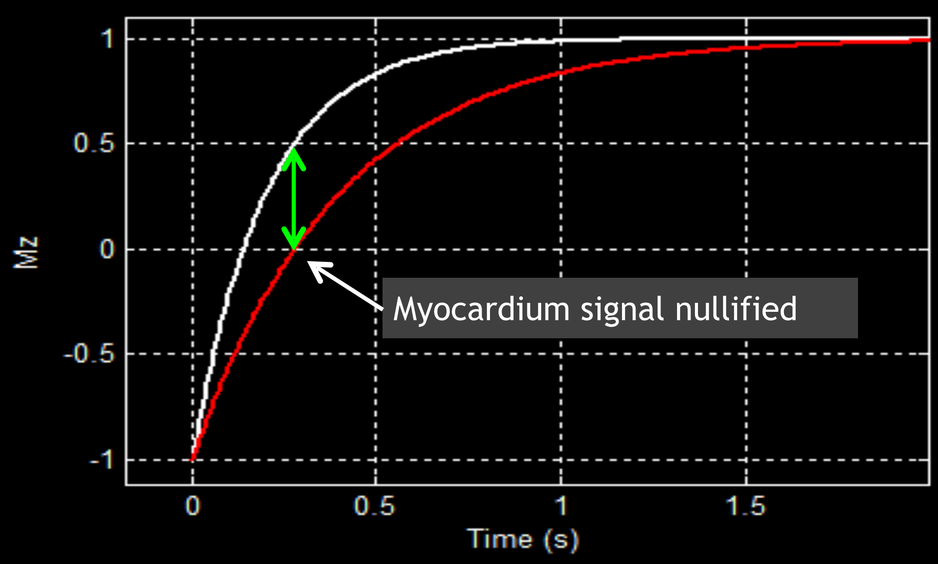
- Fat suppression (STIR)
- CSF suppression (FLAIR)
- Myocardium suppression in cardiac scar imaging (LGE)
- Black blood cardiac imaging (DIR TSE)
- T1 Mapping

Late Gadolinium Enhancement (LGE)

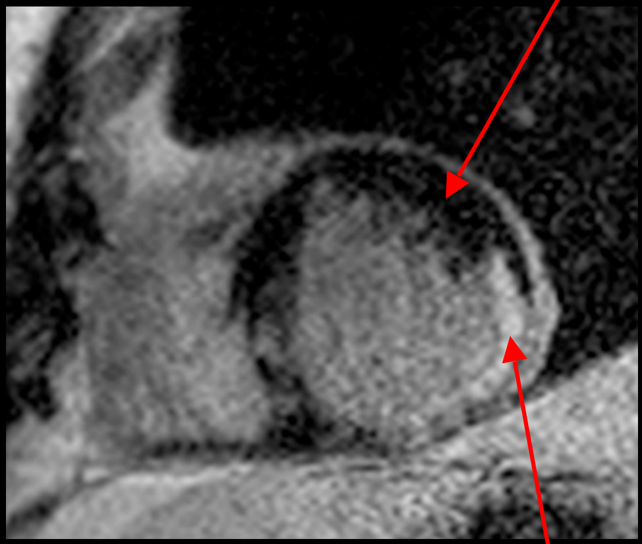
- Gold standard for detection of scar/myocardial fibrosis
- Spoiled gradient echo (SPGR) sequence with an inversion pulse (inversion recovery SPGR)
  - Inversion pulse is usually hyperbolic secant pulse
  - Healthy myocardium is nulled with the inversion pulse
  - Scar tissue (which has shorter T1 than healthy tissue) appear bright

- The conventional LGE sequence uses an RF-spoiled gradient echo (FLASH) readout with an inversion recovery (IR) pulse as a preparation pulse
- The readout is acquired at a time after inversion at which the healthy myocardium signal reaches zero

Inversion recovery curves of postcontrast scar (white) and myocardium (red)



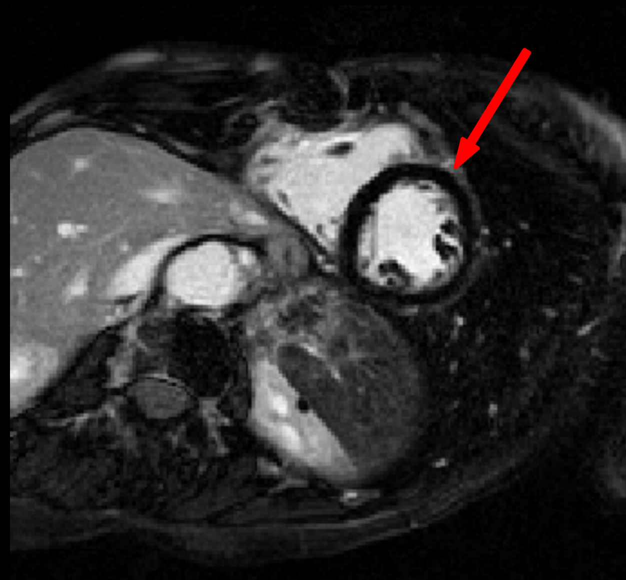
Nullified signal from healthy myocardium



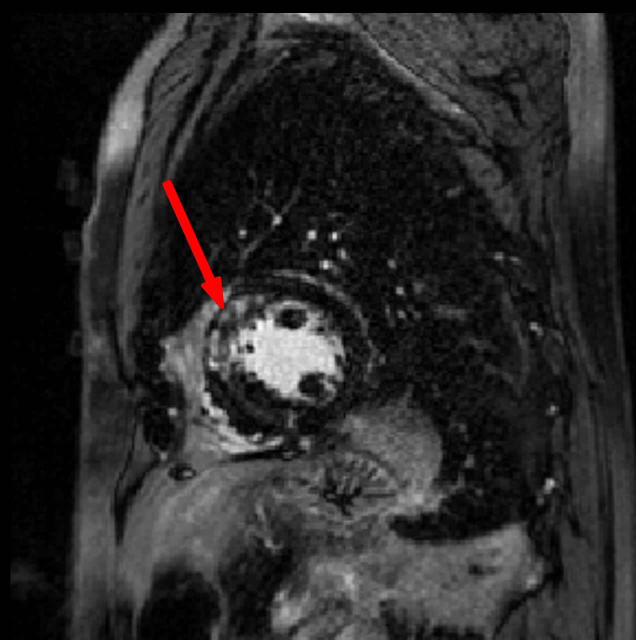
Hyper-enhanced scar region

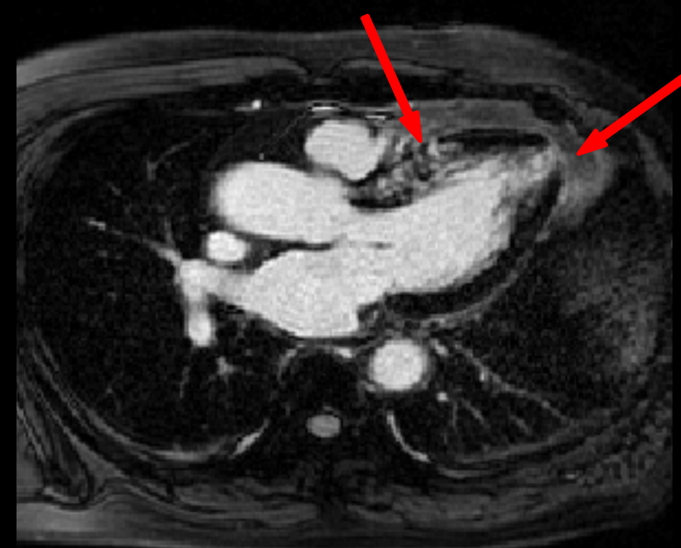
Clinical Example

Patient with healthy myocardium



Patient with scar tissue

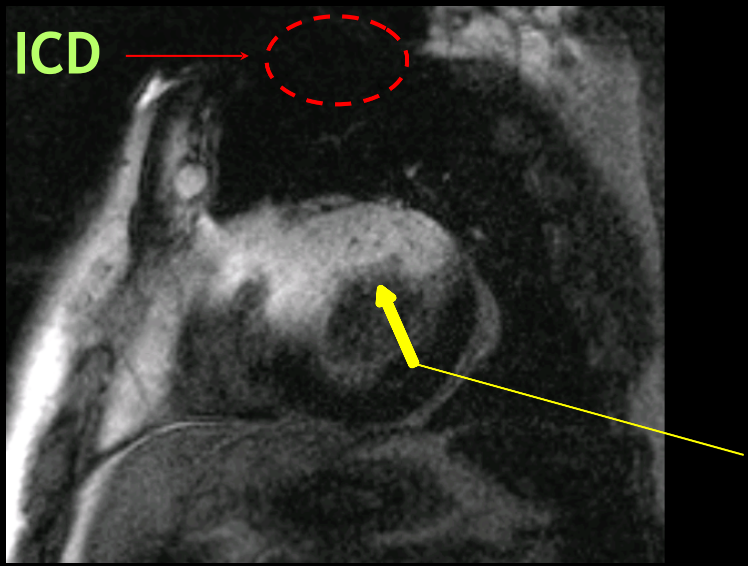




Clinical Example

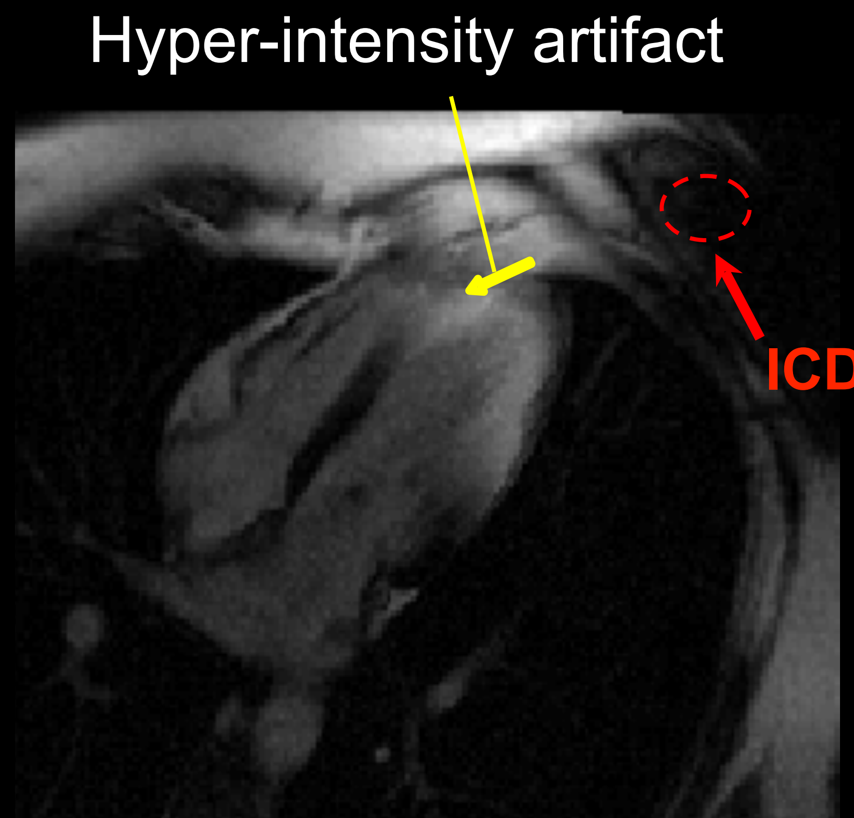
Late Gadolinium Enhancement (LGE) in patients with implantable cardiac devices

- Presence of an implantable cardiac device in the patients produces an interesting off-resonance artifact



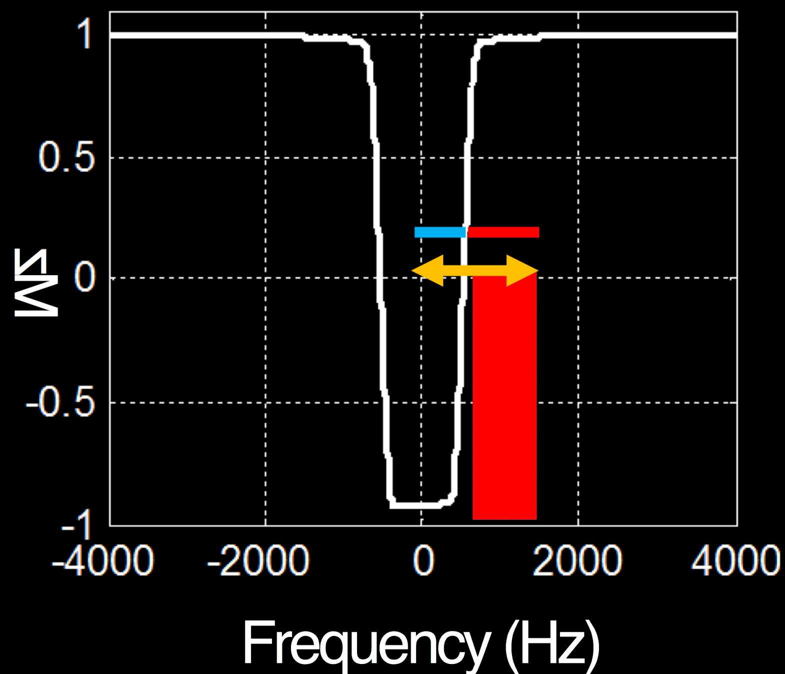
Hyper-intensity Artifacts

Conventional IR LGE Image



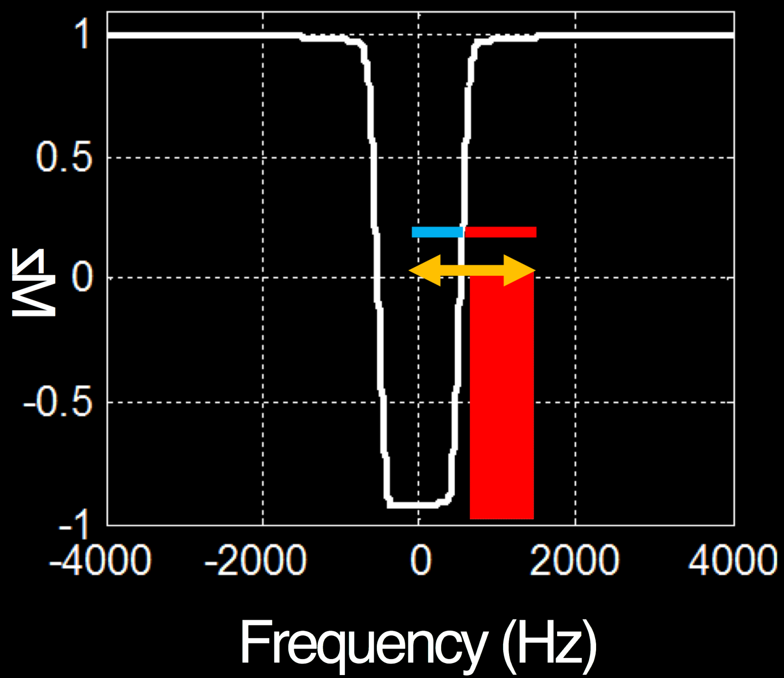
Conventional IR LGE Image

Cause of Artifact

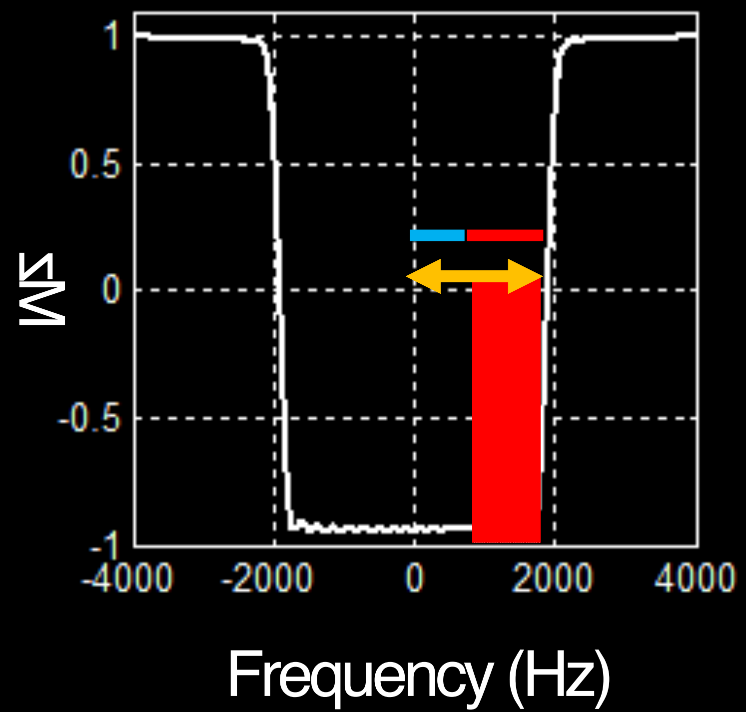


Longitudinal magnetization produced
by conventional IR pulse
BW = 1.1 kHz

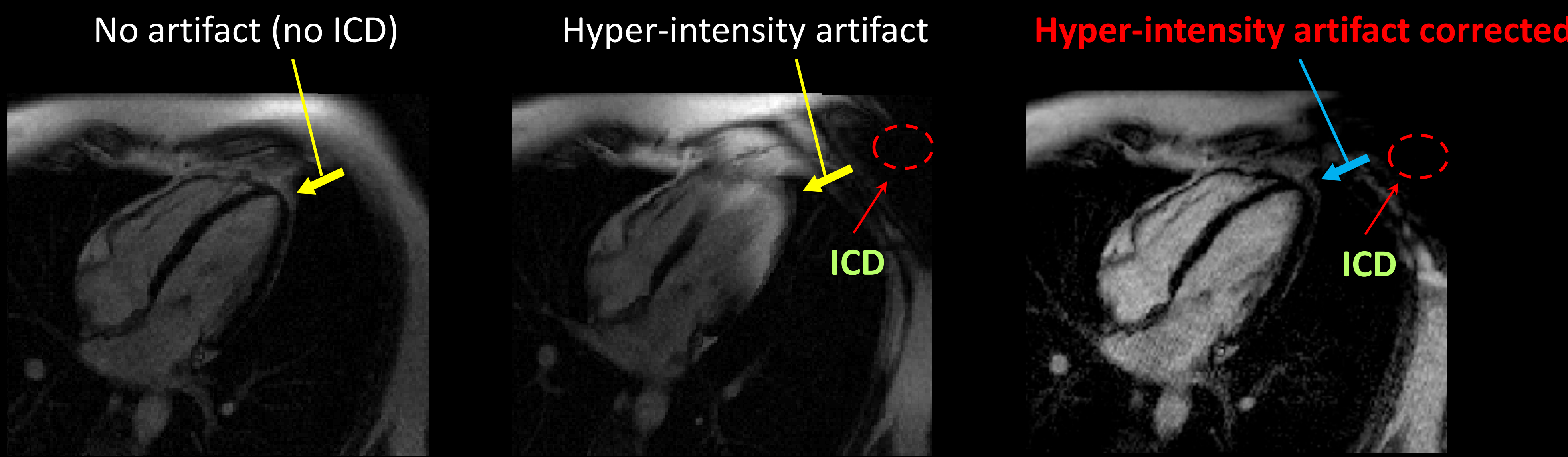
Solution: Increase Bandwidth of Inversion Pulse



Longitudinal magnetization produced by conventional IR pulse
BW = 1.1 kHz



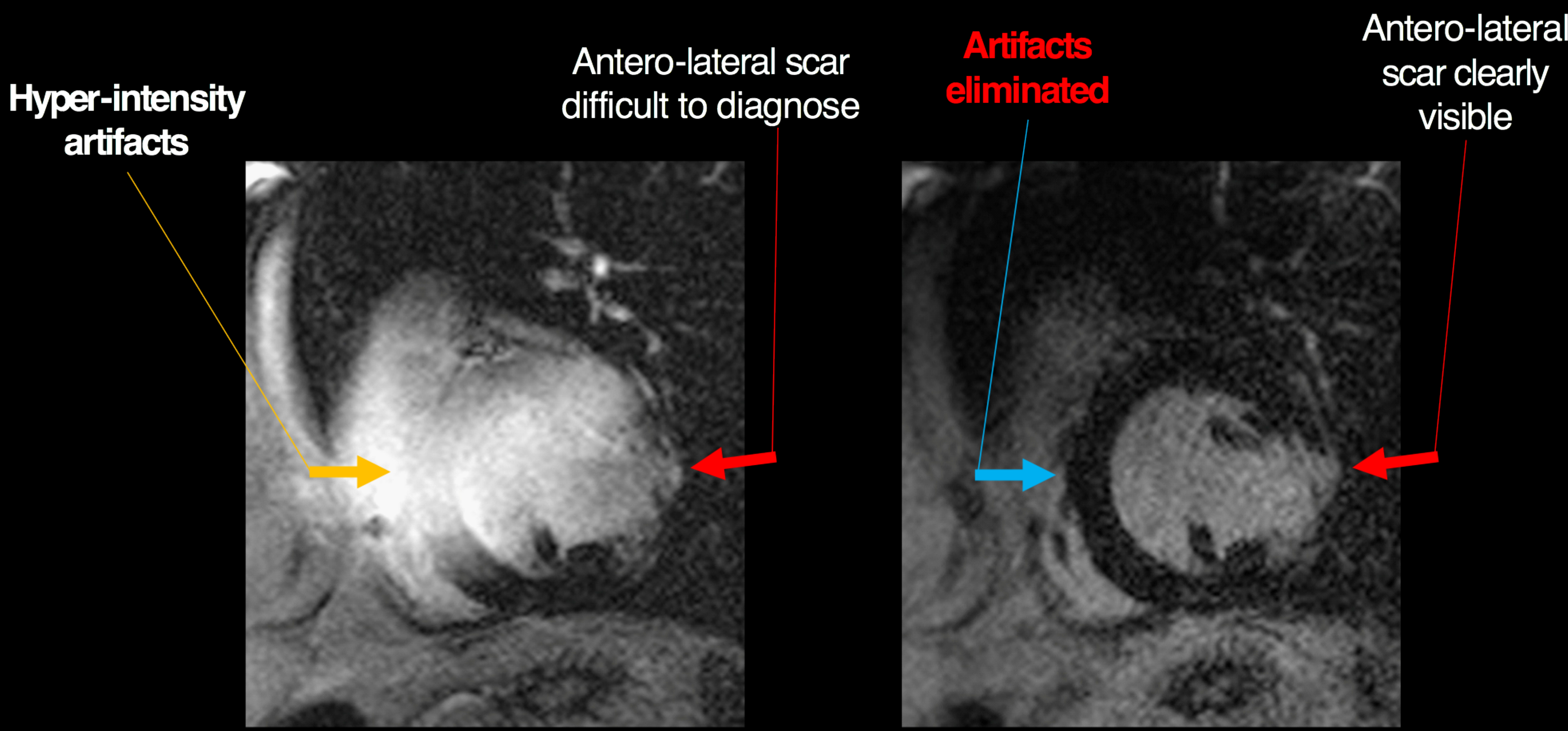
Longitudinal magnetization produced by wideband IR pulse
BW = 3.8 kHz



Conventional IR LGE Image

Conventional IR LGE Image

Wideband IR LGE Image


Hyper-intensity artifacts
Antero-lateral scar difficult to diagnose
Artifacts eliminated
Antero-lateral scar clearly visible

Small Tip Approximation

Bloch Equation (at on-resonance)

$$\frac{d\vec{M}_{rot}}{dt} = \vec{M}_{rot} \times \gamma \vec{B}_{eff}$$

where $\vec{B}_{eff} = \begin{pmatrix} B_1(t) \\ 0 \\ \cancel{B_0 - \frac{\omega}{\gamma}} + G_z z \end{pmatrix}$

When we simplify the cross product,

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$$\omega(z) = \gamma G_z z \qquad \omega_1(t) = \gamma B_1(t)$$

Small Tip Approximation

$$\frac{d\vec{M}}{dt} = \begin{pmatrix} 0 & \omega(z) & 0 \\ -\omega(z) & 0 & \omega_1(t) \\ 0 & -\omega_1(t) & 0 \end{pmatrix} \vec{M}$$

$M_z \approx M_0$ small tip-angle approximation

sin θ ≈ θ
cos θ ≈ 1
$M_z \approx M_0 \rightarrow$ constant

$$\frac{dM_z}{dt} = 0$$

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

$$M_{xy} = M_x + iM_y$$

First order linear differential equation. Easily solved.

$$\frac{dM_{xy}}{dt} = -i\gamma G_z z M_{xy} + i\gamma B_1(t) M_0$$

Solving a first order linear differential equation:

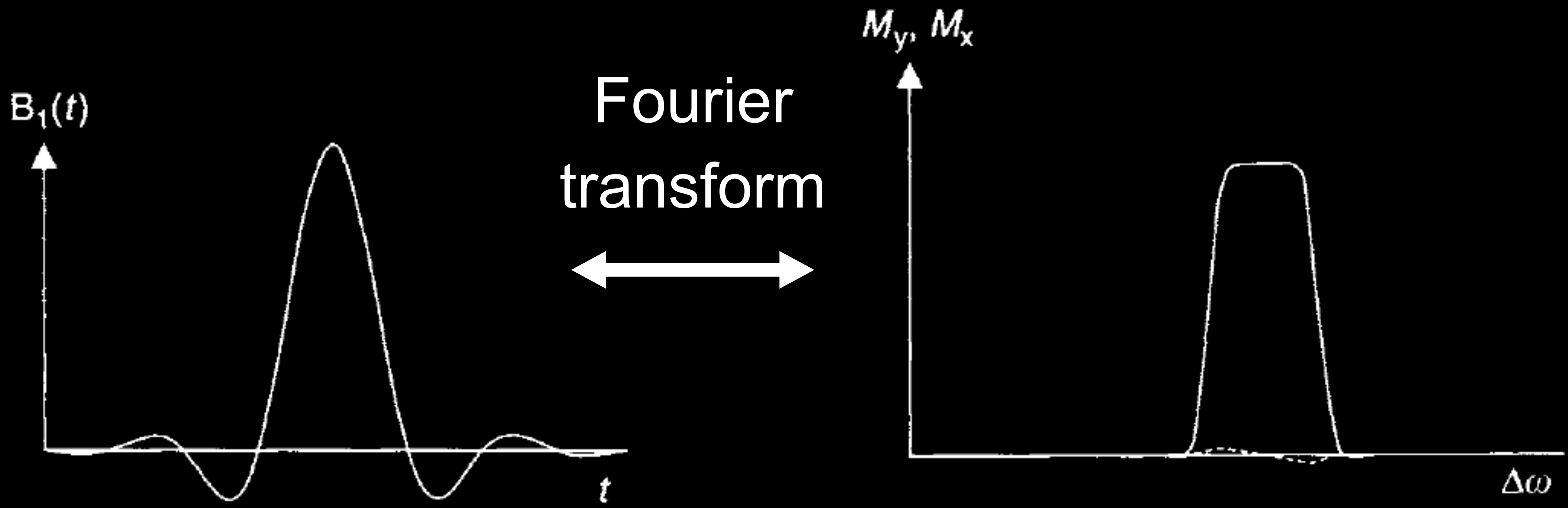
$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma G_z z \cdot (t-s)} ds$$

$$M_r(\tau, z) = iM_0 e^{-i\omega(z)\tau/2} \cdot \mathcal{FT}_{1D}\{\omega_1(t + \frac{\tau}{2})\} \,|_{f=-(\gamma/2\pi)G_z z}$$

(See the note for complete derivation)

To the board ...

Small Tip Approximation



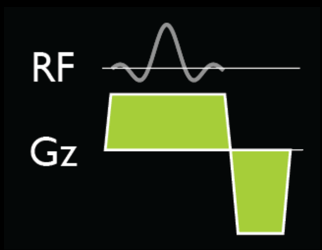
- For small tip angles, “the slice or frequency profile is well approximated by the Fourier transform of B1(t)”
- The approximation works surprisingly well even for flip angles up to 90°

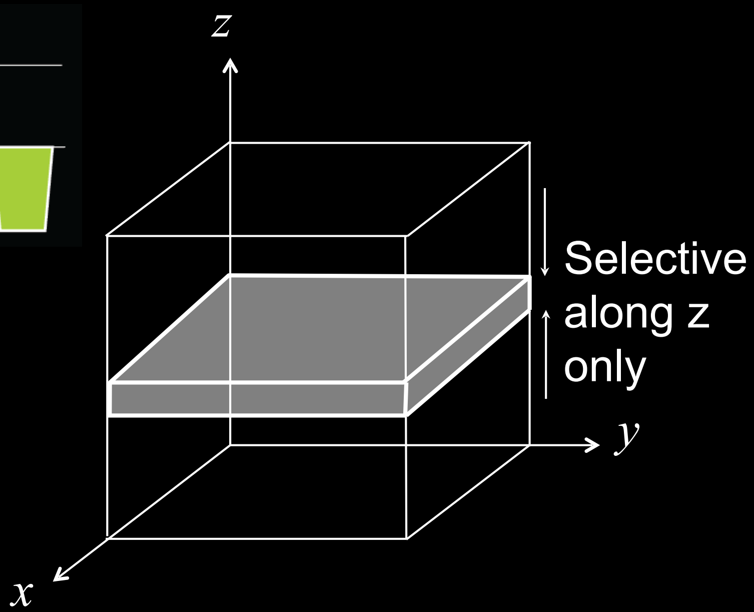
What is Multi-Dimensional Excitation?

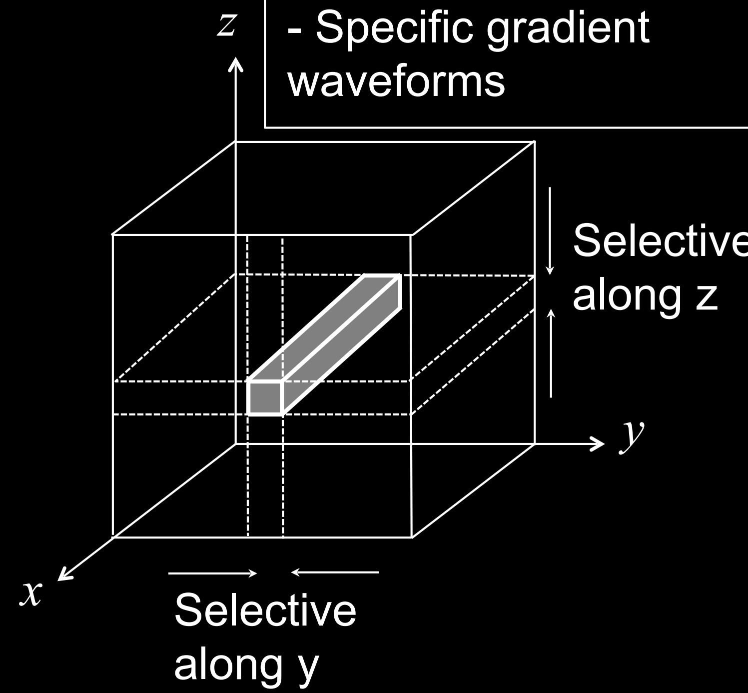
Multi-dimensional excitation occurs when using multi-dimensional RF pulses in MRI/NMR, i.e. 2D or 3D RF pulses

1D vs. N-D RF Pulses

2D/N-D Pulse Design Requires:
- Specific B1 waveform
- Specific gradient waveforms







- 1D pulses are selective along 1 dimension, typically the slice select dimension
- 2D pulses are selective along 2 dimensions
  - So, a 2D pulse would select a long cylinder instead of a slice
  - The shape of the cross section depends on the 2D RF pulse

Excitation k-space Interpretation

Small Tip Approximation

$$M_{xy}(t,z) = i\gamma M_0 \int_0^t B_1(s) e^{-i\omega(z)(t-s)} ds$$

$$\omega(z) = \gamma G_z z \quad \Rightarrow \quad \omega(\vec{r},t) = \gamma \vec{G}(t)\vec{r}$$

$$\Downarrow$$

$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma \int_s^t \vec{G}(\tau)d\tau \cdot \vec{r}} ds$$

Small Tip Approximation

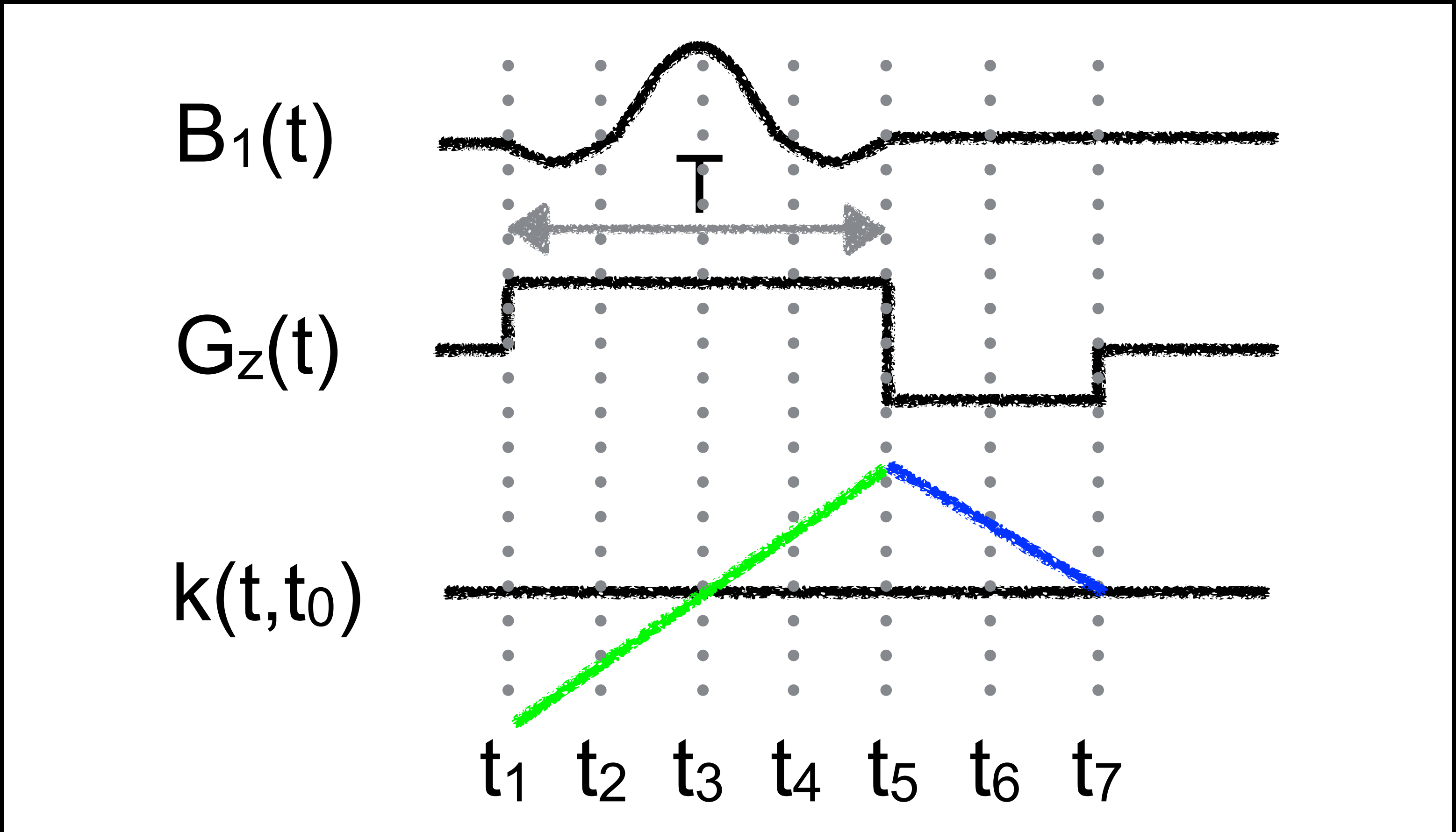
$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{-i\gamma \int_s^t \vec{G}(\tau)d\tau \cdot \vec{r}} ds$$

Let us define: $\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_s^t \vec{G}(\tau)d\tau$

$$M_{xy}(t,\vec{r}) = i\gamma M_0 \int_0^t B_1(s) e^{i2\pi \vec{k}(s,t)\cdot\vec{r}} ds$$

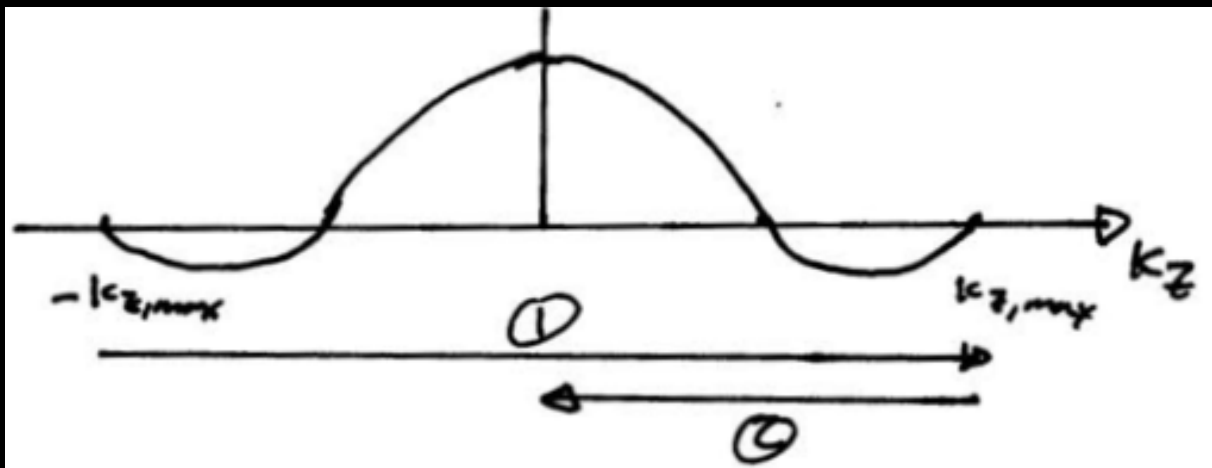
One-Dimensional Example

$$\vec{k}(s,t) = -\frac{\gamma}{2\pi}\int_s^t \vec{G}(\tau)d\tau$$



Consider the value of **k** at s = t_1, t_2, … t_7

One-Dimensional Example



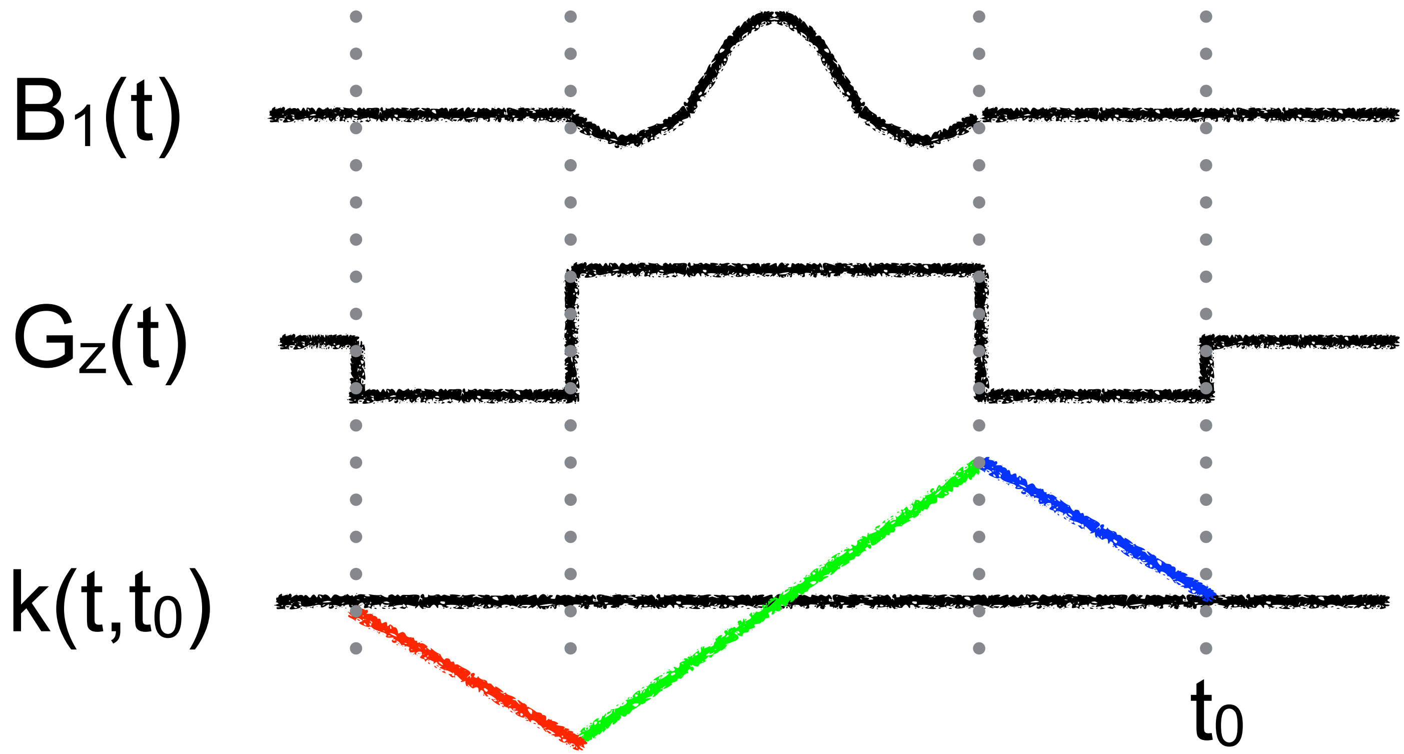
$$k_{z,max} = \frac{T}{2}\frac{\gamma}{2\pi}G_z$$

- This gives magnetization at $t = t_0$, the end of the pulse
- Looks like you scan across k-space, then return to origin

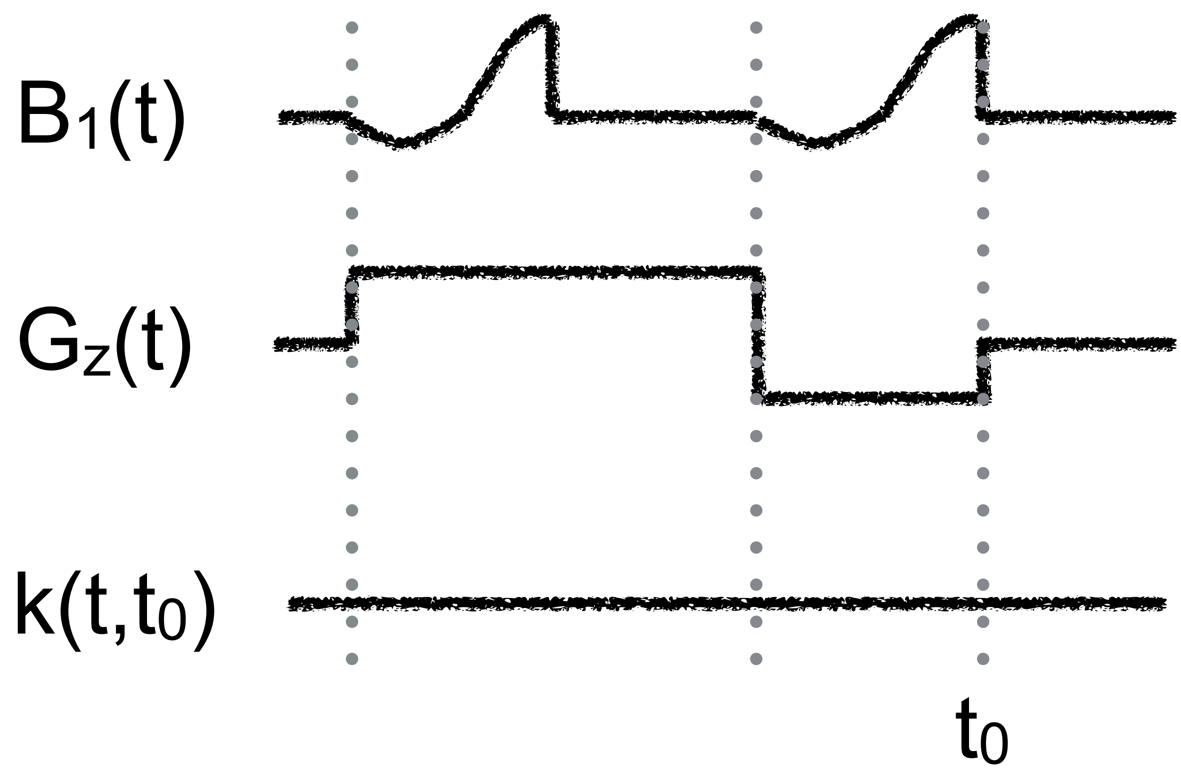
Evolution of Magnetization During Pulse

- RF pulse goes in at DC ($k_z = 0$)
- Gradients move previously applied weighting around
- Think of the RF as “writing” an analog waveform in k-space
- Same idea applies to reception

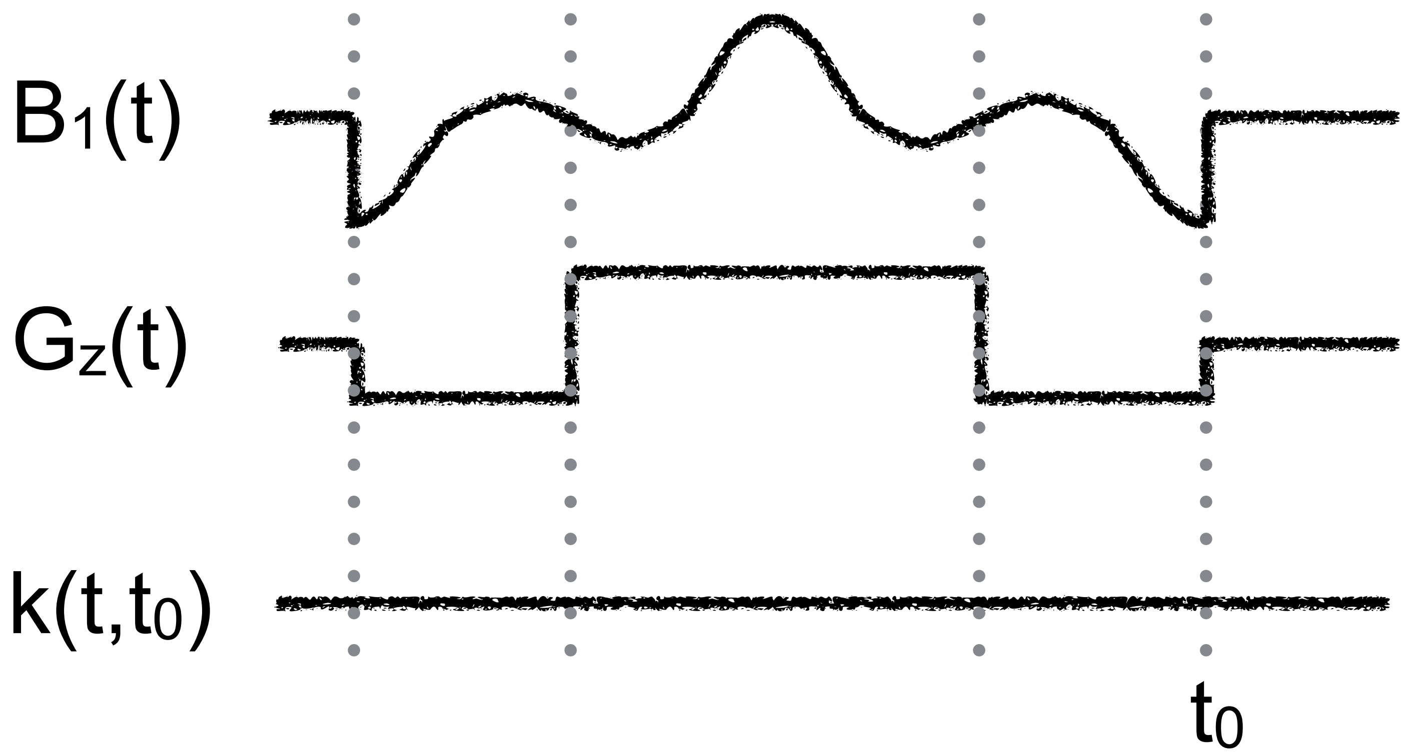
Other 1D Examples



Other 1D Examples



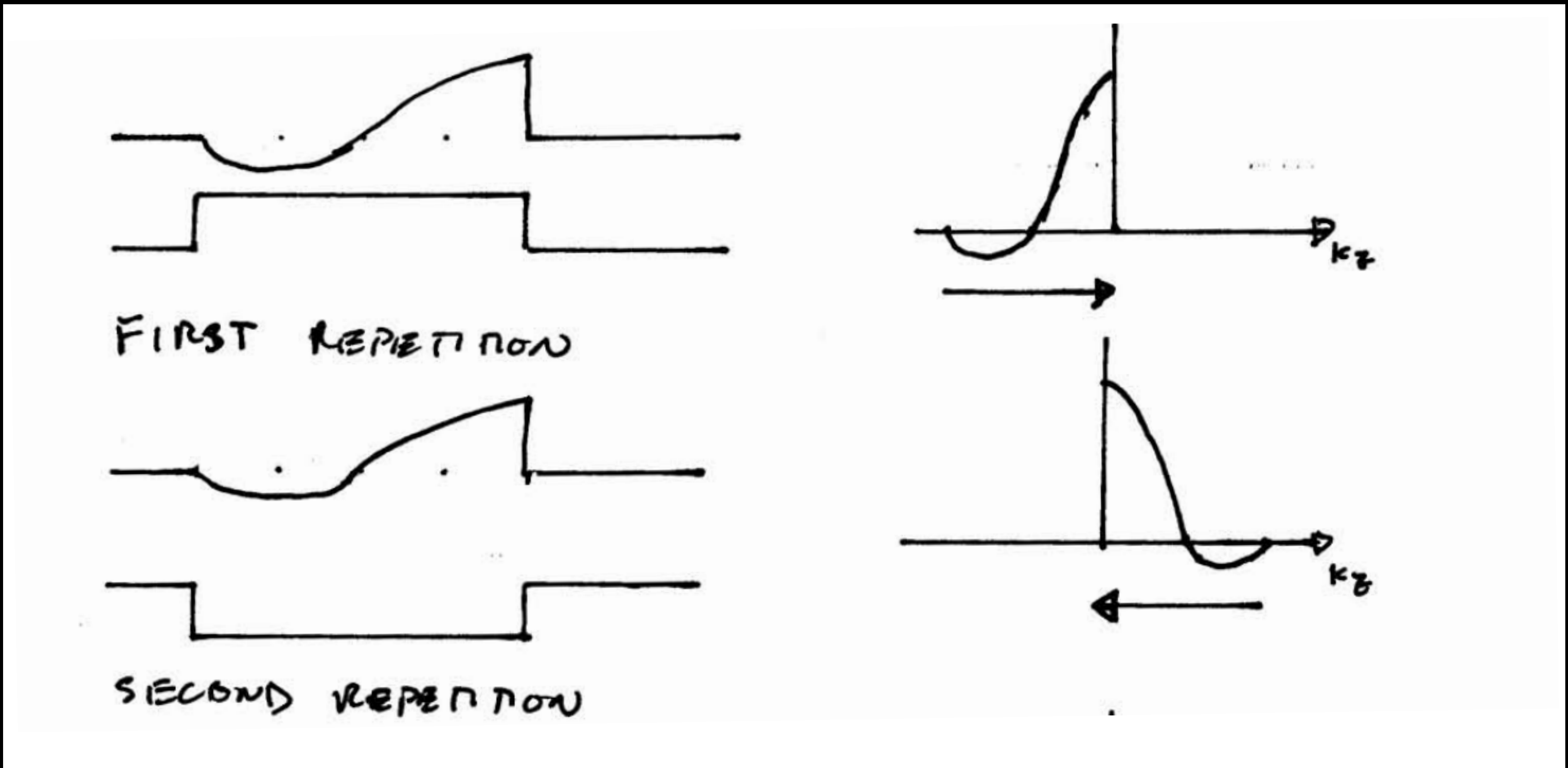
Other 1D Examples



Multiple Excitations

- Most acquisition methods require several repetitions to make an image
  - e.g., 128 phase encodes
- Data is combined to reconstruct an image
- Same idea works for excitation!

Simple 1D Example



Sum the data from two acquisitions

Same profile as slice selective pulse, but zero echo time

Thank You!

- Further reading
  - Read "Spatial-Spectral Pulses" p.153-163

- Acknowledgments
  - John Pauly's EE469b (RF Pulse Design for MRI)
  - Shams Rashid, Ph.D.

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